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RESEARCH ARTICLE

"It's easy. We got Desmos right here": The role of mathematical action technology in positioning students as mathematical explorers

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Abstract

The positive impact on student learning and continued support of mathematical action technology (MAT) in classrooms deems a need to better understand what teaching practices maximize the affordances of MATs. The purpose of this study was to better understand the technology-centered teacher moves that allow students the opportunity to be positioned as mathematical explorers and sustain mathematical authority during a MAT task. In this case study of a MAT task designed to leverage the power of sliders in Desmos to explore key characteristics of the sine function, participants were two ninth-grade students (age 14), who engaged with a task-based interview. By coding the transcript of the task-based interview, the findings identified and described the teacher's actions with the technology that resulted in meaningful mathematical activity for the two students. Along with teacher actions with the technology, evidence showed the importance of the design of the MAT task and the ability of students to troubleshoot the technology. Ultimately, we identified important considerations for teaching mathematics with technology as well as several technology-centered teaching moves, leaving room for the students to perform as mathematical explorers. Applying these research methods for future cases could help generalize these technology-centered teaching strategies that position students as mathematical explorers, thus strengthening students' mathematics identities.

Keywords: mathematical action technology, positioning, mathematical explorers, mathematics identity, mathematical authority

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I. INTRODUCTION

An important role of the mathematics teacher is to foster students' positive mathematics identities to create an equitable mathematics learning environment (AMTE, 2017; Gutiérrez, 2012). Further, the role of the teacher as students engage in learning mathematics using technology is vital for integrating the technology into the learning environment (Drijvers et al., 2010; Huang & Sutherland, 2022). One way to foster students' mathematics identities through technology use is through the use of mathematical action technologies (MATs; e.g., dynamic geometry environments), which "perform mathematical tasks and/or respond to the user's actions in mathematically defined ways" (Dick & Hollebrands, 2011, p. xii). MATs have the power to position all students as explorers of mathematics by mediating mathematical discourse regardless of access to precise language, providing multiple entry points for students, and helping students build powerful and personal ways of thinking about mathematics (McCulloch et al., 2021). These are mirrored in equitable teaching practices that provide students opportunities to analyze, compare, justify, and prove their mathematical findings (Aguirre et al., 2017). According to Su (2020), mathematical explorers can be seen as students who practice mathematical thinking and feel comfortable in mathematical environments that are driven by curiosity and the desire to understand patterns, structures, and relationships.

The way students are positioned through teachers' pedagogical practices has been shown to positively and negatively affect their learning as they develop disciplinary identities (e.g., mathematics identity, science identity; Bishop, 2012; Hazari et al., 2010; Tait-McCutcheon & Loveridge, 2016; Turner et al., 2013; Wood, 2013), suggesting teachers' practices are a crucial component of identity development. We refer to the in-the-moment enactment of teachers' practices as positioning moves, or more simply, teacher moves (Harré & van Langenhove, 1991). For example, a teacher might ask students to justify their mathematical thinking using technology, positioning the students as the mathematical explorers. Pedagogical moves such as how a technology task is structured can also be a teacher move. For instance, a teacher might assign students to work on a Desmos task consisting of an information content page and then a dynamic graphing window to make sense of the questions that follow. Depending on the context, students would be positioned in different ways through the structure of the technology task, perhaps as mathematics learners who are confident exploring mathematical phenomena. We will further define positioning moves in the theoretical framing section.

Although there has already been research done on teacher strategies that promote mathematical discourse, a gap exists in the research on the role of MAT tasks during discourse in mathematics classrooms (Huang & Sutherland, 2022). According to Huang and Sutherland, the "impact of interactive technology on education makes understanding its potential for promoting creative dialogue an important strand of research" (p. 323). They also called for further investigation into understanding the role of teacher moves to promote the use of technology artifacts jointly constructed by the teacher and the students for facilitating meaning-making discourse.

Because the teacher-student and technology-student discourse during a task has the potential to position students, we focused on both during this study. We investigated the following research questions:

1. What is the role of technology in positioning students as mathematical explorers when engaging with mathematics action technology tasks?
2. What specific teaching moves leverage mathematics action technology while position students as mathematical explorers?
3. How are these specific teaching moves positioning students to engage in mathematical thinking (i.e., what are they enabling students to do?)

II. RELATED LITERATURE

Researchers have linked identity and learning to participation in social practice (Lave & Wenger, 1991). Teachers facilitate students' mathematics identity development through their positioning moves. Specific teacher moves, which can be made to engage students in rich mathematical discourse, include revoicing students' thinking, repeating others' reasoning, asking students to apply their reasoning to others' reasoning, prompting students for further participation, and using wait time (Chapin & Anderson, 2013; Kim & Yeo, 2019a, 2019b). Students have multiple, dynamic mathematics identities in any given moment, which are influenced by how they are positioned. For example, in a mathematics classroom, a student might be positioned as someone who answers questions by the teacher, but as a know-it-all by other students in the class. Over time, the student takes up and responds to the way they are positioned, influencing how their mathematics identity changes and develops. Several researchers have described this dynamic nature of students' mathematics identities, finding that students were positioned differently from moment to moment (Esmonde & Langer-Osuna, 2013; Radovic et al., 2018; Wood, 2013). From moment to moment, students are constantly positioned through classroom discourse by the teacher, their peers, and themselves. The dynamic process of identity formation occurs through this constant positioning, as students take up how they are positioned over time. A thorough understanding of these ideas is useful for examining the role teachers play in mathematics identity formation, particularly in the context of students interacting with MATs.

Mathematical Action Technologies

A crucial consideration for this study is the widely accepted position of technology as a transformational tool in the learning and teaching of mathematics. In the U.S., the National Council of Teachers of Mathematics (NCTM) suggests that a high-quality mathematics program uses technology, specifically mathematical action tools, as an essential asset to "help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking" (NCTM, 2014, p.78). Supporting this vision, the Association for Teachers of Mathematics (AMTE) has stood behind the mission of mathematics teacher training programs ensuring that all mathematics teachers have the opportunity to gain the knowledge and practices for effectively integrating technology within the scope of teaching and learning mathematics (AMTE, 2006). NCTM places significant value on the use of mathematical action technologies, termed by Dick and Hollebrands (2011), due to their interactive nature and affordances of supporting mathematical reasoning and sense-making. AMTE (2006) also recognized the power in use of these technologies to transform student thinking.

Research shows MATs playing a role in students' conceptual agency - students' own responsibility for developing meaning and relationships between concepts and developing their own strategies to solve problems (e.g., Atabas et al., 2020; Leung, 2011). When conceptual agency was enacted, learners were empowered, and instruction was changed by engagement with dynamic technology tasks. In reverence of this research, it is critical that mathematics teachers examine the role of technology and surrounding teaching practices in support of equity to students, schools, and communities (Barlow et a., 2020; Gomez et al., 2021). McCulloch and colleagues (2021) encouraged making decisions so that mathematics action technologies are addressing inequities by positioning *all* students as explorers of mathematics. McCulloch and colleagues (2021) focused on three ways of positioning the student as mathematical explorers by using technology to: (1) enter a mathematical problem, (2) mediate mathematical discussions regardless of familiarity with mathematical language or dominant language, and (3) build personal and powerful ways of mathematical thinking.

Studies have also detailed the complex process through which MATs become tools for learning (Artigue, 2002; Drijvers et al., 2010; Huang & Sutherland, 2022; Trouche, 2004). Artigue (2002) described instrumental genesis as the process through which an artifact becomes an instrument. For instance, as students work through a MAT task, instrumental genesis would describe the process through which the task becomes a tool for the students' learning. Trouche (2004) described instrumental orchestration as "the teacher's intentional and systematic organization and use of the various artifacts available in a learning environment in a given mathematical task situation, in order to guide students' instrumental genesis" (p. 214-215). There are three elements of instrumental orchestration: didactical configuration (Trouche, 2004), exploitation mode (Trouche, 2004), and didactical performance (Drijvers et al., 2010). Didactical configuration refers to the arrangement of the teaching setting and the artifacts involved. Exploitation mode refers to how the teacher utilizes the didactical configuration to meet their objectives, and didactical performance refers to the in-the-moment decisions taken while teaching. In a study characterizing the types of instrumental orchestration occurring during a technology-rich lesson for eighth grade students, Drijvers et al. (2010) described six orchestration types enacted by the teacher and the students. Huang and Sutherland (2022) also described moves made by both teachers and students during a MAT task, examining the role of digital artifacts generated during a MAT task. They described pre-service teacher pedagogical moves for promoting high quality mathematics discussion and identified pedagogical moves that either influenced discourse characteristics or instrumentally orchestrated opportunities for student learning as mediated by mathematical artifacts. In addition, they found that different technology artifacts mediated mathematics discourse in different ways, which related to the features of the technology as well as the teacher moves during the lesson. They emphasized the important role of the teacher as students engaged in a MAT task, stating, "the artifact is only responsible for creating space and is not an adequate means in and of itself without a teacher's pedagogical support" (p. 344-345). Another study found that specific instructional moves, as enacted by the teachers, during a technology mediated task, were able to position the students as mathematical explorers (Fletcher & Fye, 2022). Though the researchers found notable teacher moves present, what was not explored was how the teachers utilized the features of the technology in their teaching moves to afford students the opportunity to be positioned as mathematical explorers.

III. THEORETICAL FRAMEWORK

As a reminder, the purpose of this study was to better understand the technology-centered teacher moves that allow students the opportunity to be positioned as mathematical explorers and sustain mathematical authority during a MAT task. Thus, it was necessary to frame our interpretation of the teachers' moves in terms of the classroom discourse, the technology's role in the discourse, and the way authority was distributed in the classroom. We drew from three theoretical frameworks to frame the study: the didactic tetrahedron (Cohen et al., 2003; Hollebrands, 2017), mathematical discourse (Sfard, 2007), and positioning theory (Davies & Harré, 1990; Harré & van Langenhove, 1991).

The Didactic Tetrahedron

To describe the classroom discourse, we first drew from Cohen, Raudenbush, and Ball's (2003) instructional triangle model to identify and describe the different dimensions of classroom discourse. This model outlined mathematics teaching practices through the dynamic interactions between three dimensions: the teacher, students, and content. The didactic triangle has been employed in previous research to characterize the interactions between an educator, their students, and the subject matter being instructed. In our case, the interactions can be thought of as the teaching practices that help the students engage in meaningful mathematics activity. We consider meaningful mathematics activity as problem solving, proving, constructing arguments, communicating reasoning, connecting representations, modeling with mathematics, and attending to precision (NCTM, 2000, 2014). Extending this framework, Hollebrands (2017) incorporated a fourth dimension, examining how technology mediates the relationships between the teacher, students, and mathematical tasks. (see Figure 1). We use this as a basis for our consideration of how the use of technology influences teaching practices, particularly in-the-moment teacher moves, and what the subsequent meaningful mathematical activity that results from those moves.

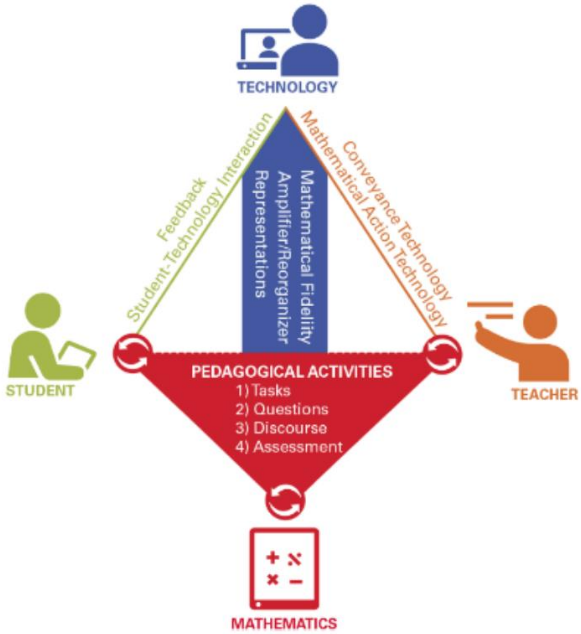

Figure 1. The didactic tetrahedron (Hollebrands, 2017)

Mathematical Discourse

Having identified and described the different dimensions involved in mathematical discourse, it was also necessary to define mathematical discourse and the role it plays in student learning. In general, for students to learn mathematics, they must modify or extend their discourse so that they can solve problems (Sfard, 2007). For students participating in mathematical discourse encompasses all communication methods used in conveying understanding, including spoken words, written symbols, images, as well as actions and gestures (NCTM, 2000). Mathematical discourse involves thinking and communicating about abstract objects such as equations, numerals, and models. According to Sfard (2007), thinking is a form of discourse, thus learners can have discourse with themselves. For example, a student may position themselves in the role of "dumb" in mathematics by mentally engaging in negative self-talk about their ability to do mathematics. As students engage in mathematical discourse through a communicational conflict, a difference in the learner's discourse and the communication of the group, incites learning. Technology can facilitate communicational conflicts by providing opportunities for students and teachers to engage with multiple representations, collaboratively create and explore interconnected mathematical objects, and exercise agency in generating mathematical discourse and practices (Acrs et al., 2008). In using technology, Gonzalez and Herbst found "the interplay between speech acts, gestures and use of tools (e.g. dragging and measuring) allowed students to act upon diagrams and participate in classroom discourse" (2009). Thus, technology plays a crucial role in the facilitation of mathematical discourse. The teachers' role in the discourse as students engage with technology should be to facilitate students' encounters with communicational conflicts. In other words, the teachers' discourse moves should be understood in light of how they facilitate or constrain students' participation in the discourse. For example, when students encounter mathematical phenomena they are not familiar with (e.g., a dynamic graph responds to a change in the algebraic representation), the teacher can ask questions to prompt students to think deeply about mathematics by further engaging with the technology to explore their ideas.

Positioning Theory

Because teachers' participation in the classroom discourse inevitably facilitates or constrains students' participation in the discourse, it is also necessary for us to frame how mathematical discourse is enacted and taken up in the classroom by the teachers and students. We drew from positioning theory to meet this goal. Positioning refers to recognizing discursive actions as social acts within storylines familiar to participants in discourse (Harré & van Langenhove, 1991). People are positioned when the community establishes the rights, duties, non-rights, and non-duties of its individual members in a particular storyline. In a mathematics classroom, a common storyline might be the teacher as the deliverer of knowledge and the student as the recipient of knowledge. For example, a teacher might position a student as a "knower of mathematics" by referring to the student's knowledge frequently during a class. Though the teacher is traditionally positioned as the "expert" in common storylines like this one, they do not have to draw from this natural authority and can justify their ideas logically instead to help students learn. The teacher's role in the discourse is to help students move out of circular discourse by facilitating instructionally effective conflicts, those in which students have a realistic communicational agreement about the leading discourse, participants' roles, and the nature of the expected change in discourse. Thus, the teachers' role is to consider how their discourse positions students to engage in mathematics in this way.

People are constantly positioned through discourse during their lives in numerous and potentially contradictory roles and storylines. An individual making sense of these different ways of experiencing themselves leads to a multiplicity of selves (Davies & Harré, 1990). According to Davies and Harré (1990), "it would be a mistake to assume that, in either case, positioning is necessarily intentional. One lives one's life in terms of one's ongoingly produced self, whoever might be responsible for its production (p. 48). In other words, positioning is a subconscious process, which is a result of having a dynamic identity. Both teachers and students engage in mathematical discourse, which results in everyone involved in the discourse being positioned in many ways over time. Applying positioning theory to understand teacher-student discourse enables the researcher to explain how in-the-moment discourse facilitates individuals experiencing themselves in different ways in relation to familiar storylines or larger social discourses.

From a positioning perspective, mathematics identity can be thought of as "the repetition of 'performances' in mathematics learning contexts that generates our recognition of ourselves in certain ways as learners of mathematics" (Darragh, 2015, p. 85). This performative perspective on identity originated from the work of Butler (1988, 1997), who described gender identity as performative. As a student performs their mathematics identity, those they are interacting with are audience members that recognize and interpret the performance (i.e., position the student) in different ways. The idea of performance calls to mind the idea of a “stage” in which learners engage in mathematical discourse. To tie performative mathematics identity to positioning, it is important to note that an individual is also a member of their own audience (Darragh, 2015). As a learner does mathematics, they engage in significant social interactions, or social acts. The learner positions them self and is positioned by those in the learning environment based on how these social acts become relevant to known roles, storylines, or issues of access (Davies & Harré, 1990). As these positions persist, the learner forms a mathematics identity. For example, a student frequently positioned as a knower of mathematics by the teacher may eventually recognize themselves as a “mathematics person.” Thus, it is essential for teachers to position students as competent doers of mathematics for students to develop positive mathematics identities. Furthermore, in the context of engaging with MATs (and as described by the didactic tetrahedron, students’ participation in mathematics also involves their discourse with the technology, so technology use can be thought of as part of their performance (Fletcher & Fye, 2022). The teachers, students, and the technology task work synergistically to make up the discourse as students work through the task.

IV. METHODS

This study was a single case study of the discourse between two students and two teachers mediated by a Desmos task focused on key characteristics of the sine function (Yin, 2009). We observed student-student, student-teacher, student-self (thinking), student-task, and task-student discourse in the video recording of the students working through the activity. This was an intrinsic case study because we were motivated to examine the recording of the students working through the task in more depth by the evidence, we noticed in the previous study of the technology playing a role in how the students were positioned (Stake, 1995). The discourse and actions of the students, teachers, and technology served as our representation of performative displays of enacting mathematics identity, thus, allowing us to find and describe how positioning was happening (Darragh, 2015).

This study is an extension of a prior study in which two second semester ninth-grade students (age 14) attended an in-person after school session to engage with a task, part of which is shown in Figure 2, designed to leverage the power of sliders in Desmos to enhance and empower their learning around key characteristics of the sine function (i.e, amplitude, midline, and period). Both students were engaged regularly in technology-enhanced tasks using MAT in their first and second semester of ninth grade mathematics and had a high comfort level with the structure of the task. As they engaged in the activity, we collected screen capture recordings of their work with Desmos. The students were encouraged to discuss with each other as they worked, and the two teachers circulated sometimes engaging in teacher moves to help students to reach the intended learning goal. The teachers participating in the study had 12 and 15 years teaching experience and have previously demonstrated a history of successfully developing and facilitating mathematics lessons with MAT.

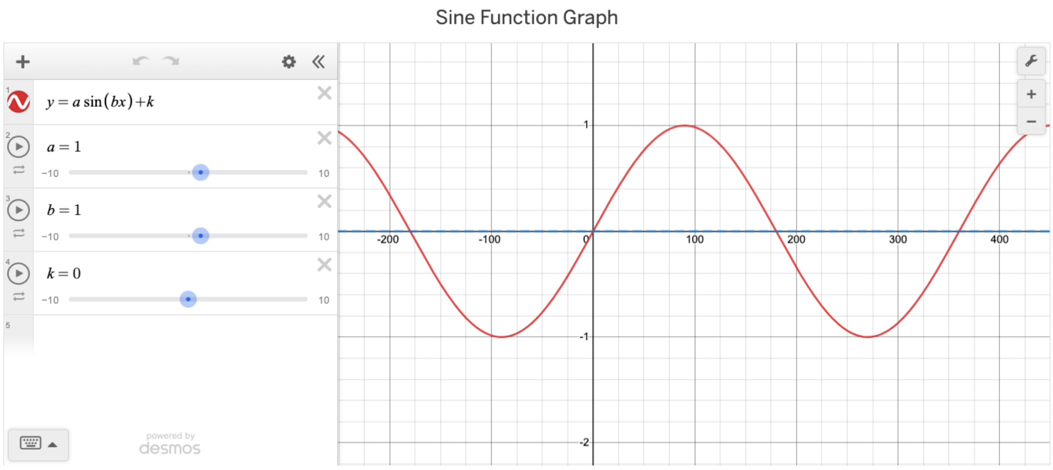

Figure 2. Sine parameter task slide in Desmos

The Desmos task consisted of the exploratory slide shown in Figure 2 as well as screens where students were given specific definitions of features of the sine function (amplitude, period, and midline), a supporting image for the definition, and a few questions. In Figure 3, you can see the task's page focused on the period feature. These feature slides contained questions asking students to consider the original position of the sine function to begin. Then students were asked to identify the slider responsible for the changes of that feature and to mathematically describe how the specific sliders impact the feature. The intended learning outcome was for students to make accurate generalizable conjectures to describe the parameters a, b, and k associated with the amplitude, midline, and period, respectively.

Our familiarity with the transcript and video allowed us to develop our first round of coding where we were trying to identify what technology centered actions (coded as *tech actions*) afforded students to engage in meaningful mathematical activity. The *tech actions* coded referred to when the discourse involved engaging or discussing the technology to position the students as mathematical explorers. Because mathematical discourse refers to communicating about abstract objects such as equations, numbers, and models, as well as using mathematical language, we considered the teachers' and students' actions with the technology to be a form of mathematical discourse (Sfard, 2007). The actions with the technology served as partners to the tech action codes, providing a subsequent code of what each tech action enabled students to do (coded as *students enabled*). In this first phase, we analyzed the transcript from a technology-focused lens in Atlas.ti by open coding instances where the affordances of the technology were highlighted through teacher direction or student-initiated interaction with the technology allowed some sort of meaningful mathematical activity. Because a difference in the students' discourse and the teachers' communication was needed to incite learning, it was important to note the students' discourse with the teacher that followed specific teacher moves (Sfard, 2007).

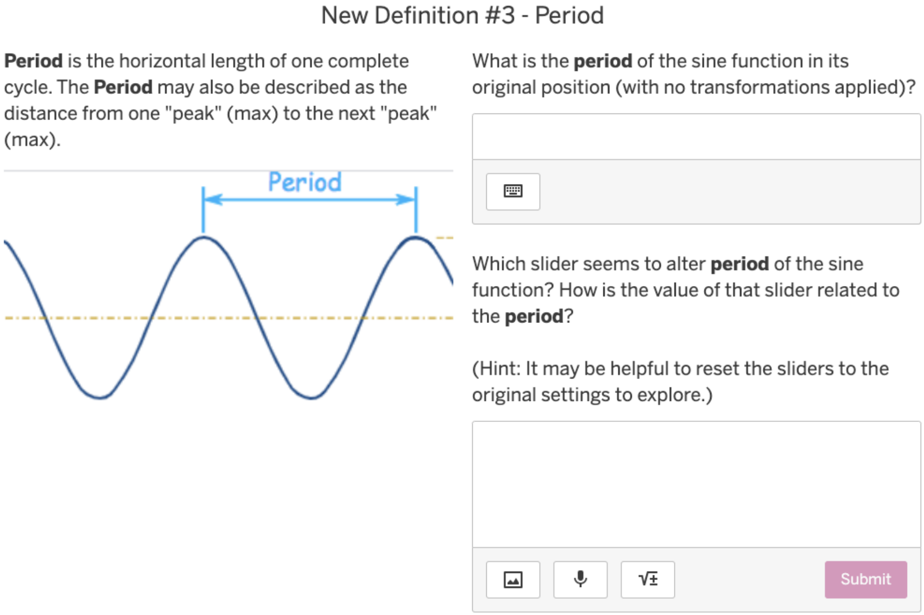

Figure 3. Sine period feature task slide.

During a discrepancy meeting for our initial codes, there were instances when students engaged in meaningful mathematical activity that was not preceded by a previously used tech action code. Therefore, additional codes for design of the technology and technology fixes emerged. The *tech design* code was used to describe the phenomena when the design of the task itself, rather than discourse or actions, was responsible for the subsequent students enabled code. The technology design code was used in instances when the students' attention was focused on non-technology content within the task. We often applied a technology design code when referring to the embedding of pencil and paper features. For instance, when students referred to questions, images, or definitions within the task. The other code that emerged in this meeting was developed to describe the phenomena of teachers or students engaging in technology troubleshooting (coded as *tech fix)*, which also served as a mechanic in enabling students in meaningful mathematical activity. The tech fix code instances were refreshing the web page and resetting the interactive features of the Desmos task.

Following this discrepancy meeting, each individual researcher returned to the transcript and coded each technology action as enacted by the teachers or students, finding that all but two occurrences in the transcript were teacher technology actions. This clarified our focus of the second and third research questions to name and describe the teaching moves in the technology mediated environment and how they sustain mathematical authority for the students to be positioned as mathematical explorers.

We began the second phase of this process with each researcher independently reading the coded data and developing short descriptions of each quotation identified by the *tech action, tech fixes, tech design,* and *students enabled* codes. We used these descriptions to help generate descriptive codes and create a preliminary codebook in Excel by meeting and discussing broad themes between the codings. We applied our preliminary descriptive codes and built new codes as we compared our descriptions with each other to ensure the data could be described by the codebook. We continued comparing our descriptive coding to develop the remaining descriptions until all of the initial codes had a descriptive code for the codebook. In our third phase of coding, we used our researcher developed Excel spreadsheet codebook to describe the technology actions, technology design statements, and technology fixes (Table 1), and the opportunities for students to recognize themselves as mathematical explorers that followed (Table 2). An example of a technology action was *Scaffolding Technology*, which referred to the teachers' discourse related to ways the technology could be manipulated to remove barriers to understanding. In this case, this discourse tended to refer to teachers' verbal instructions to students to manipulate the technology, such as suggesting the removal of the gridlines on the graph. In some cases, however, we foresee a teacher might also enact this discourse move by physically manipulating the technology if the level of scaffolding necessary for students requires it.

Table 1. Code descriptions: Technology-centered considerations and teaching moves

Technology-Centered Considerations		
Task Design	*Description*	
Explain Definition	Teacher ensures understanding of words in definition	
General Task Design	Student or teacher refers to or uses general structure and design of the task (e.g., indicating where questions are located on the page)	
Refer Definition	Student or teacher refers back to definitions or visualizations embedded in the task	
Refer Question	Student or teacher refers back to questions embedded in the task	
General Operation and Troubleshooting of Technology (Tech Fixes)	*Description*	
Reload Page	Student or teacher reloads or anticipates reloading web-browser when technology does not behave as expected	
Reset Sliders	Student or teacher uses the reset sliders feature of Desmos to reset the sliders to their original positions	
Technology-Centered Teacher Moves		
Ability Scaffolding Technology	*Description*	*Example*
Scaffolding Technology	Teacher scaffolds tech features to eliminate barriers	Teacher changes view settings, uses second tab for an empty graph to eliminate clutter on the screen, suggests using integers in exploration that make considering calculations easier, removes sliders for students to pay attention to function structure

Attending to Student Thinking Through Their Discourse & Actions	*Description*	*Example*
Direct Thinking	Teacher directs one student to another students thinking through technology	Teacher asks one student to pay attention to the other student's technology actions and make judgments about the mathematical phenomena
Follow-Up Question	Teacher asks students a follow-up question, which builds off of their thinking	Teacher restates students' thinking and asks a question to move their thinking forward
Recall Prior Knowledge	Teacher asks students to recall prior knowledge	Teacher asks students to recall function structure of a specific parent function
General Exploration	Teacher asks students to explore the task without prompting them to explore a specific feature	Teacher asks students to explore the features of the task and talk about what they notice
Technology-Specific Questioning	*Description*	*Example*
Generate Example	Teacher asks students to generate an example with the technology features	Teacher prompts students to give an example of an equation of a line
Highlighting Features	Teacher points out features of the technology	Teacher draws students' attention to the sliders, the graph, or the equation entry box to help guide their thinking
Probe for Conjecture	Teacher asks students to conjecture on generalizations	Teacher asks students to conjecture about the general relationship between values in the sine function
Probe for Justification	Teacher asks students to justify their thinking with technology	Teacher asks students to prove, show, defend and uphold how their thinking is validated with the technology explorations
Probe for Self-Assess	Teacher asks students to self-assess their thinking or responses	Teacher asks students if their conjectures or generalizations about mathematical phenomena are supported by their current explorations
Specific Exploration	Teacher encourages students to explore the task and prompts them to explore a specific feature	Teacher asks students to test a specific slider, specific feature, or extend an exploration of a feature that has not been previously considered
Test Specific Values	Teacher asks students to test specific values	Teacher asks students to test a specific value in a slider or pause and take note of a specific value that is currently being explored

Table 2. Code descriptions: Students enabled

Students Enabled	Description
Access Prior Knowledge	Students access and use prior knowledge to describe mathematical phenomena explored with the technology
Answer Expression	Students explore how to express their answers with the technology
Better Access Learning Targets	Students were able to better access the intended learning targets after teacher scaffolding technology
Eliminate Answers	Students eliminate a possible answer to a task question based on reasoning with the technology
Justification	Students justify their thinking with technology
Learning Target Deductions	Students arrive at correct deductions aligned with the learning targets
Make Conjecture	Students make conjectures or predictions about the mathematical phenomena being explored with the technology
Question Generation	Students generate their own questions about the mathematical phenomena being explored with the technology
Self-Assess	Students self-assess their thinking and are able to make adjustments in strategies using the technology
Sensemaking-Task	Students make sense of questions or definitions using the technology
Sensemaking-Thinking	Students make sense of each other's thinking using the technology
Student Language	Students describe mathematical phenomena being explored with the technology using their own language

During discrepancy meetings for this phase, we rectified any misaligned coding and discussed phenomena not covered by the codebook to generate the remaining codes in the codebook. We wrote shorthand versions of the codes and applied the codes to the transcript in Atlas.ti. Then, we made visuals for each category of codes, along with a co-occurrence table (see Appendix) to visualize relationships between the *tech actions*, *tech design*, and *tech fixes* codes when compared to the *students enabled* codes. The final part of this phase involved the researchers grouping the teacher technology action codes, given in Table 1, into two different themes - *technology-centered considerations* and *technology-centered teaching moves* - based on their proximity to the teaching versus proximity to the technology. The technology-centered considerations were meant to describe the phenomena not necessarily related to teaching such as tech design and tech fixes codes, that still managed to position students. The technology-centered teaching moves category was used to describe the specific teaching moves that were used in relation to the technology. The technology-centered teaching moves included more specific groupings of our teacher technology codes which were identified as *ability to scaffolding technology*, *attending to student thinking through their discourse and actions*, and *technology specific questioning.*

V. RESULTS

Positioning students with as mathematical explorers through teacher pedagogy has shown to be an impactful way to strengthen the mathematics identities of students (Boaler & Greeno, 2000; Dunleavy, 2015; Harrell-Levy & Kerpelman, 2010; Radovic et al., 2018). Over time, positioning students as mathematical explorers facilitates their recognition of themselves as capable of doing mathematics, and as a result development of a positive mathematics identity. In a technology mediated environment, implementation of successful teacher pedagogies becomes more complex due to the nuanced nature of teaching and learning mathematics with mathematics action technologies (Drijvers et al., 2010) The findings from this study suggests that there are some key technology-centered considerations and teaching moves that are crucial to helping students remain mathematical explorers during a task that uses mathematical action technologies. Thus, we have structured the following narrative of our findings according to those two categories.

Technology-Centered Considerations

There are two technology-centered considerations that presented themselves in the analysis and coding that were able to position students as mathematical explorers: troubleshooting technology and task design. Task design would be one of Trouche's (2004) considerations for didactical configuration - where thoughtful selection or organization of the task impacts student learning outcomes.

General Operation and Troubleshooting of Technology. In this case study, students and teachers anticipated ways in which the physical technology or app was no longer operating as expected. These were distinct from technology actions taken by the teachers or students or considerations related to task design because they were not tied to the specific task. Instead, they were technology fixes that would be helpful for users during any task created using the technology. An example of technology fixes we noted in the transcript were when the teachers or the students used the reset sliders functionality of the Desmos tool to reset the sliders to their original positions or when the teachers or students reloaded the webpage. This occurred when the Desmos activity no longer operated as the students and teachers expected; therefore, the students refreshed the browser to get it to behave in the expected manner. For instance, during the interaction below, the students reloaded the webpage when the browser windows were malfunctioning:

Teacher K: Looks like you're editing this in a different window
Kei: Just uh reload the page.
Teacher K: That's a new one
Xarielle: What
Teacher K: When in doubt, huh?
Teacher N: Yeah
Teacher K: Alright good call
Teacher N: You win
Xarielle: Okay, wait where'd it go?
Kei: It refreshed it.
Teacher K: There it is, okay

General operating considerations and anticipating troubleshooting for how the device behaves with the mathematics action technology in regards to operation (e.g., browser, applications, internet connectivity, window sizing) was evidenced to be on the teacher's radar for implementation to ensure students can continue as the authority in their mathematical exploration. Because of this, the teachers were able to quickly explain or show to students how to troubleshoot the device and MAT. In this case, it was effective for the teachers to show hidden interactive features, such as resetting sliders, to ensure that students would be able to follow the instructions embedded in the task and come to intended mathematical conceptions.

Task Design. We also coded actions occurring during the activity that related to the design of the task. The task in this study was designed to ensure that students did not necessarily need authority from the teachers to embark in exploration or understand goals of the task clearly. For instance, in this task, embedding directions, questions, images, definitions, along with the interactive features allowed students to refer to any slides of the task to ensure that the mathematical authority was sustained by the students. The teachers in this task frequently described the general features of the task and how it was designed, for instance when the teacher said, "So this is the same graph the sine function right here and it's telling you what the amplitude is and so this is your definition of amplitude." We distinguished this code from a technology action because the teacher was not prompting the students to use the technology. After the teacher had modeled effective use of the general features of the task design, the students mirrored this strategy later in the task. When they moved on to a later question in the task, Xarielle said, "Okay. Now let's do number 9." Simultaneously, they each said, "How do we find the altitude [amplitude]?" and Kei responded, "It's easy. We got Desmos right here." Xarielle's ability to progress through the questions in the task by clicking the arrows and Kei's response about using the Desmos graphing window and sliders on an earlier page to reason about the question at hand demonstrated their understanding of the general design of the task.

The teachers or students also referred back to a definition or specific question in the task. For example, when the students were reasoning about how to find the amplitude of the sine function in its original position, Xarielle said, "But it don't say which one," to which Kei responded, “Look. It’s the height from the center line to the peak, or we can measure the height from the highest to lowest points and divide that by two.”

We found that rather than needing a teacher to provide a definition or illustrate a concept, the task does this in a way that allows the students to consult the task and not the teachers. By defaulting to the task for these clarifications, even when questions arrive to the teachers, it puts the mathematical authority with the task and the students by proxy. The teachers never have to serve as “all-knowing” beings of the mathematics, and the students instead grapple with the intended mathematical concepts. Another way that the task design in this study preserved students' engagement as mathematical explorers was the type of questioning used in the task. Early in the task students were asked to perform general exploration of the task and later given targeted exploration. A question earlier in the task “What do the sliders do?” where as a later question reads “Which slider seems to alter amplitude of the sine function? How is the value of that slider related to the amplitude?” Notice the earlier question encourages general exploration and the later one provides an intended learning target embedded in the questioning. Students remained mathematical explorers due to the nature of these questions and were given opportunities to explain the mathematical phenomena in their own words first before trying to connect their exploration to intended learning outcomes. This act of allowing students to engage with mathematics formally and informally, is a way of leveraging multiple mathematics competencies by drawing upon learners' prior and current knowledge to explore and explain the world mathematically (Turner et al., 2013). Consistent with McCulloch et al. (2021), the teachers mediated the students’ mathematical discussion by pointing to particular features in the task as they tested their conjectures and justified their mathematical ideas in their own language, providing pathways for both students to reason about the sine function.

Technology-Centered Teaching Moves

In addition to important considerations centered on the technology, we also identified and described teaching moves that were crucial to helping Kei and Xarielle remain the mathematical authorities during the Desmos task. Rather than taking on the traditional role of “expert” as the students worked through the task, the teachers facilitated instructionally effective discourse with the students as they scaffolded the technology based on the students’ abilities to meet the learning targets, attended to students’ thinking beyond discussion by paying attention to technology actions performed by students, and asked questions that were specific to the actions with the technology. In these ways, they structured the students’ work through the MAT task in a way that made mathematics learning accessible to the students and positioned students as the mathematical explorers. These teacher moves serve as examples of Drijvers’ (2010) didactical performance, which refers to the in-the-moment actions taken while teaching with technology.

Ability Scaffolding the Technology. We saw instances where the ability of the teachers to change the way the technology operates was crucial in helping the students uncover the intended learning outcomes. These scaffolds extended beyond the general troubleshooting described above and were much more teacher-centered. In this context, the teachers paid attention to how the mathematics action technology potentially impeded students' ability to reach the intended learning target and made suggestions to students about the adjustments that could be made to clarify the learning target. In our case, for instance, the students had correctly identified a value for the midline of the sine function, but there was a clear difficulty for students in coming up with a proper expression of a midline as a line. The students were focused on adjusting the parameters of the sine function to create the midline and trying to express the midline with a sine as part of the expression, but not understanding why the complex expression still produced zero based on their current knowledge. So, the teacher suggested opening up a new tab with a blank graphing calculator for them to do their thinking. Almost like a scratch sheet of paper, this became a scratch tab. The scratch tab reduced the cognitive load placed on the students by the task because there was no longer the clutter of the sine function and sliders. Rather, the line of questioning focused on creating an equation of a line that ultimately helped students successfully express the equation of the midline. By reducing the students' cognitive load to make the complex mathematical ideas more accessible for the students, the teacher facilitated an opportunity for the students to perform as mathematical explorers and recognize themselves as successful mathematics learners. Other examples of scaffolding the technology included helping students adjust view settings, encouraging students to use values such as integers to make calculations easier, or removing sliders for students to allow them to closely pay attention to the structure of a function. For example, in the below interaction, the teacher instructed the students to use the Desmos settings to turn off the minor grid lines to more easily visualize the distance from the midline to the peak, or the amplitude, of the sine function. The scaffolding enabled both of the students to correctly give the value of the amplitude, and Xarielle insisted on continuing to explore the graph with this setting.

> Teacher K: Take off minor grid lines. Okay alright okay, how far apart are they? Give me one more go at it.
> Kei: [speaking gibberish] Which one we said, this one right here, and this one?
> Xarielle: Yes
> Kei: One, two, three
> Xarielle: Four
> Teacher K: Did you make it down to that other point, Kei?
> Xarielle: Why do you keep stopping right there?
> Kei: Okay, okay. One two three four
> Teacher K: Okay, Xarielle, you agree?
> Xarielle: Yes, I agree. I've been trying to…
> Teacher K: That's why we had to...We had to convince Kei though. We had to convince Kei.
> Kei: No, take those off [referring to minor gridlines].
> Xarielle: No, I need it.

In this case the teachers' abilities to note student actions with the technology as thinking along with their discourse helped in their decision making around in-the-moment moves. The scaffolds were catered moves based on what students were demonstrating in understanding without taking students out of productive struggle. These can allow student actions with the technology to be more meaningful and clear to the student, ultimately allowing them to sustain mathematical authority, reach intended learning outcomes, and recognize themselves as mathematical explorers.

Attending to Student Thinking Through Their Discourse & Actions. Student thinking can be presented in ways beyond what is spoken or written. For instance, teachers and students can engage in mathematical discourse centered around a student- or jointly-created technology artifact (Huang & Sutherland, 2022). Paying attention to student technology moves, we saw the teachers asking students follow-up questions that built upon students' thinking. It initially presented itself as teachers asking students to discuss their initial understanding of the general exploration of mathematical phenomena with their own language and prior knowledge. For example, to introduce the Desmos task, one of the teachers said, "so we're going to pay attention to some of the features of that sine function so I want you to go to the next page and just kind of talk about what you notice." In the interaction that followed, the students used their prior knowledge and experiences to describe the graph of the sine function in their own language:

Kei: It's curvy.
Teacher N: Curvy
Kei: Yeah
Teacher N: Good.
Xarielle: It looks like a heartbeat. Like in like the doctor thing.
Teacher N: It looks like a heartbeat, absolutely, okay.

This act of allowing students to engage with mathematics formally and informally, is a way of leveraging multiple mathematics competencies by drawing upon learners' prior and current knowledge to explore and explain the world mathematically (Turner et al., 2013). The action of leveraging multiple mathematics competencies includes providing multiple entry points, allowing students with varying levels of skills to engage in with the problem and make meaningful contributions (Aguirre et al., 2013). In this technology task, we found teachers attending to not only written and verbal contributions, but paying attention to students' actions with the technology.

In this case, the teachers were noticing not only what students are saying but also what they are carrying out with the technology in order to fully understand how they are thinking or why they are developing specific thinking. This was revealed in some cases when students arrived at unintended conclusions about the sine function characteristics. For instance, when Kei was counting amplitude with the cursor on the screen, the cursor was moving from a highlighted maximum to a highlighted minimum, but the verbal counting was beginning at the starting point and the ending point. Paying attention to the student's action with the technology revealed that the student was not counting distance but rather endpoints within the vertical scale of the graph. In this instance, the teacher used this as an opportunity to have students consider each other's thoughts.

Teacher K: Xarielle says it's four, you say it's three. We gotta come up to a conclusion.
Kei: I'm always right.
Teacher K: That's your reason? You better give me a better reason than that.
Xarielle: Wait, one, two, three, four. I got four. Do you not count that one at the end?
Kei: No, look okay. Between it, so like one, two, three
Xarielle: Oh, okay I see it now. You counting the line, I'm counting the block
Xarielle: Okay okay you counting the lines
Teacher K: Okay, how far apart are they?
Xarielle: Three
Teacher K: You guys are convinced it's three?
Kei and Xarielle: Yes

Contrarily, Xarielle was counting in a way that was mathematically accurate. Both students were able to verbalize and compare their strategies visually and verbally. Although this did actually result in the incorrect amplitude for the task, it was followed up with scaffolding too once the teacher recognized scale was getting in the way of achieving understanding.

The teachers' close attention to the students' thinking based on their actions on the technology allowed for them to point out those moves as thinking to peer partners. Teachers were directing one student to another student's thinking by pointing out how the other student's exploration, through the technology, was being carried out. The teachers would ask if one student was noticing what the other student was doing and ask them to consider each other's actions as thinking, an example of asking students to apply their reasoning to others' reasoning, described by several researchers as a way of facilitating mathematical discourse (Chapin & Anderson, 2013; Kim & Yeo, 2019a, 2019b). This teacher move allowed mathematical exploration to remain a focus of the students' efforts. Peers seeing each other's technology moves as mathematical thinking often helped them to try and make sense of each other's thinking and facilitate their own questions between each other. These conversations and questions being centered around the students allowed for sustained positioning of the students as the mathematical explorers and afforded them the opportunity to recognize themselves in this role over time.

Technology Specific Questioning. When considering the teacher technology actions that afforded students the opportunity to be positioned as mathematical explorers in this case study, several technology specific questions and suggestions were evidenced. NCTM (2014) suggests that posing purposeful questions can assess and advance student sense-making when exploring mathematical relationships. Our case demonstrated several specific ways in which students were asked these types of questions. One of the ways was asking students to justify their explanations with demonstrations using the technology. When a student made a claim, oftentimes the teachers would ask the student to convince them or the other student about their claim by showing where their thinking is coming from when operating the technology. In our transcript, the phrase *show me* appears twenty-two times, as well as other similar tasking words like *prove* or *justify*. As the students worked through the Desmos activity, the teachers probed them to justify their thinking a total of 15 times. For example, one of the teachers said, "Can you show me on those sliders? Show me what you worked through and you can talk through it real quick." Furthermore, this type of questioning from the teachers most often was associated with the students enabled code - providing justification for thinking. In fact, the most frequently occurring descriptive code for what students were enabled to do by the teachers' actions was justification. An example of the students justifying their thinking was,

Teacher N: You're telling me the amplitude is two?
Kei: Yes
Teacher N: Well, how could you get that from the equation right there?
Kei: Well, it says it right there basically.
Xarielle: um because since the front of the parentheses. it's the… How do I explain that? It's…I know it's like the way it is way the way it is on the axis. So, since a is the amplitude of the sine wave, uh, like it like controls the y, is the y is the is the… it's the reflection on the y-axis.
Kei: You basically can see it right there in the equation.

We see Kei & Xarielle justifying their answer with function structure as well as their prior knowledge of function transformations. This is following making generalized conjectures during general and focused exploration of the sliders.

Another specific type of technology-based question was when students were asked to demonstrate their thinking by building their own example. This did not happen frequently in our case, likely due to the nature of the task, but the power of students building their own examples truly embodies the idea of students maintaining mathematical authority during a task. One instance when the teacher made this move was when she said, "Put an equation of a line in one of those. How do you write an equation of a line? Give me an example of an equation of a line" to prompt students to express their knowledge of the location of the midline. Another type of questioning the teachers used was asking students to conjecture or generalize, sometimes with the additional task of self-assessing the conjectures through the exploration of the mathematical relationship. In this case, the teacher asked for conjecture about the relationship between the parameter, b, of the function and the period. Once student's started making generic conjectures about the relationship, they were asked to check and see if their conjecture held up with values of exploration.

Teacher K: So what does…. make that more generic for me so you say we can take the… what's the 360 represent
Xarielle: the b
Kei: the period
Xarielle: I meant the period
Teacher K: the period when we're when no transformations are applied

Kei: yes
Teacher K: and we're taking that and we're doing what with it
Kei: dividing it
Teacher K: by what
Kei: 4
Teacher K: what does that four represent
Kei: the period… the yeah period of that other one
Teacher K: the what?
Kei: the period
Xarielle: the period
Teacher K: four is the period? i thought you told me the period of that was going to be 90.

Following this interaction, the teacher asked further prompting questions, and the students were enabled to generalize the relationship between the parameter b, the period, and the period of the sine function in its original position.

Along with asking purposeful questions related to the technology, technology-specific suggestions and highlighted features helped hone the students' focus to the intended learning targets. For instance, when our students were testing ideas about period they were directed back to the b slider to specifically execute their explorations. This strategy was often used when general exploration did not clarify the learning targets of the questions being asked. In a similar way, the teachers would highlight features of the technology that required student manipulation such as sliders or clickable points. In one instance, the teacher highlighted the sliders as a useful tool for the students as they were manipulating the sine equation to understand how to find the amplitude, midline, and period from an equation.

Teacher N: So go back to where your sliders are.
Xarielle: Don't I got to change it to two?
Teacher N: So right now you've got a is one, b is one, and k is zero.
Kei: That two
Xarielle: So this has to be one because its plus one.
Kei: Thats 4
Xarielle: Oh lord. Oopsies. This one is two.
Kei: And b was what?
Xarielle: 4x
Kei: Why didn't you just type in an equation like that?
Xarielle: Uhhhhh…Oh I could have did that couldn't I
Xarielle: So, sin
Teacher N: Rather than changing that one, sliders you can type in anything

Teachers also highlighted the different representations that the manipulations changed based on student exploration. In the case of Kei and Xarielle's discourse about the amplitude of the sine function, when they each conjectured that a different parameter affected the amplitude, the teacher asked them to notice what the graph was doing in response to the movement of each slider.

> Teacher N: Okay, now move slider K to somewhere else.
> Kei: Go down.
> Xarielle: Let's see.
> Teacher N: What's the amplitude?
> …
> Teacher N: Why do you say A?
> Xarielle: Because, okay watch this, watch this, watch this, watch this
> Teacher N: What's the amplitude right now?

Here the teacher emphasized paying attention to the graph as the parameters of the function were manipulated through the sliders to help students differentiate between the role of parameter a and k in the sine function. In this case, Xarielle used it to help the Kei visualize and ultimately understand the impact of parameter a on the feature of amplitude.

Discussion

In summary, our findings demonstrated key technology-centered considerations and teaching moves, which helped students remain mathematical explorers as they worked through a MAT task. In regard to research question one, how technology positioned students as mathematical explorers when engaging with MAT tasks, rarely was the MAT acting alone to position the students. This aligned with Huang and Sutherland's (2022) suggestion that the technology's role likely opens up opportunity for teacher moves, rather than acting as a positioning agent alone and suspicions that an artifact itself could facilitate agency. We found a few instances in our case where MAT task design, without the teacher, was able to position students as mathematical explorers. More often, though, teaching moves positioned students as mathematical explorers and contained all four components of the didactic tetrahedron (Hollebrands, 2017). Due to this, our research questions shifted to account and describe how the specific teaching moves helped Kei and Xarielle remain the mathematical authorities during the MAT task. Our case study revealed that technology-centered teacher moves have the ability to position all students as explorers of mathematics.

In regard to research questions two and three we discuss what and how specific teaching moves leverage mathematics action technology in order to position students as mathematical explorers. Troubleshooting the technology, describing the design of the task, and using technology-centered teacher moves helped students reach the intended learning outcome of understanding the effect of parameters on the graph of the sine function. The teachers and students troubleshooted the technology when they used the reset sliders functionality of the Desmos tool to reset the sliders to their original positions and when they reloaded the webpage. When the discourse was centered on the design of the task, the teachers and students referred to embedded directions, questions, images, definitions, and interactive features to ensure that the students were positioned as mathematical explorers.

In addition to important technology-centered considerations centered on the technology, we also identified and described teaching moves, which positioned Kei and Xarielle as the mathematical explorers during the Desmos task. The teachers ability-scaffolded the technology, attended to students' thinking beyond discussion by paying attention to technology actions performed by students, and asked questions that were specific to the actions with the technology. Specifically, they helped students adjust view settings, encouraged students to use values to simplify calculations, or removed sliders for students to highlight a specific mathematical concept. The teachers asked students follow-up questions that built upon their thinking, asking students to discuss their understanding of mathematical phenomena using their own language and/or prior knowledge. Teachers directed students to examine each other' thinking, justify their explanations with demonstrations using the technology, build their own examples, make conjectures or generalize, and self-assess through the exploration of the MAT task. They also highlighted useful features of the technology and the different representations that the student manipulations changed during the students' exploration. The technology-centered considerations and teacher moves enabled students to engage with the mathematical concepts in a significantly diverse number of ways during a single technology task, including accessing prior knowledge, better accessing learning targets, eliminating answers, making conjectures, learning target deductions, generating questions, self-assessing, and sensemaking about each other's thinking in addition to justifying their thinking using the technology.

Limitations. Our title "It's Easy. We Got Desmos Right Here" speaks to our limitations of this research. The participants of this study were students of a teacher who regularly used inquiry learning partnered with MATs, particularly Desmos. They had been in a classroom for several months that had developed norms and expectations that facilitated the richness of the data collected and observed. Students had a positive working relationship with each other, the technology, and the one of the two teachers present during data collection. Although this didn't impede our study, it is worth noting that a recreation of this study may not present itself with the same results without these conditions or participants.

Implications & Future Studies. Based on these findings, we believe that teachers do not need to be experts in designing technology tasks. Instead, they should be able to critically analyze tasks that incorporate technology to ensure they possess the characteristics described above, positioning students as mathematical explorers. With repeated use, teachers—and eventually students—develop expectations for how MATs function and learn to troubleshoot the devices or software accordingly. Teachers must be prepared to explicitly teach troubleshooting techniques for these technologies so that students can efficiently diagnose and address issues, maintaining their engagement as mathematical explorers. Similarly, being aware of the affordances of MATs means allowing space for productive struggle to facilitate meaningful mathematical discourse between students (Dick & Hollebrands, 2011; Acrs et al., 2008). Facilitating learning through MATs is a complex process requiring numerous in-the-moment teaching decisions (Fletcher & Fye, 2022). To sustain students' participation as mathematical explorers, teachers need to recognize students' actions and gestures with the technology as forms of mathematical thinking, alongside their verbal discourse (Gonzalez & Herbst, 2009). Teachers should provide scaffolding suggestions that remove barriers to intended learning targets, making students' interactions with the technology more meaningful and clear. This approach helps students sustain mathematical authority, achieve learning outcomes, and see themselves as mathematical explorers. The teaching moves paired with MATs described in this study serve as a starting point for replicable ways to inform teachers how to engage with students during a MAT task that prioritizes the ownership of the learning being centered on the students. We can see these teacher moves and technology considerations as helping students continue their mathematical explorations without being robbed of opportunities for their own thinking. These teacher decisions ultimately helped this pair of students maintain mathematical authority throughout the task. Engaging students in these tasks while facilitating these moves with fidelity has the potential to ultimately strengthen students' mathematics identity over time. As a student is frequently positioned as a mathematical explorer, they may eventually recognize themself as a "mathematics person," a phenomenon Darragh (2015) described as performative identity.

In the future, researchers should carry out similar analyses across additional cases to determine the validity of the claims made from this case. This further exploration would allow for explanation of if these mathematical action technology teaching strategies re-emerge in other cases or if new teaching moves are established that position students as mathematical experts. More cases could potentially allow generalizable technology-centered teaching strategies that position students as mathematical explorers to emerge, thus strengthening students' mathematics identities. We are still considering the question: *Is the student transformation attributed to the actions of their teachers with MAT, or could it occur independently through their independent interaction with MAT?* To explore this question, researchers should analyze new cases where the teacher does not interject during the task to see if the technology itself would be capable of sustaining positioning students as mathematical explorers.

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Appendix. Co-Occurrence Table

	Access Prior Knowledge	Answer Expression	Better Access Learning Targets	Eliminate Answers	Justification	Learning Target Deductions	Make Conjecture	Question Generation	Self-Assess	Sensemaking-Task	Sense-Making-Thinking	Student Language
Direct Thinking	1	0	1	1	1	0	2	0	2	0	3	0
Follow-Up Question	3	1	0	0	4	4	3	1	0	1	3	0
General Exploration	1	0	0	0	0	0	0	0	0	0	0	1
Generate Example	1	0	0	0	1	0	0	1	2	0	0	0
Highlighting Features	2	1	1	0	0	1	2	1	1	0	3	1
Precision	0	1	1	0	0	0	1	0	0	0	1	0
Probe for Conjecture	0	0	0	0	0	1	1	1	0	0	0	0
Probe for Justification	2	0	1	1	14	1	2	1	2	0	4	0
Probe for Self-Assess	0	0	0	0	0	0	0	0	2	0	0	0
Recall Prior Knowledge	1	0	0	0	0	0	0	0	1	0	0	0
Scaffolding Tech	2	2	7	0	1	1	3	0	3	0	3	0
Solution Path	1	0	0	0	0	0	0	1	0	1	1	0
Specific Exploration	6	0	1	2	3	1	2	1	1	3	2	1
Test Specific Values	0	0	0	0	2	1	1	2	1	1	1	0
Reload Page	0	0	1	0	1	1	1	0	0	0	0	0
Reset Sliders	0	2	1	1	2	2	3	0	1	1	2	0
Explain Definition	0	1	0	0	0	1	0	0	0	1	0	0
General Task Design	4	1	0	0	1	3	1	1	0	5	1	1
Refer Definition	0	0	0	0	1	0	0	2	0	4	1	0
Refer Question	0	1	0	0	2	2	0	0	1	1	0	0