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RESEARCH ARTICLE

Examining the enactment of learning technologies to support learners' access, power, and achievement in elementary school mathematics

Drew Polly[1], Christie S. Martin[2]

[1] Professor, Department of Reading and Elementary Education, University of North Carolina at Charlotte
[2] Associate Professor, Department of Teacher Education, University of South Carolina

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Abstract

Technology has potential to support mathematics teaching and learning when technology is used in specific ways. This study examines how the use of mathematics learning technologies (MLTs) promotes students' Access, Power, and Achievement, as defined by Gutiérrez' (2009, 2012) equity-based framework. The article includes two cases that were collected during the authors' time engaging with students in mathematics classrooms through work in elementary school classrooms. The inductive qualitative analysis of data conducted during teaching episodes concluded that teachers' launch of the activities that used MLTs and their support during MLT use influenced students' Access, Power, and Achievement. Specifically, the more support that a teacher provided with direct telling was associated with decreases in Access and Power. There was also evidence of student engagement and Achievement based on teachers' actions during MLT activities. The article concludes with implications to support teachers' enactment of specific pedagogies during the use of MLTs in order to promote Access, Power, and Achievement.

Keywords: educational game, educational technology, elementary education, elementary school, learning technology, mathematics education, mathematics learning technology, technology integration, virtual manipulatives

• Corresponding Author: Drew Polly, email: drew.polly@charlotte.edu

I. INTRODUCTION

Importance of Equity-based Mathematics Practices

Data on large-scale national and international assessments continue to identify empirical and statistically significant differences between students of varying ethnicities and races (National Center for Educational Statistics [NCES], 2022; Organization for Economic Cooperation and Development [OECD], 2022). There is a consistent trend that children who identify with historically-marginalized groups are at a disadvantage and continue to have to fight against the odds to become flourishing, successful mathematics learners (Gutiérrez, 2009; NCES, 2022).

In response to these data, scholars have advanced ideas related to culturally sustaining pedagogy, which advocates that learners have learning opportunities that are relevant to their cultural assets and backgrounds and make clear connections between academic content and their backgrounds (Paris & Alim, 2017; Zaretta-Hammond, 2014). These critical ideas are content agnostic and cut across all content areas (Paris & Alim, 2017).

In mathematics education, scholars have advanced the idea of equity-based mathematics practices with mathematics specific references (e.g., Bakker et al., 2021; Chao et al., 2015; Gutiérrez, 2009, 2012). In the United States, the National Council for Teachers of Mathematics coupled equity and access together in some of their recommendations for mathematics teaching (NCTM, 2014). This connection between equity and access promotes the idea that equity-based mathematics practices give students access to multiple aspects of effective mathematics teaching, including grade level content, research-aligned teaching practices, relevant mathematics activities, and appropriate support (NCTM, 2014). In Gutiérrez' framework for equity-based mathematics instruction (2009, 2012), she advances four distinct dimensions which are described in Table 1. In this study, we are focusing on Access, Achievement, and Power. While Identity is a critical aspect of the framework, it was not clearly evident in the MLTs that were examined.

Table 1. Dimensions to address equity in mathematics (Adapted from Gutiérrez, 2009, 2012)

Dimension	Description
Access	Access to rigorous and updated curriculum and tasks
	Access to high-quality mathematics teachers
	Access to mathematical tools (e.g., manipulatives, technology)
Achievement	Engagement in mathematics
	Scores on assessments
	Preparation for STEM-based fields
Identity	Incorporation of personal and cultural backgrounds in mathematics
	Opportunity to use their own language
	Students see themselves as able to do mathematics
Power	Allowed to choose their own strategies
	Students are given opportunities to participate in task exploration and discussions

In Gutiérrez' framework (2009, 2012) of equity-based mathematics practices teachers can meet aspects of each of the four dimensions by situating learners in engaging, rigorous activities that are aligned to grade-level Standards (Achievement and Access). Additionally, learners need access to research-based scaffolds and mathematics tools as they choose their own strategies to explore mathematics concepts in these activities (Access and Power). Buchheister and colleagues (2019) used Gutiérrez' framework to provide specific examples about how the framework can be applied in classrooms. We have also used Gutiérrez' framework in earlier work about how the four dimensions play out in elementary mathematics classrooms (see Polly, 2021; Polly & Holshouser, 2021; Polly & Martin, 2024; Polly, 2024). In this study, we are focused on examining the extent to which teachers provided students with opportunities related to Access and Power while using mathematics learning technologies (MLTs). Scholars have found that technology best supports students' mathematics learning (Achievement) when technology supports higher-level thinking and rigorous tasks (Access), students have ownership of their own use of mathematics strategies (Power) (Polly & Martin, 2024; Urbina & Polly, 2017; Moyer-Packenham & Suh, 2012; NCTM, 2014; Rich, 2021; Zbiek et al., 2007). Students' ownership of their own their learning has been linked to increased motivation and achievement (Kim, 2024).

The work of equity-based mathematics practices has also been extended by other groups. UnboundEd, a non-profit organization, has made a more practitioner-focused framework for equity-based practices as well using the acronym GLEAM (UnboundEd, 2021). The acronym posits that instruction should be grade level appropriate (GL), engaging (E), affirming (A), and meaningful (M). They situate the idea of engaging, affirming, and meaningful in educational activities that are culturally relevant and build off of students' interests and cultural assets (Paris & Alim, 2017), and argue that mathematics learners need to have experiences that align with all aspects of the GLEAM framework in order to be successful.

Mathematics Learning Technologies

When considering equity-based mathematics practices it is critical to consider teachers' access to resources. Teachers are frequently using technology-based tools nearly daily in most elementary school mathematics classrooms. In this article we use McCulloch's and Lovette's (2023) term about mathematics learning technologies (MLTs) to categorize learning technologies that are programs, websites, and tools that are mathematics specific and not content agnostic. Technology-based mathematics manipulatives such as pattern blocks, number racks, and place value blocks are examples of MLTs. Meanwhile, general non-mathematics specific learning technologies such as Kahoot or tools in the Google Suite would not be included since they can be used in multiple content areas.

MLTs provide students with opportunities to explore mathematics concepts and deepen their understanding of concepts and connections between concepts (NCTM, 2011, 2014). Some of these MLTs are open-ended in which learners have complete control over the technologies, manipulate them on the screen and have the responsibility of making sense of the activity and then using the technology to successfully complete it (Polly & Hannafin, 2010; Hannafin et al., 1994; Land, 2000).

Based on the potential that MLTs have to support mathematics teaching and learning this study examined two different cases using MLTs. The study was grounded in the following research questions:

1) How did teachers enact the MLT during the activity?
2) To what extent did the enactment of MLTs include aspects of Access, Power, and Achievement?

II. OVERVIEW OF CASES

The cases occurred in two different educational settings. The first took place with the second author (Martin) enacting the MLT, the NCTM Fraction Game (https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Fraction-Game/), with five sixth grade students in the suburbs of a major city in the southeastern United States. The second author enacted the activity with the MLT and wrote reflective notes after the activity. The second case took place with the first author (Polly) observing a second grade teacher enacting the MLT, Thinking Blocks Addition and Subtraction (https://www.mathplayground.com/tb_addition/index.html). The intent of these cases was not to provide generalizations about the use of MLTs, but rather to provide descriptive examples about teachers' enactment of MLTs and examine the extent to which there was evidence of Access and Power in the enactments.

Case 1: Fraction Game with Sixth Graders

Context. This case was observed by the second author when a teacher taught this to a group of students to provide additional learning opportunities with adding and subtracting fractions. The case took place in the suburbs of a mid-size city in the southeastern United States.

Description of activity. The Fraction Game (https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Fraction-Game/) is included in the National Council of Teachers of Mathematics (NCTM) classroom resources provides students with a game in which they have to move multiple game pieces along number lines from 0 to 1 (Figure 1). The MLT is played by clicking on the deck of cards to flip one revealing a fraction and then the student must move up to that amount on the number line. For example, if the card says $\frac{1}{2}$, the student could move the game marker on the half number line $\frac{1}{2}$ or move an amount equivalent to $\frac{1}{2}$, on a different number line, such as $\frac{2}{4}$, $\frac{3}{6}$, etc. Students can also move a total of $\frac{1}{2}$ on two different number lines, such as $\frac{2}{8}$, and $\frac{1}{4}$.

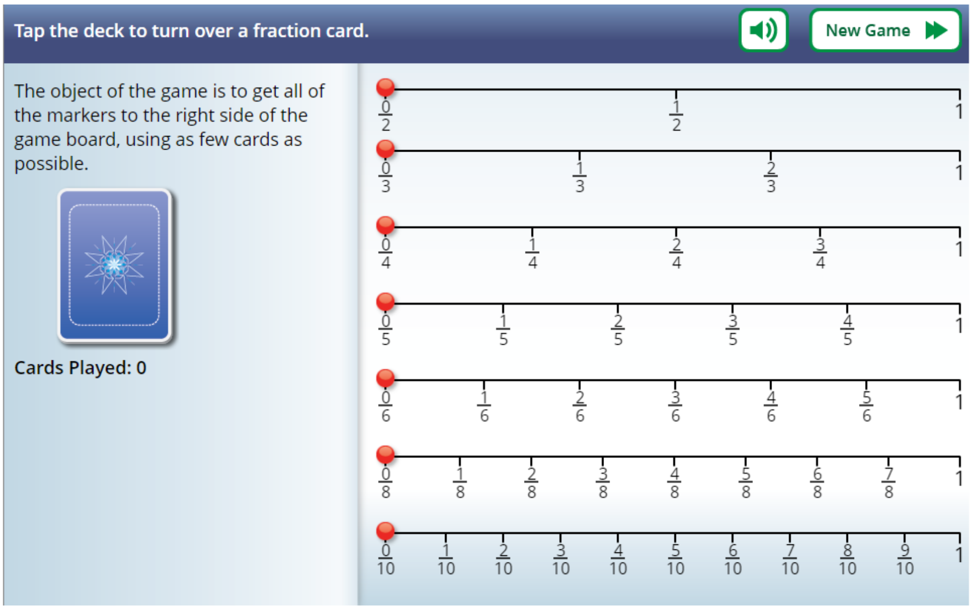

Figure 1. Screenshot of the Fraction Game

This activity has the potential for extensions and rich discourse about strategies such as using benchmark fractions and personal connections. The teacher began this lesson by playing the game as a number talk activity with the whole class. The students engaged in mental math to make suggestions and were not expected to write anything down. The teacher would flip the card in the game and ask the students to suggest how to play the card and share their reasoning.

Teacher: Our first card is $\frac{1}{5}$. How should we play it?
Student A: We should move it to $\frac{1}{5}$.
Teacher: Would you share your reasoning?
Student A: Well, I was thinking $\frac{1}{5}$ or $\frac{2}{10}$, the sixths and the eighths wouldn't get to use all the $\frac{1}{5}$. I just decided to just use the $\frac{1}{5}$ as is.
Teacher: Do we agree with this placement or have another idea? (Everyone nods) The next card is $\frac{2}{3}$, where should we place it?
Student B: I think we should use it as $\frac{2}{3}$ because those are bigger jumps to make and if we wait on the bigger jumps, we may end up wasting cards later.
Teacher: Would anyone share their thoughts about this strategy?
Student C: I see what she is saying, later in the game if we move the smaller fractions over to one first and we get a card like $\frac{1}{6}$ or $\frac{1}{8}$ we won't be able to use them. I think that might be a good idea, but I want to see how the game goes.
(Game continues…)

Teacher: What other strategies are you using?
Student D: I am using a combination of things, I am using the number lines to help me, I sometimes create common denominators for fractions, and I think about how close to half or one.
Teacher: What helps you decide on the strategy to use to relate the fractions?

The first-round ends with the whole group needing 16 cards to be flipped to complete the game. The conversation was guided by the teacher asking purposeful, high-quality questions that serve in making the thinking and reasoning of students available to the whole class (NCTM, 2014).

Next, the teacher asks the students to turn to a partner and discuss the game and their strategies. The students then find a partner different from the one they just spoke to and are instructed to use a recording sheet as they play in pairs for three rounds (Figure 2). The teacher gave directions to play each round differently.

In round one, students played in pairs, recorded strategies, including an instance where you and your partner thought differently about card placement, and the total number of cards needed to finish the game. During the second round, students played separately on two computers and they recorded the total number of cards as well as one strategy that was used. During round 3, students discussed round 2 and their strategies, and then played together recording their total number of cards as well as strategies that they used.

During this time, the teacher circulated the room, asking open questions about how the game and the strategies that students were using. The lesson includes some structure and guidance; however, the students had a lot of autonomy to explore the game and work together to make decisions on which strategies to use and how to move the game pieces on the board.

Round 1 with a Partner	Round 2 Individual Round	Round 3 with a Partner
Total Cards Used _____	Total Cards Used______	Total Cards Used____
Strategies Used	Strategies Used	Strategies Used
When did you and your partner disagree?		Reflection

Figure 2. Fraction game recording sheet

Role of the mathematics learning technology. The role of mathematics learning technology (MLT) in this lesson is key to the engagement of students in mentally building their reasoning skills with fractions. It allows students to use their own understanding and references for fractions, visual number lines, and move efficiently through the game with the technology catching errors as well. The technology provides visual number lines that make the game accessible for all students. Students can start the game at the level of using the cards at face value and the number lines and continue to advance in their thinking and strategies. Technology aids in this process and development.

Connections to equity-based mathematics practices. The Fraction Game MLT included aspects of Access, Achievement, and Power from Gutiérrez' (2009) framework of equity-based mathematics practices. In terms of Access, the design of the NCTM Fraction game is a high-quality resource designed to build number sense with fractions that aligns to sixth grade state standards. Additionally, the design aligns to the research-based approach of adding and subtracting fractions using a visual such as moving on a fraction number line. The teachers' enactment of the Fraction Game with their questioning techniques and allowing students to make their own decisions of their thinking were evidence of giving students Access to effective mathematics teaching.

Related to Achievement, the Fraction Game MLT kept students engaged in the activity and students were able to successfully play the game and determine the sums of fractions. Students were able to orally describe the strategies that they used and wrote down notes about their strategies on their recording sheet.

Related to the Power dimension of the equity-based mathematics practices framework Gutiérrez' (2009), there was ample evidence of students making their own decisions and choices about how to move the game pieces across the board. This was evidenced by the conversations between students and the recording sheets. The teachers' enactment of the Fraction Game MLT allowed for students to use their own words to vocalize their strategies and the connection they are making when deciding how a card should be used.

Case 2: Thinking Blocks Addition and Subtraction in Second Grade

This case focused on a second grade classroom teacher working with 5 students about solving addition and subtraction word problems using an internet-based MLT on iPads called Thinking Blocks Addition and Subtraction.

Description of context. The case took place in a school in a suburban school 15 miles from a major city in the southeastern United States. The first author (Polly) observed a second grade teacher working with four students on the activity while the classroom teacher worked with a different group of students, and the rest of the children completed independent assignments. Three of the children were Black and 2 were Caucasian. The teacher indicated that all of the students in the group needed support with solving word problems based on recent data from the school districts' commercially-made diagnostic mathematics assessment.

Description of activity. On a laptop computer, the teacher displayed the Thinking Blocks Addition and Subtraction website (https://www.mathplayground.com/tb_addition/index.html) and read the word problem aloud to students (Figure 3). The blue labels on the right and the green and purple rectangles can be moved on the screen to set up a representation that helps students determine how to solve the problem. Providing students with support on how to use bar models or similar representations has been shown to support students' problem solving (Howerton & Polly, 2023; Morin et al., 2017; Polly & Howerton, 2023; Powell et al., 2020).

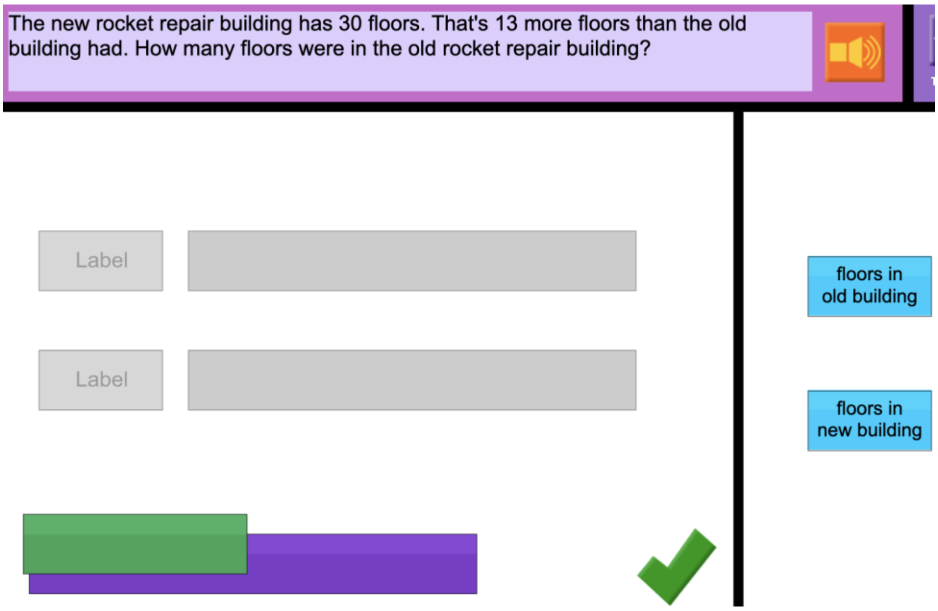

Figure 3. Thinking blocks addition and subtraction work space

The teacher then started the 3 Reads Protocol to help the students understand the word problem. First, she asked them to retell the problem in their own words to a classmate at the table. Second, she asked them to identify and describe what each number means in the problem with a classmate.

T: What is a number that you see?
S1: I see 30.
T: What does 30 mean in the problem?
S1: 30 is how many floors are in the new building.
S2: I see 13. There is 13 more in the new building than the old building.
T: What is the question asking us to find?
S3: We need to find the number of floors in the old building.
T: Let's move the bars. Where should we put the larger purple bar and the smaller green bar?
S4: Where are we moving the bars? I don't get it.
S2: The larger number should have the larger bar. I think the larger purple bar needs to go with the floors in the new building.

All students nod their head and agree.

T: Why do you all agree?

S1: It says that the rooms in the new building is 13 more than the rooms in the old building so the new building should get the larger purple bar.

The teacher then had a student move the labels and the rectangles into their locations. Clicking on the check mark gave them feedback that they were correct and then numbers appeared on the screen. The teacher asked students where they should place the 30, the 13, and the question mark. All of the students correctly identified where to put the numbers (Figure 4).

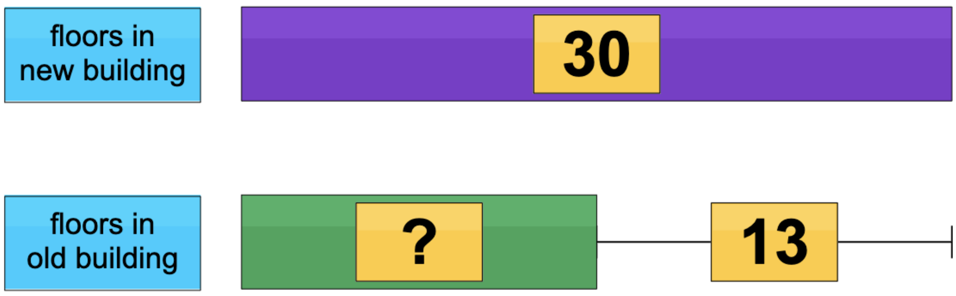

Figure 4. Thinking blocks website after the representation

After the check mark was clicked the screen then displayed a keypad to type in the answer. The teacher then had a conversation with the students:

T: Based on what we see, what should we do to find the answer?

S4: Let's put 30 and 13 together to get 43.

S1: I think it's smaller than 30 so we need to subtract.

S2: That makes sense. Let's subtract.

T: Why should we subtract?

S4: I think we actually do need to subtract. The floors in the old building is smaller than the floors in the new building so the old building will have less than 30 rooms.

T: Okay. Work with your neighbor and find the answer.

All of the students found that the answer was 17. The teacher then gave each pair of students an iPad that already had the website loaded. Students worked together then on the next problem with the instructions of pausing to show their iPad to the teacher after they had moved the labels, rectangles, and numbers into the work space.

While the teacher observed and asked questions, the MLT provided students with feedback on the placement of the labels and rectangles before allowing them to move the numbers into the workspace. This affordance allowed students to get feedback without having to wait for the teacher. After students correctly moved everything into the workspace they used whiteboards and markers to find the answer to their word problem. Students completed three different problems in pairs during the activity.

Role of the mathematics learning technology (MLT). In the case of the MLT Thinking Blocks Addition and Subtraction, the second grade students benefited in two specific ways from the MLT. First, the work space and the tools provided them with a step-by-step process to setting up a bar model representation of the word problem using the labels and rectangles. The MLT required students to make sense of the word problem before moving on to the part of the program where they could solve the problem; specifically, the program required students to make a correct mathematical representation with specific labels and rectangles that matched the quantities that were in the problem. This feature of the MLT acted like a mandatory checkpoint since it required students to make a correct representation before moving on to the part of the program where they solved the problem and entered an answer.

Additionally, students benefited from the feedback that was embedded in the MLT. While the teacher guided the group of students through a word problem and was at their table to ask questions, the MLT included feedback built in where the teacher was not needed. Students who did not correctly make the mathematical representation with matching labels and values were not allowed to progress on and were given a visual cue on the screen about which labels or parts of the representation were incorrect. While an argument could be made that the MLT's feedback could have been more specific with a hint, the program at least gave students a clear indication of what aspects of the labels and/or representation were incorrect. The fact that this feedback is embedded has led researchers to posit that this tool could be used by students independently without requiring the presence of a teacher (Howerton & Polly, 2023; Polly & Howerton, 2023).

Connections to equity-based mathematics practices. The enactment of the Thinking Blocks Addition and Subtraction MLT included various aspects of Gutiérrez' framework of equity-based mathematics practices, especially Access and Achievement. In terms of Access, there was evidence based on the design of the MLT as well as the way that the teacher enacted the activity. In terms of the design of the MLT, students had access to the research-based approach of using bar models to make sense of word problems as well as feedback to help students think about any misconceptions or mistakes. In terms of the enactment of the MLT, students had access to a teacher who used research-based approaches of the 3 Reads Protocol and asking questions to elicit students' mathematical thinking. Additionally, the MLT included word problems that aligned to both the types of word problems and the number sizes in second grade.

Related to Achievement, students were engaged in doing mathematics during the entire activity. There was also evidence of learning; students correctly solved each word problem in pairs with minimal support from the teacher. Related to Power, the MLT is highly structured and does not give choices in how students set up the bar model representation. While students had little Power as they used the MLT and the specific process to set up the bar model representation, there was evidence of Power when students chose their strategy to find the answer. While the teacher provided access to hands-on manipulatives and hundreds charts, all students used whiteboards and markers to solve the tasks using drawings of base ten blocks, number lines, or specific written strategies or algorithms for addition and subtraction.

Looking Across the Cases

The enactment of both MLTs provided examples of how the design of MLTs and teachers' enactment of MLTs can promote teaching and learning and the use of equity-based mathematics practices. Table 2 displays connections between the two cases and dimensions of Gutiérrez' (2009, 2012) equity-based mathematics framework. As stated previously, the focus was on Access, Achievement, and Power, since these MLTs were not embedded in real-world contexts, limiting the potential for Identity to be embedded within the activities.

Table 2. Connections between cases and equity-based mathematics teaching practices

	Fraction Game	**Thinking Blocks Addition and Subtraction**
Access	Students had access to the visual number line that was on the screen. The program allowed for the quick and easy generation and manipulation of mathematics represe-ntation as students moved game pieces on the fraction number lines. Students had access to a teacher who used effective moves, such as posing questions about students' thinking about the use of technology and the mathematics concepts.	Students had access to the workspace where they created a representation and used it to make sense of the word problem. Students had access to a teacher who asked questions using the 3 Reads Protocol and about students' thinking.
Achievement	Students were engaged the entire time while playing an open-ended technology-based mathematics game that was aligned to grade-level content. Students demonstrated an understanding of fractions as they talked with their partner about the mathematics representations generated in the program and the fractional amounts on the game cards.	Students used the website which had grade-level appropriate second grade word problems. Students demonstrated that they could work with a classmate to successfully solve word problems.
Power	Students chose how they wanted to move their game pieces in the MLT as they tried to move all of their game pieces along the game board to win the game. Students talked about the mathematical representations on the MLT as well as the fractional values on the game cards. Students talked about strategies and possible moves on the game board while using the MLT.	Students could choose how they solved the problem using whiteboards and markers. Manipulatives and hundreds charts were available but no one used them.

While the two cases varied by grade level and included primary learners (second grade) and upper elementary/middle grades learners (sixth grade), there were some similarities noted in the table above. The enactments of both Mathematics Learning Technologies (MLTs) included aspects of Access as students completed activities that were aligned to grade level standards, and the technology provided visuals with fractions on a number line (Case 1) and bar diagrams (Case 2). Additionally, the MLT used in Case 2 provided feedback to students and scaffolds by providing suggestions if students made a mistake. Additionally, in terms of access, teachers' enactments with the MLTs also provided aspects of Access as teachers used three specific research-based approaches: 1) questions about students' mathematical thinking (Case 1), 2) writing about strategies (Case 1), and 3) the use of the 3 Reads Protocol (Case 2). While these three strategies are research-based they do not necessarily require the use of technology or MLTs. What is noteworthy, is that the use of MLTs in conjunction with research-based, non-technology specific pedagogies increases aspects of equity-based pedagogies.

Achievement was evident as students remained engaged in and demonstrated that they were able to successfully complete the activity. While student learning was not measured in these cases, students remained engaged and participated at all times, which is foundational aspect of Achievement (Gutiérrez, 2012). Meanwhile, Power was evident in Case 1 as students had total autonomy on how they were moving their game pieces on the number lines. In Case 2 there were no aspects of Power during the use of the structured MLT, but there was Power when students chose which strategy to use to find the answer.

III. DISCUSSION AND IMPLICATIONS

There is great potential for mathematics learning technologies (MLTs) and teachers' enactments to support mathematics teaching and learning while simultaneously including aspects of Gutiérrez' (2009, 2012) framework for equity-based mathematics practices.

MLTs Support Equity-based Mathematics Practices

Both cases included examples of MLTs that included aspects of equity-based mathematics practices. This was evident by the design of the MLT as well as teachers' enactments. In this study we focused on Access, Achievement, and Power. These were supported during the lesson by both the design and teachers' enactments of the MLTs.

MLT design supports equity-based mathematics practices. In terms of Access, both MLTs in the cases provided access to research-based approaches to grade level mathematics concepts. Both MLTs in the cases included visuals that supported students' mathematical work. In Case 1, the Fraction Game MLT included the visual of a number line to explore ways to move their game pieces from 0 to 1 using fractions with different denominators. In Case 2, the Thinking Blocks Addition and Subtraction MLT provided students with work space to create their technology-based representation of bar models and then use that to determine if they should add or subtract the numbers in the problem.

Additionally, both MLTs provided feedback and scaffolding. In Case 1, the Fraction Game would not let students move their game pieces an amount greater than the fraction that was on their card. In Case 2, the Thinking Blocks activity required students to make a correct bar diagram representation with the size of the bars and the labels correct before working with the numbers. These technology-based scaffolds support students' Access to high-quality mathematics experiences and support achievement and students' learning (Moyer-Packenham & Suh, 2012; Park et al., 2022).

In terms of Power, the design of the Fraction Game supported Power since students were able to decide how to move their game pieces across the multiple number lines from 0 to 1. Students used different moves and strategies and spent time talking as a class and in pairs about which strategies may be the best in terms of moving all of the game pieces in the fewest moves possible. Power was not embedded in the Thinking Blocks MLT since students had a set way to make the bar model representation before they worked on finding the answer.

Teacher enactment of MLTs and equity-based mathematics practices. The two cases provided examples of teachers providing students with Access to a high-quality mathematics activity that supported students' Achievement. This was done by teachers enacting research-based strategies during their enactment of the MLTs. In Case 1, the teacher provided the opportunity to play the game as a whole class and asked open-ended questions about the various ways that students could move their game pieces. The teacher allowed students to drive the class-wide conversation and helped students to make connections between the written fraction, equivalent fractions, and the denominators on the number line. The teacher then continued to give students even more Power as she allowed students to play the game individually against the computer or in pairs with a focus on the possible moves that they could make and which strategy could be the best move. In terms of Achievement, while there was no formal measure of learning, evidence of student understanding was observed as students shared their strategies and reasoning about why they made certain moves and why certain moves would not work.

During Case 2, the teacher enacted the Thinking Blocks Addition and Subtraction MLT by using the research-based 3 strategies such as the Reads Protocol and asking questions about students' thinking. Additionally, the teacher provided students with support by guiding them through the first problem together and facilitating a discussion about how they should use the MLT to make a bar diagram representation that matched the word problem. These teacher moves provided Access to a high-quality mathematics activity for students which in turn was associated with students' being able to demonstrate Achievement by successfully solving word problems in Pairs.

Case 2 allowed students to have Power when students had ownership about what strategy to use to solve the problem. The first author (Polly) observed students using pictures of base ten blocks, number lines, and the U.S. standard algorithm to solve the problem. Had the teacher not allowed students to choose the strategy, the whole activity would have had no evidence of Power since the Thinking Blocks MLT was very structured and only had 1 way to set up the bar diagram representation.

Future Directions for Research and Practice

In light of the cases provided here and the current literature there is a need to consider future directions that will provide more guidance about how MLTs can support teaching and learning and the use of equity-based mathematics practices.

Implications for research. The goal for all mathematics teachers is to support students' learning. In that spirit, it is important to consider the intersection between equity-based mathematics practices and student learning. Scholars have shown that when learners are actively engaged in grade-level specific activities (Achievement) that are supported by effective teachers and resources such as manipulatives or technology (Access), learning is likely to occur (Hattie et al., 2018; NCTM, 2014; U.S. Department of Education, 2008). Findings from a large-scale meta-analysis also indicate that allowing students to explore a mathematics concept prior to a discussion or explicit teaching leads to learning (Sinha & Kapur, 2021), which supports the Power dimension.

Still, though, there is a body of research from the fields of Special Education and Educational Psychology citing that explicit teaching can lead to learning in individual and small group settings for students whose data suggests they need more mathematics support (Hughes et al., 2017; Kong et al., 2021). This form of explicit teaching has not been linked to whole class settings and does not align with the Power dimension. More research is needed that examines effective ways to support students' learning in whole class settings, including how equity-based mathematics practices intersect with those settings.

Additional research is needed specifically on the influence of MLTs. In the case of these two cases, students engaged in technology-based open-ended activities. Students were able to successfully use the MLTs to complete the task. Follow up research studies should examine how learners using the MLTs compare to learners who do not use technology to explore the same mathematics concepts. Additionally, work is needed to consider how learners make sense of the mathematics in these open-ended MLTs and then transfer that understanding to other tasks that are not technology-based. Lastly, we acknowledge that MLTs are vastly different; research on student learning using the Fraction Game from Case 1 may have a different impact than the Thinking Blocks MLT from Case 2 or other MLTs. While educational researchers want to generalize about technology's impact on student learning, the vast differences between MLTs may limit the ability to make broad claims between technology and student learning.

Lastly, as stated earlier, these MLTs provided meaningful practice of mathematics concepts without a lot of real-world examples. While the MLT in Case 1 included word problems there was evidence that these problems were embedded in contexts that may be appealing to children (e.g., sports, toys, cars), but there was no evidence that these problems were specifically related to diverse cultures. As a result, the Identity aspect of Gutiérrez' framework (2009, 2012) was not clearly evident. More research is needed regarding how the presence or absence of identity may influence students' interest, participation, and therefore their Achievement, when using MLTs. Experts in culturally responsive teaching would posit that when students do not see themselves reflected in the real-world contexts of mathematics that they may lose interest and become less engaged (Gutiérrez, 2012; Paris & Alim, 2017).

Implications for practice. Based on the cases presented, we consider two types of individuals to consider related to implications for practice: designers of learning activities and mathematics teachers.

Designers of learning activities. Designers of learning activities would include companies, organizations, and individuals who design and develop MLTs or curricula resources that include MLTs. In the case of the cases in this paper, the Virtual Number Frames program included only the technology, and required the teacher to come up with the word problem and various scaffolds of the Three Reads protocol and questions posed while students explored the word problem. Meanwhile, the Fraction Game included a website with directions and the activity, so while the teacher did have the class play a round of the game together, students could have been given the game and the opportunity to figure out how to use it with minimal support. One thing that must be considered is that all teachers may not know how to support or scaffold learners in the ways described in these cases. Therefore, designers of MLTs or curricula that use MLTs need to look for ways to embed scaffolds or ideas that teachers can use to scaffold and support their students. This may include directions on how to introduce the activity or MLT or possible questions that teachers can ask to support students.

In this study, a limitation is that there was no formal measure of student learning associated with the use of the MLTs. For each of these cases future work could examine students' learning while using the MLT as evidenced by students' written work, their conversations, and an assessment of the mathematics concepts given after the MLT activity. While some studies have started to more formalize links between MLTs and student learning, most of these are in individual or very small groups that lack the authenticity and logistical complexity of large classroom-size implementation (Morin et al., 2017; Powell et al, 2020).

Supporting teachers' enactment of MLTs. Related to mathematics teachers, the cases bring to light the connection between teacher actions and equity-based mathematics teaching practices. In both episodes, the authors chose to use these specific MLTs which aligned to grade-level content (Achievement) and then decided to let learners use the MLTs to explore mathematics concepts to support their learning (Access). Teachers need support to determine which MLTs they should choose to support their students' learning of grade-level content as well as how the MLTs should be used in ways that promote learning. Professional learning (aka professional development) may be as formal as workshops or as informal as conversations with colleagues or mathematics leaders about specific MLTs and/or ways to use MLTs in ways to promote equity-based teaching. Research has indicated that teachers tend to focus on surface-level features of mathematics-specific technologies such as the graphics or visuals rather than the mathematical tasks (Polly & Hannafin, 2011; Yeo, 2023). Future work is needed to consider how to best support teachers' learning about and use of MLTs in ways that promote equity. MLTs clearly have potential to promote equity-based mathematics teaching practices and support learning. However, in order for equity-based mathematics teaching practices to be enacted, teachers need to be supported in the selection and enactment of MLTs.

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