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Compensate and analyze of Optical Characteristics of AR display using Zernike Polynomials

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Abstract

Aberration is still a problem for making augmented reality displays. The existing methods to solve this problem are either slow and inefficient, consume too much battery, or are too complex for straightforward implementation. There are still some problems with image quality, and users may suffer from eye strain and headaches because the images provided to each eye lack accuracy, causing the brain to receive mismatched cues between the vergence and accommodation of the eyes. In this paper, we implemented a computer simulation of an optical aberration using Zernike polynomials which are defocus, trefoil, coma, and spherical. The research showed that these optical aberrations impact the Point Spread Function (PSF) and Modulation Transfer Function (MTF). We employed the phase conjugate technique to mitigate aberrations. The findings revealed that the most significant impact on the PSF and MTF comes from the influence of spherical aberration and coma aberration.

Keywords: *Holographic optical elements, Shack-Hartmann wavefront sensor, Zernike polynomial analysis, Optical characteristics, Wavefront aberrations, Near-eye display, Virtual reality.*

1. Introduction

Head-mounted displays have become an increasingly important area of research and development, especially with the rise of virtual reality (VR) and augmented reality (AR) technologies. These displays offer a highly immersive experience by placing virtual content directly in the user's field of view, either overlaying it in the real world (AR) or completely replacing the real world with virtual environments (VR). Near-eye displays have great potential in various fields such as education, teleconferencing, scientific visualization, entertainment, military, and training [1]. However, those commercialized products still have several problems. Research groups worldwide are developing augmented reality (AR) techniques to blend computer-generated

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images with the real world. Aberration plays a significant role in the quality of images produced by Augmented Reality (AR) displays, impacting the overall user experience in AR applications [2]. Aberration in optical systems refers to the deviation of light from its intended path, resulting in image blur, distortion, and color fringing (chromatic aberration). The optical imperfections can compromise the clarity, sharpness, and realism of computer-generated images when overlaid in the real world. Addressing aberrations is crucial for creating immersive and effective AR experiences [3]. As adaptive optics technology continues to evolve, further innovations in the analysis and correction of distorted wavefronts are expected. These advancements could lead to even more effective compensation for aberrations in real-time, pushing the boundaries of what can be achieved in imaging and optics. The principle of using the complex conjugate for wavefront correction has wide-ranging applications beyond improving the quality of images [4]. In AR, for instance, it can be used to ensure that virtual images are rendered sharply and clearly, enhancing the user experience. In this research, we study several types of common aberrations using Zernike polynomials and present how point spread functions (PSFs) and modulation transfer functions (MTFs) can be utilized to assess wavefront aberrations.

2. Background Theory

2.1. Method of aberration analysis using Zernike polynomials

Wavefront aberration analysis plays a crucial role in understanding the performance of images in AR display. The wavefront aberration is defined as the optical path difference between the reconstructed wavefront and the reference wavefront at the exit pupil of the element. An analytical expression for the optical path difference can be formulated as a function of the pupil coordinates P) denoted as $w = (x, y)$ This wavefront error can be quantified by fitting Zernike polynomials, where each term of these polynomials represents a specific type of aberration such as spherical aberration, coma, astigmatism, etc. The Zernike polynomial functions are convenient ways to characterize intuitive optical defects such as defocus, astigmatism, coma, distortion, etc. While Zernike polynomials are orthogonal over the interior of a unit circle, they may not be orthogonal over a discrete set of data points. Despite their widespread use, caution is advised in blindly applying Zernike polynomials, as improper usage can lead to misleading results. In the analysis of wavefront aberrations in HOEs, the wavefront error $w(x, y)$ and the real amplitude distribution $A(x, y)$ across the exit pupil are utilized to represent the pupil function of the HOE, expressed as [5].

$$P(x, y) = A(x, y)exp\big(ikW(x, y)\big) \tag{1}$$

where:

- λ represents the wavelength of the beam.
- $W(x, y)$represents the wavefront aberration.
- $k = 2\pi/\lambda$.

To express $w(x, y)$ using Zernike polynomials, we first need to convert $w(x, y)$ to polar coordinates (ρ, θ) and then expand it into a series of Zernike polynomials [6].

$$W(x, y) = \sum_{n=0}^{N} \sum_{m=-n}^{n} a_{nm} Z_n^m(\rho, \theta) \tag{2}$$

where:

- N is the maximum order of Zernike polynomial used in the expansion.
- a_{nm} are the coefficients representing the amplitude of each Zernike polynomial term.

$$Z_j(\rho,\theta) = Z_{mn}(\rho,\theta) = \begin{cases} \sqrt{2(n+1)}R_n^m(\rho)cosm\theta, \text{if } m \neq 0, j \text{ is even,} \\ \sqrt{2(n+1)}R_n^m(\rho)sinm\theta, \text{if } m \neq 0, j \text{ is odd,} \\ \sqrt{2(n+1)}R_n^m(\rho)cosm\theta, \text{if} m = 0. \end{cases} \quad (3)$$

where n is a non-negative integer, m is an integer, $n - |m| \geq 0$ and is even, j is a mode-ordering number starting from 0.

The general form of a Zernike polynomial $Z_n^m(\rho,\theta)$of radial order n and azimuthal frequency m is given by:

- ρ is the radial distance from the center of the unit circle.
- θ is the azimuthal angle.
- $R_n^m(\rho)$ is the radial polynomial, which depends only on ρ and is given by a specific formula.
- The radial polynomial, $R_n^m(\rho)$, is defined as:

$$R_n^{|m|}(\rho) = \sum_{s=0}^{(n-|m|)/2} \frac{(-1)^s(n-s)!}{s!\left(\frac{n+|m|}{2}-s\right)!\left(\frac{n-|m|}{2}-s\right)!}\rho^{n-2s} \quad (4)$$

Table 1. First 15-term orthogonal Zernike circle polynomials

OSA/ANSI Index j	Radial order n	Azimuthal order m	Zernike polynomials $Z_j(\rho,\theta)$	Aberration
1	0	0	$Z_1 = 1$	Piston
2	1	-1	$Z_2 = 2\rho\sin(\theta)$	x-tilt
3	1	1	$Z_3 = 2\rho\cos(\theta)$	y-tilt
4	2	0	$Z_4 = \sqrt{3}(2\rho^2 - 1)$	Defocus
5	2	-2	$Z_5 = \sqrt{6}\rho^2\sin(2\theta)$	45^0 Primary astigmatism
6	2	2	$Z_6 = \sqrt{6}\rho^2\cos(2\theta)$	0^0 Primary astigmatism
7	3	-1	$Z_7 = \sqrt{8}(3\rho^3 - 2\rho)\sin(\theta)$	Primary y-coma
8	3	1	$Z_8 = \sqrt{8}(3\rho^3 - 2\rho)\cos(\theta)$	Primary x-coma
9	3	-3	$Z_9 = \sqrt{8}\,3\rho^3\sin(3\theta)$	Trefoil x
10	3	3	$Z_{10} = \sqrt{8}\,\rho^3\cos(3\theta)$	Trefoil y
11	4	0	$Z_{11} = \sqrt{5}\,(6\rho^4 - 6\rho^2 + 1)$	Primary spherical aberration
12	4	-2	$Z_{12} = \sqrt{10}(4\rho^4 - 3\rho^2)\sin(2\theta)$	2nd astigmatism
13	4	2	$Z_{13} = \sqrt{10}(4\rho^4 - 3\rho^2)\cos(2\theta)$	2nd astigmatism (vertical)
14	4	-4	$Z_{14} = \sqrt{10}\rho^4\sin(4\theta)$	Quadrafoil y
15	4	4	$Z_{15} = \sqrt{10}\rho^4\cos(4\theta)$	Quadrafoil x

Table 1 [7] shows common aberration associated with each Zernike polynomial up to the 15th term, based on the OSA/ANSI indexing scheme.

That while certain low-order Zernike polynomials correspond well to classical aberrations, higher-order terms don't always have a straightforward physical interpretation [8]. The physical interpretations of these aberrations apply to wavefronts in optical systems, where deviations from the perfect wavefront shape cause various types of image quality degradation.

3. Proposal Method

Point Spread Function (PSF) and Modulation Transfer Function (MTF) are important tools in optical system analysis and provide insight into system performance in terms of spatial resolution and contrast. (PSF) represents the distribution of light from a point source through the optics of the system, providing insight into its spatial resolution capabilities. Given a wavefront aberration represented by Zernike polynomials, the PSF can be calculated as the squared magnitude of the Fourier transform of the pupil function P, y). The PSF is delineated by Equation 5 [9].

$$\mathrm{PSF}(x', y') = |F\{P(x, y)\}|^2 \tag{5}$$

Where $F\{\}$ denotes the Fourier transform, and (x', y') are coordinates in the image plane.

The MTF describes how the contrast in an image varies with the spatial frequency of the pattern being imaged. It is a measure of the optical system's ability to preserve the visibility of fine details in the object. The MTF can be derived from the PSF as the normalized magnitude of the Optical Transfer Function (OTF), which is the Fourier transform of the PSF:

$$\mathrm{MTF}(f_{x,}\, f_y) = \frac{|F\{P(x', y')\}|}{max|F\{P(x', y')\}|} \tag{6}$$

Where (f_x, f_y) are spatial frequencies in the image plane. The MTF provides a frequency-based representation of the optical system's performance, with values ranging from 0 (no transmission of spatial frequency) to 1 (perfect transmission), by systematically analyzing the PSF and MTF derived from Zernike polynomial-based wavefront aberration models. In optical imaging systems, the process of image formation can be mathematically represented by a convolution operation that integrates the object intensity with the system's point spread function (PSF). This convolution characterizes the inherent blurring introduced by the imaging system and is essential for understanding and simulating the optical imaging process. The foundational equation for this convolution is expressed as:

$$G(x, y) = O(x, y) * PSF(x, y) \tag{7}$$

In this study, common aberrations in optical systems are aided by computer simulation through Zernike polynomials using MATLAB software. The study investigated how these optical aberrations affect the system's Point Spread Function (PSF) and Modulation Transfer Function (MTF). Figure 3 illustrates the four distinct categories of Zernike polynomials.

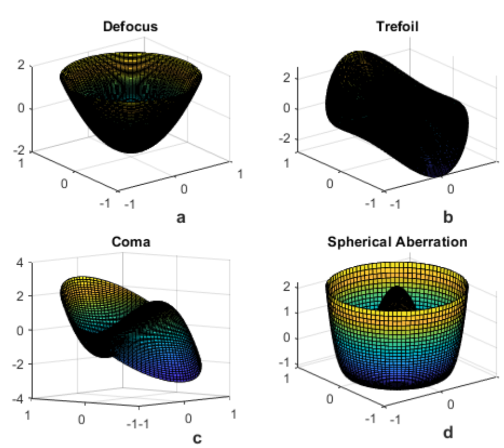

Figure 3. (a) Defocus aberration, (b) Trefoil aberration, (c) Coma aberration, (d) Spherical aberration

Equation 5 is employed to simulate the Point Spread Function (PSF) by conducting a Fourier transform on the pupil function that includes optical phase anomalies defined by Zernike polynomial terms. Figure 4 illustrates the PSF simulations for the aberration categories that were analyzed in this research. The graph shows the x-axis represents angular distance while the y-axis represents irradiance. The graph describes how the light's intensity is distributed across the image plane. In the experiment parameters below those would respectively be a wavelength is $550nm$, pupil radius is $10mm$, and a clear circular aperture of $10mm$ diameter with $a_{nm=0.1}$.

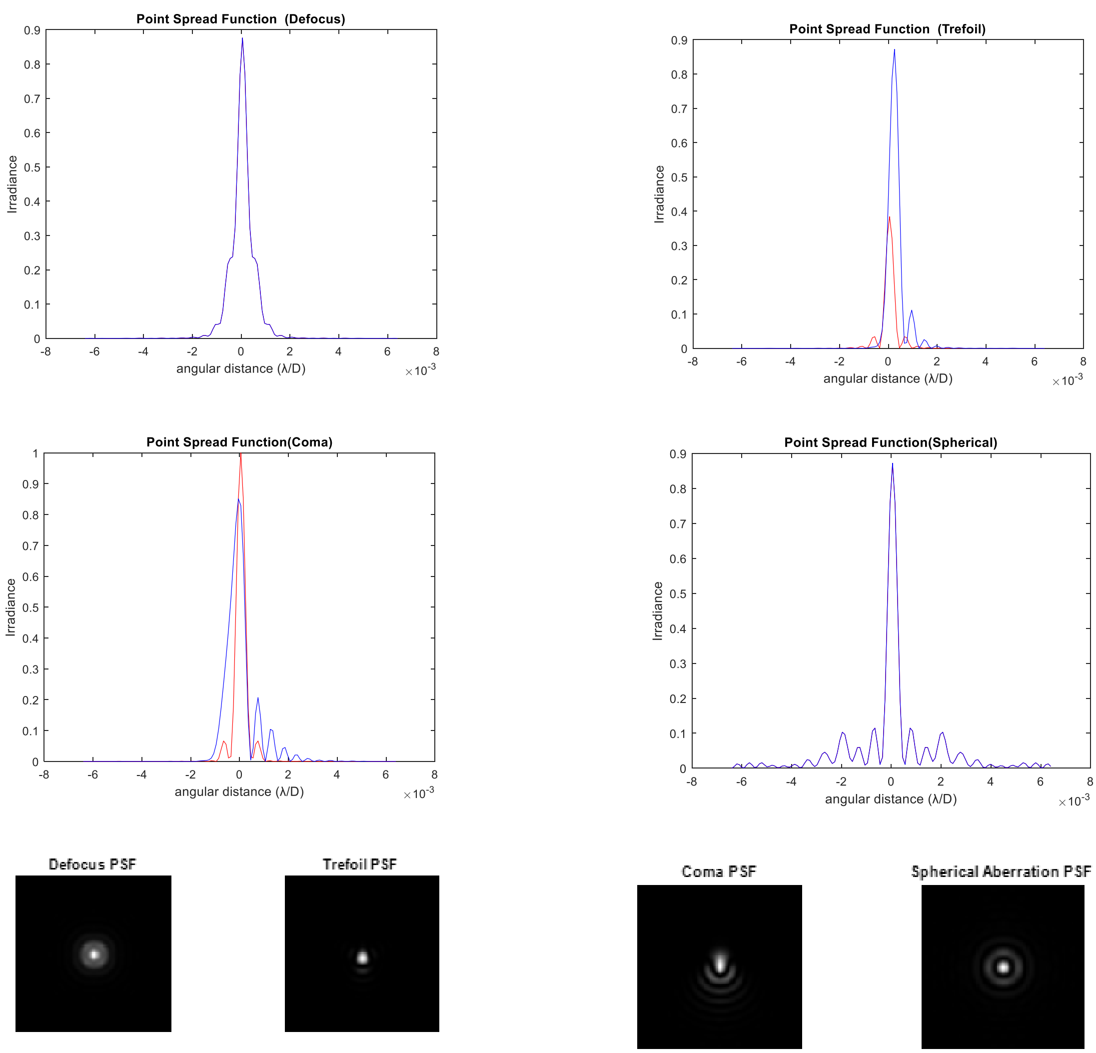

Figure 4. The Point Spread Function (PSF) for wavefronts with aberrations: defocus, trefoil, coma, and spherical aberration

MTF measures the image contrast of an optical system at different frequencies. A high MTF value means the system's ability to provide high contrast and sharp images. Because of aberration, the MTF is low, which reduces the image contrast of the system at different frequencies. As a result, small details around the edges of the image look worse, and the image is generally blurry.

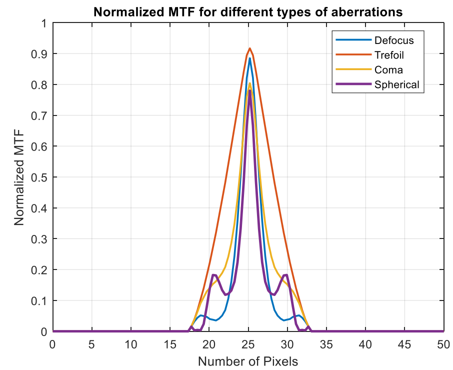

Figure 5. *MTF* for different types of aberrations

Figure 5 provides insight into how each type of aberration impacts the image quality of an optical system. To eliminate the aberration, we used optical phase conjugation. Phase conjugation is one of the techniques of adaptive optics that can be used to correct optical phase errors [10]. The phase conjugation could be computed by multiplying the phase error by its complex phase conjugation. Therefore, the pupil function could be written in term phase conjugation as:

$$P(x,y) = A(x,y)e^{(-iw(x,y))} \tag{8}$$

Equation 8 represents the pupil function where $A(x,y)$ is the amplitude distribution across the pupil, $w(x,y)$ represents the wavefront error function at a point (x,y)in the pupil plane. The terms $e^{(iw(x,y))}$ and $e^{(-iw(x,y))}$ represent the phase error and its complex conjugate, respectively.

After correction, the PSF becomes sharper, and its shape approaches the ideal, significantly improving image resolution and quality. This allows for more detailed and accurate representations of the object being imaged, which is crucial in fields such as AR display.

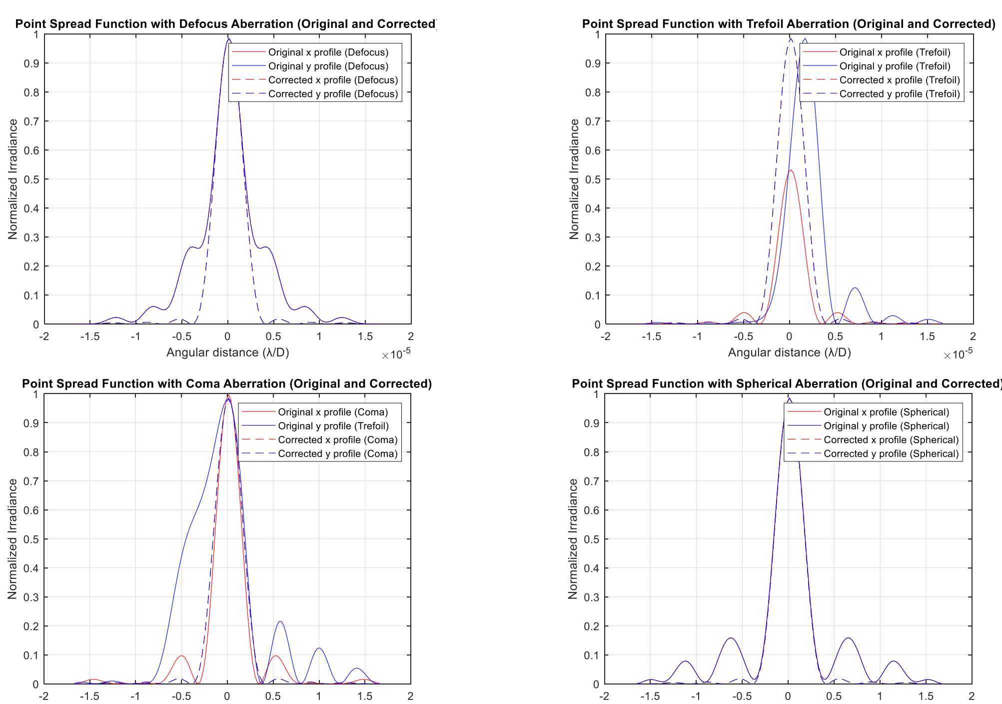

Figure 6. *PSF* for different types of aberrations after correcting

Figure 7 shows the graph of the Modulation transfer function (MTF) after correcting for 4 types of aberrations.

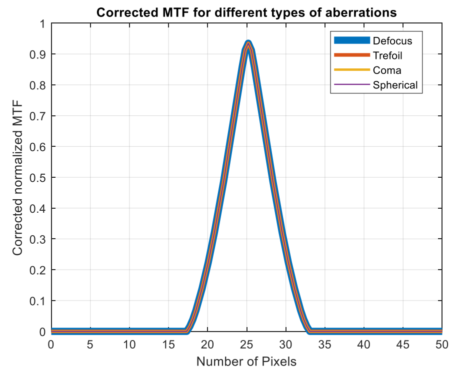

Figure 7. *MTF* for different types of aberrations after correcting

4. Result and discussion

In AR displays, implementing optical phase conjugation via adaptive optics can significantly improve image clarity and fidelity, as it corrects aberrations introduced by the optical components of the AR system or by the user's eye imperfections. In our experiment, Zernike polynomials are the best tools. Zernike polynomials are mathematically orthogonal over the unit circle, meaning that each polynomial represents a unique aspect of the wavefront error with minimal overlap with others. Zernike polynomials are well-suited for adaptive optics because they can model the wavefront error with high accuracy using a relatively small number of coefficients. By analyzing the effect of optical aberrations on the point spread function (PSF) and modulation transfer function (MTF) of the system, the quality and efficiency of optical systems can be evaluated. We used a phase conjugate technique to eliminate aberration. This process requires an adaptive optical system capable of measuring the wavefront distortion and dynamically adjusting it to provide the conjugate phase correction in real-time. Adaptive optics traditionally employs complex designs with multiple lenses to correct aberrations, which can make the systems bulky and heavy. A promising alternative is optical phase conjugation, which effectively reduces aberrations and enhances image quality. Our findings reveal that applying the phase conjugation technique alters the contribution of spatial frequency to contrast differently for each type of aberration. Specifically, the technique has the least impact on trefoil aberration, while it shows the most significant improvement in correcting spherical aberration and coma aberration.

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