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RESEARCH ARTICLE

Guidance offered to teachers in curriculum materials for engaging students in proof tasks: The case of Korean grade 8 geometry

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Abstract

Researchers and curricula continue to call for proof to serve a central role in learning of mathematics throughout kindergarten to grade 12 and beyond. Despite its prominence and recognition gained during past decades, proof is still a stumbling block for both teachers and students. Research efforts have been made to address issues related to teaching and learning of proof. An area in which such research efforts have been made is analysis of curriculum material (i.e. textbook analysis) with a focus on proof. This study is another research effort in this area of research through investigating the guidance offered in curriculum materials with the following research question: What is the nature (e.g., kinds of content knowledge, pedagogical content knowledge) of guidance is offered for teachers to implement proof tasks in grade 8 geometry textbooks? Results indicate that the guidance offered for proof tasks are concerned more with content knowledge about the content-specific instructional goals than with pedagogical content knowledge which supports teachers in preparing in-class interactions with students to teach proof.

Keywords: guidance, proof, teacher learning, textbook analysis

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I. INTRODUCTION

Researchers and curricula continue to call for proof to serve a central role in learning of mathematics throughout kindergarten to grade 12 and beyond (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2015; Department of Education [DoE], 2014; Kim, 2022; Ministry of Education [MoE], 2015; National Council of Teachers of Mathematics [NCTM], 2000, 2009; Stylianides et al., 2016; Stylianides et al., 2017; Stylianides, 2008; Stylianou et al., 2010). Despite of its prominence and recognition gained during past decades, proof is still a stumbling block for both teachers and students (Chazan, 1993; Schoenfeld, 1994; Stylianides et al., 2017). Research efforts have been made to address issues related to teaching and learning of proof. An area in which such research efforts have been made is analysis of curriculum material (i.e. textbook analysis) with a focus on proof (Cai & Cirillo, 2014; Kim, 2023; Kim et al., forthcoming; Stylianides, 2014). By curriculum material, I refer to instructional resources in form of textbook (student edition or teacher edition), game, worksheet, and etc (Schneider & Krajcik, 2002). Researchers pursued this line of research to address an important and focal research question: how *educative* the guidance is for teachers in the area of proof and proving? (Stein et al., 2007). Assuming that textbooks might have impact on student learning (Fan, 2013) as teachers consult with textbooks when preparing their instruction and implementing tasks contained in textbooks (Ball & Cohen, 1996; Bergwell & Hemmi, 2017; Charalambos et al., 2010; Kim, 2018; Matić & Gracin, 2021; Stein et al., 2007), a primary reason why researchers endeavor to address the research question is due, in part, to the fact that teachers need much support in teaching proof (Bieda, 2010; Herbst & Brach, 2006; Kim, 2021, 2022; Knuth, 2002) and, in large part, to the fact that there is still a persistent problem in student’s tendency to accept an empirical argument as a proof (Basturk, 2010; Kim et al., forthcoming; Knuth et al., 2009). This study is another research effort in this area of research through investigating the guidance offered in curriculum materials with the following research question: What is the nature (e.g., kinds of content knowledge, pedagogical content knowledge) of guidance is offered for teachers to implement proof tasks in grade 8 geometry textbooks? By proof tasks, I refer to tasks with which students are expected to provide proofs as a result of engagement. By addressing the research question, this study would provide insight into the nature of guidance offered to teachers for implementing proof tasks and suggestions are made for future revisions of the guidance.

II. LITERATURE REVIEW

Textbook analysis as a research methodology has recently been given much attention in the literature (Bergwall, 2021; Davis et al., 2014; Fujita & Jones, 2014; Han, 2005; Hong & Choi, 2014; Hummer, 2016; Jung & Lee, 2016; Miyakawa, 2017; Otten et al., 2014; Park & Lee, 2016; Stylianides, 2008, 2009; Thompson et al., 2012; Zhang & Qi, 2019). Researchers have made research efforts in analyzing textbooks, aiming at achieving various goals to the best of interest to research teams (Cai & Cirillo, 2014; Stylianides, 2014). For example, Stylianides (2008) examined the teacher guide of *Connected Mathematics Project* and provided an analytic approach to characterize the guidance offered for teachers to implement proof tasks with the intent to make suggestions for what would support teachers' instruction of proof while others have sought to investigate the nature of proving-related tasks available in textbook using student textbook: for instance, Otten et al. (2014) analyzed the introduction of proof in secondary geometry; Charalambos et al. (2010) took an analysis of the addition and subtraction of fractions. Regardless of particular goals that researchers sought in their studies, the research rests on a common assumption that textbooks might have impact on student learning promoting teacher learning (Ball & Cohen, 1996; Bergwell & Hemmi, 2017; Charalambos et al., 2010; Fan, 2013; Kim, 2018; Matić and Gracin, 2021; Stein et al., 2007).

During past decades, curriculum materials have been considered as important as a source of teacher's implementation that occurs in class. Though some researchers consider curriculum materials only as one of sources for teachers to prepare daily instruction of mathematics with limited effect on teacher's instruction (Ball & Cohen, 2002; Coburn, 2001; Collopy, 2003) and document that teacher's use of curriculum materials varies with respect to dependency on textbook (Lepik et al., 2015), other studies (e.g., Begle, 1973; Bergwall & Hemmi, 2017; Grouws et al., 2004; Kim, 2013) highlight that teachers primarily use curriculum materials to prepare daily instruction. In this regard, researchers (e.g., Davis & Krajcik, 2005; Stylianides, 2008) argue that curriculum materials should be designed to be educative, promoting teacher learning through curriculum materials. If teachers are required to teach content or mathematical practices (e.g., proof, reasoning, problem solving) that are challenging for them, substantial support must be offered for teachers to be geared with required knowledge package to implement curriculum materials.

Research continues to document that proof and proving is a hard-to-teach and hard-to-learn topic through K-16 education. Earlier work on student understanding about proof (Schoenfeld, 1983) and students' performance in proof writing tasks (Senk, 1985) documented that students' understanding about proof and proof writing need to be more developed to discern deductive reasoning from inductive reasoning. Research findings in recent years (Basturk, 2010; Kim, 2022) suggest that students' appreciation of proof needs to be more developed to be able to distinguish an empirical argument from a valid proof. Issues that teachers have for teaching proof include difficulties in finding a balance between surface features of mathematical arguments and the substance of a proof when asked to evaluate given mathematical arguments (Coe & Ruthven, 1994; Dickerson & Doerr, 2014; Kim, 2022), need of more breadth in knowledge base concerned with various types and forms of proof (e.g., Harel & Sowder, 2007), and need of a broader understanding about the role of proof in mathematics (Bieda, 2010; Dreyfus, 1999; Kim, 2022; Kotelawala, 2016). In efforts to promote teacher learning in proof concerned with the aforementioned issues, researchers have taken different approaches. One of such approaches is investigation into the nature of teacher guide (Matić and Gracin, 2021; Stylianides, 2008) with particular attention given to its impact on teacher practices in class (Kim, 2018; Remillard, 2000). This study is another research effort in examining the nature of teacher guide in proof and proving to provide insight into the nature of guidance offered for teachers to implement proof tasks in grade 8 geometry where Euclidean geometry is covered and two-column proofs are present (Kim, 2023; MoE, 2015).

Research has documented that there are several areas of support required for teachers to teach proof and proving. Support likely to enhance implementations of proof tasks includes *explanations about why students' engagement in a proof task matters*, *cautious points on how to manage student approaches to a proof task*, and *discussions that teachers' content knowledge of proof* (Stylianides, 2008). He developed the aforementioned constructs (*Forms of Additional Guidance* [FAGs]) based mainly on Ball & Cohen (1996) and Davis & Krajcik (2005) which are concerned with teacher's knowledge for teaching, not being specific to proof and proving (see Stylianides, 2008, pp. 197-198 for detail). However, I value the work since his constructs provide an analytic way to investigate guidance and are used to make suggestions for how guidance might be improved in future revisions of curriculum materials. More recent work by Lesseig (2016) enables a more microscopic view on curriculum materials than Stylianides' work (2008) in that FAG 3 of Stylianides' (2008) work is further discussed using the types of content specifically. In this regard, Lesseig's (2016) work is deemed as appropriate to further analyze instances categorized as FAG 3.

III. METHODOLOGY

Data Collection

The data for this study is an excerpt from the teacher guide of all nine Korean grade 8 textbooks authorized for use by the governmental agency. The excerpt was consisting of chapters that cover geometry at grade 8. The content in the excerpt includes: similarity and congruence of shapes and Pythagorean theorem (MoE, 2015). From the teacher guide, I only took the guidance offered for teachers to implement proof tasks for analysis. Here and thereafter, acronyms for the names of textbook series are used: KH for Kyohaksa (Kang et al., 2018), GS for Geumsung (Joo et al., 2018), DA(K) for Donga (Kang et al., 2018), DA(P) for Donga (Park et al., 2018), MR for MiraeN (Hwang et al., 2018), VS for Visang (Kim et al., 2018), SS for Sinsago (Kim et al., 2018), JH for Jihaksa (Jang et al., 2018), CJ(R) for Chunjae (Lew et al., 2018), and CJ(L) for Chunjae (Lee et al., 2018).

Data Analysis

The unit of analysis for this study is *a task*. A task is referred to as a problem or an exercise with a distinct marker. Tasks with different (i.e. mathematically inequivalent) markers are taken as separate (Davis et al., 2014). With the unit of analysis, I conducted an initial path of coding tasks and the coding reliability was compared with the second coder who had about nine years of teaching at the secondary level in Korea and held a master's degree in mathematics education at the time being. The second coder did the same coding with a sample of ten percent of pages from each textbook. The coding reliability of the initial coding was deemed substantial with Cohen's kappa of 0.76. The next path of the coding was conducted to identify proof tasks. For this coding, the author used the answer keys corresponding to the tasks that the textbook authors provided. The reason why I based the analysis of the data on the answer keys provided by the textbook authors was due, in some part, to the fact that expected student response to a task may vary from one to another, in large part, to the fact that (if any) such variations in expectations on student responses to proof tasks are beyond the reach of the author. Similar to the coding for tasks, I first coded all the tasks to decide which task is a proof task and the result of the coding was compared with the result of the second coder's coding for a sample of ten percent of the tasks. Our coding agreement for whether a task is indeed a proof task was near perfect agreement with Cohen's kappa of 0.87. Then, the third round of coding to identify which proof task provides additional guidance was conducted and the coding reliability for this coding was deemed near perfect agreement with Cohen's kappa of 0.91. After conducting the third path of the coding, proof tasks under *additional guidance* were further coded with the coding scheme (see Table 1).

Table 1. The coding scheme for proof tasks with additional guidance

Code	Definition
Knowledge about the Goals of Student's Engagement with a Proof Task [KG]	Knowledge that informs of the reader the goal of student's engagement with a proof task.
Knowledge about Why Student's Engagement with a proof task matters [KW]	Knowledge that provides the reason why it is important for students to engage with a proof task.
Knowledge of Student Conceptions [KS]	Knowledge about (mis-)conceptions that students may possess at the time of engagement with a proof task.
Knowledge of Practices for Supporting Students [KP]	Knowledge about instructional practices that are potentially implemented in class to support student's engagement with a proof task.
Knowledge of Mathematical Content [KM]	Knowledge about mathematical content that is concerned with the range of definitions or theorems useful in a proof task or the role that language and defined terms serve in a proof task.
Knowledge about Logical Aspects of Proof [KL]	Knowledge about logical aspects of proof that is concerned with logical relationships between assumptions, hypotheses, conclusions, and partial arguments of proof or with proof methods (e.g., proof by exhaustion, modus ponens) that are considered to be sufficient and efficient for a proof or with recognizing the defining characteristics of empirical arguments from deductive arguments vice versa.
Knowledge about Mode of Representation [KMR]	Knowledge about representations (e.g., visual, symbolic) which are used to provide a proof or about connections between various representations.
Knowledge about Evaluation of Student Proof [KE]	Knowledge about how a student proof can be evaluated frequently in the form of a rubric.

The coding scheme was developed through the synthesis of relevant frameworks and inductive coding. Styliandes (2008) provides an analytic approach to investigate FAGs of curriculum materials. FAGs are concerned with discussions about the importance of student's engagement with proof tasks (FAG 1), cautious points in managing student responses to a proof task (FAG 2), and discussions concerned with teacher's content knowledge for proof (FAG 3). With the mathematical knowledge for teaching proof frameworks (Buchbinder & Cook, 2018; Lesseig, 2016) considered, codes were developed as *a priori* and several codes emerged after the initial round of inductive coding. Table 1 is the resulting coding scheme used for the analysis of this study.

V. RESULTS

An Overview of The Guidance Offered for Teachers to Implement Proof Tasks

The guidance offered in proof tasks was primarily concerned with goals set for proof tasks. In other words, the vast majority of the guidance offered for proof tasks in all the textbook series were coded for KG ranging from sixty-eight to hundred percent. The guidance coded for KG communicates instructional goals which textbook authors intend for teachers to achieve through engaging students with the tasks. Figure 1 is an example guidance of this nature.

문제 8 문제 9	합동인 두 직각삼각형을 찾아보고, 직각삼각형의 합동 조건을 확실히 이해할 수 있도록 지도한다.

Translation: Find two congruent right triangles and be sure to have students to understand the conditions for congruent right triangles.

Figure 1. An example guidance coded for KG (Adapted from Koh et al., 2018, p. 348)

As seen above in Figure 1, textbook authors communicate with teachers about the instructional goal set for the proof task and teachers may find it helpful in making decisions when implementing the task. The instructional goal is not specific to proof but to the target content of point. Other instances of the KG guidance were also concerned with the target content rather than the practice of proof or the conception of proof. There were several codes with no relevant instances fallen under the codes. These codes include KW, KL, and KMR. Not all the textbooks contained relevant instances fallen under codes, KS, KP, KM, and KE. For example, the instances coded for KS were found in the geometry chapters only from the textbook series KH, DA(K), MR, and JH. The only two instances coded for KMR were found from SS. A summary of the result is shown below in Table 2.

Table 2. A summary of the coding

	KG	KW	KS	KP	KM	KL	KMR	KE	AO
KH	17 (68%)	0	2 (8%)	2 (8%)	1 (4%)	0	0	3 (12%)	0
GS	1 (50%)	0	0	0	1 (50%)	0	0	0	0
DA(K)	21 (87.5%)	0	1 (4.2%)	0	2 (8.3%)	0	0	0	2
DA(P)	3 (100%)	0	0	0	0	0	0	0	1
MR	22 (95.7%)	0	2 (8.7%)	0	1 (4.3%)	0	0	0	1
VS	18 (100%)	0	0	0	0	0	0	0	3
SS	21 (100%)	0	0	0	1 (4.8%)	0	0	1 (4.8%)	0
JH	58 (100%)	0	7 (12.1%)	3 (5.2%)	15 (25.9%)	0	0	3 (5.2%)	0
CJ(R)	8 (100%)	0	0	0	0	0	0	0	0
CJ(L)	5 (100%)	0	0	0	0	0	0	0	0

Note: AO stands for answer only. The percentages were calculated dividing the number of instances by the aggregate number of all the proof tasks excluding the number of those coded for AO.

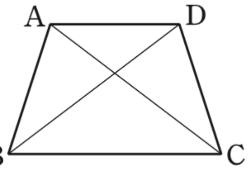
The instances coded for KS were concerned with the content of point. In the guidance shown below in Figure 2, potential misconception that might arise in class was presented: students might think quadrilaterals under conditions such as ‘two diagonals are of the same length’ or ‘an inner angle is 90 degree’ are rectangles. Relating to the misconception, the textbook authors speak to teachers asking to place emphasis on the fact that the conditions only make parallelograms (rather than quadrilaterals) rectangles. This instance is a representative instance of those coded for KS. Other instances coded for KS were also concerned with (mis-)conceptions which students might possess with the content being discussed in the proof tasks rather than student conceptions in close relevance to proof.

The guidance coded for KP provided general guide for teachers to find useful in preparing the implementation of the proof tasks. The guidance in Figure 3 makes connection to the previous activity and provides a general guide which teachers may find useful in preparation of the implementation of the task. However, the guidance is generally stated in the sense that general terms are used and lacks relevance to practices specific to proof. Also, other instances coded for KP were of this nature lacking relevance to practices specific to proof.

2 오개념 진단 · 지도

평행사변형이라는 조건이 없을 때, '두 대각선의 길이가 서로 같다.' 또는 '한 내각이 직각이다.'의 조건만으로 직사각형이 된다고 생각하는 오개념을 가질 수 있다. 따라서 반드시 평행사변형이라는 조건이 있어야 함을 강조한다.

예 두 대각선의 길이가 같은 사각형을 모두 직사각형이라고 말할 수는 없다.
다음 그림의 □ABCD는 $\overline{AC}=\overline{BD}$인 사각형이지만 직사각형이 아닌 등변사다리꼴이다.



Translation: Without the hypothesis that the given quadrilateral is not a parallelogram, students might think as rectangles quadrilaterals under the conditions, 'two diagonals are of the same length' or 'an inner angle is 90 degree'. Thus, make sure to emphasize the fact that those conditions will only make parallelograms (rather than quadrilaterals) rectangles.
Example) It does not hold true that quadrilaterals with diagonals of the same length are always rectangles. □ABCD in the figure seen below is a quadrilateral with AC and BD of the same length, however, it is not a rectangle but an isosceles trapezoid.

Figure 2. An example of the guidance coded for KS (Koh et al., 2018, p. 358)

함께 풀기 2	활동하기에서 알아본 성질을 논리적으로 설명하여 쉽게 이해할 수 있도록 지도한다. 이때 학생들의 수준을 고려한 수업 방법을 적용한다.

Translation: Have students to understand the property learned from the previous activity with ease by logically explaining it. Implement pedagogy considering student's level of understanding.

Figure 3. An instance of the guidance coded for KP (Koh et al., 2018, p. 328)

The instances coded for KE provided rubrics that might be used to qualitatively assess student proofs. Figure 4 is an instance of point. The rubric is broken down into two parts which constitute the proof expected of students to provide as a result of students' engagement with the proof task. This rubric may help teachers to effectively and readily assess student's arguments (citations- proof validation/evaluation).

채점 기준	배점
(i) △OHC≡△OID임을 보인 경우	3점
(ii) 겹쳐진 부분의 넓이가 □ABCD의 넓이의 $\frac{1}{4}$임을 설명한 경우	2점

Criteria	Point
(i) Successfully showing that triangles OHC and OID are congruent	3
(ii)The area of the shaded area is a quarter of the area of the rectangle ABCD	2

Figure 4. An instance of the guidance coded for KE (Jang et al., 2018, p. 331)

V. DISCUSSION

This study investigated the nature of the guidance offered in the Korean textbooks for teachers to implement proof tasks in geometry at grade 8 and conducted an in-depth analysis on representative instances of the guidance. Some of the results of this study are worth noting: the guidance offered for proof tasks are concerned more with content knowledge which promotes teacher learning about the content-specific instructional goals than with pedagogical content knowledge which supports teachers in preparing in-class interactions with students to teach proof; and the guidance is stated generally in the sense that goals of implementation of proof tasks are stated while particular questions or questioning strategies to address issues which might arise interacting with students are not specifically stated. Given the previous research on challenges which teachers encounter with teaching proof (e.g., Bieda, 2010; Buchbinder & McCrone, 2019), the results of this study suggest that more support for teachers in the instruction of proof must be provided in teacher guide.

There were several assumptions which need clarification in relation to this study on textbook analysis. First, for this study, the relationship between teacher and curriculum material was not assumed. Among various relationships between a teacher and a curriculum material (Remillard, 2000), I did not position teachers are only "messengers" between the intended curriculum and students: Rather, they participate with curriculum materials, thus not assuming the tasks as represented in curriculum materials are implemented as intended by textbook writers. Furthermore, research documents that individual teacher's task selection and implementation (Stein & Lane, 1996), authority and positioning (Wilson & Lloyd, 2000), positioning of students (Steele & Rogers, 2012) might account for individual teacher's transformations as a result of teachers' interactions with curriculum materials and implementations in class. Second, it was not assumed to hold true that curriculum materials alone have impact on teacher's teaching practices in a classroom and student's learning practices of proof. Though curriculum materials certainly have an impact on teacher learning (Ball & Cohen, 1996; Matić & Gracin, 2021) and teacher's implementation in class (Davis, 2009), Rezat et al. (2021) contend that mathematical knowledge for teaching and implementation of proof are not alone influential in teacher practices. Also, another study documented that teacher's use of a curriculum material is impacted by the relationship between a teacher, a curriculum material, and a school (Matić & Gracin, 2021) and a case study on a teacher's change in teaching how to prove revealed that teacher's pedagogical content knowledge for teaching proof has impact on teacher's beliefs about teaching students how to prove (Cirillo, 2011). Thus, a question on how tasks represented in curriculum materials are implemented in a classroom still remain unaddressed through this study and the question itself is beyond the scope of this study. Third, it was not assumed that the content providing in the teacher guide need not directly lead to promoting teacher learning in that content. Although this is beyond the scope of this study, it should be noted that the thrust of pursuing this study was to draw textbook writers' and researchers' attention to the nature of the guidance offered for teachers to implement proving-related tasks so that the guidance may be more supportive for teachers who seek support for their teaching proof. Fourth, I did not assume that the coding scheme used for this study is not a comprehensive content knowledge package required for teaching proving. As Lesseig (2016) acknowledged, the framework provides a snippet of content knowledge for teaching proving. One of content knowledge that I think an analytic framework for analyzing the teacher guide to contain is support for teachers who struggle in adjusting their language to the level for students while maintaining the cognitive demand of the tasks (Buchbinder & McCrone, 2019). Lastly, it was not my intention to criticize the guidance per se. As Stylianides (2008, 2014) and Miyakawa (2017) noted, the analysis on the guidance offered for teachers and the text is not aimed at criticizing curriculum materials *as is* but intended to provide insights into the nature of the guidance with a focus on support for teachers to implement proof tasks, drawing attention of textbook writers to particular issues related to the instruction of proof in geometry for future revisions of their textbooks. I also acknowledge the fact that, given the constraint of producing a manageable volume of teacher guide, textbook authors are prone to include elements in their teacher guides that are of their prioritization and some of lengthy elements might be truncated or excluded to fit the set volume, leaving little room to be comprehensive in all areas.

The results of this study necessitate future research on several grounds. The particularity of the data and the context enables conjecture formulation for future research of which data or context is different from this study. Other variations in future research in this area would be made through use of another conceptualization of proof and analytic framework. As seen from the special issue by International Journal of Educational Research on textbook analysis with a focus on proof and reasoning, the conceptualization of proof for a study and the analytic framework used for analysis allow for different results and interpretations of the results (Stylianides, 2014). Furthermore, viewing from the tripartite model of curriculum (Valverde et al., 2002), curriculum materials may account, in part, for teacher's implementations in class at the level of *implemented curriculum,* thus leaving a room for investigations at the levels of *enacted curriculum* and *attained curriculum* on how teachers interpret, select, and implement proof tasks in a classroom and how students engage in proof tasks and how they acquire content knowledge for proof.

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