

## **RESEARCH ARTICLE**

# Flourishing as an aim of mathematics education

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Received: May 3, 2024 / Revised: June 7, 2024 / Accepted: June 23, 2024

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### **Abstract**

What is the aim of mathematics education? Current aims of mathematics education often lack the multidimensionality needed to account for a successful experience in mathematics. In this short paper, we argue for a multidimensional aim of mathematics education via the construct of *flourishing*. Flourishing is derived from the notion of *eudaimonia*, which broadly refers to achieving the “highest good,” or living a well-lived life. Building on prior research, we operationalize flourishing as an aggregate of several positive affective, behavioral, cognitive, and social traits, all of which contribute to students’ propensities to achieve the “highest good” in mathematics. In particular, we propose five traits which contribute to students’ propensities to achieve the “highest good” (i.e., flourish) in mathematics: (1) positive emotions toward mathematics; (2) engagement in mathematics; (3) community in mathematics; (4) meaning in mathematics; (5) perceived competence in mathematics. Thus, we argue that one productive aim of mathematics education is to support students in fulfilling each of these traits, which ultimately leads to flourishing in mathematics. To supplement our theoretical stance, we offer suggestions for measuring flourishing as an aim. We close this short paper by describing the implications that such an aim might suggest for pedagogy, policy, and research.

**Keywords:** flourishing, mathematics education, aims of mathematics education

## I. INTRODUCTION

What is the aim of mathematics education? This is a vital question that deserves significant attention. Aims are important because they dictate a host of educational decisions, including pedagogical choices, policy initiatives, assessment protocols, and external funding priorities. Perhaps the most prominent aim of mathematics education is for students to attain mathematical knowledge. The relative influence of this aim is evidenced by global policy initiatives<sup>1</sup> and pedagogical debates<sup>2</sup> that prioritize students' mathematical performance. Yet, there are other potential aims of mathematics education (e.g., critical mathematics learning, increasing mathematical affect), all of which have the potential to influence research, policy, and practice. Thus, it is vital to clearly articulate the aim(s) of mathematics education as a starting point for offering strategies for improvement.

In this short paper, we argue for a multidimensional aim of mathematics education via the construct of *flourishing*. Flourishing is derived from the notion of *eudaimonia*, which broadly refers to achieving the "highest good," or living a well-lived life (Duignan, 2023). Recently, Francis Su (2020) applied flourishing to mathematics in his landmark book *Mathematics for Human Flourishing*, arguing that the proper practice of mathematics can support individuals to live flourishing lives. We build upon Su's work, along with research in positive psychology (Seligman, 2011), to operationalize flourishing in the context of mathematics education. We, then, argue that flourishing offers a multidimensional aim of mathematics education—an aim that encompasses the multifarious components of being successful in mathematics. Along the way, we offer suggestions for measuring flourishing in mathematics education. Finally, we discuss how flourishing, if accepted as an appropriate aim of mathematics education, might significantly influence mathematics education pedagogy, policy, and research.

## II. AIMS OF MATHEMATICS EDUCATION

There has been a recognizable evolution in the focus of mathematics education research since the 1900s. To some extent, the research emphasis within the mathematics education community can be taken as an implicit indication of the objectives established by the field. One aim that has remained constant over the last century is for students to attain mathematical knowledge, which is usually measured by standardized achievement tests. In the 1970s, a paradigm shift occurred within the mathematics education community, to include social, cultural, and political dimensions of mathematics as aims of mathematics education (Furinghetti et al., 2012). For example, D'Ambrosio (1979) proposed that the mathematics education community should aim to develop citizens with comprehensive mathematical skills that can be used to address global, societal, and cultural issues. This

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<sup>1</sup> See, for example, *No Child Left Behind* (U.S.) (Ohnemus, 2002); *The Importance of Teaching* (UK) Department for Education, 2010)

<sup>2</sup> See, for example, the debate on dialogic vs. direct instruction (Munter et al., 2015)

call was echoed by other scholars in the field (e.g., Skovsmose, 1994). As cognitive, societal, and cultural aims revolutionized mathematics education research, establishing objectives related to equity in mathematics education (e.g., Apple, 1992; Battista, 1994; Ladson-Billings, 1997) and students' affect towards learning mathematics (e.g., Macnab, 2000; McLeod, 1989) also emerged.

Reflecting this shift in paradigm, in the late 90s, Robitaille (1997) proposed the following aims for mathematics education: performance skills in mathematics; underlying understanding of mathematical skills and processes; understanding and using mathematics in everyday life; improvisation skills in the application of mathematics; organizational and logical skills; initiative and resourcefulness; self-esteem and confidence; working effectively with others; historical and cultural understanding. In a similar vein, Ernest (2002) suggested that the field should strive for *empowerment*, or the “gaining of power” (p. 1), as an aim for mathematics education. He further posited three categories of empowerment: mathematical, social and epistemological empowerment. Later, in the 21<sup>st</sup> century, Clarkson et al., (2010) argued for mathematical wellbeing as an aim of mathematics education, suggesting that emotional, affective, and cognitive dimensions of learning are vital to students' overall disposition as lifelong learners.

Contributing to the evolving aims of mathematics education over the last century, Su (2020) recently proposed the construct of *flourishing* as a holistic aim of mathematics education. In many ways, Su's work consolidated the prior articulated aims to create a multidimensional aim of mathematics education that considers students' overall state of being while doing mathematics. In the next section, we build on Su's work and other research in positive psychology to operationalize flourishing as a multidimensional aim of mathematics education.

### III. FLOURISHING AS AN AIM OF MATHEMATICS EDUCATION

Flourishing is conceptualized in a variety of ways in the field. De Ruyter et al. (2020) characterized flourishing as “both the optimal continuing development of human beings' potentials and living well as a human being, which means being engaged in relationships and activities that are meaningful, i.e. aligned with both their own values and humanistic values, in a way that is satisfying to them” (p. 2). Similarly, Su (2020) suggested, “*Human flourishing* refers to a wholeness—of being and doing, or realizing one's potential and helping others to do the same, of acting with honor and treating others with dignity, of living with integrity even in challenging circumstances” (p. 10, emphasis theirs). As illustrated by these quotes, flourishing involves a sense of “good” that is quite difficult to put into words. This is because flourishing transcends emotions (e.g., happiness)—it is a holistic state of being.

There has been much work in positive psychology to operationalize flourishing as a construct that can be measured. In particular, Seligman (2011) operationalized flourishing as consisting of five pillars: (1) positive emotions (i.e., experiencing love, joy, excitement, and other positive emotions); (2) engagement (i.e., being totally absorbed by meaningful

activities); (3) relationships (being socially connected to people); (4) meaning (i.e., feeling a sense of purpose); and (5) accomplishment (i.e., experiencing success). Seligman suggested that individuals flourish when they experience fulfillment in each of these categories. This research was groundbreaking because it offered a robust conceptualization of flourishing that allowed researchers to measure it (e.g., see Goodman et al., 2018; Kun et al., 2017).

Some scholars in mathematics education have built upon Seligman's work to apply it to mathematics education. For instance, Hill et al. (2021) and Hill and Hunter (2024) leveraged Seligman's research to explore how students experience well-being in mathematics. They explored how students experienced accomplishment, positive emotions, engagement, and other indicators related to flourishing (Hill et al., 2021). They found that flourishing can be meaningfully applied to mathematics education as evidenced by students' responses to open-ended survey questions. Other scholars have similarly sought to explore how mathematics can be taught in ways that promote enjoyment and love of mathematics (Kim, 2023; Yeo, 2024), which is related to the concept of flourishing.

We draw principally on Seligman's (2011) research in positive psychology and Hill et al.'s research in mathematics education (Hill et al., 2021; Hill & Hunter, 2024) to apply flourishing to doing mathematics. We view flourishing as an aggregate of several positive affective, behavioral, cognitive, and social traits, all of which contribute to students' propensities to achieve the "highest good" in mathematics. In particular, we propose five traits which contribute to students' propensities to achieve the "highest good" (i.e., flourish) in mathematics: (1) positive emotions toward mathematics; (2) engagement in mathematics; (3) community in mathematics; (4) meaning in mathematics; (5) perceived competence in mathematics. Thus, one productive aim of mathematics education is to support students in fulfilling each of these traits, which ultimately leads to flourishing in mathematics. Importantly, we note that these five traits are very similar to the traits that Hill et al. (2021) identified in their research. We affirm their research and seek to bring more attention to the concept of flourishing as an aim in mathematics education. Such an aim considers the multifarious dimensions of a successful experience in mathematics—the type of multidimensionality that is missing in prior articulated aims of mathematics education. We describe each of these traits in detail, followed by suggesting ways that researchers can measure flourishing in mathematics.

Positive emotions toward mathematics is an affective trait that refers to students' propensities to experience satisfying or "good" emotions while doing mathematics. There are several positive emotions that students may feel when doing mathematics, including enjoyment, pride, relief, enthusiasm, happiness, curiosity, and determination (Bieleke et al., 2023; Greensfeld & Deutsch, 2016; Liu et al., 2018; Villavicencio & Bernardo, 2016). These emotions are important because they promote overall wellbeing (e.g., Seligman, 2011), encourage creativity (e.g., Greensfeld & Deutsch, 2016), and may support mathematics achievement (e.g., Villavicencio & Bernardo, 2016). Thus, positive emotions are a critical component for supporting students to achieve the "highest good" in mathematics.

Engagement in mathematics is an affective, behavioral, and cognitive trait that is

conceptualized in a variety of ways in the field (e.g., Middleton et al., 2017). For our purposes, we conceptualize engagement as an intense state of captivation, referred to as “flow” (Czíkszentmihályi, 1990). Flow is comprised of three components: concentration, interest, and enjoyment (e.g., Shernoff et al., 2003). To illustrate “flow,” consider a mathematician writing a proof for an important theorem in their field of study. The mathematician tunes out all external stimuli (e.g., people talking in the hallway, the phone ringing, time of day) and places an intense focus on thinking about the proof. The mathematician is in flow. Engagement (i.e., flow) supports individuals to be creative and perform optimally (e.g., Czíkszentmihályi, 1990). Thus, “flow” is important to students’ propensities to flourish.

Community in mathematics is a social trait that refers to students’ tendencies to be socially connected within a mathematics setting (e.g., mathematics classroom). Seligman (2011) suggested that social relationships are vital to overall wellbeing. Students need to be socially connected to their teachers and classmates to feel welcome in mathematics settings. In addition to overall wellbeing, Su (2020) stated, “Community serves an important function in bringing people together in mathematical exploration—helping them to grow in the virtues promoted by socialization” (p. 190). Community increases creativity and expands exploration, which are important aspects of mathematics. Some research has shown that community can support students’ grades in math (e.g., Dohyoung, 2009). Community, therefore, is an integral to flourishing in mathematics.

Meaning in mathematics is an affective and cognitive trait that refers to students finding meaning in their mathematical work. Thompson (2013) wrote, “...if we intend that students develop mathematical understandings that will serve them as creative and spontaneous thinkers outside of school, then issues of meaning are paramount” (p. 61). Similarly, Su (2020) wrote, “...when you want to grasp the meaning of something, you are always asking about its relationship to other things” (p. 36). As illustrated by these quotes, “meaning” is measured by the extent to which mathematics is relevant to students outside of the specific context of the classroom. Unfortunately, mathematics is often taught as a list of rules and procedures with few opportunities to explore its relevance. Students need opportunities to understand the relevance of mathematics in order to flourish in mathematics.

Finally, perceived competence is an affective/cognitive trait. Perceived competence refers to one’s perceptions about their ability. One’s beliefs about their mathematical ability impacts their performance (Boaler, 2016; Dweck, 2006). This phenomenon has sparked much research into “growth mindset” interventions, wherein students are supported in improving their mindset toward mathematics (e.g., Samuel & Warner, 2021). Perceived competence aligns with the prominent aim of mathematics education: students need to achieve at high levels to be successful. Yet, importantly, perceived competence accounts for just one of the five flourishing traits.

In summary, we believe flourishing offers a constructive aim of mathematics education by considering the multifarious dimensions related to achieving the “highest good” in mathematics.

#### IV. MEASURING FLOURISHING

One of the most compelling features of flourishing as an aim is its ability to be measured. First, from a self-report lens, there are validated flourishing scales that can be appropriately modified to mathematics education to assess student flourishing in mathematics (e.g., Butler & Kern, 2016; Diener et al., 2010). In fact, two scales already exist that measure students' flourishing in mathematics (Campbell et al., under review; Hill et al., 2024). Hill et al. (2024) modified an existing wellbeing scale in psychology to create a scale that measures students' flourishing with 17 items. Campbell et al. (under review) created a brief 7-item scale to measure mathematical flourishing by modifying Diener et al.'s (2010) Flourishing Scale. These scales contain items that align closely with the five traits we identified in the prior section.

Outside of these existing self-report flourishing scales, mathematics education researchers have created survey instruments and observation protocols to measure each of the traits that are central to flourishing. Table 1 shows a non-exhaustive sample of instruments that can be used to measure each trait. Using these instruments, scholars can explore specific aspects of flourishing, or aggregate instruments to create a robust measure of student flourishing. Of course, scholars may need to carefully modify existing instruments to ensure alignment with the conceptual underpinnings of flourishing.

**Table 1.** Measurements for traits of flourishing

| Flourishing Trait                    | Measurement Tool  |
|--------------------------------------|---|
| Positive emotions toward mathematics | Achievement Emotions Questionnaire-Mathematics (AEQ-M) (Bieleke et al., 2023; Frenzel et al., 2007)   |
| Engagement in mathematics            | Experience Sampling Form (ESF) (Shernoff et al., 2003)  |
| Community in mathematics             | Attitudes to my maths partner instrument (Thurston et al., 2020); Mathematics Classroom Observation Protocol (MCOP2) [certain items] (Gleason et al., 2017) |
| Meaning in mathematics               | MCOP2 [certain items] (Gleason et al., 2017)  |
| Perceived competence in mathematics  | Self-Description Questionnaire (SDQ II) [math items] (Marsh, 1988)  |

Our goal in this section was to provide a (very) brief argument that mathematical flourishing *can* be measured. Creating robust instruments will require significant work, but we believe prior research has provided an appropriate baseline for this work.

#### V. INFLUENCES ON PEDAGOGY, POLICY, AND RESEARCH

Assuming that we have convinced the reader that flourishing is a constructive *and* measurable aim of mathematics education, we now discuss the influence that such an aim might have on pedagogy, policy, and research. From a pedagogical standpoint, there has

been much debate on the most “effective” instructional practices. For instance, consider the popular pedagogical debates regarding the amount of guidance teachers should provide to students (e.g., Campbell et al., 2022; Clark et al., 2012; Munter et al., 2015). Clark et al. (2012) boldly stated, “Decades of research clearly demonstrate that *for novices* (comprising virtually all students), direct, explicit instruction is more effective and efficient than partial guidance” (p. 6; emphasis theirs). As illustrated by this quote, researchers have argued for specific ways of teaching based on its “effectiveness” and “efficiency” for learning.

Now, let us consider how measures of pedagogical “effectiveness” change if we take flourishing to be the aim of mathematics education. Maximally guided instruction, as suggested by Clark et al., (2012), is unlikely to support students in experiencing *all* the traits required for flourishing (e.g., positive emotions, engagement, and community). Rather, pedagogical practices that center student dialogue and active engagement are more likely to elicit positive emotions, engagement, and community. Flourishing as an aim, therefore, changes the topic of current pedagogical debates and allows teachers to view “effectiveness” from a multidimensional lens.

In addition to pedagogical changes, flourishing as an aim of mathematics education could initiate global policy changes. One area of transformation would relate to assessment. Students, schools, and teachers are regularly evaluated by students’ performance on standardized achievement tests, while traits such as engagement and community are overlooked. If we take flourishing as an aim, then it requires policymakers to create multidimensional assessments of students’ experiences in mathematics. There has been some mobility in multidimensional assessments of students’ knowledge (e.g., see PISA collaborative problem-solving assessment [OECD, 2017]). Policymakers would need to build upon current momentum in assessment and other domains to enact policy changes that are consistent with flourishing as an aim of mathematics education.

Lastly, changes in research could necessarily follow from pedagogical and policy changes. Flourishing provides researchers with leverage to explore how teaching practices influence a variety of outcomes. Rather than being fixated on student achievement, researchers might interpret the efficacy of instructional practices based on their potential to improve engagement, positive emotions, and other traits that support students in achieving the “highest good” in mathematics. These changes in research could offer a more holistic understanding of student success in mathematics.

## VI. CONCLUSION

To summarize our argument, we believe that flourishing is a constructive aim of mathematics education. Flourishing provides a measurable aim of mathematics education that views traits such as positive emotions, engagement, and community as critical to experiencing the “highest good” in mathematics. Notwithstanding the aforementioned implications for pedagogy, policy, and research, we believe that upholding flourishing as an aim could improve society’s perceptions and willingness to engage in mathematics. Unfortunately, few students pursue math-related careers due to their prior experiences.

Flourishing as an aim could improve students' perceptions about mathematics.

While our long-term hopes for flourishing as an aim of mathematics education are lofty, we note that this work is preliminary. We hope that scholars engage with the ideas presented here toward continued discussions regarding an essential question in mathematics education: What is the aim?

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