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RESEARCH REVIEW

Understanding the developmental process of a mathematics teacher's competencies in mathematical modeling: A study conducted by Jung (2023)

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Abstract

Mathematics educators have examined mathematical modeling, where students tackle authentic real-life problems and develop problem-solving strategies with a sense of agency. However, few empirical studies have been conducted to illuminate the developmental process of teachers' competencies in mathematical modeling, particularly for elementary school teachers. Scholars have noted that elementary mathematics teachers can effectively teach mathematical modeling by designing tasks that consider students' abilities and preferences. In this vein, this review paper introduces a study conducted by Jung (2023), which examines the developmental process of an elementary school mathematics teacher's competencies in mathematical modeling and how she overcame related challenges.

Keywords: mathematical modeling, teacher education, professional development, task development

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I. INTRODUCTION

Mathematics is considered a foundational subject for studying other disciplines, such as science, technology, and engineering, and it serves as a crucial tool for solving real-world problems in our complex modern society (Kavaz & Kocak, 2024). Despite its applicability in addressing various industrial and societal issues, several students fail to recognize the importance and necessity of learning mathematics (Maaß et al., 2018). This lack of appreciation leads to low motivation and negative attitudes toward mathematics, particularly among students in East Asia (Mullis et al., 2020). Moreover, educators have raised concerns about traditional mathematics teaching methods, which often rely on repetitive problem-solving drills and lack opportunities for student autonomy and agency (National Council of Teachers of Mathematics [NCTM], 2014, 2020; Suh et al., 2021). In response, NCTM (2020) emphasized the need for a focus on "deep mathematical understanding, reasoning, and sense-making, to ensure the highest-quality mathematics education" (p. 1). These concerns have driven mathematics educators worldwide to examine mathematical modeling, where students tackle authentic real-life problems and develop problem-solving strategies with a sense of agency (Flavin & Hwang, 2024). This approach has been adopted in various regions, including the United States (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), Europe (Bardy & Fehlmann, 2023; Schleicher, 2019), South Korea (Han & Hwang, 2023; Ministry of Education, 2015), and China (Yang et al., 2022).

Scholars have proposed various mathematical modeling perspectives based on their goals and definitions (Abassian et al., 2020; Blum, 2015; Jung & Brady, 2023). For example, based on a review of related literature, Abassian et al. (2020) suggested five perspectives on mathematical modeling including realistic, educational, models and modeling, socio-critical, and epistemological modeling: (a) the goal of realistic modeling is to acquire skills to model and understand the real world (e.g., Pollak, Ferri), (b) educational modeling aims to develop an understanding of the real world and the skills to model it (e.g., Niss, Blum), (c) the models and modeling perspective uses modeling contexts to develop an understanding of mathematics (e.g., Lesh, Doerr), (d) socio-critical modeling focuses on acquiring modeling skills to take action in society (e.g., D'Ambrosio, Skovsmose), and (e) epistemological modeling aims to develop formal mathematical reasoning (e.g., Freudenthal, Gravemeije). As mathematical modeling encompasses a broad spectrum of perspectives, it is distinct from problem-solving and proving processes, which focus solely on learning mathematical contents.

Researchers commonly agree that mathematical modeling consists of mathematics, the real world (extra-mathematical world), and the links between them (a mapping from the real world to mathematics; Niss et al., 2007). Kohen and Orenstein (2021) described mathematical modeling as "the process of building a mathematical model for solving real-world problems" (p. 72). The mathematical modeling process involves simplifying the real-world problem, mathematizing (transforming the real model into a mathematical model), working with mathematics (solving the mathematical problem), interpreting outcomes, and validating the interpretation (Maaß, 2006). Thus, mathematical modeling experiences help students not only acquire mathematical knowledge but also develop various mathematical skills (e.g., metacognitive, communication, critical thinking, and reasoning skills) and foster a positive attitude toward mathematics (Lesh et al., 2013; Wei et al., 2022). These experiences also help students understand the value and relevance of learning mathematics (e.g., why we need to study mathematics), as they are encouraged to construct mathematical models based on their life experiences and knowledge (Hernandez-Martinez & Vos, 2018; Suh et al., 2021).

Teachers' capacity for mathematical modeling is crucial for the development of students' modeling competencies (Barquero et al., 2018). Teachers design and implement modeling tasks, interpret and evaluate students' modeling processes, and provide feedback. Their knowledge and skills in mathematical modeling form the foundation for the mathematical modeling experiences of their students. Thus, scholars have emphasized the importance of teachers' mathematical modeling competencies, including knowledge of mathematical content, mathematical modeling, pedagogical content, and curriculum (Barquero et al., 2018; Lesh et al., 2013; Maaß, 2006; Yang et al., 2022).

However, few empirical studies have been conducted to illuminate the developmental process of teachers' knowledge in mathematical modeling. For instance, Çetin et al. (2023) reviewed articles on mathematical modeling published in Social Science Citation Index journals and reported that only five out of 42 articles (12%) focused on mathematics teachers. Similarly, after analyzing 54 studies published in South Korean educational journals, Hwang and Han (2023) found that research has examined various mathematical modeling topics (theory, task, lesson, and teacher education), with less attention given to teacher education, particularly for elementary school teachers (n = 2). In this context, Zbiek et al. (2024) highlighted the need for more studies to understand "how teachers come to know and facilitate MM [mathematical modeling]" (p. 57).

Moreover, previous studies have focused on secondary mathematics education rather than elementary mathematics education (Chang et al., 2019). Some educators believe that mathematical modeling requires high-order thinking skills, which they assume elementary students still need to be ready to learn. However, scholars have noted that elementary mathematics teachers can effectively teach mathematical modeling by designing tasks that consider students' abilities and preferences (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Thus, teachers need to develop competencies in mathematical modeling to teach elementary students, helping them develop positive attitudes toward mathematics and acquire higher-order thinking skills.

Many elementary school teachers lack sufficient knowledge to teach mathematical modeling effectively. Due to their broad responsibilities in teaching various subjects, they often do not receive adequate training in mathematical modeling during their teacher preparation programs. Therefore, there is a critical need for professional development programs tailored to their needs. Most studies on mathematical modeling among Korean elementary school teachers have focused on their perceptions, instructional practices, or competency rather than their experiences with professional development (Hwang & Han, 2023), with the exception of Jung's research in 2023. As a result, there is limited information available on how to support teachers and effectively implement professional development initiatives. This review paper introduces a study conducted by Jung (2023), which examines the developmental process of an elementary school mathematics teacher's competencies in mathematical modeling during the professional development program.

It is important to note that this paper did not intend to claim that traditional teaching methods are problematic or that mathematical modeling is the only way to teach mathematics. Rather, mathematical modeling is part of the broad range of options teachers can use to teach mathematics. Mathematical modeling encompasses teaching and learning practices as well as curriculum development practices (Niss et al., 2007). Therefore, it can be used to achieve various goals (Abassian et al., 2020; Jung & Brady, 2023), such as realistic goals, educational goals, socio-critical goals, and epistemological goals. Given these potentials, this review study on the professional development program for mathematics teachers aims to provide teacher educators with an understanding of the developmental process of mathematics teachers' competencies in mathematical modeling. This review paper also offers valuable insights for mathematics teacher educators on how to support preservice teachers during their preparation periods.

As a theoretical framework, this review paper employed transformative learning theory (Mezirow, 2000). According to transformative learning theory, individuals adopt new perspectives or schemas and revise existing ones when they confront dilemmas that cannot be resolved using their current perspectives (Zbiek et al., 2024). Therefore, we assumed that challenges encountered by teachers prompt them to critically examine their current understanding of mathematical modeling and compel them to embrace new knowledge or revise their perspectives to address these challenges. In this review, we examined Jung's (2023) study, which focuses on identifying the challenges faced by a participating teacher and exploring how she overcame these challenges through the acquisition of new knowledge.

II. THE SUMMARY OF ARTICLE

Among the various competencies teachers need to develop, Jung (2023) focused on task design, as most Korean elementary mathematics textbooks contain simple computation or word problems, rather than mathematical modeling tasks (Jung et al., 2020). To address this, Jung (2023) implemented a detailed professional development program for one elementary school mathematics teacher. Specifically, Jung examined the challenges the teacher faced in transforming traditional mathematical tasks into mathematical modeling tasks and how the teacher's knowledge of designing mathematical modeling tasks developed.

To address the research questions, Jung (2023) recruited the participation of a female mathematics teacher with over 10 years of teaching experience, Mrs. Kim, who harbors a passion for implementing modeling tasks, lacked prior experience in designing and executing mathematical modeling tasks. Dr. Jung tasked Mrs. Kim with transforming a conventional mathematical task from textbooks into a mathematical modeling task. Mrs.

Kim focused on ‘‘finding averages’’ as a modeling topic, believing it to be relevant to real-life contexts and conducive to her students' active engagement in the modeling process. After modifying the traditional task into a modeling task, Mrs. Kim convened meetings with three mathematics educators specializing in mathematical modeling, constituting a professional teacher research community. They evaluated the modified modeling task and offered suggestions for improvement across four meetings. Mrs. Kim subsequently revised the modeling task based on the feedback from these sessions.

Jung (2023) gathered four types of data: (1) pre- and post-interview data and four discussions of a professional teacher research community, (2) the mathematical modeling tasks modified by Mrs. Kim, (3) the notes taken by Mrs. Kim and three educators during the meetings, and (4) the author's (Dr. Jung) observation notes written during the meetings.

Dr. Jung analyzed the collected data in four steps. First, she decomposed all utterance data (interview and discussion data) into sentences and categorized each sentence into one of the main ideas: mathematical content knowledge, mathematical modeling cycle, reality of the task, complexity of the task, openness of the task, cognitive level of the task, students’ cognitive level, and so forth. Second, the time spent on discussing individual ideas was measured to determine which ideas the professional teacher research community focused on. Third, the proportional changes of the individual ideas across the four meetings were calculated. Fourth, the author’s interpretations of the development of Mrs. Kim’s mathematical modeling competency were validated based on other data sources, including the modified modeling tasks, the notes taken by the professional teacher research community, and Dr. Jung’s observation notes.

As presented in Table 1, the primary focus of each meeting shifted from the reality of the task (first meeting) to the complexity of the task (second meeting), and finally to the mathematical modeling cycle (third and fourth meetings). Before the first meeting, Mrs. Kim transformed the task in the textbook (“Find the average number of clips in clip boxes”) into a mathematical modeling task (“Predict the level of fine dust in the next week”). She believed that masks, playgrounds, and fine dust were directly relevant to students' daily lives. During the first meeting, the three educators concurred that the fine dust task was more familiar to students than the original clip task. However, they assessed that the topic exceeded the cognitive level of elementary school students. Consequently, Mrs. Kim modified the first task into a sandwich task (“Distributing ingredients to make a sandwich”) that maintained alignment with the cognitive level of the original clip task while still relating to students' daily lives.

At the second meeting, the three educators agreed that the reality and cognitive level of the sandwich task were appropriate. However, they expressed the need for teachers to provide more guidance to support students’ mathematical modeling activity. Mrs. Kim also recognized the necessity for detailed guidance tailored to the mathematical modeling cycles. However, lacking sufficient knowledge in this area, the majority of discussions at the third and fourth meetings revolved around the mathematical modeling process. Specifically, Mrs. Kim, through discussions with the three experts, refined the type and quantity of information that students needed to collect at various stages of mathematical modeling (see Table 2).

Table 1. The development of mathematics modeling tasks

	First meeting	Second meeting	Third and fourth meetings
Developed task	Predict the level of fine dust in the next week	Distributing ingredients to make a sandwich	Distributing ingredients to make a sandwich with additional guidance
Evaluation on the task	It relates to student daily lives. However, it does not align with the cognitive levels of students.	It aligns with the cognitive levels of students. However, guidance needs to be provided.	Detailed guidance needs to be provided according to the mathematical modeling process.
Major focus of the discussion	Reality of the task	Complexity of the task	Mathematical modeling process
Minor focus of the discussion	Complexity of the task	Mathematical modeling process	

Note. The original task in the mathematics textbook was "Find the average number of clips in clip boxes."

Table 2. Comparison between mathematical modeling tasks 3 and 5

	Task 3	Task 5
Developed task	Distributing ingredients to make a sandwich	
Sample guidance for modeling process	1) Research the ingredients needed to make a sandwich 2) Gather information about the ingredients needed to make the sandwich 3) Divide the ingredients according to how to divide them fairly	1) Research the ingredients needed to make a sandwich. 2) Select three kinds of information about the ingredients needed to make a sandwich. 3) Collect data based on the selected three kinds of information and organize them using mathematical representations. 4) Select two kinds of information about the ingredients that should be considered to divide the sandwich ingredients fairly and justify your selection. 5) Check the data for the selected two information about the ingredients to make a sandwich. 6) Divide the ingredients fairly for groups and explain how you calculate the fair values

The final modeling task (the fifth version task) encompasses topics related to the concept of average and its application in finding averages. This task aligns with recommendations from professionals and literature, emphasizing that modeling tasks should not only help students grasp real-world contexts but also reinforce the mathematical concepts they are expected to learn. By prompting students to consider fairness (e.g., "Select two kinds of information about the ingredients that should be considered to divide the sandwich ingredients fairly and justify the selection"), the task naturally leads them to understand the concept of average. Furthermore, this open-ended task encourages students to explore various methods for calculating averages (e.g., "Divide the ingredients fairly for groups and explain how you calculate the fair values"). This approach contrasts with traditional textbook exercises that typically ask students to find averages solely to determine the central value within a dataset (e.g., "Find the average number of clips in clip boxes").

In conclusion, Jung (2023) reported that teachers encounter various challenges when transforming traditional mathematical tasks into mathematical modeling tasks. These difficulties transition from concerns regarding the task's relevance to reality, to its complexity, and ultimately to understanding and implementing the mathematical modeling process. However, through collaborative discussions with experts, these challenges were surmounted, leading to the development of appropriate modeling tasks and an acquisition of knowledge on mathematical modeling. These findings reveal the interconnected nature of teachers' knowledge in developing mathematical modeling tasks, which encompasses understanding mathematical content (e.g., finding averages), pedagogy (e.g., comprehending students' cognitive levels), mathematical modeling (e.g., grasping the modeling process), and real-world applicability (e.g., understanding the context). Furthermore, this knowledge base is cultivated and expanded through collaboration with experts in mathematical modeling.

III. CONCLUSIONS AND IMPLICATIONS

Given the scarcity of research on teacher education in mathematical modeling, Jung's (2023) study examined the challenges encountered and growth experienced by an elementary school mathematics teacher within a professional teacher research community. It's notable that many elementary school teachers lack adequate knowledge of mathematical modeling, as this concept is relatively novel in Korean mathematics education. The 2015 revision of the Korean mathematics curriculum marked the introduction of mathematical modeling as a sub-domain within problem-solving competency (Ministry of Education, 2015). Consequently, some teachers may have not been exposed to mathematical modeling during their teacher education programs.

This study provides valuable insights for teacher education. While Jung's (2023) study focused on a single teacher, the professional development process it employed could be adapted for larger-scale implementation. Moreover, teacher educators could apply a similar approach to smaller professional development programs, such as mentor-mentee initiatives. For instance, educators can employ a sequential task development mentoring process—comprising designing, evaluating, and modifying tasks—to enhance mathematics teachers' proficiency in creating modeling tasks. These interventions help teachers gain essential knowledge about mathematical modeling, including the task's reality and complexity, and the mathematical modeling cycle. Such initiatives could not only elevate the quality of teachers' instructional practices but also foster positive attitudes toward mathematics among students (Maaß et al., 2018). Additionally, addressing cognitively challenging problems with authenticity helps students acquire high-level cognitive thinking skills and improve mathematics achievement (Schleicher, 2019; Suh et al., 2021; Yang et al., 2022; Wei et al., 2022). Therefore, I recommend this article to teacher educators and teachers alike who are considering the implementation of professional teacher development programs.

However, while this study offers important insights, it is essential to acknowledge its limitations. Certainly, Jung's (2023) study primarily focused on the impact of participation in a professional teacher research community on the development of Mrs. Kim's mathematical modeling competency. While a single study cannot explain all the complexities of educational phenomena, Jung's research could offer more comprehensive insights into the development of teachers' modeling competencies by exploring various educational activities beyond participation in such communities. For instance, in a review conducted by Kwakman (2003), five types of teacher learning activities were identified, including reading (e.g., reading teacher guidance), experimenting (e.g., trying out new teaching methods with students), reflecting (e.g., evaluating instructional practices), collaborating (working with other experts), and other activities not directly related to the curriculum (e.g., implementing extracurricular activities). Moreover, Thoonen et al. (2011) argued that factors such as school conditions, school leadership, teacher motivation, and professional learning experiences contribute to teacher change. Given these insights, Jung's (2023) study could have provided a more comprehensive understanding by exploring other elements that might have influenced Mrs. Kim's competency in mathematical modeling. Thus, more research is needed to examine the impacts of various elements on the development of teachers' mathematical modeling competency.

In conclusion, this study found that cooperative professional development enhances the competency of an elementary school mathematics teacher in developing modeling tasks, which constitutes the initial step in teaching mathematical modeling to students. Jung's (2023) findings demonstrate that elementary school teachers can effectively design modeling tasks by integrating various components of mathematical modeling as identified in the literature. Furthermore, the challenges faced by participants and their methods of overcoming them provide valuable insights for mathematics researchers seeking to design empirical studies on mathematical modeling. These findings underscore the ongoing challenge of acquiring modeling competencies through professional development.

Building on Jung's (2023) emphasis on competency in designing modeling tasks, researchers should examine strategies to support teachers in implementing these tasks within mathematics classrooms. This includes examining the roles of teachers and students, as well as assessing student proficiency in mathematical modeling. Additionally, future studies could explore how the distribution of high-quality modeling tasks as educational resources impacts the development of teachers' modeling competencies.

REFERENCES

Abassian, A., Safi, F., Bush, S., & Bostic, J. (2020). Five different perspectives on mathematical modeling in mathematics education. *Investigations in Mathematics Learning, 12*(1), 53-65. https://doi.org/10.1080/19477503.2019.1595360

Bardy, T., & Fehlmann, R. (2023). Mathematical modeling of the tennis serve: Adaptive tasks from middle and high school to college. *Research in Mathematical Education, 26*(3), 167-202. https://doi.org/10.7468/jksmed.2023.26.3.167

Barquero, B., Bosch, M., & Romo, A. (2018). Mathematical modelling in teacher education: Dealing with institutional constraints. *ZDM-Mathematics Education, 50*(1), 31-43. https://doi.org/10.1007/s11858-017-0907-z

Blum, W. (2015). Quality teaching of mathematical modelling: What do we know, what can we do? In *The proceedings of the 12th international congress on mathematical education: Intellectual and attitudinal challenges* (pp. 73-96). Springer International Publishing. https://doi.org/10.1007/978-3-319-12688-3_9

Çetin, İ., Aydın, M., & Bilgiç, Ş. (2023). Review of mathematical modeling research: A descriptive content analysis study. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 17*(2), 994-1025. https://doi.org/ 10.17522/balikesirnef.1321365

Chang, H. W., Choi, H. Y., Kang, Y. J., & Kim, E. H. (2019). Development and application of mathematical modeling task for the lower grade elementary school students. *Journal of Elementary Mathematics Education in Korea, 23*(1), 93-117.

Flavin, E., & Hwang, S. (2024). U.S. and Korean teacher candidates' approaches to mathematical modeling on a social justice issue. *Research in Mathematical Education,* 27(1), 25-47. https://doi.org/10.7468/jksmed.2024.27.1.25

Han, S., & Hwang, J. (2023). Exploring mathematical modeling in classrooms: Insights from diverse studies. *Research in Mathematical Education, 26*(3), 121-125. https://doi.org/10.7468/jksmed.2023.26.3.121

Hernandez-Martinez, P., & Vos, P. (2018). “Why do I have to learn this?” A case study on students’ experiences of the relevance of mathematical modelling activities. *ZDM - Mathematics Education, 50*(1), 245-257. https://doi.org/10.1007/s11858-017-0904-2

Hwang, S. Y., & Han, S. Y. (2023). A study on mathematical modeling trends in Korea. *Journal of Educational Research in Mathematics, 33*(3), 639-666. https://doi.org/10.29275/jerm.2023.33.3.639

Jung, H., & Brady, C. (2023). Modeling actions foregrounded in whole-class modeling discourse: A case study of a model-eliciting activity and a three-act task. *Mathematical Thinking and Learning,* 1-24. https://doi.org/10.1080/10986065.2023.2180849

Jung, H. Y. (2023). Analyzing an elementary school teacher’s difficulties and mathematical modeling knowledge improvement in the process of modifying a mathematics textbook task to a mathematical modeling task: Focused on an experienced teacher. *The Mathematics Education, 62*(3), 363-380. https://doi.org/10.7468/mathedu.2023.

62.3.363
Jung, H. Y., Lee, K. H., & Jung, J. H. (2020). Analyzing real world tasks of 6th grade textbook from a mathematical modeling perspective: Focused on the curriculum for revised 2011 and 2015. *The Journal of Learner-Centered Curriculum and Instruction, 20*(18), 1313-1340. http://doi.org/10.22251/jlcci.2020.20.18.1313

Kavaz, S., & Kocak, O. (2024). The effect of the online flipped learning model on secondary school students' academic achievement, attitudes towards their mathematics course, and cognitive load. *International Journal of Science and Mathematics Education*, 1-29. https://doi.org/10.1007/s10763-024-10455-5

Kohen, Z., & Orenstein, D. (2021). Mathematical modeling of tech-related real-world problems for secondary school-level mathematics. *Educational Studies in Mathematics, 107*(1), 71-91. https://doi.org/10.1007/s10649-020-10020-1

Kwakman, K. (2003). Professional learning throughout the career. *International Journal of Human Resources Development and Management, 3*(2), 180-190. https://doi.org/10.1504/ijhrdm.2003.002419

Lesh, R., English, L., Sevis, S., & Riggs, C. (2013). Modeling as a means for making powerful ideas accessible to children at an early age. In S. J. Hegedus & J. Roschelle (Eds.), *The SimCalc vision and contributions* (pp. 419–436). Springer. https://doi.org/10.1007/978-94-007-5696-0_23

Maaß, J., O'Meara, N., Johnson, P., & O'Donoghue, J. (2018). *Mathematical modelling for teachers*. Springer. https://doi. org/10.1007/978-3-030-00431-6.

Maaß, K. (2006). What are modelling competencies? *ZDM - Mathematics Education, 38*(2), 113–142. https://doi.org/10.1007/BF02655885

Mezirow, J. (2000). Learning to think like an adult: Core concepts of transformation theory. In J. Mezirow and Associates (Eds.), *Learning as transformation: Critical perspectives on a theory in progress* (pp. 3–34). Jossey-Bass.

Ministry of Education. (2015). *2015 revised mathematics curriculum*. Author.

Mullis, Ina V.S., Martin, Michael O., Foy, Pierre, Kelly, Dana L., Fishbein, B. (2020). *TIMSS 2019 international results in mathematics and science*. TIMSS & PIRLS International Study Center, Boston College. Retrieved from https://dlib.bc.edu/islandora/object/bc-ir:109272/datastream/PDF/view

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Retrieved from https://www.thecorestandards.org/Math/

NCTM. (2014). *Principles to actions: Ensuring mathematical success for all*. Author.

NCTM. (2020). *Catalyzing change in early childhood and elementary mathematics*. Author.

Niss, M., Blum, W., & Galbraith, P. L. (2007). Introduction. In W. Blum, P. L. Galbraith, H. W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 3–32). Springer. https://doi:10.1007/978-0-387-29822-1_1

Schleicher, A. (2019). *PISA 2018: Insights and interpretations*. OECD Publishing

Suh, J., Matson, K., Seshaiyer, P., Jamieson, S., & Tate, H. (2021). Mathematical modeling as a catalyst for equitable mathematics instruction: Preparing teachers and young learners with 21st century skills. *Mathematics, 9*(2), 162. https://doi.org/10.3390/math9020162

Thoonen, E. E., Sleegers, P. J., Oort, F. J., Peetsma, T. T., & Geijsel, F. P. (2011). How to improve teaching practices: The role of teacher motivation, organizational factors, and leadership practices. *Educational Administration Quarterly, 47*(3), 496-536. https://doi.org/10.1177/0013161x11400185

Wei, Y., Zhang, Q., & Guo, J. (2022). Can mathematical modelling be taught and learned in primary mathematics classrooms: A systematic review of empirical studies. *Education Sciences, 12*(12), 923. https://doi.org/10.3390/educsci12120923

Yang, X., Schwarz, B., & Leung, I. K. (2022). Pre-service mathematics teachers' professional modeling competencies: A comparative study between Germany, Mainland China, and Hong Kong. *Educational Studies in Mathematics, 109*(2), 409-429. https://doi.org/10.1007/s10649-021-10064-x

Zbiek, R. M., Peters, S. A., Galluzzo, B., & White, S. J. (2024). Secondary mathematics teachers learning to do and teach mathematical modeling: A trajectory. *Journal of Mathematics Teacher Education, 27*(1), 55-83. https://doi.org/10.1007/s10857-022-09550-7