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RESEARCH ARTICLE

Using evidence of student thinking as resources in a digital collaborative platform

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Abstract

Learning mathematics in a student-centered, problem-based classroom requires students to develop mathematical understanding and reasoning collaboratively with others. Despite its critical role in students' collaborative learning in groups and classrooms, evidence of student thinking has rarely been perceived and utilized as a resource for planning and teaching. This is in part because teachers have limited access to student work in paper-and-pencil classrooms. As an alternative approach to making student thinking visible and accessible, a digital collaborative platform embedded with a problem-based middle school mathematics curriculum is developed through an ongoing design-based research project (Edson & Phillips, 2021). Drawing from a subset of data collected for the larger research project, we investigated how students generated mathematical inscriptions during small group work, and how teachers used evidence of students' solution strategies inscribed on student digital workspaces. Findings show that digital flexibility and mobility allowed students to easily explore different strategies and focus on developing mathematical big ideas, and teachers to foreground student thinking when facilitating whole-class discussions and planning for the next lesson. This study provides insights into understanding mathematics teachers' interactions with digital curriculum resources in the pursuit of students' meaningful engagement in making sense of mathematical ideas.

Keywords: curriculum, digital resources, student thinking, teacher learning

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I. INTRODUCTION

> *[I like using the digital platform] because you could see each other's work without having to bend over their desks, and also the teacher can see our work without having to go by it at everyone's table. And the graphs, like making graphs, are a lot easier. So, that opens up new windows for strategies* (Student M during a focus group interview).

When asked about their learning experiences in the digital collaborative platform, a group of students shared their thoughts in comparison with a usual paper-and-pencil classroom environment. As Student M mentioned, these students were not new to small group collaborations where group members had to "bend over their desks" to see each other's work. For decades, small group work to solve cognitively demanding mathematics problems has been widely known as high-leverage, effective teaching practices that promote students' conceptual understanding (e.g., Horn, 2012). In addition, integrating technology into classrooms is not new either as mathematics educators increasingly examine the potential of educational technology to support teaching and learning (e.g., Pepin et al., 2017). Nevertheless, Student M confidently stated that the digital platform opened "new windows for strategies" when they were grappling with challenging mathematics problems. What aspects of the digital collaborative platform made these students see new opportunities, different from their small group work using the printed version of the curriculum?

Learning mathematics in a problem-based, inquiry-oriented classroom requires students to develop mathematical understanding and reasoning collaboratively with others. The learning process involves making sense of and taking up mathematical problems, constructing solution strategies, discussing mathematical ideas in small groups, and considering multiple strategies during whole-group conversations. Such learning environments emphasize building a new understanding by drawing on student thinking (Jacobs et al., 2010). Despite its critical role in students' collaborative learning in groups and classrooms, evidence of student thinking has rarely been perceived and utilized as a resource for planning and teaching (Remillard & Heck, 2014). In part, it is due to limited access to student work during and after students' small group work in paper-and-pencil environments.

The purpose of this study is to examine how students engaged in challenging mathematics problems when their digital workspaces were connected with peers' workspaces and how teachers utilized student work generated on the platform in their planning and teaching. In contrast to other classroom technologies where individual student work is visible only to the teacher (e.g., Fahlgren & Brunström, 2020), the digital collaborative platform utilized in this study allows students to share their work with classmates and pull from others' work in real time. With access to these novel digital collaborative features, how do students develop and express their mathematical thinking? Further, how do teachers draw on this digitally inscribed student thinking in their instruction?

II. RELATED LITERATURE

Collaborative Learning in Problem-Based Mathematics Classrooms

The set of curriculum materials embedded in the digital collaborative platform discussed in this paper is the Connected Mathematics Project's middle grades problem-based curriculum, *Connected Mathematics Project 4* (CMP: The Connected Mathematics Project, 2023; Phillips et al., in press). Its emphasis on student thinking and conceptual understanding is well aligned with the perspective of curriculum resources as *thinking devices*, which differs from those as *delivery mechanisms* where students memorize facts and practice demonstrated procedures in a direct instruction classroom (Choppin et al., 2015; McDuffie et al., 2018). The overarching goal of CMP is to help students and teachers develop mathematical knowledge, understanding, and problem-solving competence along with an awareness of and appreciation for the rich connections among mathematical strands and between mathematics and other disciplines. Through connected sequences of contextualized problems, students are engaged in exploring big mathematical ideas over time. These problems were designed to support some or all of the following characteristics: they (a) have important, useful mathematics embedded in them, (b) promote both conceptual and procedural knowledge, (c) build on and connect to other important mathematical ideas, (d) require higher-level thinking, reasoning, and problem-solving, (e) provide multiple access points for students, (f) engage students and promote classroom discourse, and (g) create an opportunity for teachers to access student learning (Lappan & Phillips, 2009, p. 8).

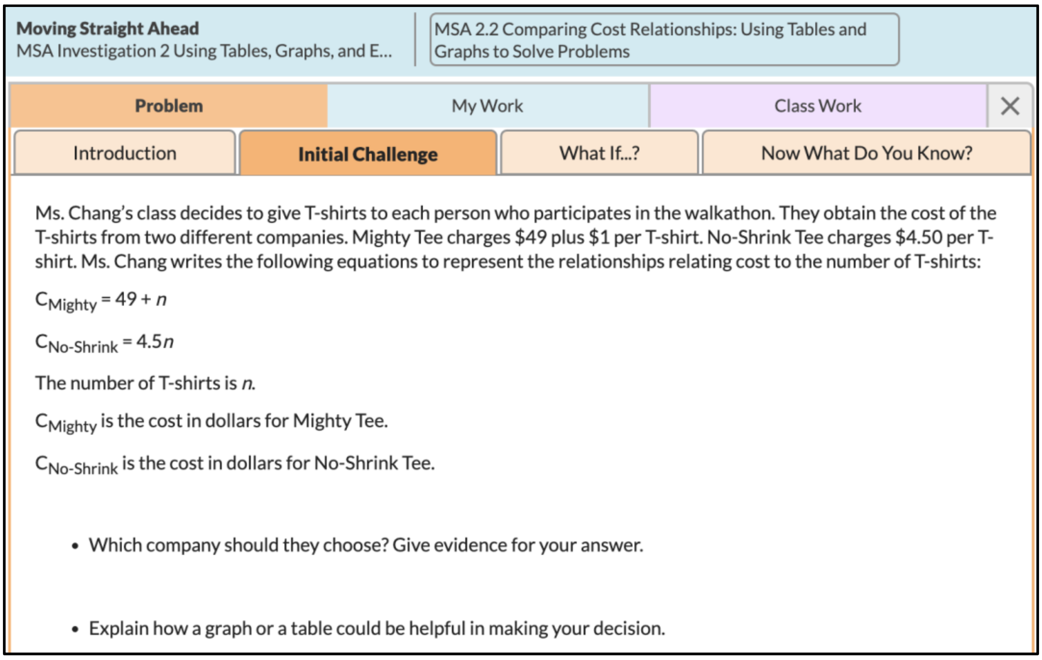

Figure 1. *MSA* 2.2: Example mathematics problem in CMP4 (Phillips et al., in press) embedded in the digital platform (Reprinted with permission)

Each CMP4 problem consists of three parts: *Initial Challenge*, *What If…?* and *Now What Do You Know?* The *Initial Challenge* engages students in tackling contextualized problems from diverse perspectives; the *What If…?* unpacks the embedded mathematics of the problem; and the *Now What Do You Know?* connects learning to prior knowledge. In this paper, we focus on the *Initial Challenge*, which typically takes one class period. Figure 1 shows an example of a seventh-grade mathematics problem from CMP4 field-test materials embedded in the digital collaborative platform. This problem is in the *Moving Straight Ahead* (*MSA*) unit that focuses on understanding linear relationships. The *Initial Challenge* part of this problem engages students in exploring linear relationships within a real-life situation, building on the previous problem about comparing linear relationships in a different situation.

CMP problems provide individual students, or groups of students, with opportunities to discover diverse solution paths which in turn enrich the collaborative problem-solving in the class (Harris et al., 2001; Moyer et al., 2018). Through both small-group and whole-class collaboration, students discuss different ways of mathematical thinking and further abstract mathematical ideas and problem-solving strategies. When small group discussion is combined with subsequent whole-class discussions, students can enhance and solidify their mathematical understanding more effectively (Wester, 2021). In CMP classrooms, thus, individual and collaborative learning is operationalized through the *Launch-Explore-Summarize* instructional model (see Table 1 for students' and teachers' roles in each phase). As mathematics learning entails both communication and social relations (e.g., Greeno, 1989, 1991; Lave & Wenger, 1991; Rogoff, 1994), this instructional model underscores the importance of creating a learning environment where students take ownership and agency in their mathematical problem-solving while teachers pay careful attention to student thinking throughout a lesson.

Table 1. The Launch-Explore-Summarize instructional model

Launch	Explore	Summarize
• **Students** are introduced to the context and challenge of the problem. They make predictions and ask clarifying questions about the problem situation. • **The teacher** engages the students in the challenge and helps position the problem within prior understandings. The teacher must be careful not to tell too much and consequently lower the challenge of the problem to something routine, or to be so directive that the rich array of strategies that may evolve from a more open launch of the mathematics problem is lost.	• **Students** work collaboratively to explore and solve the problem. They gather data, share ideas, look for patterns, make conjectures, develop strategies, and create arguments to support their reasoning and solutions. • **The teacher** moves around the classroom, observing and interacting with individuals and small groups. The teacher helps students by asking appropriate questions and providing confirmation and redirection where needed.	• **Students** present and discuss their solutions and strategies, discuss the embedded or encoded mathematics of the problem, and connect learning to prior and future knowledge. • **The teacher** facilitates discussion to reach the problem's mathematical goals and connect their new understandings to prior mathematical goals. The teacher uses evidence from the *Explore* phase that can be used to support student understanding.

Collaborative learning in a problem-based classroom can be enhanced by using inscriptions. In our work, inscriptions refer to external representations of thinking that exist in material form where meanings are developed in social settings. Inscriptions are constantly changing and improving (Latour & Woolgar, 1986), and eight characteristics are common among inscriptions: they are (1) easily able to be sent and received, (2) do not change when being sent or received, (3) easily embedded into different contexts, (4) easily modified, (5) easily combined and superimposed with other inscriptions, (6) reproduced at low economic, cognitive, and temporal cost, (7) easily merged with geometry, and (8) often translated into other inscriptions (Roth & McGinn, 1998, pp. 37-38). Research has overwhelmingly shown that the use of inscriptions fosters collaborative learning (e.g., Dillenbourg & Hong, 2008; Lehrer & Schauble, 2000, 2004, 2012; Schwarz et al., 2011; Squire & Jan, 2007; van Amelsvoort et al., 2007). This is because collaborative learning is "not merely a matter of communication or information sharing but also involves knowledge-construction phenomena that take place at the group level" (Medina & Suthers, 2013, p. 34). For example, Enyedy (2005) found that students' individual inventions of inscriptions could be translated and integrated through social interactions into the conventions of the classroom community. The use of inscriptions also supports collective conceptual understanding and problem solving. Barab and colleagues (2010) found students gain conceptual and ethical understandings of issues related to water quality in environments where the "most involved and meaningful interactions occurred when students were required to interrogate the narrative and the relations of discovered inscriptions to the narrative" (p. 403). Therefore, inscriptions are powerful for collaborative learning because students develop and use practices that emerge over time in a classroom setting (Medina & Suthers, 2013).

Attending to Evidence of Student Thinking

A critical aspect of problem-based, inquiry-oriented teaching and learning centers on eliciting evidence of student thinking and using it to make instructional decisions (Jacobs et al., 2010; Leahy et al., 2005; van Es, 2011). Evidence of student thinking plays a vital role as students coordinate and navigate their development of understanding in pairs, small groups, and whole class settings (Dillenbourg & Hong, 2008; Lehrer & Schauble, 2000, 2004, 2012; Schwarz et al., 2011; Squire & Jan, 2007; van Amelsvoort et al., 2007). In face-to-face classrooms, evidence of student thinking can be publicly accessible and directly available to others through oral communications, gestures, and written representations. Extensive research has investigated the importance of examining student work and reported on how teacher knowledge can be developed (e.g., Bautista et al., 2014; Leatham et al., 2015), how it can positively influence student learning outcomes (e.g., Hill et al., 2005) or their practices (e.g., Fennema et al., 1996), and how teachers' positioning of student thinking in mathematics classroom can influence student identity (Wagner & Herbel-Eisenmann, 2014). Moreover, the core teaching practices, particularly teacher noticing and leading class discussions, heavily draw on evidence of student thinking (Jacobs & Spangler, 2017). Research efforts have focused on identifying indicators of what is important to notice in student thinking—planning for ways to elicit that information, interpreting what the evidence means with respect to their learning, and deciding how to respond to student understanding (e.g., Sherin et al., 2011; van Es, 2011).

To support teachers in purposefully attending to student thinking during mathematics lessons, Smith, Stein, and colleagues (2008, 2011) proposed the five practices that help whole-class discussions to foreground student-generated mathematical representations, which involve anticipating, monitoring, selecting, sequencing, and connecting students' different strategies. While these practices provide teachers with explicit expectations in preparing for fruitful mathematical discussions, utilizing student work as a resource to support student learning is not straightforward. For example, Evans and Dawson (2017) observed that teachers found it more challenging to use student-generated strategies in whole-class discussions compared to using pre-designed student work examples. Acknowledging few studies, despite its importance, have focused on how teachers select student work for whole-class discussions, Dunning (2023) provided a framework that encourages teachers to consider mathematics within the strategy, the author of the strategy, and the potential class engagement with the strategy when selecting student work. Although increasing attention has been paid to identifying effective teaching practices for eliciting and using student thinking (e.g., National Council of Teachers of Mathematics[NCTM], 2014), teachers face significant challenges when facilitating discussions that build on student thinking (Ball, 2001; Brown & Campione, 1996; Chazen & Ball, 2001; Lampert, 2001; Leinhard & Steele, 2005) in part because they have limited access to student work visible and available during lessons (Bieda et al., 2020).

The CMP Digital Collaborative Platform

Because of their potential benefits to promote instructional changes, mathematics education literature has been paying increasing attention to digital resources (Pepin et al., 2017; Rezat et al., 2021). Recent studies have focused on investigating student use of digital platforms or systems, including various types of resources: e-textbooks (Rezat et al., 2018), digital curriculum materials (Edson, 2014, 2016, 2017; Edson et al., 2018), or digital assessments (Cusi et al., 2016; Naftaliev & Yerushalmy, 2013). While resources do matter for instructional changes, an underlying assumption in the literature is that the impact of resources relates to access and use within the classroom (Cohen et al., 2003). This perspective shifts attention from resources themselves to how they are used and changed in mathematics teaching and learning (Adler, 2000). While abundant online resources are readily available to teachers, however, they tend to exist as isolated, discrete activities that do not necessarily lend themselves to cohesively developing mathematical big ideas (Remillard, 2016). As for digital curriculum resources, which maintain the cohesive nature of curriculum materials, Choppin et al. (2014) found that most of the cases fall into either digital individual learning programs or digitized versions of textbooks.

The digital curriculum resources used in this study were embedded with curriculum programs identified as *thinking devices* to allow problem-solving rather than *delivery mechanisms* to directly transmit knowledge (Choppin et al., 2015; McDuffie et al., 2018). For teachers, teaching mathematics with a *thinking device* curriculum calls for developing *pedagogical design capacity* to flexibly and purposefully interact with resources to make them responsive to their own student thinking (Adler, 2000; Brown, 2009). When curriculum materials are designed to serve as *thinking devices*, corresponding digital features would not be limited to checking individual students' mathematical understanding by multiple-choice questions or filling in the blank but expand to encourage multiple problem-solving strategies and adaptive reasoning (Edson & Phillips, 2021). Acknowledging that such practice is complex and challenging and requires teacher learning effort over time (Choppin, 2011; McDuffie et al., 2018), the CMP digital collaborative platform is designed, developed, and researched to enhance teaching and learning experience with a problem-based, inquiry-oriented curriculum.

The CMP digital collaborative platform aims to help students make their mathematical thinking visible and enable teachers to be more responsive to student thinking. Individual students have their own workspaces where they can create mathematical inscriptions using various digital tools and keep records of their ideas. Whenever they feel comfortable with sharing, students can turn the "share" button on to allow groupmates to view and copy their real-time work. When they click the "four-up" view button, they can access groupmates' shared workspaces (see Figure 2). For teachers, the digital platform allows them to monitor students' workspaces at the individual, group, and class levels on the teacher dashboard (see Figure 3) as well as navigate curriculum resources, such as textbook problems, solutions, or teacher guide materials.

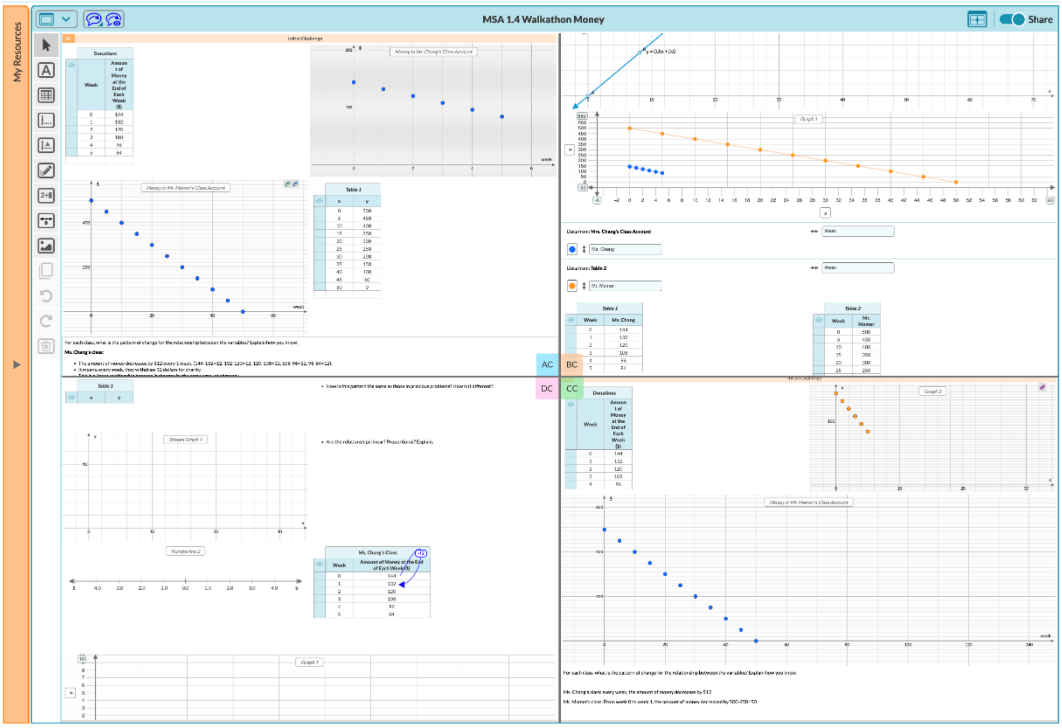

Figure 2. Student "four-up" view

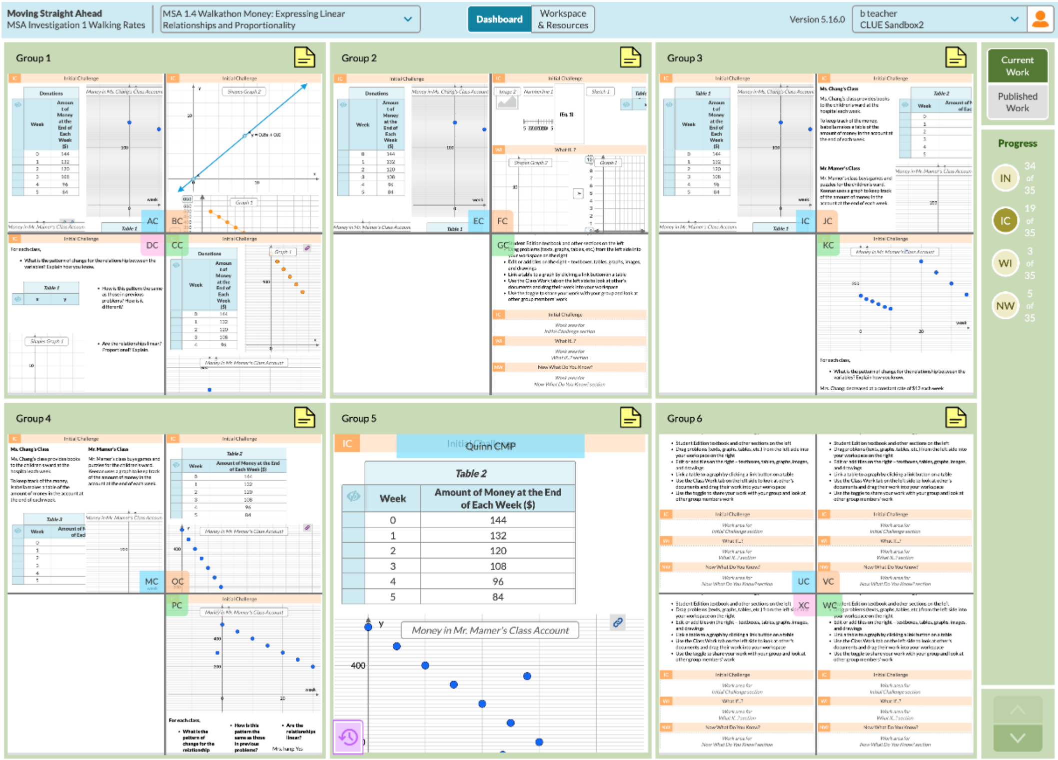

Figure 3. Teacher dashboard view

Taken together, the CMP digital collaborative platform is designed to help transform classroom teaching and learning by making student thinking visible and accessible while engaging students in challenging mathematics problems. Given that little is known about how students and teachers use digital curriculum resources embedded with a *thinking device* curriculum, more empirical study can contribute to understanding how students and teachers draw on students' mathematical thinking in their learning and teaching in digitally connected classrooms. This study is guided by two research questions:

1. How is evidence of student thinking of mathematical big ideas produced on a digital collaborative platform?
2. How do teachers incorporate evidence of student thinking into daily planning and teaching?

III. METHODS

This study was conducted in the context of a larger design-based research project that began in 2016. The goal of the larger project was to iteratively design and develop a digital collaborative platform embedded with the CMP curriculum materials. The digital platform was designed for face-to-face classroom instructions with the CMP materials where each student has one-to-one access to laptops connected to the internet, and the teacher can access students' digital workspaces. Over time, based on the feedback from the field-testing teachers, the digital platform and its features continue to evolve with more features and capabilities for both students and teachers (see Edson et al., 2018, 2019; Edson & Phillips, 2021, 2022). Using a subset of data sources collected for the larger design-based research project, this study focused on gaining a deeper understanding of how evidence of student thinking is generated and utilized within the context of digitally enhanced mathematics classrooms. For data triangulation, a variety of data sources were analyzed by mixed methods—quantitatively and qualitatively. To ensure reliability and credibility, two or three researchers were independently involved in the first cycle of analysis regarding each research question, then they discussed the relationships among codes to come to a consensus. Drawing on multiple data sources, the second cycle of our analysis process was not linear but holistic to account for the multifaceted, complex nature of teaching and learning.

Data Sources and Data Analysis

We drew on both quantitative and qualitative methodologies and relied on data sources collected for the larger project before the conceptualization of the research reported in this study. Specifically, we drew on data sources from two school years (2019-2020 and 2022-2023) because the corresponding data sources focused on unique aspects of the project. In 2019-2020, the project focused on teaching and learning within the seventh-grade mathematics classrooms, particularly around small group student explorations of mathematics; thus, data sources included individual students' screen recording videos. In 2022-2023, the project focused on networks or teams of teachers and their interactions with the digital platform for planning, teaching, and reflection on student thinking; the data sources included weekly teacher reflections, classroom observations, interviews, and log files. Drawing on these data sources across project years from a variety of different school contexts allows the research team to examine evidence of student thinking from both student and teacher perspectives, which is the focus of this study.

Participating schools and districts were located across multiple states in the United States with a wide range of diversity in terms of gender, race, ethnicity, disability, and/or family's economic status. Screen recordings, classroom observation videos, and student focus group interviews were collected in the seventh-grade mathematics classrooms from schools in the Midwestern suburban area. Seventh-grade students are typically at the age between 11 and 13 years old. Teachers in this study had a varying number of years in teaching the CMP curriculum materials (between 8 and 30), including several years of experience using the CMP digital platform. Table 2 provides an overview of the data sources and related analysis approaches to answer the research questions in this particular study.

Table 2. Data sources and analysis procedures regarding each research question

Research Question	Data Sources	Related Analysis Procedures
How is evidence of student thinking of mathematical big ideas produced on a digital collaborative platform?	• Students' screen recordings (two students from one school, 2019-2020)	• Identified inscriptional practices applying the Digital Inscriptions Framework
	• Student focus group interview videos and transcripts (two groups from two schools, 2022-2023)	• Open-coded students' perce-ptions of using the digital platform
	• Student documents on the digital platform and log files (two classes from two schools, 2022-2023)	• Quantitative analysis of log files and content analysis of digital documents
How do teachers incorporate evidence of student thinking into daily planning and teaching?	• Weekly reflection survey responses (seven teachers and two coaches from five schools, 2022-2023)	• Open-coded ways teachers used student work
	• Class observation videos and transcripts (two teachers from two schools, 2022-2023)	• Identified classroom episodes of using student work; Compared the two teachers' use of student work
	• Post-unit teacher interview videos and transcripts (two teachers from two schools, 2022-2023)	• Open-coded teachers' perce-ptions of using the digital platform
	• Teacher documents on the digital platform and log files (nine teachers from five schools, 2022-2023)	• Quantitative analysis of log files and content analysis of digital documents

To answer the first research question that involved investigating how evidence of student thinking is produced, we drew from three main data sources: (a) screen recordings of individual students' digital workspaces during small group work, (b) student focus group interview videos and transcripts, and (c) students' digital documents saved on the digital platform and log files. First, we applied the Digital Inscriptions Framework (Bowers et al., 2019; Edson, Park, et al., 2023) to identify two students' inscriptional practices—*Constructing*, *Communicating*, and *Circulating*. The Digital Inscriptional Framework allowed us to generate diagrams to visualize the complex process of how student work was produced during small group collaboration. Given that a situated perspective of learning shifts attention from focusing on representations of the individual mind towards viewing representations as a social practice, representing activities are "part of networks of social practices that take their characteristic shape and meaning from the contexts, purposes, and functions of their use" (Roth & McGinn, 1998, p. 46). The *Constructing* codes capture students' actions that physically make changes to mathematical inscriptions to see them from a new perspective. Within each *Constructing* code, we further identified representation tools used, such as Text, Graph, or Table. The *Communicating* codes involve students' verbal interactions with themselves, classmates, or the teacher about mathematical inscriptions, such as making mathematical claims or asking questions. The *Circulating* codes describe students' actions in making their work visible to others and accessing others' work both physically and digitally. Multiple researchers independently coded for the inscriptional practices, and then we used a consensus model to attend to any disagreements. Second, applying an open coding approach to qualitative analysis (Saldaña, 2016), we investigated student focus group interview videos and transcripts to identify students' perceptions of collaborative learning on the digital platform. From two student groups in two different schools, we looked for similarities and differences in the codes, which were used to provide rich illustrations of the patterns identified in students' inscriptional practices. Third, we analyzed students' digital documents saved on the platform and log files to identify diverse ways to use digital resources to solve problems collaboratively. Students' digital documents on the same problem, *MSA* 2.2 (see Figure 1), were examined from two teachers' classes to identify both the types of digital tools students used in their mathematical work and the number of students who used each type. Using log files, which give a record of "button clicks" in the platform, we applied descriptive statistics about the counts of different types of tools and actions on the platform. This provided insight into which students used different digital resources (e.g., creating inscriptions, making their work visible to their group, viewing published work from peers) and how often.

Regarding the second research question that focused on understanding how teachers utilized evidence of student thinking, we first analyzed weekly reflection survey responses completed by seven teachers and two coaches. Their responses were open-coded to identify when and how teachers used student work inscribed on the digital platform. After this descriptive coding, we categorized common themes among codes: teachers perceived digital evidence of student thinking as a powerful resource because they could (a) display student work during whole-class summary discussions, and (b) reflect on student understanding during planning. Also, we analyzed the classroom video recordings of two teachers teaching the same problem (*MSA* 2.2) to identify episodes where teachers and students shared and discussed student work, capturing instructional moves around students' digital inscriptions. Within each of the Launch, Explore, and Summarize phases, we compared the two teachers' instructional moves (e.g., projecting individual students' workspace with their names, projecting a collection of student inscriptions without names, probing, revoicing, and connecting student strategies). These two teachers' interview videos and transcripts were used to gain further information about their intentions and decisions on instructional moves. Lastly, we examined nine teachers' digital documents and log files for the entire *MSA* unit to identify which features of the digital platform they engaged and how frequently they used each feature. Specifically, as we focused on the features that allowed teachers to view and copy student work for whole-class discussions and reflections, we extracted the numbers of counts for the log file events related to student work.

IV. FINDINGS FOR RESEARCH QUESTION 1: EVIDENCE OF STUDENT THINKING ON A DIGITAL COLLABORATIVE PLATFORM

In this chapter, we report on two main themes regarding the first research question, "How is evidence of student thinking of mathematical big ideas produced on a digital collaborative platform?" Our analysis showed that during the Explore phase, students constructed various mathematical inscriptions by flexibly utilizing digital representation tools and collaborative features on the platform. First, students navigated various approaches to tackle challenging problems while interacting with different digital tools. Sometimes they quickly changed their approaches from using one tool to another, and other times they used multiple tools simultaneously. This digital flexibility allowed students to explore multiple strategies based on their own thinking rather than following a particular path to solving a problem. Second, collaborative features on the platform helped students' small-group interactions center around their mathematical inscriptions. As they gave permission to their groupmates to view their work progress in real-time, they shared accountability for each other's learning. Through digital sharing features, they influenced each other's mathematical thinking and built on others' work. The following sections describe evidence of student thinking in individual, group, and class levels to elaborate on (a) how students used various digital tools in their problem-solving processes, and (b) how students used collaborative features in their small group work.

Various Digital Tools Supported Students' Problem-Solving Process

We found that students used a variety of digital tools available on the platform as they explored different strategies to solve problems. First, two students' examples (Student A and Student B in the same group) provide close-up illustrations of how individual students use digital tools in different ways. Figure 4 and Figure 5 present the diagrams of two students' inscriptional practices on the platform during their small group work on *MSA* 2.2 (see Figure 1 for the problem). The diagram displays students' inscriptional practice codes and their durations chronologically, generated based on the analysis of student screen recordings. Student A created and deleted digital tiles on his workspace whenever he wanted a different approach (see Figure 4). He started by creating a text tile (*Constructing–Text*) to write his claim, "It depends on how many people participate in the walkathon. This is because," After a short pause, he verbally expressed his intention for the next step, "OK, I need to make a graph" (*Communicating*), and he clicked a new graph tile button (*Constructing–Graph*). Then again, he changed his approach when stating, "I'm just going to make a table. […] You know what? I'm going to make two tables" (*Communicating*). He deleted the graph tile, created two tables, and spent a longer time working on these tables than on other tiles (*Constructing–Table*). Later, he revisited the text tile that he had stopped at "This is because," and continued typing to finish the sentence: "No-Shrink is cheaper until person #14 where it costs the same as Mighty which is cheaper from then on" (*Constructing–Text*).

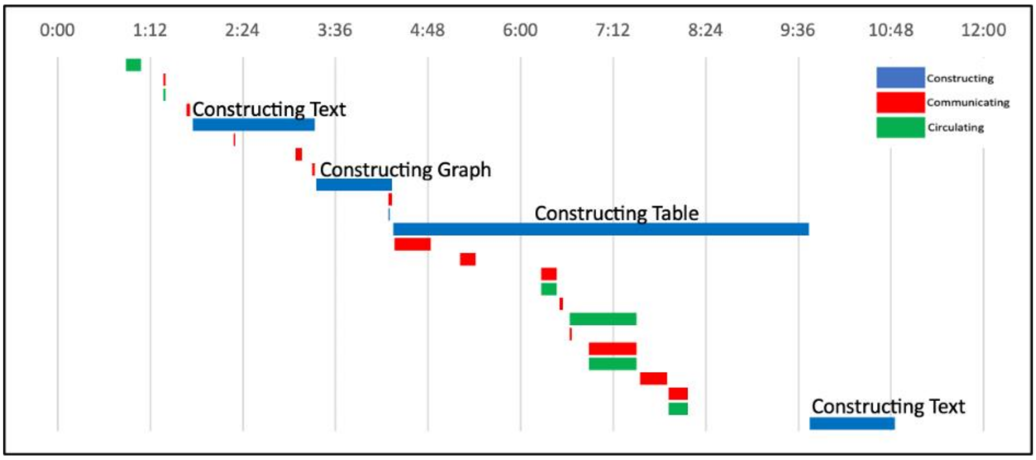

Figure 4. Student A's inscriptional practices during small-group work for *MSA* 2.2

Student A attempted to make a graph to provide evidence for his claim in the text tile. Adjusting the axes on the graph tile, however, he might have recognized that he needed to get coordinates before plotting points on a graph because he discarded the graph and created two tables instead. Even if his intention was to get coordinates to create a graph, he might have thought that he could get the answer using the tables because he worked on the tables until he figured out "where it costs the same" and did not make a graph. Student A's inscriptional practices demonstrate how easily he could shift from one strategy to another as he produced a novel approach to solve the problem. Also, he could decide to change (or not to change) his approach based on his interaction with the digital tools.

While Student A used tables, Student B primarily worked on graphs (see Figure 5). Student B first created a graph tile (*Constructing–Graph*) and subsequently created a table (*Constructing–Table*). Upon filling in the table with five x-y pairs, he dragged the table tile and dropped it onto the graph to see that five points automatically appeared. He clicked the straight-line button in the graph tile and changed the line's position so that it went through all five points (*Constructing–Graph*). Then, he created another table with three ordered pairs (*Constructing–Table*), dragged and dropped the table onto the same graph tile to get three new points, and added another straight line to fit them (*Constructing–Graph*). Digitally zooming in on the graph, he placed a point on the intersection of two lines (*Constructing–Graph*), which showed the coordinate (14, 63). He went back to his second table to put 14 in the x column, stating "14 times 4.5 (pause) 63" (*Communicating*), and put 63 in the y column (*Constructing–Table*). He scrolled up his digital workspace to find the text tile that he had copied from the curriculum and typed his answer in: "It depends on how much shirts they buy, up until 14 shirts No-Shrink is cheaper but after 14 shirts Mighty will be less expensive look at my graph for evidence" (*Constructing–Text*).

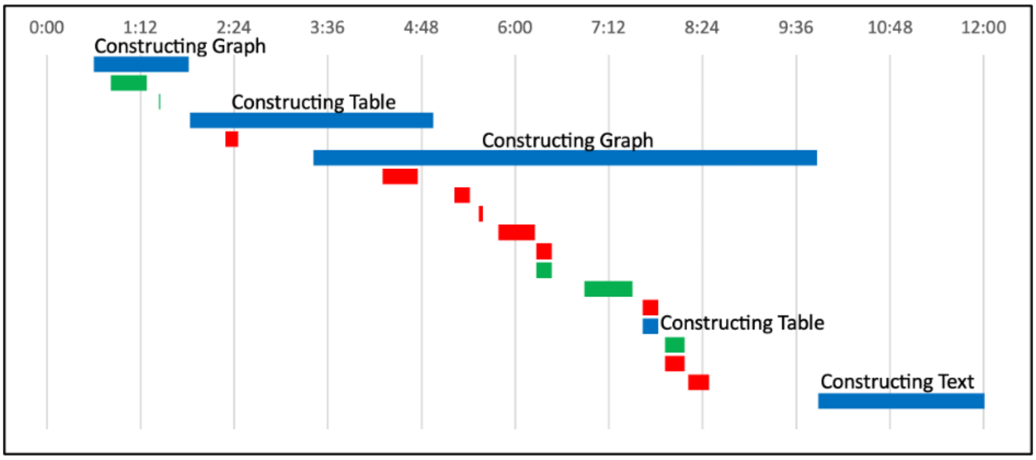

Figure 5. Student B's inscriptional practices during small-group work for *MSA* 2.2

Student B viewed tables as a vehicle to make a graph easily in this problem. This intention was apparent when he put only three ordered pairs (x=0, 1, and 2) in the second table before dragging it onto his graph. He did not plot points because the digital feature linking tables and graphs allowed him to generate points on a graph quickly. Also, he did not use many points to see patterns because he could add straight lines. Student B's inscriptional practices demonstrate how students can benefit from interconnected digital features—linking tables and graphs or adding straight lines. With these affordances, students could focus on exploring mathematical big ideas, such as linear relationships, rather than spending time plotting points on a graph, which was not the learning goal of this lesson.

Table 3. Digital tools used by students in two classrooms for MSA 2.2

Digital Tool Used by Students	Ms. Clark's Class (25 students)	Ms. Davis's Class (20 students)
Text		
- Text describing equation approach	3	3
- Other text explanation	14	11
Table		
- Table generated by equation	13	0
- Table with variable intervals	6	5
- Table with sequential intervals	2	9
Graph		
- Graph generated manually	3	1
- Graph generated by tables	12	2
- Graph generated by equations	5	0
Total	58	31

Similar to the two students' examples, flexible digital tool usages in the process of problem-solving were frequently observed in students' work across multiple classrooms. Our analysis of students' digital documents from two teachers' classrooms (Ms. Clark and

Ms. Davis) confirms that students used various inscriptions to represent their thinking as well as they used interconnected digital features. Table 3 shows the number of students in each class who used each type of digital tool in their work for the problem (*MSA* 2.2, *Initial Challenge*) and the method they used to create it. The text tool was one of the main ways students expressed their thinking, and most students in both classes used text tools in their work (17/25 in Ms. Clark's class and 14/20 in Ms. Davis's class). Text allowed students to describe their solutions to the problem or include their interpretation of their inscriptional work in other tools.

Like Student A and Student B used tables with different purposes, even when students created the same type of inscription, they did not all use the same method to produce it. For example, when manually creating tables, students chose which data points to include. Some students chose data points in sequential, predictable intervals (e.g., increasing the independent variable by 1 for each row on the tables), whereas others used variable intervals to closely examine data points as they got close to the intersection. Additionally, students in Ms. Clark's class generated tables by putting equations in them for automatic calculation. Each of these methods allowed students to examine the cost of T-shirts from both companies in a table, but each method reflects different strategies and ways that students might think about the problem. It was also common for students to include multiple strategies and inscriptions in their work on the problem. 22 out of 25 students in Ms. Clark's class and 9 out of 20 students in Ms. Davis's class used more than one type of tool while solving the problem. Figure 6 shows one student's digital document in Ms. Clark's class. This student created graphs using equations and used the text tool to describe the equation approach and solution.

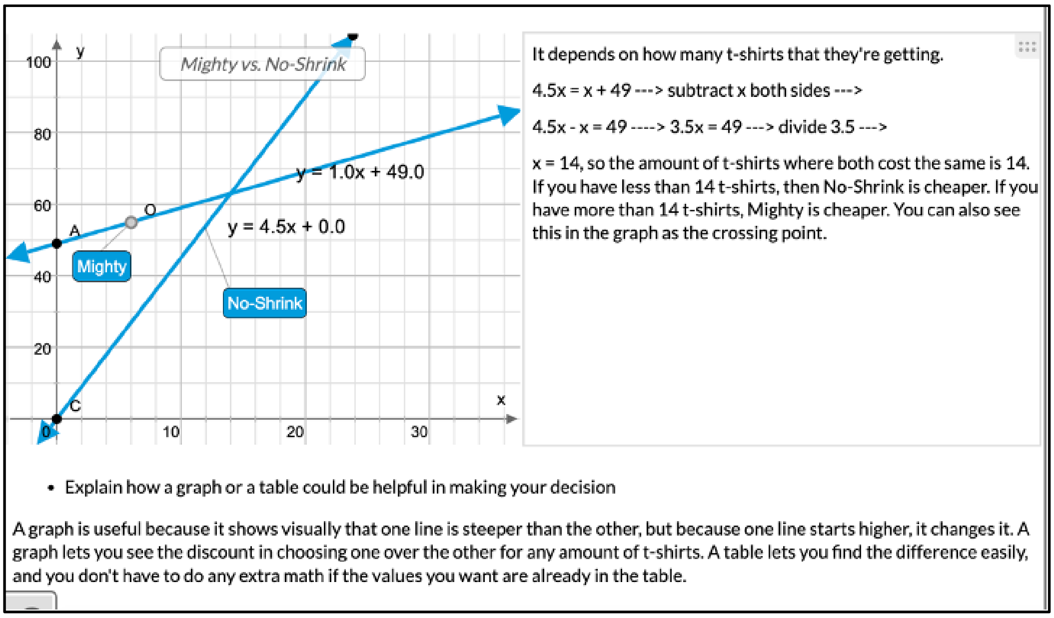

Figure 6. Student digital document from Ms. Clark's class using both graph and text

While students in both classes showed some similarities in the ways they used the digital tools, one significant difference was that Ms. Clark's students used built-in features within digital tools more often than Ms. Davis's students. In Ms. Clark's class, the built-in equation tool in graphs and tables had been discovered by students during a previous problem, so the use of these tools had become more widespread in the classroom. In contrast, students in Ms. Davis's class had not yet been introduced to the equation feature embedded within tables and graphs and did not use them on this problem. This difference highlights how students' use of digital resources can influence ways the classroom community uses the same tools.

Collaborative Features Supported Students' Small Group Interactions

We found that students influenced each other's problem-solving process by utilizing collaborative features embedded in the digital platform. Recall that Student A first attempted to create a graph, and he did not go back to the graph idea but used the tables to find the solution and the text tile to explain his reasoning. While it is possible that he simply changed his approach from graphs to tables, his inscriptional practice codes also suggest that his interaction with groupmates influenced his work. He constantly looked at his partner Student B's workspace (*Circulating*) and talked about Student B's graphs (*Communicating*), which could explain why he found it unnecessary to draw graphs on his own workspace. We now turn to *Communicating* and *Circulating* practices that explain how students utilized collaborative features on the digital platform to access and build on work from their group mates.

Accessing other's work was usually coupled with *Communicating* codes as the diagram shows that many of *Communicating* and *Circulating* bars appear at the same time (see Figures 4 and 5). While Student A was working on his tables, he noticed that Student B was using a graph. He accessed Student B's screen by both clicking the "four-up" view on the platform and physically leaning over to Student B's screen (*Circulating*). He asked, "OK, so, where did they meet?" (*Communicating*). A few minutes later, looking over Student B's screen (*Circulating*) again, Student A asked, "Where did you have them meet?" and stated, "One, two, three, you have them meet at 14" (*Communicating*). Then, he brought his attention back to his workspace and deleted 15 from his tables and put 14 instead (*Constructing–Table*). This episode shows that Student A's revision of his inscriptions was influenced by Student B's inscriptional work—the intersection of the two lines. Such influence, however, does not mean that he took others' work without critical thinking. Rather, he accessed Student B's graph with the clear intention that he wanted to know where the two lines meet. Even if he did not create a graph on his own, his thinking process was shaped by both his own work on tables and his partner's work on graphs. In other words, he utilized his groupmates' work as a resource and evidence to revise his problem-solving process.

Consistent with the two students' cases, log files of students' work in the digital platform for the same problem indicated that students' digital actions influenced the classroom community. For example, in Ms. Clark's class, all 25 students chose to make their work visible to their groupmates in the "four-up" view, and 6 students chose to publish their document to the whole class. In addition to their usual collaboration in the classroom (e.g., looking at a peer's computer screen, physically turning the computer to show work to a peer), sharing features on the digital platform allowed for increased collaboration where students could easily view and adapt others' work. For this problem, 22 of 25 students viewed work from their groupmates, and 10 of 25 students viewed published documents from the larger classroom community. 18 of 25 students copied and adapted work from peers into their own mathematical work. With increased availability and easy access to peers' work, digital collaborative features helped students explore multiple mathematical strategies, which is potentially powerful for student learning.

Moreover, student focus group interviews provided further insights into their perspectives on collaborative learning on the digital platform. Even though students could instantaneously copy others' work, they seemed to believe that these features were to support individuals' learning rather than having the right answer and getting things done. They used the "four-up" view to gain ideas on which they could build their own solution strategies because they thought that the problem could be solved by many different approaches. One student described that the benefit of using the digital platform was getting "inspiration" from others' work:

> [In the digital platform,] you get a lot of different thoughts and different methods. And you can like, get inspiration from each other's work. […] When you're going off on different methods, it's kind of like, you are working individually, but then, for the one you share, and all those different methods, [you] just got more inspiration for the text answers.

As students appreciated that they could get inspiration from others' different strategies, which they found a lot easier on the platform than in a paper-and-pencil group work environment, they collectively wished to have their workspace always visible to groupmates by default: "I wish it was automatically on 'share,' so I didn't have to click it every single time." As such, our analysis of multiple data sources revealed that digital collaborative features allowed students to share ideas and inspirations while grappling with challenging mathematics problems and developing mathematical understanding. During this Explore phase, teachers monitored all groups' workspaces and planned to showcase student work during the summary discussion. In different classrooms, students produced and refined their work in different phases of a lesson, depending on the teachers' practices and the classroom community. That is, the flexibility of the digital platform allowed teachers to incorporate student work into their daily teaching practices in their own ways. In the next chapter, we shift to the findings about how teachers used evidence of student thinking.

V. FINDINGS FOR RESEARCH QUESTION 2: TEACHER USE OF EVIDENCE OF STUDENT THINKING

Drawing from multiple data sources, our analysis revealed that teachers recognized the efficiency of the digital platform that enabled them to be more responsive to their students' mathematical thinking than in non-digital environments. Specifically, we found that using the digital platform, (a) teachers displayed student work as they facilitated whole-class summary discussions, and (b) teachers reflected on student understanding as they planned the next lesson.

Teachers Displayed Student Work During Whole-Class Discussions

In teachers' weekly reflection surveys, the most frequently reported way of using student work in the digital environment was to display students' different strategies to discuss similarities and differences among them. Teachers displayed student-generated mathematical inscriptions on the classroom projection screen, which could provide students with opportunities to compare and connect different strategies and unpack embedded mathematical ideas. For example, Mr. Irwin wrote:

> We conducted a virtual gallery walk from Problem 2.1 (*Stretching and Shrinking*) since the students all made different figures. We then discussed what we learned about stretched vs. similar figures, and how it related to the coordinate rules. For Problem 2.2, we looked at student graphs and the coordinate rules students constructed (Dec. 9, 2022).

Teachers' emphasis on multiple strategies was consistent across problems in three units. This tendency was influenced by the curriculum design, which means that the teachers would have attended to multiple strategies regardless of using the digital platform. However, when using the digital platform, teachers pointed out the convenience of displaying student work during whole class discussions as Ms. Davis put it, "It was easier and better to show student-created work for these problems. It was quick and easy to display and click through different student artifacts that they wanted to show" (Mar. 3, 2023).

Log files of teachers' digital platform use, which provide records of each "button-click" teacher taken in the platform, can give further insight into how teachers interacted with student work. The platform allows teachers to see the "live" work for an entire class at once (e.g., log events: VIEW_GROUP, DASHBOARD_TOGGLE_TO_CLASS), and also to zoom in on or select individual students' work for closer examination (e.g., log events: VIEW_FOUR_UP_RESIZED, DASHBOARD_SELECT_STUDENT). Table 4 shows the total number of log file events for each teacher during the *MSA* unit and the number of events to view student work. We found that on every problem they taught in the digital platform during this unit, teachers viewed students' group workspaces, either embedded in the teacher dashboard to show the whole class or by viewing groups one at a time. Although the prevalence of viewing students' group workspaces differed across teachers, interactions with students' group workspaces made up a significant portion of teachers' digital activity. Further, viewing actions occurred at multiple points throughout teachers' work in the digital platform, indicating that teachers could view student work flexibly when planning or implementing. Between viewing students' group workspaces, teachers typically viewed resources from the curriculum (e.g., Problems and Problem Solutions, Teacher Guide) or used platform tools to create and adapt their own documents. As teachers described in interviews, they projected student work and other connected resources as needed during whole-class discussions.

Table 4. Log file event counts on viewing student work

Teacher	Number of Problems Taught on the Platform in *MSA* (out of 14)	Total Log File Events	Events Viewing Student Workspaces
Ms. Clark	14	2881	864 (~30%)
Ms. Davis	10	1710	892 (~52%)
Ms. Evans	9	1705	245 (~14%)
Ms. Foster	5	793	234 (~30%)
Ms. Gonzales	7	258	51 (20%)
Ms. Harris	10	1338	136 (~10%)
Mr. Irwin	7	2181	933 (~43%)
Ms. Jordan	14	452	107 (~24%)
Ms. Knowles	2	391	194 (~50%)

When displaying student work on the classroom projection screen for whole-class summary discussions, teachers used not only 'viewing student workspace events' but also 'copying student work events.' Some teachers interacted with student work through published documents and copied student work into their own teaching and planning documents in the digital platform. The teacher who utilized this to the greatest extent was Ms. Clark, who copied work from multiple students into a class summary document which she published to students for each problem. Other teachers also copied from student work for problems, including Ms. Davis (3 problems), Ms. Harris (2 problems), and Ms. Knowles (1 problem). Once copied, student work could be displayed or published to the whole class during summary discussions or saved for future reflection on teaching. Copying directly from student work allowed teachers to have more significant interaction with students' inscriptions (e.g., resizing, editing) and to potentially build on student work during discussions or future problems.

Ms. Clark's classroom observation provided an example of how she incorporated copied student work into their summary discussions. During the Explore, she encouraged students to think about various approaches and observed what strategies students were working on. After checking in with each group, she went to her desk and opened a new document in her digital workspace to copy a few different text tiles, table tiles, and graph tiles from various students' workspaces. Then, she began the whole-class discussion by acknowledging that she found it "super interesting" to observe "at least four different strategies" and that all different approaches were valid. She projected the class summary document she just created on the class smartboard and said, “OK, now, I don’t know who I got this graph from, but I want to use [this because] [student name] pointed something out to me. [student name], what did you say?” Even though it was not her own graph, the student explained her thoughts about making a graph, and Ms. Clark led the class discussion about scales in graphs. Then, she scrolled down the class document to display “a couple of people who did it with graphs,” and she had students verbally share their different approaches to making graphs. After the discussion about graphs, she asked for volunteers to explain how they used tables and equations. As such, Ms. Clark organized the class discussions to move through different strategies. After asking some students to verbally share how they found the answer by solely using equations without graphs or tables, she gave students reflection time: “Go ahead and make sure, maybe if anything in our discussion might add to your answer, […] if you got an idea from a way that a different group did it.”

In contrast, with less frequent copying events, Ms. Davis displayed student work for whole-class discussions by asking for volunteers to show their work: “Who would like for me to share something that they created?” While one student raised his hand and said he made a table, Ms. Davis pulled up his workspace on the platform projecting on the whiteboard so everyone could see his table. She asked the student, “So, you did for 5, 10, 15, and how did that help you decide who you should buy from?” After this conversation, she pulled up another student’s workspace who volunteered to share a “big table,” and she asked the class, “Anybody else make a big table, too?” By quickly moving through different students’ workspaces to display different tables, she showed that there were multiple ways to make tables for the same problem. When one student spoke, “And I also dragged it onto a graph,” she shifted the conversation to graphs by saying, “Oh, yeah, I saw quite a few of us make graphs” and displayed a couple of examples. She continued to share her observation across students’ workspaces: “So, a lot of us probably said in our answer that it depends. In one case, one is more expensive versus the other case, the other one is more expensive.” This might be because she noticed that from the dashboard view, some students left their answers vague by typing, “It depends.” She called for another volunteer who used equations without making a table or a graph, and then the class discussed how graphs, tables, and equations could help them answer the question. For the last few minutes of the lesson, she encouraged students to reflect on the class discussion and “write something from today, something that you think is useful and important” in their mathematics notebooks.

Although the two teachers used the platform in different ways (i.e., Ms. Davis displayed volunteers’ workspaces one at a time and Ms. Clark showed a collection of copied student work), both teachers similarly emphasized the importance of using multiple strategies and put student-created inscriptions at the center of whole-class discussions. That is, both teachers facilitated the whole-class discussion in ways that students could see a variety of approaches and recognize how each tool or strategy could be helpful in solving the problem. Regarding the copying of student work events, Ms. Clark claimed that the digital version of class summary documents was “a game changer” for its efficiency. The digital accessibility and mobility reduced her difficulties in selecting and sequencing student work in a paper-and-pencil classroom where she typically asked her students to bring their notebooks to the document camera in front of the classroom one after another. In the digital environment, she could pre-select student work of "an example that is correct, another one that's incorrect, one that's close" in order to facilitate the whole-class discussions about "what's right, what's wrong, what needs to be fixed to make things correct if they are not" (post-*MSA* interview, May 12, 2023). She advocated for digital class summary documents by saying, "It's much more efficient than having kids bring their notebooks and also easier for kids to read and easier to change." Further, she highlighted that her "kids feel a little bit more ownership in the summary" because they could see their own work included in the digital summary documents.

It is important to note that limited class time can influence teachers' use of student work. Ms. Clark compared the two scenarios where she facilitated the summary discussion on the same day after the Explore versus on the next day, highlighting the difference being "a factor of time." Sometimes she did not have sufficient time during the Explore to look through all students' different strategies; thus, she had to "just quickly grab something or even use what's in the answer [Teacher Guide digital materials] provide." When she planned the whole class summary discussions for the next day, she could take time to "look at pretty much everybody's work and put in a variety of different answers." In both scenarios, she found the digital platform helpful for her summary discussions: (a) even if she runs out of class time, she can quickly copy student-created inscriptions and curriculum resources, and (b) even if students leave the classroom after the lesson, she can take her time to look at student work closely. Other teachers also reported that the digital accessibility to student work after lessons helped them reflect and plan for the next lesson based on student understanding.

Teachers Reflected on Student Understanding During Planning

Teachers reported that they perceived evidence of student thinking inscribed on the digital platform as a reflection tool that could inform their planning. Such formative assessment of student understanding can be conducted both during and after lessons. During lessons, teachers made in-the-moment decisions to provide individual support when they noticed "who needed more support with graphing" (Ms. Davis, Dec. 2, 2022). Sometimes teachers decided to adjust their plans for the next lesson based on what they noticed about student thinking. In her reflection on teaching problems in the unit *Comparing and Scaling*, Ms. Davis explained how she drew on student understanding for her planning:

> For both problems, student work and strategies were important to the summary. Also, since I noticed a lot of student work was not making the connection to proportions and percentages, I used that information to change how I launched the lesson in future days (Jan. 27, 2023).

After lessons, teachers evaluated whether their students achieved the learning goals for the lesson. For example, Ms. Evans and Ms. Foster, who were working at the same school and typically reflected on and planned lessons together, wrote that they assessed student understanding by looking at their digital workspaces after each lesson:

> We used the students' work to see if we needed to review any of the ideas (which we did not need to do). We look at the students' work to determine what their level of understanding is and how we can advance that (Jan. 27, 2023).

Similarly, other teachers working at the same school tended to use student work on the platform as resources for their collaborative planning meetings. For example, one mathematics coach explained, "We looked at 2-3 pairs' responses to discuss what students were taking away from the problem and how they were thinking about the relationships shown in the picture and the real-world scene" (Mr. Irwin, Dec. 6, 2022). As such, the digital platform helped teachers focus on understanding what students were thinking and building a new mathematical idea based on it. The digital accessibility to student work at any time during and after lessons allowed teachers to look through student work more closely whenever they wanted as well as to make adequate instructional decisions. Throughout their weekly reflection surveys and interviews, teachers expressed that they could get a better sense of student understanding in their daily lessons because they had easy access to their student work without putting the effort into collecting students' notebooks. One teacher further highlighted that in the following year, she would come back to the platform to see student work.

Log files can also give insight into other features of the digital platform to center their planning and teaching around student work. Teachers focused on using features of the digital platform based on their school context and mathematical goals for problems, and not all teachers used the same features. For example, during *MSA*, two teachers in the same school (Ms. Evans and Ms. Foster) used the digital platform to support their existing co-planning by creating digital teaching documents together, and then sharing and copying those inscriptions to use in their own classes. These documents included templates for tables and graphs to support students in exploring problems and prompts in "kid-friendly" language. In another case, Ms. Jordan used the platform with her coach on one problem, where she requested assistance with planning. Her coach used the comment feature to highlight parts of the problem and teacher guide materials for Ms. Jordan to attend to and provide insight into how to support student thinking (e.g., questions to expose mathematical aspects of the problem and connections to future problems). In these ways, the digital platform supported teachers to work together to access and create mathematical resources for students and center their planning on students' mathematical thinking.

VI. DISCUSSION

The findings of this study provide empirical evidence to support the claim that the design of digital resources should consider coherence in developing mathematical ideas (Remillard, 2016). When digital resources are provided as discrete activities, it is difficult for teachers to select and sequence them in a way that supports students in developing mathematical ideas over time (Pepin et al., 2017; Remillard, 2016). To this point, the participating teachers did not need to change their usual curriculum use to take advantage of digital resources. While using the platform for three consecutive units, teachers navigated their own ways to incorporate technology into daily teaching practices and further hoped to continue using the platform for all units. Having a complete set of problem-based curriculum resources on the digital platform will become even more powerful because teachers can focus on developing conceptual understanding over time rather than coordinating isolated worksheets from various online sources (Edson, Phillips, et al., 2019).

In concert with platform and tool design, the nature of the mathematics problems in the curriculum plays a critical role in engaging students in using multiple strategies and reasoning their mathematical thinking. For example, because *MSA* 2.2 was not set up for students to use a specific approach, students freely experimented with their ideas to solve the problem. Also, the problem explicitly asks for "evidence for your answer," which led students to type in text tiles to explain their solutions. We observed students exploring multiple tools and strategies to examine relationships between equations, whereas, in a more procedural problem, students might use one strategy only or use tools in an order specified by the curriculum. The nature of the mathematics problems being open, contextualized, and cognitively demanding influenced students to use multiple digital tools and a variety of inscriptional practices to express their mathematical thinking. This is an important consideration when designing digital curricula and tools as thinking devices (Choppin et al., 2015; McDuffie et al., 2018) that support students' development of multiple mathematical strategies and can empower teachers to attend to student thinking (Edson & Phillips, 2021).

Our findings suggest that the real-time accessibility under student authors' permissions and the mobility of mathematical inscriptions can engage each student in collaborative group work. Each student has ownership of their own workspace but can also use digital tools to get "inspiration" from others' different approaches and build up their ideas. In our study, students could share or un-share work at any time, but we found that most students freely circulated their work digitally (e.g., making their work visible to others, accessing groupmates' work) and in-person (e.g., turning their computer screen to peers, gathering around the same computer, and asking peers questions to understand their work) throughout their group's work on a problem. These types of collaborative practices are important since students' interactions with others and their mathematical work impact individuals' mathematical sensemaking (Cobb & Yackel, 1996) and their identities as learners (Wagner & Herbel-Eisenmann, 2014). This might not occur, however, in classrooms where teachers and students had not yet developed productive classroom norms around sharing and discussing in-progress mathematical work. Further work is needed to explicate how student accountability and authority can be supported in digital settings and in other classroom contexts. For example, what features of the digital platform can support students who might be anxious about mathematics to share their own work and consult others? What features of the digital platform can support students to use inscriptions they were inspired by or found helpful while maintaining authority and ownership of their own work? The insight and feedback from our research participants (e.g., students who suggested “tagging” features to indicate when they use peers’ inscriptions) continues to inform our development of platform features. Such digital information can also help teachers as they attend to student collaboration and group dynamics.

For teachers, the accessibility and mobility of student work on the digital platform can bring student thinking to the fore. The digital platform can elevate teachers’ capacity to quickly monitor different strategies and prepare for summary discussions. Using the teacher dashboard overview and group views, teachers can foreground students’ mathematical ideas. This can have positive influences on classroom culture to focus on ideas rather than people. Our finding about the tendency for teachers to display student-generated inscriptions challenges the claim that teachers would find pre-designed student responses more effective than authentic student responses during whole-class discussions (Evans & Dawson, 2017). This might be because, on the digital platform, students and teachers can view digital inscriptions with better readability than students’ handwriting and drawings on paper (Dunning, 2023). It is important that the digital features support and empower teachers to make sense of student thinking at individual and group levels, so the larger research project is continuing to develop and enrich these digital tools. For example, when viewing individual student work, teachers now have access to a history feature to scroll through the timeline of how that work was created. We are also developing tools to allow teachers to sort and categorize student work using flexible metrics to give insight into the inscriptions and mathematical strategies that students use in the digital platform.

It is worth emphasizing that the teachers reported in this study had used the digital platform for many problems throughout the year prior to the *MSA* unit and in some cases multiple years. As critical collaborators in our larger project, these teachers worked with the platform extensively and provided constructive feedback from users’ perspectives. While the digital platform has been improved to be more intuitive and user-friendly based on their feedback, we have also found that the ways teachers interact with the digital platform could develop over time. As teachers took time to become familiar with digital tools and navigate possibilities to incorporate them into their practices, they found it easier to attend, interpret, and respond to students’ mathematical thinking (Jacobs et al., 2010). In part, this may be because they have developed confidence and competence in utilizing digital curriculum resources over time (Sherin & Drake, 2009). In so doing, various digital tools in the platform gave teachers the freedom to use technology in ways to support their own teaching practices. From those flexible interactions, teachers might have recognized that they had the authority to decide when and how to incorporate technology into their practices rather than thinking technology would determine their teaching (Pepin et al., 2017). Across participating teachers, the features of the digital platform allowed for a wide variety of planning and teaching practices. This mirrors our prior work, where we found that teachers develop distinct ways of using digital resources and choose resources based on their classroom needs and goals (Edson, Fabry, et al., 2023). The distinct ways that teachers use digital resources can, in turn, shape the ways that students use the digital platform, for example by exposing students to particular tools and ways of representing mathematical ideas digitally, or by establishing classroom practices for sharing work and collaborating digitally.

As teachers use the digital platform for multiple units (or even years of teaching), it is important for teacher educators to consider how to support teachers in learning about features of the digital platform and integrating useful features into their own practice. Throughout the data collection, the research team provided teachers with professional learning workshops and individualized, informal support sessions. In addition, teachers found the weekly reflection surveys helpful for them to reflect on and learn from their own teaching practices. Perhaps, survey questions asking the benefits and challenges of using (or not using) student work on the digital platform encouraged teachers to pay attention to student thinking in their daily lessons. The teachers who participated in this study were committed to spending out-of-school time sharing their reflections with us through interviews and weekly survey forms, but teachers faced significant demands on their time, and we cannot assume that they would be able to continue such reflection beyond the scope of the project. Based on the teachers' insightful survey responses, we suggest that teachers need reflection time built into their school hours. Teachers' reflections as a regular practice can certainly solidify their knowledge about how to maximize the potential of evidence of student thinking to achieve their instructional goals.

Take Ms. Evans and Ms. Foster's co-planning hours as a good example of having designated time for reflection. These two teachers met daily after teaching the same problem and reflected on student learning, which consequently informed their planning for the next lesson. Since they were connected on the digital platform as colleagues, they could view each other's teaching documents and student work examples. Just like students in a group communicated about and circulated their mathematical inscriptions both digitally and physically, these two teachers collaboratively reflected on and planned lessons drawing on evidence of student thinking. Because they had already established such close colleagueship before joining our research project, they were willing to test the digital platform, knowing that they would help each other in the process of learning about using new technology. Given the situated nature of teaching, they could focus on discussing how to utilize the platform within their own school context. This leads us to wonder about to what extent the digital platform can serve as a space for teachers, especially teachers with few in-person colleagues, to reflect, plan, and teach mathematics collaboratively with other teachers. Further research can investigate digitally enhanced teacher collaboration strategies or support systems, which can combat the limitations of sharing concrete examples of student thinking and time constraints during school hours.

VII. CONCLUSION

In this paper, we investigated how students utilized digital collaborative tools to produce inscriptions that reflected their mathematical thinking and how teachers used such evidence of student thinking in their planning and teaching. As digital technology cannot automatically advance teaching and learning experiences, our findings provide important insights into using digitally inscribed student thinking as resources in teaching and learning mathematics. First, the developed digital resources were designed to be open, connected, and collaborative for both students and teachers. This provides students and teachers with a "low-floor, high-ceiling" digital environment to access mathematics in the curriculum at different levels and use different representations to show their thinking and communicate their understandings in different ways. The implication for digital curriculum developers is to consider providing various tools so that students can explore multiple strategies as opposed to suggesting one specific tool to solve a given problem. Second, we found that in the digital platform, students were able to focus on exploring mathematical ideas as they flexibly interacted with digital inscriptions. This finding suggests that when planning for a problem, teachers should consider digital affordances that can leverage student inquiry. Third, on the platform, teachers could monitor student work at any time to formatively assess student understanding and incorporate it into their planning and teaching more efficiently compared to their non-digital classes. Beyond simply checking for completion or correct answers, teachers used the digital platform to gain insight into students' problem-solving processes and mathematical understandings. As a result, these findings provide evidence that the digital collaborative platform enhances student mathematical inquiry and helps to empower teachers to foreground student thinking in their mathematical teaching practices.

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