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Contagion in Global Bond Markets*

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Abstract

Purpose: The paper analyzes for detecting unexpected shocks such as global financial crisis and COVID-19 pandemic, and contagion between countries by capturing in the mean-shift, variance-covariance-shift, and skewness-coskewness-shift parameters of interest rates. **Research design, data and methodology:** A flexible multivariate model of interest rates is provided by allowing for regime switching and a joint skewed normal distribution. The model is applying to the structural breaks of crisis and contagion between the US and the selected global bond markets during the global financial crisis and COVID-19 pandemic, respectively. Inspection of the moment statistics weakly suggests a flight to safety to the US during the global financial crisis and to Canada during the COVID-19 pandemic. **Results:** The results indicate that risk averse investors had a higher risk appetite for the US and Canada assets during the crisis regimes, compared to their counterparts. **Conclusions:** The results show that coskewness contagion dominates correlation contagion, and coskewness contagion is significant for the Korea and Japan-US pairs for the global financial crisis and the Euro-US pair for the COVID-19 pandemic. All channels of structural breaks of crisis and contagion are significant when considered jointly, reinforcing the need to consider contagion and structural breaks during crises in a multivariate setting.

Keywords : Contagion, Gibbs sampling, Global Financial Crisis, COVID-19 Pandemic, Regime Switching Skewed Normal Distribution

JEL Classification Code : C11, C34, G15

1. Introduction

The recent sharp increases in interest rates both in the US and throughout the world pose a significant threat to the economic welfare of global economies. It first classifies changes in the US interest rates into those caused by changes in inflation expectations, changes in perceptions of the Federal Reserve's reaction function, and changes in real activity. The rapid rise in the US interest rates, along with the associated rise in the foreign exchange value of the dollar, exerts notable spillovers on borrowing costs in global economies. These spillovers are substantially exacerbating debt burdens, rendering it more difficult to finance debt repayments, and heightening the likelihood of debt distress and financial crises. Anticipation of such developments, in turn, are disrupting financial markets, discouraging capital inflows, and leading to financial market strains.

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Many different types of contagion models are all based on identifying significant changes in structures of the dependence between financial asset returns during crisis compared to non-crisis periods. A well-known stylized fact in financial crises is that asset returns are characterized by asymmetric and fat-tailed distributions, implying that for modelling asset returns the traditional mean and variance framework with the Gaussian distribution is not appropriate. Policymakers and financial market participants are very interested in the contagion effect between markets during a financial crisis for reasons such as monetary policy, risk management, and asset pricing.

However, most of the existing tests of contagion effect proposed in the literature focus on correlations, coskewness, cokurtosis and covolatility, respectively. Correlations suggested by Forbes and Rigobon (2002) focus on the interaction between the expected returns in financial markets, whereas coskewness by Fry-McKibbin et al. (2010, 2014) focusses on the interaction between the expected return and volatility in markets. Fry-McKibbin and Hsiao (2018) suggest the contagion effect through the cokurtosis channel, which measures the interaction effects between expected returns and skewness in financial markets, and covolatility channel that focusses on contagion from volatility spillovers. However, a common feature of these tests of contagion focuses on a single channel and do not necessarily consider the possibility of contagion operating simultaneously through multiple channels.

In contrast to these previous approaches, the aim of this paper is to propose joint tests of contagion that allow for a range of contagious channels simultaneously. The approach is to construct Lagrange multiplier tests which are based on the likelihood associated with the multivariate generalized normal distribution of Fry-McKibbin et al. (2010). Working with this class of distributions provides a convenient framework as the role of higher order moments including coskewness, cokurtosis and covolatility are explicitly included in the form of the joint distribution. The joint contagion testing framework also shares a broader relationship with the joint tests of multivariate normality originally proposed by Mardia (1970), and Bera and John (1983), and extended by Doornik and Hansen (2008), Zhou and Shao (2014), and Kim (2016). This earlier work tends to focus on joint multivariate tests of skewness and kurtosis combined with covolatility. However, this literature does not focus on testing for changes in these higher order moments which is desired in testing for contagion and is the focus of the present paper.

A regime switching skewed normal model by Chan et al. (2019), and Fry-McKibbin et al. (2019) is considered, which is an extension of Hamilton (1989), as the multivariate skewed normal distribution assumption allows for non-normality, better reflecting the characteristics of high frequency financial market data. First, a feature of the regime switching skewed normal model is able to estimate the emergence of linear and non-linear comovements between asset returns that are likely to emerge during a switch to a crisis regime. The second contribution is that the model simultaneously tests for a range of linear and non-linear channels in the same model. A third contribution of the model provides a general framework for examining different types of transmission channels of financial market crisis and contagion through changes in the model parameters including of the mean, variance, skewness, covariance and coskewness. We are careful to distinguish between a crisis where own moments experience a shift in a regime specific parameter of the regime switching skewed normal model during a crisis regime, and a channel of contagion where a cross market moment experiences a shift in a regime specific parameter of the model during the crisis period. The crisis is captured in the mean-shift, variance-shift and skewness-shift parameters of an asset, whereas contagion is captured through the covariance-shift, and coskewness-shift of an asset. The final contribution is that the joint tests are applied to studying global contagion in bond markets during the global financial crisis in 2008-09 and the COVID-19 pandemic from 2020-22. The COVID-19 pandemic that substantially affected almost all sectors is different from other financial and economic crises (Chopra & Mehta, 2022). Using daily data, the empirical results highlight the importance of higher order moment channels in transmitting contagion across bond markets globally.

The rest of the paper is organized as follows. Section 2 presents the multivariate regime switching skewed normal model of asset returns in which five types of crisis and contagion channels are developed. Section 3 outlines the hypotheses and testing methods for the contagion and structural break tests. Section 4 presents the empirical analysis and Section 5 concludes.

2. The Regime Switching Skewed Normal Model

The regime switching model of Hamilton (1989) is considered by assuming that under each regime, y_t is assumed to have a multivariate specification with skewed normal distribution. This is useful for capturing the typical behavior of asset returns, including asymmetry, heteroskedasticity, fat tails, time-varying linear and nonlinear comoments across markets, as well as controlling for parameters that allow for differences across states. In this regard, the multivariate skewed normal distribution is considered for a set of asset returns y_t, but allows for the model parameters to be state dependent as follows;

$$y_t = X_t \beta_{s_t} + \varepsilon_t \tag{1}$$

$$\varepsilon_t \overset{iid}{\rightarrow} N(0, \Sigma_{s_t}) \quad (2)$$

$$L_t \overset{iid}{\rightarrow} N(c \cdot 1_m, I_m)1(L_{it} > c, i = 1, \dots, m) \quad (3)$$

where $y_t = (y_{1t}, \dots, y_{mt})'$ is an m-dimensional random vector with $t = 1,..,T$, $X_t = (I_m, I_m \otimes L_t')$ where $L_t = (L_{1t}, \dots, L_{mt})'$ is an m-dimensional random vector of latent variables, $\beta_{s_t} = (\mu_{s_t}', \omega_{s_t}')'$, where μ is an $m \times 1$ vector of constants, $\omega_{s_t} = vec(\Omega_{s_t}')$. ε_t is an $m \times 1$ innovation vector, Σ is an $m \times m$ variance-covariance matrix, 1_m is an $m \times 1$ column of ones, I_m is the identity matrix, and 1(.) is a scalar indicator function which takes a value of '1' if all L_{it} are greater than 'c' and '0' otherwise. The constant term c is set to be $-\sqrt{2/\pi}$, so that the latent variables, L_t, do not affect the unconditional expectation of y_t. The state dependent variable, s_t needs to be specified to complete the model. The standard Markov transition is assumed to have $Pr(s_t = 1|s_{t-1} = j) = p_{jt}$ for j = 0 and 1, where the probabilities p_{jt} are fixed time varying constants. Suppose that $\{y_t\}$, t = 1, …, T, are generated by the model (1)-(3) and the parameters of the regime switching skewed normal model are $\Theta = (\beta_{s_t}, \Sigma_{s_t})$ for $s_t = 0, 1$ in the mean and variance. For Bayesian estimation, stack $y = (y_1', \dots, y_T')'$, $L = (L_1', \dots, L_T')'$ and $s = (s_1', \dots, s_T')'$ and let μ_{i,s_t}, Σ_{ij,s_t} and Ω_{ij,s_t} denote the i-th element of μ_{s_t}, s_t = 0, 1. Notably, the correlation coefficient, ρ_{ij,s_t}, can also be estimated as follows; $\rho_{ij,s_t} = \left(\Sigma_{ij,s_t}/\sqrt{\Sigma_{ii,s_t}}\sqrt{\Sigma_{jj,s_t}}\right)$ for $s_t = 0,1$. Here, the state s_t at time t is a binary variable, i.e., $s_t \in \{0,1\}$, where the state $s_t = 0$ is called a non-crisis period and $s_t = 1$ a crisis period to focus the discussion on modeling crisis and contagion. In other words, there are two sets of state dependent parameters: $(\mu_{s_t=0}, \Omega_{s_t=0}, \Sigma_{s_t=0})$ and $(\mu_{s_t=1}, \Omega_{s_t=1}, \Sigma_{s_t=1})$.

The regime switching skewed normal model allows the parameters including the means, μ_{s_t}, coskewness, Ω_{s_t}, and the error cross-covariances, Σ_{s_t} and it is possible to analyze the contagion effect when there is a crisis, state $s_t = 1$ compared to non-crisis, state $s_t = 0$. The crisis is captured in the mean-shift, variance-shift and skewness-shift parameters of an asset during the second regime, whereas contagion is captured through the covariance-shift, and coskewness-shift of an asset in the second regime. Changes in the moment parameters of the mean, variance and skewness in the second regime are structural breaks.

The states, s and the model parameters, Θ are estimated by a Bayesian approach. More specifically, in order to obtain draws from the posterior distribution for the analysis Markov Chain Monte Carlo methods are used. The likelihood function of the regime switching skewed normal model in equations (1) and (2) is given by

$$f(y|L, \Theta, s) = (2\pi)^{-\frac{mT}{2}} \prod_{t=1}^{T} \left|\Sigma_{s_t}\right|^{-\frac{1}{2}} exp\left\{-\frac{1}{2}\sum_{t=1}^{T}\left[y_t - X_t\beta_{s_t}\right]'\Sigma_{s_t}^{-1}\left[y_t - X_t\beta_{s_t}\right]\right\}, \quad (4)$$

where $\Theta = (\beta_{s_t}, \Sigma_{s_t})$ and $s_t \in \{0,1\}$. The priors for the model parameters are specified as

$$\beta_{s_t} \sim N(\bar{\beta}, \bar{V}_\beta), \quad (5)$$

$$\Sigma_{s_t} \sim IW(\bar{\tau}_\Sigma, \bar{S}_\Sigma), \quad (6)$$

$$\begin{cases} Pr(s_t = 1|s_{t-1} = j) = p_{jt} \\ Pr(s_t = 0|s_{t-1} = j) = 1 - p_{jt} \end{cases} \quad (7)$$

where $IW(\bar{\tau}_\Sigma, \bar{S}_\Sigma)$ represents the inverse-Wishart distribution with degree of freedom, $\bar{\tau}_\Sigma$ and scale matrix, $\bar{S}_\Sigma$. The prior mean for β_{s_t} is set to $\bar{\beta} = (\bar{\mu}', \bar{\omega}')'$ and the prior covariance matrix for β_{s_t} is set to $\bar{V}_\beta = \begin{bmatrix} \varphi_\mu I_m & 0 \\ 0 & \varphi_\omega I_k \end{bmatrix}$, where $k = m^2$.

The model parameters are estimated by the Gibbs sampler, which follows from Bayes rule that the joint posterior distribution is proportional to the product of the likelihood function and the joint prior density defined by

$$\pi(\Theta, L, s|y) \propto f(y|L, \Theta, s)f(L)f(s|\Theta)\pi(\Theta), \quad (8)$$

where $f(y|L, \Theta, s)$, $f(L)$ and $f(s|\Theta)$ are given in equations (4), (3) and (7), respectively and the notation π denotes the prior and posterior density functions. By assuming prior independence between β and Σ, the joint prior density is given by $\pi(\Theta) = \pi(\beta_0)\pi(\beta_1)\pi(\Sigma_0)\pi(\Sigma_1)$. Geweke (1991), Robert (1995), Kroese et al. (2011), Chib (1996), Frhwirth-Schnatter (2006), and Chan et al. (2019) provide the full conditional distributions and their derivations for β_{s_t}, Σ_{s_t}, Z_t, and s_t.

3. Empirical Example

3.1. Data and Descriptive Statistics

The time series characteristics of the data used in the estimation and our methodology are briefly described in this section. The daily interest rates for the global bond markets from the IMF database are used for measuring financial market crisis and contagion, covering September 7, 2004 through February 29, 2024 for Euro area, Canada, Korea, Japan and USA. The first difference of the daily interest rates, multiplied by 100 is used. For the selected USA, Figure 1 plots the first difference of the daily interest rates, clearly showing that the series in both variables are rather stable around the mean. However, the volatility clusters are more severe around the global financial crisis in October 2008 and during the COVID-19 pandemic period from March 2020 to January 2022. Therefore, market volatility is changing over time, which suggests that a suitable model for the data should have a time varying volatility structure as suggested by the ARCH-type model. A similar pattern of the mean and volatilities is observed in the other countries, but the graphs are not shown for space reasons.

By motivating the use of the skewed normal distribution, Table 1 provides summary statistics including higher order moments and comoments of the daily data for the US and selected global markets. For the global financial crisis, the periods are from September 2004 to July 2007 before the crisis and from March 2008 to the end of 2013 after the crisis, respectively. The periods for the COVID-19 pandemic crisis are from January 2014 to March 2020 before the crisis and from February 2022 to February 2024 after the crisis, respectively. The table shows that the statistics are very different, providing evidence of non-normality in the first difference of the daily interest rates of each country in both periods.

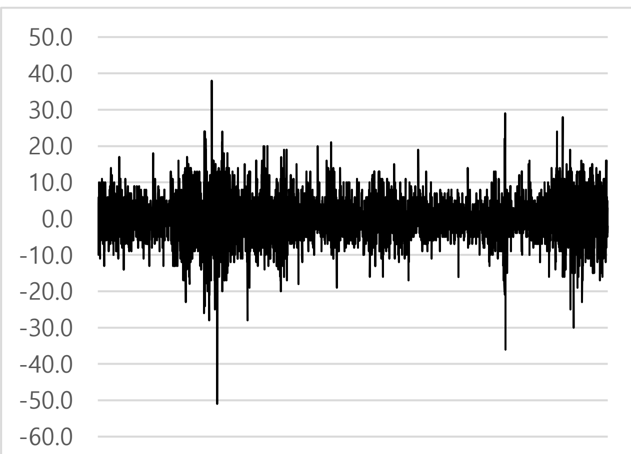

Figure 1: First Differences of the Interest Rates for the Selected USA

The mean returns for the period after the global financial crisis are lower and negative for all countries compared to those for the period before the crisis. From the summary statistics of skewness and kurtosis, it is evident that the series are symmetric, while the dispersion of a large number of observed values is very small, implying a leptokurtic frequency curve. This means that the series do not follow a normal distribution, instead presenting a sharp peak and fat tail distribution. Additionally, Table 1 shows that all return series investigated suffer from long-run dependencies from the Ljung and Box (1978) $Q(10)$ statistics, which evaluates the independence between the series in conditional mean.

Table 1: Summary Statistics

	Mean	Skewness	Kurtosis
A. Global financial crisis			
Before the crisis: September 2004 to July 2007			
Euro	0.035	-0.061(0.235)	3.818(0.000)
Canada	-0.128	0.049(0.282)	3.572(0.000)
Korea	0.192	0.211(0.006)	5.849(0.000)
Japan	0.020	0.057(0.249)	3.914(0.000)
USA	-0.033	-0.057(0.251)	3.942(0.000)
After the crisis: March 2008 to December 2013			
Euro	-0.186	0.223(0.001)	4.849(0.000)
Canada	-0.070	0.043(0.266)	4.613(0.000)
Korea	-0.175	0.497(0.000)	9.049(0.000)
Japan	-0.084	-0.080(0.123)	5.795(0.000)
USA	-0.074	-0.169(0.007)	6.209(0.000)
B. COVID-19 pandemic crisis			
Before the crisis: January 2014 to March 2020			
Euro	-0.206	0.621(0.000)	7.019(0.000)
Canada	-0.118	0.015(0.406)	3.848(0.000)
Korea	-0.168	0.191(0.002)	6.660(0.000)
Japan	-0.063	0.602(0.000)	11.990(0.000)
USA	-0.139	0.055(0.200)	3.820(0.000)
After the crisis: February 2020 to February 2024			
Euro	0.346	-0.510(0.000)	6.254(0.000)
Canada	0.287	-0.127(0.061)	3.945(0.000)
Korea	0.240	0.273(0.001)	7.009(0.000)
Japan	0.094	0.885(0.000)	14.145(0.000)
USA	0.368	-0.240(0.001)	5.143(0.000)

	Q(10)	$Q^2(10)$	Correlation	Coskewness
A. Global financial crisis				
Before the crisis: September 2004 to July 2007				
Euro	0.038(1.000)	0.2810(1.000)	0.417	1.085
Canada	0.132(1.000)	0.442(1.000)	0.095	1.397
Korea	14.447(0.153)	4.159(0.939)	0.329	0.814
Japan	1.666(0.998)	0.015(1.000)	0.468	0.845
USA	1.335(0.999)	6.546(0.767)	-	-
After the crisis: March 2008 to December 2013				
Euro	6.103(0.806)	6.119(0.805)	0.484	1.053
Canada	3.880(0.952)	21.806(0.016)	0.129	1.589
Korea	0.580(1.000)	34.607(0.000)	0.253	1.066
Japan	1.222(0.999)	51.328(0.000)	0.485	0.963
USA	0.090(1.000)	9.749(0.462)	-	-
B. COVID-19 pandemic crisis				
Before the crisis: January 2014 to March 2020				
Euro	1.788(0.997)	63.662(0.000)	0.541	1.288
Canada	0.366(1.000)	3.344(0.972)	0.373	1.461
Korea	0.897(0.999)	54.273(0.000)	0.324	1.048
Japan	2.886(0.984)	94.960(0.000)	0.591	1.153
USA	1.069(0.999)	4.383(0.928)	-	-
After the crisis: February 2020 to February 2024				
Euro	0.613(1.000)	37.780(0.000)	0.580	1.376
Canada	0.189(1.000)	20.370(0.025)	0.354	1.582
Korea	0.281(1.000)	10.984(0.358)	0.260	1.199
Japan	0.200(1.000)	46.725(0.000)	0.614	1.113
USA	0.151(1.000)	19.192(0.037)	-	-

Note: The sample is daily observations from September 7, 2004 to February 29, 2024 for global bond markets. The Q(10) and $Q^2(10)$ are the Ljung-Box statistics for tenth-order serial correlation in the residuals and squared residuals, respectively. The standard errors for skewness and kurtosis are $(6/T)^{0.5}$ and $(24/T)^{0.5}$, respectively where T is the number of observations. The critical values of the rejection of null hypothesis of normal distribution for skewness and kurtosis statistics at 5% level are $\pm$ 0.091 and $\pm$ 0.182, respectively. The comoment statistics of correlation and coskewness are calculated for each country with the US. The coskewness statistic is assumed to be symmetric.

On the while from Ljung and Box (1978) $Q^2(10)$ statistics, it is clear that there is a significant, nonlinear temporal dependence in the squared adjusted returns series, suggesting that the volatility of adjusted returns follows an ARCH-type model. Of note are the statistics of market comovement for the global interest rates with those of the US, showing that not only does correlation with the US rise in the period after the global financial crisis for all global bond markets, but also coskewness becomes more positive. This is commonly referred to as shift-contagion that emphasizes the change component. However, the reason for changes in the coskewness statistics is that the linear correlation coefficient may not reflect all changes in the inter-market dependence. Comparing the summary statistics before and after the COVID-19 crisis, the means after the crisis show a positive sign in all countries, and the remaining statistics were similar to those of the global financial crisis.

For the daily level series of the interest rates, they have been continuously decreasing from the global financial crisis in 2008 until COVID-19 and then have been rapidly increasing after the end of COVID-19, providing evidence of structural breaks. The graph for the daily interest rates that are not reported here for reasons of spaces illustrates the potential arbitrariness of choosing a crisis date and how this could allow for weak conclusions, showing the statistics of a bivariate test of contagion as a function of changes in the correlation between the US and each selected global bond markets shown in Table 1. In this regard, Table 2 provides the results of multiple structural breaks for global financial crisis using Bai and Perron (2003) test, which is interested in the presence of abrupt structural changes in the mean of the series. They consider the multiple linear regression model with M breaks and two different independent covariates of $x_t(p \times 1)$ and $z_t(q \times 1)$. Since we are interested in the presence of abrupt structural changes, we apply our procedure with only a constant as regressor, i.e., $z_t = \{1\}$ and q=1, p=0 and M=5. The asymptotic distribution depends on a trimming parameter via the imposition of the minimal length h of a segment for a trimming $\varepsilon = 0.15$.

To that effect we apply the procedure with only a constant as regressor (i.e. $z_t = \{1\}$) and account for potential serial correlation via nonparametric adjustments. We allowed up to 5 breaks and used a trimming $\varepsilon = 0.15$, hence each segment has at least 15 observations. We also allowed serial correlation in the errors and different variances of the residuals across segments. The first issue is how to determine the number of breaks, indicating that the $SupF_T(k)$, $UDmax$ and $WDmax$ tests are all highly significant for k between 1 and 5, and so at least one break is present. The most of $supF_T(2|1)$ tests are not significant and for any $l \geq 2$, $supF_T(l+1|l)$ is also not significant. Therefore, according to the results such as SEQ, BIC, and LWZ, there is at least one break overall, as shown in Tables 2. According to the studies by Yao (1998), Liu et al. (1997) and Perron (1997), the information criteria are biased downward, and the sequential procedure and the $supF_T(l+|l)$ perform better in this case, we conclude in favor of the presence of 2 breaks for Canada, Japan and USA, and 1 break Euro and Korea during the global financial crisis, but in favor of the presence of 2 breaks for Japan only, and 1 break for the others during the COVID-19 pandemic crisis. The estimation results of multiple structural breaks for the COVID-19 pandemic crisis respectively are available from authors on request but are not reported here for reasons of spaces.

Table 2: Empirical Results of Multiple Structural Breaks

Specification	$z_t = \{1\}$ $q = 1$ *M=5* $p = 0$ *h=646*	
	Euro	Canada
$SupF_T(1)$	19.578**	31.540**
$SupF_T(2)$	45.628**	59.831**
$SupF_T(3)$	91.972**	35.441**
$SupF_T(4)$	134.606**	27.688**
$SupF_T(5)$	127.544**	17.042**
$UDmax$	134.606**	59.831**
$WDmax$	279.881**	78.561**
$SupF_T(2\|1)$	4.111	44.246**
$SupF_T(3\|2)$	5.325	1.296
$SupF_T(4\|3)$	3.051	0.158
$SupF_T(5\|4)$	0.000	0.000
SEQ	1	2
LWZ	5	3
BIC	5	3
$\widehat{\delta_1}$	3.746(0.306)	4.195(0.083)
$\widehat{\delta_2}$	2.190(0.177)	3.316(0.102)
$\widehat{\delta_3}$	-	2.112(0.163)
$\widehat{T_1}$	2011.8.2 [2011.7.16 -2013.11.5]	2008.2.27 [2007.8.10-2008.6.25]
$\widehat{T_2}$	-	2011.7.8 [2011.6.10-2011.10.6]

Specification	$z_t = \{1\}$ $q = 1$ *M=5* $p = 0$ *h=646*		
	Korea	Japan	USA
$SupF_T(1)$	24.813**	18.333**	14.992**
$SupF_T(2)$	26.517**	25.344**	25.990**
$SupF_T(3)$	26.110**	24.997**	24.615**
$SupF_T(4)$	31.591**	89.396**	16.300**
$SupF_T(5)$	24.799**	79.209**	13.043**
$UDmax$	31.591**	89.396**	25.990**
$WDmax$	62.724**	198.266**	39.805**
$SupF_T(2\|1)$	2.155	23.147**	9.316*
$SupF_T(3\|2)$	0.413	9.775	3.575
$SupF_T(4\|3)$	1.492	11.185*	1.397
$SupF_T(5\|4)$	0.000	0.000	0.000
SEQ	1	2	2
LWZ	5	5	4
BIC	5	5	4
$\widehat{\delta_1}$	5.071(0.213)	1.510(0.078)	4.383(0.235)

$\widehat{\delta_2}$	3.504(0.230)	1.079(0.059)	3.219(0.182)
$\widehat{\delta_3}$	-	0.748(0.035)	2.075(0.327)
$\widehat{T_1}$	2011.5.20 [2010.10.25-2012.10.26]	2010.6.17 [2010.3.18-2012.10.5]	2008.11.17 [2008.4.7-2010.11.15]
$\widehat{T_2}$	-	2012.4.26 [2012.2.27-2013.1.9]	2011.8.2 [2009.4.9-2012.3.14]

Notes: The $SupF_T(k)$ tests allow for the possibility of serial correlation in the disturbances using the standard errors and confidence intervals. Andrews and Monahan (1992) provide the heteroscedasticity and autocorrelation consistent covariance matrix, which is constructed using a quadratic kernel with automatic bandwidth selection based on an AR(1) approximation. The residuals are pre-whitened using a VAR(1). The significant level of 5% is used for the sequential test, $SupF_T(l+1|l)$. In parentheses are the standard errors with robust to serial correlation for $\hat{\delta}_i$, i=1, …, 4 and the 95% confidence intervals for $\hat{T}_i$, i= 1, 2, 3. * and ** denote significance at the 5% and 1%, respectively.

3.2. Estimation Results

In order to estimate posterior means of the regime switching skewed normal distribution, the prior hyper-parameters in equations (5) to (7) are assumed to be $\bar{\beta}=0$, $\varphi_\mu = 0.01$, $\bar{\tau}_\Sigma = 20+m+10$, $\bar{S}_\Sigma = (\bar{\tau}_\Sigma - m - 1)\times I_m$ with m = 5. The prior variances are chosen to be relatively small to make the prior distributions proper and relatively informative. The regime dependent beliefs about the likelihoods of a change in regime for the global financial crisis and COVID-19 pandemic, respectively are incorporated formally by the following prior probabilities; $p_{it} = Pr(s_t = 1) = 1 - Pr(s_t = 0)$ for i=0, 1.

For incorporating information about the timing of a regime change, the initial value for the probability of being in regime 0 is set to $Pr(s_t = 0) = 0.99$ during the period from September 7, 2004 to December 1, 2006 for the global financial crisis (January 6, 2014 to December 2, 2019 for the COVID-19 pandemic). The probability of being in regime 1 is set to $Pr(s_t = 1) = 0.01$ during the period between June 2, 2008 and December 30, 2013 for the global financial crisis (February 3, 2022 to February 29, 2024 for the COVID-19 pandemic). The probability of being in regime 0 decreases linearly from 0.99 on December 4, 2006 to 0.01 on May 30, 2008 for the global financial crisis and December 3, 2019 to January 28, 2022 for the COVID-19 pandemic, by a margin of $\left[\frac{1}{327}(0.99-0.01)\right]$ and $\left[\frac{1}{471}(0.99-0.01)\right]$ per day, respectively. For estimation purposes, the coskewness matrix Ω in equation (1) is assumed to be a symmetric matrix, so that the dimension of ω changes from $k=m^2$ to $k=m(m+1)/2$. Furthermore, the constant term c in equation (3) is set to '$-\sqrt{2/\pi}$', which means $E(Z_t)=0$ and $V(Z_t)=(\pi-2)/\pi$, and the inclusion of the latent variables, L_t does not affect the unconditional expectation of y_t.

The regime switching skewed normal model is estimated by using the procedure for the Gibbs-sampling, where the first 20,000 draws are discarded to allow the Markov Chain to converge to a stationary distribution. For reducing sample autocorrelation and avoiding biased Monte Carlo standard errors, posterior means of the regime-switching parameters in Table 3 for the global financial crisis are calculated by the procedures that every 10 draws for the next 200,000 iterations are recorded for a total of 20,000 draws. The estimation results of posterior means of the switching parameters for the COVID-19 pandemic crisis are available from authors on request but are not reported here for reasons of spaces.

The panel A of the Table 3 presents the results for regime $s_t = 0$ where there is no crisis, while the panel B presents the results for regime $s_t = 1$ where there is crisis. The parameters for correlation and coskewness appear to change across the regimes. The correlation of the pairs of markets is lower in the financial crisis than when no financial crisis for all pairs of markets except the pairs of Euro and USA, whereas in the case of the COVID-19 crisis, the correlation increases significantly after the crisis for all pairs of markets. For coskewness, skewness statistics vary between countries before and after the global financial crisis, but increased in all pairs between the US and other countries except the Euro-US pair. On the other hand, skewness statistics increased in all country pairs before and after the COVID-19 crisis, reflecting the preference of risk averse investors for positive coskewness in a crisis period; see Guidolin and Timmerman (2008) and Fry et al., (2010) for more detailed. The anomaly is for the value of coskewness for the Korea-Japan pair which falls from 0.5277 to 0.3256. Before and after the global financial crisis and the COVID-19 crisis, covariances are found to have increased significantly after the crisis, implying that the bond markets are becoming more synchronized after the crisis.

Table 3: Posterior Means of the Switching Parameters for the Global Financial Crisis Period

Parameters	Markets	Euro	CAN	KOR	JPN	USA
A. Before the crisis ($s_t = \mathbf{0}$)						
Covariance ($\Sigma_{ij,0}$)	CAN	5.674	-	-	-	-
	KOR	2.495	5.038	-	-	-
	JPN	1.022	2.028	4.562	-	-
	USA	7.793	10.230	5.810	2.524	-
Correlation ($\rho_{ij,0}$)	CAN	0.701	-	-	-	-
	KOR	0.318	0.198	-	-	-
	JPN	0.810	0.502	0.321	-	-
	USA	0.845	0.682	0.490	0.335	-
Coskewness ($\omega_{ij,0}$)	CAN	1.224	-	-	-	-
	KOR	0.917	-2.282	-	-	-
	JPN	-1.691	0.174	-0.326	-	-
	USA	1.109	0.411	-2.189	1.953	-

Mean ($\mu_{i=0}$)		-0.004	-0.057	0.043	0.003	0.045
Variance ($\Sigma_{ii,0}$)		6.446	10.179	11.452	4.110	14.352
Skewness ($\omega_{ii,0}$)		-1.529	0.670	4.262	-0.212	0.486
B. After the crisis ($\boldsymbol{s_t = 1}$)						
Covariance ($\Sigma_{ij,0}$)	CAN	13.483	-	-	-	-
	KOR	3.677	7.881	-	-	-
	JPN	3.256	1.653	2.345	-	-
	USA	16.749	27.214	16.173	3.790	-
Correlation ($\rho_{ij,0}$)	CAN	0.642	-	-	-	-
	KOR	0.262	0.505	-	-	-
	JPN	0.603	0.494	0.259	-	-
	USA	0.870	0.474	0.756	0.431	-
Coskewness ($\omega_{ij,0}$)	CAN	1.144	-	-	-	-
	KOR	-0.154	-1.892	-	-	-
	JPN	2.013	-1.819	-0.213	-	-
	USA	-0.615	1.421	-4.406	0.159	-
Mean ($\mu_{i=1}$)		-0.033	-0.012	-0.018	-0.032	-0.022
Variance ($\Sigma_{ii,1}$)		18.671	23.625	11.692	2.204	41.399
Skewness ($\omega_{ii,1}$)		-0.124	0.897	5.168	0.101	2.882

Notes: The posterior means of the covariance, correlation, coskewness, mean, variance and skewness are provided for the period before the global financial crisis and after the crisis regimes for the interest rates of selected global markets and the US. The sample period is September 7, 2004 through December 30, 2014.

4. Empirical Analysis for Structural Breaks of Crisis and Contagion

Tables 4 and 5 present the empirical results for the tests of contagion and structural breaks of crisis that are summarized in Section 4 of Chan et al. (2019) between the US and the selected global bond markets during the global financial crisis and the COVID-19 Pandemic crisis, respectively. The tables consist of three panels: the first examines the evidence of the moment structural breaks of crisis in the mean, variance and skewness parameters, the second examines the evidence of contagion between the US and the selected global bond markets through the correlation and coskewness parameters, and the third considers a joint test of all of the contagion and structural break of crisis parameters.

4.1. During the Global Financial Crisis

The first panel of Table 4 shows that there is evidence for a structural break in all of the moments of the mean, variance and skewness for all countries considered in the global financial crisis period. There is decisive evidence of a structural break for the US in the skewness parameter with the value of natural logarithm of the Bayes factor of -3.0022. The probabilities of a structural break of crisis in the mean are 97.23%, 98.28%, 85.62%, 87.07% and 95.46% for Euro, Canada, Korea, Japan and USA, respectively. Further, there is no evidence of a structural break of crisis in skewness for all global bond markets considered individually. Although all countries are not affected by a structural break in the second order moment, they are affected by structural breaks in the first or third order moments.

Considering all m markets jointly, there is evidence for a structural break in the mean in the global financial crisis regime compared to the period before the crisis with a probability of 86.41%. The higher order moment breaks are also evident jointly. The probability of the joint structural break in the variance is 100%; and there is decisive evidence of a structural break in skewness with the value of natural log of the Bayes factor of -13.46. The joint test of the mean, variance and skewness structural breaks for each country and for the countries considered jointly show decisive evidence of structural breaks. Overall, the results for the moment break tests show that it is the structural break in the variance which is most important for all markets, followed equally by the mean and skewness break just for the US. However, when considered jointly all moment break tests are significant.

Table 4: Empirical Results of the Structural Break Tests of Crisis and Contagion for the Global Financial Crisis

Tests	Methods	Euro	CAN	KOR	JPN	USA	∀i
Test 1: Structural break tests of Crisis (i)							
Mean	p	0.972	0.982	0.856	0.870	0.954	0.864
Variance	p	1.000	0.600	1.000	0.524	1.000	1.000
Skewness	BF	-1.105	-0.757	-0.104	-0.237	-3.002	-13.46
Mean, var., & skew	BF	-314.0	-111.02	-4.716	-4.797	-349.6	-691.8
Test 2: Contagion tests (i ≠ j)							
Correlation	p	0.279	0.940	0.389	0.680	-	0.000
Coskewness	BF	-1.248	-0.506	-4.034	-3.079	-	-499.9
Corr.& Coskew.	BF	-1.951	1.237	-4.334	-2.912	-	-498.7
Test 3: Joint structural break tests of crisis (i) and contagion (i ≠ j)							
All	BF	-325.6	-305.8	-13.92	-8.527	-	-1190.6

Notes: The sample period is September 7, 2004 through December 30, 2014. Contagion is measured with respect to the US. The method of hypothesis evaluation for each test is indicated in the table. *p* denotes that a decision is probability based for inequality hypothesis constraint. *BF* for equality hypothesis constraint denotes that a decision is based on the Bayes Factor using the model selection following Jeffrey's rule (Jeffrey, 1961)); Evidence in favor of model M_r if the value of natural logarithm of the Bayes factor, i.e., *ln(BFru)* = (0,∞), Very slightly evidence in favor of model M_u if *ln(BFru)* = (-1.15,0), Slightly evidence in favor of model M_u if *ln(BFru)* = (-2.30,-1.15), Strong evidence in favor of model M_u if *ln(BFru)* = (-4.60,-2.30), Decisive evidence in favor of model M_u if *ln(BFru)* = (-∞,-4.60).

The second panel of Table 4 provides that the probability of contagion as reflected by an increase in the traditional correlation coefficient between all combinations of the US and global markets is different for Euro (27.93%), Canada (94.03%), Korea (38.98%) and Japan (68.08%) in the period after crisis compared to the period before the global financial crisis. The correlation channel of contagion dominates the coskewness channel as the coskewness change is not significant for the US-Euro and the US-Canada country pairs. However, there are decisive support for coskewness contagion occurring between the US and selected global markets of Korea and Japan, with the value of *ln(BFru)* of -4.0347 and -3.0791, respectively, implying that the preference of risk-averse investors in the crisis regime has shifted from selected global countries of Korea and Japan to the U.S. bond market. The joint tests for contagion between the US and the selected global countries through each of the correlation and the coskewness comoments are contained in the last column of the second panel in Table 4. The probability of contagion occurring jointly through the correlation channel is very low, while there is decisive evidence of contagion through coskewness with a value of *ln(BFru)* of -499.9792.

The third panel of Table 4 provides evidence on the significance of the operation of structural breaks of crisis and contagion simultaneously for each market *i* as well as for all of the markets jointly. The bottom row, and particularly the last column of the bottom row can be thought of as a test of all channels of comoment and moment change and can be thought of as an overall test for a crisis and contagion. For the individual markets the evidence of joint contagion and structural breaks is decisive in all cases with the value of *ln(BFru)* ranging between -8.5274 for Japan, to -325.6476 for Euro. For the combined markets, *ln(BFru)* is -1190.6, indicating the importance of examining structural breaks of crisis and contagion jointly.

4.2. Comparison to the COVID-19 Pandemic Crisis

In the aftermath of the 2008 global financial crisis, the Federal Reserve implements years of quantitative easing to stimulate economic recovery, allowing interest rates to a near-zero, where they remain for the next six years. In order to drag the U.S. economy out of recession, the idea was to spur investments along with consumer spending and in the years that followed, the economy actually does begin to recover. Following the COVID-19 pandemic and the significant increase in inflation, the Federal Reserve is raising interest rates once again in 2022. Historically, an increase in U.S. interest rates has increased the value of the U.S. dollar increases, which in turn has had an impact on various economic aspects of the domestic and global economy, particularly credit markets, goods markets, stock markets, and investment opportunities. Table 5 presents the empirical results of structural breaks of crisis and the contagion. Since the first U.S. interest rate increase in February 2022, the regime change is evident in Figure 1 and is consistently in the crisis regime by the time of the speculative attack. Like the case for the global financial crisis period, transitioning between the two regimes is by no means smooth, reflecting the uncertainty in financial markets even before the speculative attack.

Table 5: Empirical Results of the Structural Break Tests of Crisis and Contagion for the COVID-19 Pandemic

Tests	Methods	Euro	CAN	KOR	JPN	USA	∀i
Test 1: Structural break tests of Crisis (i)							
Mean	p	0.528	0.980	0.964	0.162	0.986	0.378
Variance	p	1.000	1.000	1.000	1.000	1.000	1.000
Skewness	BF	0.212	-15.41	-1.453	-2.732	-0.172	-240.52
Mean, var., & skew	BF	-245.1	-712.0	-455.7	-578.4	-474.9	-2004
Test 2: Contagion tests (i ≠ j)							
Correlation	p	1.000	0.339	0.394	0.534	-	0.000
Coskewness	BF	-62.85	0.274	0.308	0.843	-	-1310
Corr.& Coskew.	BF	-78.00	2.371	1.461	1.657	-	-1331
Test 3: Joint structural break tests of crisis (i) and contagion (i ≠ j)							
All	BF	-1228	-1417	-822.9	-182.0	-	-3336

Notes: Notes: As for the table 4.

Table 5 shows that the results for the COVID-19 pandemic are different from those for the US and selected global bond markets during the global financial crisis. The channels of structural breaks of crisis and contagion that are significant during the COVID-19 pandemic crisis are similar to those for the global financial crisis. Canada has a similar role of being the destination of a flight to safety of risk averse investors to that of the US in the first application as shown by the significance of each moment of the mean, variance and skewness. Changes in these parameters are significant with 98% and 100% probability respectively for the mean and the variance, while there is decisive evidence of a structural break in skewness with a value of *ln(BFru)* of -15.4148. As it is for the application to the global financial crisis the structural break in the variance is significant across the board, with less evidence for a structural break in the mean with the exception of Euro in addition to Japan.

Unlike the global financial crisis, the contagion effect based on correlation is significant only for the euro area and does not dominate coskewness based contagion which is only significant for Euro. Hence, the abrupt changes to policies are made by Euro area, affecting international investor by instituting capital controls as a way to contain the crisis. The change in policies affecting investors is perhaps a reason for the significance of coskewness for Euro area with respect to the US. For the individual markets the evidence of joint structural breaks of crisis and contagion is decisive in all cases with the value of *ln(BFru)* ranging between -182.0 for Japan, to -1417.1 for Canada, which is very similar to the global financial crisis case. For the combined markets, *ln(BFru)* is -3336.1, indicating the importance of examining structural breaks of crisis and contagion jointly.

5. Concluding Remarks

In a period of crisis, policymakers and investors are particularly challenged by the need to understand how interest rate movements might change compared to normal times. The information on portfolio allocation and mitigation of shocks occurring domestically, internationally, or simultaneously are the key to reach appropriate decisions for policy makers and investors. The regime switching skewed normal model of crisis and contagion is considered by building upon the pioneering paper of Hamilton (1989) by relaxing the assumption of multivariate normality with a multivariate skewed normal distribution.

The structural breaks test of crisis in the moments of the mean, variance and skewness are specified. On the while contagion is working under the condition that there are changes in the comoments of correlation and coskewness in the non-crisis regime compared to a crisis regime. Many different previous studies argue that the characteristics of financial data observed in both crisis periods and normal times are better reflected by including higher order moments and comoments. In this regard, the model in this paper is extend to including coskewness based contagion and structural breaks of crisis in skewness, which are important as risk averse investors prefer positive coskewness and positive skewness, showing that these moments are very relevant to the crisis regime. Compared to other frameworks of contagion which are often conducted on a bivariate basis, the paper provides evidence of joint contagion or structural breaks across the global bond markets.

The parameters of the model considered in this study are estimated by the Bayesian model estimation technique, and various hypotheses are evaluated. In addition, this model is applied to the regimes of the period before the global financial crisis and the period after the crisis, and is compared to the COVID-19 pandemic crisis for the US and selected global bond markets, respectively.

The main findings from the empirical results for the model are in order. First, whether during the global financial crisis or the COVID-19 pandemic, the transition from a non-crisis regime to a crisis regime has been different across countries. The bond market shows evidence of a transition into a crisis before the major triggers, the collapse of Bear Stearns in March 2008 and a sharp rise in interest rates from February 2022, respectively. Second, inspection of the moment statistics weakly suggests a flight to safety to the major markets of the US during the global financial crisis and to Canada during the COVID-19 pandemic. While all markets except Canada and Japan for the global financial crisis are not affected by a significant change in volatility in the crisis periods, only the US for the global financial crisis and Canada for the global financial crisis show evidence of breaks in the mean and skewness moments. The results indicate that risk averse investors had more risk appetite for the US and Canada assets during the crisis regimes, compared to their counterparts. The flight to safety is true even when the US is a source of crisis in the first place. Third, contagion measured through the traditional correlation coefficient is not significant in all cases except Canada for the global financial crisis and Euro area for the COVID-19 pandemic, invalidating the use of the correlation coefficient as a first measure of contagion. Fourth, coskewness contagion is significant for the Korea and Japan-US pairs for the global financial crisis and the Euro area-US pair for the COVID-19 pandemic. The policy response of Korea, Japan and the euro area changed the overseas investment environment of each country, with Korea through capital market restructuring, Japan through ultra-low interest rate policies, and the euro area through the aftermath of various types of sovereign defaults from 2010 to 2011. Finally, all channels of structural breaks of crisis and contagion are significant when considered jointly, reinforcing the need to consider structural breaks of crisis and contagion during crises in a multivariate setting.

There are several issues beyond our study. First, the methodology used in this study is suitable for financial markets and may have problems in extending to other fields such as labor markets, commodity markets and etc. Second, one of the most important time series characteristics of prices in financial markets is that the conditional variance is time-varying, but this study assumes that all higher moments and comoments are not time-varying. Regarding future research areas using the methodology of this study, it is possible to apply crises and contagion effects to stock markets, foreign exchange markets, and etc. in addition to bond markets. Additionally, it is possible to analyze the occurrence and spillover effects of crises between markets, not only between countries but also within the country.

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