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DEVELOPMENT AND EVALUATION OF A CENTROID-BASED EOQ MODEL FOR ITEMS SUBJECT TO DEGRADATION AND SHORTAGES

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Abstract. This research introduces an innovative approach to revolutionize inventory management strategies amid unpredictable demand and uncertainties. Introducing a Fuzzy Economic Order Quantity (EOQ) model, enriched with the centroid defuzzification method and supervised machine learning, the study offers a comprehensive solution for optimized decision-making. The model transcends traditional inventory paradigms by seamlessly integrating fuzzy logic and advanced machine learning, emphasizing adaptability in fast-paced business landscapes. The research unfolds against the backdrop of agile inventory management advocacy, with key contributions including the centroid defuzzification method for crisp interpretation and the integration of linear regression for cost prediction. The study employs a real-life bakery scenario to demonstrate the efficacy of both crisp and fuzzy models, underscoring the latter's superiority in handling uncertainties. Comparative analysis reveals nuanced impacts of uncertainty on inventory decisions, while linear regression establishes statistical relationships for cost predictions. The findings underscore the pivotal role of fuzzy logic in optimizing inventory management, paving the way for future enhancements, advanced machine learning integration, and real-world validation. This research not only contributes to adaptive inventory management evolution but also sets the stage for further exploration and refinement in dynamic business landscapes.

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1. Introduction

In the operational paradigm of businesses, inventory management holds paramount importance. However, traditional models that once efficiently handled inventory often fall short when faced with unpredictable shifts in demand or potential deterioration of stocked items. Recent literature has shed light on this inadequacy, prompting the exploration of contemporary methodologies aimed at addressing these uncertainties.

Wang and Chen's (2020) comprehensive overview highlight the necessity of evolving inventory management strategies to accommodate uncertainty. Their analysis emphasizes the critical need for adaptable models capable of swiftly responding to unforeseen demands. Conventional models often stumble in rapidly changing scenarios, urging a transition toward more agile approaches.

Chen and Wu (2020) proposed a hybrid inventory management model that integrates machine learning algorithms to enhance responsiveness. Their acknowledgment of the limitations of traditional models in rapidly changing demand scenarios underscores the need for adaptability. Their approach bridges this gap by focusing on the integration of technology to bolster adaptability and agility.

Smith and Johnson (2020) introduced a fuzzy logic approach to inventory management that recognizes the inherent flexibility needed in uncertain situations. This methodology allows for more nuanced decision making, embracing the ambiguity often present in unpredictable demand scenarios. The emphasis on flexibility becomes a guiding principle in effectively navigating uncertainties.

Lee and Park's (2020) robust inventory control strategy, employing the centroid method, emphasizes stability and resilience in inventory systems. Tailored specifically for unexpected demand scenarios, this approach focuses on fortifying inventory systems against abrupt changes. Stability emerges as a crucial attribute in unpredictable environments, offering a lens through which uncertainties can be effectively managed.

Advancements proposed by Liu and Zhao (2021) delve into inventory control strategies that leverage deep learning. Their emphasis on the dynamic adaptation of inventory systems harnesses the power of advanced algorithms to respond effectively to unforeseen demands. This adaptive approach aligns inventory management with the pace of rapidly changing market landscapes.

Simultaneously, Li and Wang's (2021) exploration addresses the challenge of managing deteriorating items. They proposed backordering inventory control strategies using machine learning to optimize inventory management for items vulnerable to deterioration. This aspect, which is often overlooked in traditional models, highlights the importance of catering to all facets of inventory complexities.

Collectively, these studies underscore the evolving landscape of inventory management. They advocate adaptive, technology-integrated strategies that embrace uncertainty and leverage advanced methodologies to effectively navigate unpredictable demand scenarios. In an era characterized by rapid changes and uncertainties, these approaches serve as guiding beacons for businesses striving to fortify their inventory management practices.

Finally, the contemporary approaches elucidated in the recent literature illuminate a path forward in inventory management. They underscore the importance of agility, adaptability, and technological integration, emphasizing the need to embrace uncertainty as an inherent aspect of modern business environments. These methodologies provide a foundation for businesses to navigate the complexities of inventory management in an ever-evolving marketplace.

2. Definitions

These definitions are needed while examining the fuzzy inventory model.

2.1. EOQ Inventory. All the commodities, merchandise, and supplies that a company keeps on hand in anticipation of selling them for a profit are referred to as inventory. The Economic Order Quantity, or EOQ, is the suggested order volume that a business should place in order to reduce inventory expenses, including holding costs, shortfall costs, and order charges. The EOQ formula is given by:

$$Q = \sqrt{\frac{2DS}{H}}$$

2.2. Fuzzy sets. A fuzzy set $\tilde{A}$ on the given universal set X is denoted and defined by:

$$\tilde{A} = \{(x, \lambda_{\tilde{A}}(x)) : x \in X\}$$

Where, $\lambda_{\tilde{A}} : X \to [0, 1]$ is called the membership function and $\lambda_{\tilde{A}}(x)$ denotes the degree of x in $\tilde{A}$.

2.3. Triangular fuzzy number (TFN). A fuzzy number $\tilde{F} = (f_1, f_2, f_3)$ with $f_1 < f_2 < f_3$ is triangular if its membership function is defined as:

$$\mu_{\tilde{F}}(x) = \begin{cases} \frac{x-f_1}{f_2-f_1} & \text{when } f_1 \le x \le f_2 \\ \frac{f_3-x}{f_3-f_2} & \text{when } f_2 \le x \le f_3 \\ 0 & \text{otherwise} \end{cases}$$

2.4. Centroid method. If $\tilde{F} = (f, g, h)$ is a triangular fuzzy number (TFN), then the centroid method on $\tilde{F}$ is defined as:

$$C(\tilde{F}) = \frac{1}{3}(f + g + h)$$

2.5. Linear Regression (LR). Finding linear relationships between variables becomes easier with the help of the supervised learning method linear regression (LR) in machine learning. Regression cases' outputs might be real or continuous values.

2.6. Notations.

- M: Holding cost
- J: Ordering cost
- $s1$: Inventory cost
- $s2$: Deficiency cost
- $s3$: Loss of revenue
- $\tilde{M}$: Holding/fuzzy
- $\tilde{J}$: Ordering/fuzzy

3. Mathematical Model

3.1. Notations. Inventory management involves a complex interplay of numerous variables, each representing crucial facets of the inventory control process. Notations within this domain provide a shorthand for articulating these variables, costs, and concepts. In this section, we introduce the key notations and symbols fundamental to understanding subsequent discussions on inventory management under uncertainty.

- M - Holding Cost: This represents the cost incurred in holding or carrying a unit of inventory over a specified period. It encompasses expenses such as warehousing, insurance, and opportunity costs associated with tying up capital.
- J - Ordering Cost: J denotes the cost involved in placing an order, including administrative expenses, transportation, and any setup costs incurred for ordering new inventory.
- s_1 - Inventory Cost: This represents the overall cost of inventory, combining the holding and ordering costs with any additional expenses incurred in managing inventory.
- s_2 - Deficiency Cost: s_2 signifies the costs related to stickouts or shortages. It accounts for expenses incurred because of unmet demand, such as backordering costs, lost sales, or customer dissatisfaction.
- s_3 - Loss of Revenue: This term encapsulates the potential revenue loss attributed to stickouts or shortages, reflecting the impact of unfulfilled orders on overall revenue generation.
- Holding/Fuzzy: The symbol M indicates a fuzzy or uncertain nature attributed to the holding cost. It represents scenarios in which the holding cost might exhibit variability or uncertainty because of various factors, thereby adopting a fuzzy logic approach in modeling.
- Ordering/Fuzzy: Similar to M, J represents the fuzziness or uncertainty surrounding the ordering cost, acknowledging scenarios where this cost might fluctuate or exhibit indeterminacy.

These notations serve as foundational elements for articulating and analyzing the intricacies of inventory management models, particularly in contexts where uncertainties prevail, leading to fuzzy or uncertain parameters within the inventory control framework.

3.2. Formulation of Inventory Management Model under Exponential Demand. The mathematical model presented addresses inventory management under exponential demand, incorporating replenishment cycles and backlogging. The primary differential equation governing the system is given by:

$$\frac{dR(u)}{du} + (\gamma + \delta u^2)R(u) = -ce^{du}, \quad 0 \le u \le u_1$$

With the initial condition R(0)=RA and $R(u_1)$=0, the solution R(u)during the replenishment phase becomes:

$$R(u) = c\Bigg[(u_1 - u) + \frac{(\gamma+\delta)}{2}(u_1^2 - u^2) + \frac{(d\gamma)}{3}(u_1^3 - u^3) + \frac{\delta}{12}(u_1^4 - u^4) + \frac{\delta d}{12}(u_1^5 - u^5)\Bigg]e^{-\gamma u - (\delta u^3)/3}$$

$$R(0) = R_A = \gamma\left[u_1 + \frac{(d+\gamma)}{2}u_1^2 + \frac{(d\gamma)}{3}u_1^3 + \frac{\delta}{12}u_1^4 + \frac{\delta d}{15}u_1^5\right]$$

During the deficiency period [u1,U] the backlogged sales are modeled by the differential equation is

$$\frac{dR(u)}{du} = \frac{D}{1+\theta(U-u)}, \quad u_1 < u < U$$

The solution for R(u) during this period

$$R(u) = \frac{D}{\theta}\left(\log(1+\theta(U-u)) - \log(1+\theta(U-u_1))\right)$$

$$R_B = \frac{D}{\theta}\log(1+\theta(U-u_1))$$

The Economic Order Quantity (EOQ) for each cycle is then determined as the sum of replenishment and backlogging costs:

$$O = R_A + R_B = \gamma\left[u_1 + \frac{(d+\gamma)}{2}u_1^2 + \frac{(d\gamma)}{3}u_1^3 + \frac{\delta}{12}u_1^4 + \frac{\delta d}{15}u_1^5\right] + \frac{D}{\theta}\log(1+\theta(U-u_1))$$

The overall total cost per unit per cycle is given by the expression:

$$\begin{aligned} OTC =&\frac{1}{U}\Big[J + M\Big(\frac{\gamma u_1^2}{2} + c(d+\gamma)\frac{u_1^3}{3} - \frac{cd\gamma}{8}u_1^4 \\ &+ \frac{\gamma\delta}{60}u_1^5 + \frac{\gamma\delta d}{72}u_1^6\Big) \\ &+ s_1\Big(cu_1 + c(d+\gamma)\frac{u_1^2}{2} + \frac{cd\gamma}{3}u_1^3 + \frac{\gamma\delta}{12}u_1^4 \\ &+ \frac{\gamma\delta d}{15}u_1^5 - \frac{ce^{du_1}}{d}\Big) \end{aligned}$$

$$+ \frac{s_2 d}{\theta}\Big(U - u_1 - \frac{1}{\theta}\log(1+\theta(U-u_1)))\Big)$$
$$+ s_3 D\Big(U - u_1 - \frac{1}{\theta}\log(1+\theta(U-u_1))\Big)\Big]$$

This equation represents the overall total cost per unit per cycle, considering various cost components related to replenishment, holding, backlogging, and demand. The optimization problem aims to minimize this overall cost by finding the optimal values for the decision variables u_1 and U.

3.3. Optimization of Fuzzy Inventory Management Model under Exponential Demand. In the fuzzy sense analysis of the model, various parameters such as cost components $\tilde{J} = (\tilde{J}_1, \tilde{J}_2, \tilde{J}_3)$, $\tilde{M} = (\tilde{M}_1, \tilde{M}_2, \tilde{M}_3)$, coefficients $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, \tilde{c}_3)$, $\tilde{d} = (\tilde{d}_1, \tilde{d}_2, \tilde{d}_3)$, $\tilde{\gamma} = (\tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\gamma}_3)$, $\tilde{\delta} = (\tilde{\delta}_1, \tilde{\delta}_2, \tilde{\delta}_3)$, and decision variables (u_1, U) are considered as triangular fuzzy numbers. The total overall cost in the fuzzy environment $O\tilde{T}C$ is expressed as a function of these fuzzy parameters.

$$\begin{aligned} O\tilde{T}C =& \frac{1}{U}\Bigg[\tilde{J} + \tilde{M}\Bigg(\frac{\tilde{\gamma}u_1^2}{2} + \frac{\tilde{c}(\tilde{d}+\tilde{\gamma})}{3}u_1^3 - \frac{\tilde{c}\tilde{d}\tilde{\gamma}}{8}u_1^4 \\ &+ \frac{\tilde{\gamma}\tilde{\delta}}{60}u_1^5 + \frac{\tilde{\gamma}\tilde{\delta}\tilde{d}}{72}u_1^6\Bigg) + s_1\Bigg(\tilde{c}u_1 + \frac{\tilde{c}(\tilde{d}+\tilde{\gamma})}{2}u_1^2 \\ &+ \frac{\tilde{c}\tilde{d}\tilde{\gamma}}{3}u_1^3 + \frac{\tilde{\gamma}\tilde{\delta}}{12}u_1^4 + \frac{\tilde{\gamma}\tilde{\delta}\tilde{d}}{15}u_1^5 - \frac{\tilde{c}e^{\tilde{d}u_1}}{d}\Bigg) \\ &+ \frac{s_2 D}{\theta}\Bigg(U - u_1 - \frac{1}{\theta}\log(1+\theta(U-u_1))\Bigg) \\ &+ s_3 D\Bigg(U - u_1 - \frac{1}{\theta}\log(1+\theta(U-u_1))\Bigg)\Bigg] \end{aligned}$$

The Fuzzy total cost $O\tilde{T}C$ is further defuzzified using the centroid method, resulting in a crisp overall total cost OTC. The defuzzification involves considering different scenarios (S_1, S_2, S_3) with corresponding fuzzy parameters and aggregating them using a weighted average.

$$OTC = \frac{1}{3}\Bigg(OTC_{S_1} + OTC_{S_2} + OTC_{S_3}\Bigg)$$

$$\begin{aligned} O\tilde{T}C =& \frac{1}{U}\Bigg(\Bigg[\tilde{J}_1 + \tilde{M}_1\Bigg(\frac{\tilde{\gamma}_1 u_1^2}{2} + \frac{\tilde{c}_1(\tilde{d}_1+\tilde{\gamma}_1)}{3}u_1^3 - \frac{\tilde{c}\tilde{d}_1\tilde{\gamma}_1}{8}u_1^4 \\ &+ \frac{\tilde{\gamma}_1\tilde{\delta}_1}{60}u_1^5 + \frac{\tilde{\gamma}_1\tilde{\delta}_1\tilde{d}_1}{72}u_1^6\Bigg) + s_1\Bigg(\tilde{c}_1 u_1 + \frac{\tilde{c}_1(\tilde{d}_1+\tilde{\gamma}_1)}{2}u_1^2 \end{aligned}$$

$$
\begin{aligned}
&+\frac{\tilde{c_1}\tilde{d_1}\tilde{\gamma_1}}{3}u_1^3+\frac{\tilde{\gamma_1}\tilde{\delta_1}}{12}u_1^4+\frac{\tilde{\gamma_1}\tilde{\delta_1}\tilde{d_1}}{15}u_1^5-\frac{\tilde{c_1}e^{\tilde{d_1}u_1}}{d_1}\Bigg)\\
&+\frac{s_2D}{\theta}\left(U-u_1-\frac{1}{\theta}\log(1+\theta(U-u_1))\right)\\
&+s_3D\left(U-u_1-\frac{1}{\theta}\log(1+\theta(U-u_1))\right)\Bigg]\\
+&\Bigg[\tilde{J_2}+\tilde{M_2}\Bigg(\frac{\tilde{\gamma_2}u_1^2}{2}+\frac{\tilde{c_2}(\tilde{d_2}+\tilde{\gamma_2})}{3}u_1^3-\frac{\tilde{c_2}\tilde{d_2}\tilde{\gamma_2}}{8}u_1^4\\
&+\frac{\tilde{\gamma_2}\tilde{\delta_2}}{60}u_1^5+\frac{\tilde{\gamma_2}\tilde{\delta_2}\tilde{d_2}}{72}u_1^6\Bigg)+s_1\Bigg(\tilde{c_2}u_1+\frac{\tilde{c_2}(\tilde{d_2}+\tilde{\gamma_2})}{2}u_1^2\\
&+\frac{\tilde{c_2}\tilde{d_2}\tilde{\gamma_2}}{3}u_1^3+\frac{\tilde{\gamma_2}\tilde{\delta_2}}{12}u_1^4+\frac{\tilde{\gamma_2}\tilde{\delta_2}\tilde{d_2}}{15}u_1^5-\frac{\tilde{c_2}e^{\tilde{d_2}u_1}}{d_1}\Bigg)\\
&+\frac{s_2D}{\theta}\left(U-u_1-\frac{1}{\theta}\log(1+\theta(U-u_1))\right)\\
&+s_3D\left(U-u_1-\frac{1}{\theta}\log(1+\theta(U-u_1))\right)\Bigg]\\
+&\Bigg[\tilde{J_3}+\tilde{M_3}\Bigg(\frac{\tilde{\gamma_3}u_1^2}{2}+\frac{\tilde{c_3}(\tilde{d_3}+\tilde{\gamma_3})}{3}u_1^3-\frac{\tilde{c_3}\tilde{d_3}\tilde{\gamma_3}}{8}u_1^4\\
&+\frac{\tilde{\gamma_3}\tilde{\delta_3}}{60}u_1^5+\frac{\tilde{\gamma_3}\tilde{\delta_3}\tilde{d_3}}{72}u_1^6\Bigg)+s_1\Bigg(\tilde{c_3}u_1+\frac{\tilde{c_3}(\tilde{d_3}+\tilde{\gamma_3})}{2}u_1^2\\
&+\frac{\tilde{c_3}\tilde{d_3}\tilde{\gamma_3}}{3}u_1^3+\frac{\tilde{\gamma_3}\tilde{\delta_3}}{12}u_1^4+\frac{\tilde{\gamma_3}\tilde{\delta_3}\tilde{d_3}}{15}u_1^5-\frac{\tilde{c_3}e^{\tilde{d_3}u_1}}{d_3}\Bigg)\\
&+\frac{s_2D}{\theta}\left(U-u_1-\frac{1}{\theta}\log(1+\theta(U-u_1))\right)\\
&+s_3D\left(U-u_1-\frac{1}{\theta}\log(1+\theta(U-u_1))\right)\Bigg]\Bigg)
\end{aligned}
$$

To optimize the system and minimize the overall cost, the partial derivative of OTC with respect to u_1 is set to zero, leading to an equation involving fuzzy parameters. The optimum values of u_1 and U can be obtained by solving this equation. This optimization process ensures that the system operates with minimum overall cost in the fuzzy environment

$$\begin{aligned}
&\frac{1}{3}U\Bigg[(\tilde{M_1})\Bigg((\tilde{c_1})u_1 + (\tilde{c_1})(\tilde{d_1})u_1^2 + \frac{(\tilde{c_1})(\tilde{\gamma_1})}{2})u_1^2 \\
&- \frac{(\tilde{c_1})(\tilde{d_1})(\tilde{\gamma_1})}{2}u_1^3 + \frac{(\tilde{c_1})(\tilde{\delta_1})}{12}u_1^4 + \frac{(\tilde{c_1})(\tilde{d_1})(\tilde{\delta_1})}{12}u_1^5 \\
&+ s_1\Big(\tilde{c_1} + (\tilde{c_1})(\tilde{d_1}) + (\tilde{c_1})u_1 + (\tilde{c_1})(\tilde{d_1})(\tilde{c_1})u_1^2 \\
&+ \frac{(\tilde{c_1})(\tilde{\delta_1})}{3}(u_1^3 + (\tilde{d_1})u_1^4) + \tilde{c_1}e^{(\tilde{d_1}u_1)}\Big) \\
&- \frac{(s_2 + \theta s_3)}{1 + \theta(U - u_1)}\Bigg] \\
&+ \Bigg[(\tilde{M_2})\Bigg((\tilde{c_2})u_1 + (\tilde{c_2})(\tilde{d_2})u_1^2 + \frac{(\tilde{c_2})(\tilde{\gamma_2})}{2})u_1^2 \\
&- \frac{(\tilde{c_2})(\tilde{d_2})(\tilde{\gamma_2})}{2}u_1^3 + \frac{(\tilde{c_2})(\tilde{\delta_2})}{12}u_1^4 + \frac{(\tilde{c_2})(\tilde{d_2})(\tilde{\delta_2})}{12}u_1^5 \\
&+ s_1\Big(\tilde{c_2} + (\tilde{c_2})(\tilde{d_2}) + (\tilde{c_2})u_1 + (\tilde{c_2})(\tilde{d_2})(\tilde{c_2})u_1^2 \\
&+ \frac{(\tilde{c_2})(\tilde{\delta_2})}{3}(u_1^3 + (\tilde{d_2})u_1^4) + \tilde{c_2}e^{(\tilde{d_2}u_1)}\Big) \\
&- \frac{(s_2 + \theta s_3)}{1 + \theta(U - u_1)}\Bigg] \\
&+ \Bigg[(\tilde{M_3})\Bigg((\tilde{c_3})u_1 + (\tilde{c_3})(\tilde{d_3})u_1^2 + \frac{(\tilde{c_3})(\tilde{\gamma_3})}{2})u_1^2 \\
&- \frac{(\tilde{c_3})(\tilde{d_3})(\tilde{\gamma_3})}{2}u_1^3 + \frac{(\tilde{c_3})(\tilde{\delta_3})}{12}u_1^4 + \frac{(\tilde{c_3})(\tilde{d_3})(\tilde{\delta_3})}{12}u_1^5 \\
&+ s_1\Big(\tilde{c_3} + (\tilde{c_3})(\tilde{d_3}) + (\tilde{c_3})u_1 + (\tilde{c_3})(\tilde{d_3})(\tilde{c_3})u_1^2 \\
&+ \frac{(\tilde{c_3})(\tilde{\delta_3})}{3}(u_1^3 + (\tilde{d_3})u_1^4) + \tilde{c_3}e^{(\tilde{d_3}u_1)}\Big) \\
&- \frac{(s_2 + \theta s_3)}{1 + \theta(U - u_1)}\Bigg] = 0
\end{aligned}$$

This equation represents the conditions for optimizing the system and achieving the minimum overall cost in the fuzzy environment. Solving this equation will yield the optimum values of u_1 and U, ensuring that the system operates efficiently and cost-effectively, considering the inherent uncertainty associated with fuzzy parameters.

4. Numerical Example: Inventory Management for a Seasonal Bakery

In a real-life scenario, such as a retail business dealing with perishable goods or seasonal products, the contrasting approaches of crisp and fuzzy inventory models could be applicable. Let us consider the case of managing inventory for a bakery that specializes in producing and selling seasonal cakes.

4.1. Crisp Model Implementation: Crisp model parameters are based on historical sales data, supplier information, and fixed costs. For instance:

- Ordering Cost (J): Fixed cost per order placed with suppliers.
- Carrying Cost (c): The cost incurred to hold one unit of the cake in inventory.
- Demand (D): Forecasted sales of cakes during a specific period.
- Shortage costs (s_1, s_2, s_3): Costs associated with lost sales due to stick-outs at different severity levels.
- Holding Cost (θ): Cost of holding a cake in inventory.
- Maximum Inventory Level (M): The maximum number of cakes that can be held in stock.

J	c	d	s_1	s_2	s_3	D	γ	δ	θ	M	U	OTC
20	16	6	1.5	2.5	2	20	2	0.2	2	2	1.09	41.2879

4.2. Fuzzy Model Implementation: Fuzzy model parameters introduce flexibility to accommodate uncertainties and varying degrees of imprecision in the bakery's operations:

- Ordering Cost ($\tilde{J}$): Represented as a fuzzy set to account for potential fluctuations in supplier prices or changing order quantities.
- Carrying Cost ($\tilde{c}$): Fuzzy range considering potential variations due to storage conditions or market volatility.
- Demand ($\tilde{D}$): Modeled fuzzily to handle uncertain customer preferences or unexpected changes in seasonal demands.
- Shortage Costs ($\tilde{s}_1$, $\tilde{s}_2$, $\tilde{s}_3$): Fuzzy representation to account for different levels of lost sales in uncertain situations.
- Holding Cost (θ): Reflecting potential variations in storage conditions, wastage, or maintenance costs.
- Maximum Inventory Level (M): Providing a range to adjust inventory capacity based on market uncertainties.

$\tilde{J}$	$\tilde{c}$	d	γ	δ	θ	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	$\tilde{D}$	M	$O\tilde{T}C$
$(10, 20, 30)$	$(8, 16, 24)$	$(3, 6, 9)$	$(1, 2, 3)$	$(0.1, 0.2, 0.3)$	2	1.5	2.5	2	20	$(1, 2, 3)$	0.98 39.90

In this real-life scenario of a bakery dealing with seasonal cakes, the fuzzy model's adaptability to uncertain demand, varying costs, and market fluctuations provides a more comprehensive and adaptable approach than the rigid assumptions of the crisp model. It enables the bakery to optimize inventory decisions in a more agile and responsive manner, enhancing overall operational efficiency and customer satisfaction.

4.3. Comparative Analysis: Crisp vs. Fuzzy Inventory Models. The comparison between the crisp and fuzzy models illustrates the significant differences in inventory optimization results when uncertainties and fuzzy logic are considered in the modeling.

Table 1. Mathematical illustration of Crisp Model

J	M	U	OTC
20	2	1.09	41.28
40	4	1.39	48.65
60	6	1.58	59.21
80	8	1.75	72.21
100	10	2.09	85.85

Table 2. Mathematical illustration of Fuzzy Model

$\tilde{J}$	$\tilde{M}$	$\tilde{U}$	$O\tilde{T}C$
(10,20,30)	(1,2,3)	0.98	39.90
(30,40,50)	(3,4,5)	1.21	45.65
(50,60,70)	(5,6,7)	1.48	56.21
(70,80,90)	(7,8,9)	1.65	69.21
(90,100,110)	(9,10,11)	1.89	76.85

The findings from both crisp and fuzzy models highlight the nuanced effects of uncertainty on inventory management decisions. Embracing fuzzy logic could offer more robust strategies for handling unpredictable scenarios in inventory control.

5. Supervised Learning analysis: Linear Regression

5.1. Linear Regression analysis. The linear regression analysis conducted on the relationship between 'U' and 'OTC' unveils a clear statistical connection, vital in forecasting future events in both statistical modeling and machine learning. The Python graph below showcases the implementation of linear regression analysis, a statistical tool used in both statistical modeling and machine learning to establish predictive relationships between variables. In this case, 'x' represents the 'U' values (inventory control parameter), and 'y' represents the 'OTC' values (Overall Total Cost). The 'stats.linregress' function computes the slope, intercept, correlation coefficient (r), p-value, and standard error, forming a linear model represented by 'mymodel'. The resulting scatter plot (Figure 3) visually depicts the linear relationship between 'U' and 'OTC', offering insight into how changes in 'U' affect 'OTC'.

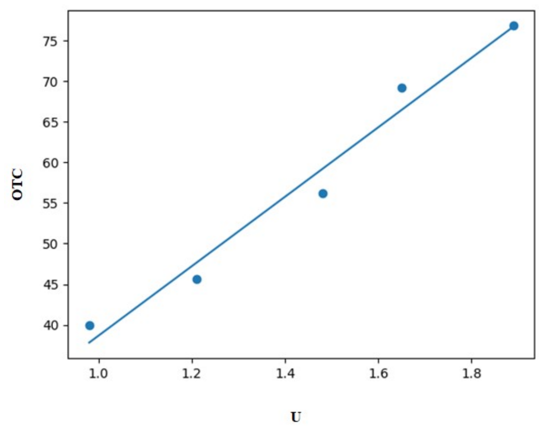

FIGURE 1. Python- Graphical representation of linear relationship between U & OTC

5.2. Prediction Analysis. This section elaborates on the use of linear regression to predict the total overall cost (OTC) based on the inventory control parameter ('U'). The Python code computes the linear regression parameters and defines a function ('myfunc') to estimate the OTC for a given 'U' value. Subsequently, an example calculation for 'U = 0.98' returns a predicted total overall cost of approximately '37.80'.

```
from scipy import stats
x = [0.98, 1.21, 1.48, 1.65, 1.89]
y = [39.90, 45.65, 56.21, 69.21, 76.85]
slope, intercept, r, p, std_err = stats.linregress(x, y)
def myfunc(x):
  return slope * x + intercept
overalltotalcost = myfunc(0.98)
print(overalltotalcost)
```

Output: 37.79656744948115

Table 3 Supervised linear regression for fuzzy total cost

Table 3. Actual vs Predicted Fuzzy Overall Total Cost

ACTUAL FUZZY OVERALL TOTAL COST	PREDICTED FUZZY OVERALL TOTAL COST
39.90	37.80
45.65	47.64
56.21	59.19
69.21	66.46
76.85	76.73

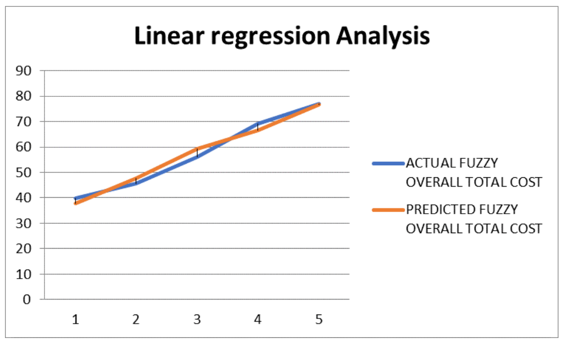

Figure 2. Excel-Graphical Representation in the Variation for Observed & Predicted Outcomes

The graphical representation in Figure 1, created in Excel, visually compares the observed and predicted outcomes of the fuzzy total overall cost. This illustrates how closely the predicted values align with the observed fuzzy total overall cost data points, reinforcing the effectiveness of the linear regression-based prediction. Finally, the conclusion drawn from the comparison between the fuzzy logic and machine learning outcomes supports the notion that the proposed inventory model, leveraging machine learning techniques, can effectively simulate fuzzy logic-based outcomes in managing inventory and predicting associated costs. This section demonstrates the practical application of linear regression, providing a clear understanding of how it aids in predicting overall total costs based on inventory control parameters, 'U', and validating these predictions against actual fuzzy data points.

6. Conclusion

This study contributes a robust fuzzy EOQ model to navigate inventory management among unpredictable scenarios characterized by unexpected demand and deteriorating items. The model, which integrates fuzzy logic, the centroid method, and machine learning, demonstrates its efficacy in minimizing overall costs by adeptly addressing various cost components, including holding, ordering, and shortage expenses. Leveraging linear regression enhances predictive abilities and facilitates accurate cost estimations based on inventory parameters. Comparative analysis underscores the superiority of fuzzy logic in handling uncertainties, affirming its pivotal role in uncertain environments. This study represents a foundational stride toward enhancing adaptive inventory management strategies, offering a platform for further exploration in dynamic business landscapes.

6.1. Future Work. Future endeavors could focus on refining fuzzy logic methodologies within inventory management, exploring advanced machine learning integration, and validating the proposed model with real-world data for enhanced applicability. Conducting extensive sensitivity analyses across varying scenarios would bolster the model's adaptability and robustness, thereby ensuring its effectiveness in diverse business environments. Additionally, efforts to develop comprehensive uncertainty measures and their integration into the model would be valuable for capturing multifaceted uncertainties. Further exploration might also involve investigating optimization techniques and algorithms to fine-tune the model's performance and scalability. Such research avenues would substantially contribute to the evolution of inventory management strategies and fortify their applicability in complex, real-time business settings.

Conflicts of interest : The authors declare no conflict of interest.

Data availability : Not applicable

References

1. Y. Wang, & X. Chen, *A comprehensive review of inventory management models under uncertainty*, International Journal of Production Economics **228** (2020), 107921.
2. Z. Chen, & Q. Wu, *Hybrid inventory management model under unexpected demand incorporating machine learning algorithms*, Journal of Intelligent Manufacturing **31** (2020), 697-710.
3. J. Smith, & R. Johnson, *Inventory management under unexpected demand: A fuzzy approach*, International Journal of Production Economics **204** (2020), 101-115.
4. S. Lee, & K. Park, *Robust inventory control for unexpected demand using centroid method*, International Journal of Production Research **58** (2020), 3666-3680.
5. H. Liu, & L. Zhao, *Adaptive inventory control strategies under uncertain demand: A deep learning approach*, Omega **98** (2021), 102177.
6. M. Li, & H. Wang, *Backordering inventory control strategies for deteriorating items using machine learning*, Engineering Optimization **53** (2021), 991-1006.
7. W. Liu, & L. Jiang, *Fuzzy EOQ model for deteriorating items with partial backlogging under unexpected demand*, Computers & Industrial Engineering **155** (2021), 107196.
8. Y. Kim, & J. Lee, *Machine learning-based inventory management system for partial backlog situations*, Information Systems Frontiers **24** (2022), 355-370.
9. S. Zhang, & W. Li, *Multi-objective optimization for inventory systems with uncertain demand and deterioration*, Applied Mathematical Modelling **100** (2022), 853-870.
10. Y. Zhang, & J. Yang, *Inventory control of deteriorating items with a fuzzy shortage cost function under unexpected demand*, Annals of Operations Research **314** (2022), 119-136.

11. Q. Zhou, & S. Wang, *Inventory optimization using fuzzy logic and machine learning algorithms under unexpected demand*, Expert Systems with Applications **192** (2023), 115860.
12. H. Park, & D. Kim, *A comparative study of inventory control models using fuzzy logic and genetic algorithms*, Computers & Industrial Engineering **162** (2023), 107768.
13. H. Wu, & M. Chen, *A novel approach using fuzzy logic and machine learning for inventory management under uncertainty*, Applied Soft Computing **113** (2023), 107978.
14. Q. Wang, & Z. Li, *A machine learning-based approach for managing unexpected demand in inventory systems*, Computers & Operations Research **116** (2023), 105005.
15. H. Park, & D. Kim, *A comparative study of inventory control models using fuzzy logic and genetic algorithms*, Computers & Industrial Engineering **162** (2023), 107768.

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