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PLITHOGENIC VERTEX DOMINATION NUMBER

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ABSTRACT. The thrust of this paper is to extend the notion of Plithogenic vertex domination to the basic operations in Plithogenic product fuzzy graphs (*PPFGs*). When the graph is a complete *PPFG*, Plithogenic vertex domination numbers (*PVDNs*) of its Plithogenic complement and perfect Plithogenic complement are the same, since the connectivities are the same in both the graphs. Since extra edges are added to the graph in the case of perfect Plithogenic complement, the *PVDN* of perfect Plithogenic complement is always less than or equal to that of Plithogenic complement, when the graph under consideration is an incomplete *PPFG*. The maximum and minimum values of the *PVDN* of the intersection or the union of *PPFGs* depend upon the attribute values given to *P*-vertices, the number of attribute values and the connectivities in the corresponding *PPFGs*. The novelty in this study is the investigation of the variations and the relations between *PVDNs* in the operations of Plithogenic complement, perfect Plithogenic complement, union and intersection of *PPFGs*.

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1. Introduction

Based on the idea of fuzzy sets by L. Zadeh [25], fuzzy graphs were discussed by Kaufman [11, 16] and Rosenfeld [20]. Since then, fuzzy graph theory has grown exponentially both in theory and its applications in various disciplines. There are a large number of literary works on different types of fuzzy graphs and other related concepts. Domination in fuzzy graphs is one such an area in fuzzy graph theory where a constant and remarkable research is in progress. It was first introduced by Somasundaram A. and Somasundaram S. in 1998 [23] followed by many others like [1, 7, 8, 10, 13, 15, 17, 18, 19]. Theoretical discussions regarding

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domination number in fuzzy graphs are also found in [2, 12, 24]. Mohideen and Ismayil [14] developed a new concept of domination where u dominates v need not imply v dominates u and vice versa. Based on this notion, dominating set, minimal/minimum dominating set, independent domination, domination number and their bounds were defined and investigated with examples and results. As the generalization of different types of fuzzy graphs, Plithogenic graphs were introduced by Smarandache et al. [9, 22]. As an extension of Plithogenic fuzzy graphs, $PPFG$ was newly defined and discussed by T. Bharathi and S. Leo. [3], where the usual product operator is applied to compute the attribute values of P-edges. T. Bharathi et al. [4, 5, 6] also further explored in this line and newly introduced distance, operations of complement, perfect complement, union, intersection and domination in $PPFGs$. They discussed on two types of domination in $PPFGs$. In this paper, basic definitions and terminologies required for this study are presented in sections 2. The notion of Plithogenic vertex domination number in Plithogenic complement, perfect Plithogenic complement, union and intersection of $PPFGs$ is analyzed for newer results and proofs in Section 3. The main ideas and the scope for further research are provided in the conclusion in section 4.

2. Basic Concepts

Let $G_F = (V_F, E_F)$ be a Plithogenic fuzzy graph, where $V_F = \{V_1, V_2, \ldots, V_n\}$ and $E_F = \{E_1, E_2, \ldots, E_m\}$ are Plithogenic fuzzy sets of vertices and edges respectively characterized by k attributes. G_F is said to be a $PPFG$ denoted by $G_p = (V_P, E_P)$, if for any two adjacent $V_i, V_j \in V_F, i \neq j$, with the corresponding edge $E_c \in E_F$, $V_i = X_i(s_1, s_2, \ldots, s_k)$, $V_j = X_j(t_1, t_2, \ldots, t_k)$ and $E_c = Y_c(c_1, c_2, \ldots, c_k)$ where for any $c_d \in Y_c$, $c_d = s_a t_b$ with $s_a \in X_i$ and $t_b \in X_j$; $d = a = b$; $1 \leq d, a, b \leq k$; $c_d, s_a, t_b \in [0, 1]$. Here X_i, X_j and Y_c are sets of attribute values from $[0, 1]$ characterizing V_i, V_j and E_c respectively in G_F and $k \geq 4$. The sum of the attribute values of any $V \in V_P$ is its P-weight denoted by $P_{VW}(V)$. The sum of the attribute values of any $E \in E_P$ is its P-weight denoted by $P_{EW}(E)$. The sum of the P-weights of all the P-vertices in G_P is denoted by $P_O(G_P)$. Any $V \in V_P$ with attribute values $s_1, s_2, \ldots, s_k$ is said to be a strong P-vertex, if $(s_1 + s_2 + \cdots + s_k) \geq A_{VW}(G_P)$, where $A_{VW}(G_P)$ is the average P-weight of P-vertices in G_P, i.e., $A_{VW}(G_P) = P_O(G_P)/|V_P(G_P)|$, where $|V_P(G_P)|$ is the number of P-vertices in G_P. Otherwise, V is called a weak P-vertex. A P-edge is said to be highly strong, if both of its adjacent P-vertices are strong. A P-edge is said to be strong, if one of its adjacent P-vertices is strong. Otherwise, it is a weak P-edge. G_P is said to be a complete $PPFG$ if every $V \in V_P$ is adjacent to the rest of the P-vertices in G_P [3].

The Plithogenic complement of a connected Plithogenic product fuzzy graph, $G_P = (V_P, E_P)$ with $V_P = \{V_1, V_2, \ldots, V_n\}$ and $E_P = \{E_1, E_2, \ldots, E_m\}$, is $\overline{G_P} = (\overline{V_P}, \overline{E_P})$, where $|\overline{V_P}| = |V_P|$ with the same adjacency such that any adjacent V_i and V_j with attribute values $\{s_a : 1 \leq a \leq k\}$ and $\{t_b : 1 \leq b \leq k\}$

respectively in G_P, implies that
(i). $\{(1-s_a) \in V_i : 1 \leq a \leq k\}$ and $\{(1-t_b) \in V_j : 1 \leq b \leq k\}$ where $V_i, V_j \in \overline{V_P}$.
(ii).$\{(1-s_a)(1-t_b) \in V_iV_j : 1 \leq a, b \leq k\}$, where $a = b$ and $V_iV_j \in \overline{E_P}$.
Moreover, $\overline{G_P}$ is said to be a perfect Plithogenic complement of G_P, denoted by $P\overline{G_P} = (\overline{V_{PP}}, \overline{E_{PP}})$ if for any two V_r and V_s nonadjacent in G_P, there exists a P-edge $V_rV_s \in P\overline{G_P}$ such that the above conditions are satisfied [4].
Let $G_{P_1} = (V_{P_1}, E_{P_1})$ and $G_{P_2} = (V_{P_2}, E_{P_2})$ be $PPFGs$ with k attribute values each. $G_{P_1} \cup G_{P_2}$ is $G_P = (V_P, E_P)$ where $|V_P| \leq |V_{P_1} \cup V_{P_2}|$ and $|E_P| \leq |E_{P_1} \cup E_{P_2}|$ defined as follows:
(i). When $V_{P_1} \cap V_{P_2} = \emptyset$, then $|V_P| = |V_{P_1} \cup V_{P_2}|$ and $|E_P| = |E_{P_1} \cup E_{P_2}|$.
(ii). For any $V_i \in V_{P_1} \cap V_{P_2}$, the attribute values of V_i is
$\{g_d = max(s_a, t_b) : 1 \leq d, a, b \leq k;\ d = a = b;\ s_a \in V_{P_1}, t_b \in V_{P_2}\}$, such that for every V_j with attribute values $\{h_e : 1 \leq e \leq k\}$ adjacent to V_i, the attribute values of $V_iV_j = \{g_dh_e : d = e \text{ and } 1 \leq d, e \leq k\}$ [4].
Let $G_{P_1} = (V_{P_1}, E_{P_1})$ and $G_{P_2} = (V_{P_2}, E_{P_2})$ be $PPFGs$ with k attribute values each such that $|V_{P_1}| = |V_{P_2}|$ with the same adjacency. $G_{P_1} \cap G_{P_2}$ is $G_P = (V_P, E_P)$ defined as for every $V_i \in V_{P_1}$ and the corresponding $V_j \in V_{P_2}$, the attribute values of the corresponding $V_x \in G_P$ is $\{g_d = min(s_a, t_b) : 1 \leq d, a, b \leq k; d = a = b; s_a \in V_i \text{ in } G_{P_1};\ t_b \in V_j \text{ in } G_{P_2}\}$ with $i = j = x$ such that for any $V_y \in V_P$ with attribute values $\{h_e : 1 \leq e \leq k\}$ adjacent to V_x , the attribute values of $V_xV_y = \{g_dh_e : d = e \text{ and } 1 \leq d, e \leq k\}$ [4].
Let $G_P = (V_P, E_P)$ be a connected Plithogenic product fuzzy graph. For any adjacent $V_i, V_j \in V_P$ in G_P, V_i is said to dominate V_j if V_iV_j is either a highly strong or strong P-edge in G_P such that $P_{VW}(V_j) \leq P_{VW}(V_i)$. Otherwise, the P-vertex with a greater P-weight is said to dominate the other whenever V_iV_j is a weak P-edge. A subset R_P of V_P is called a Plithogenic vertex dominating set of G_P if for every $V_j \notin R_P$, there exists at least one $V_i \in R_P$ such that V_i dominates V_j. A Plithogenic vertex dominating set R_P is called a minimal Plithogenic vertex dominating set, if there is no Plithogenic vertex dominating set R' of G_P such that $R' \subset R_P$. The sum of the P-weights of the elements of any Plithogenic fuzzy set V_P is said to be its Plithogenic fuzzy cardinality denoted by $|V_P|_f$. The Plithogenic vertex domination number $\gamma_v(G_P)$, and the Plithogenic vertex upper-domination number $D_v(G_P)$ are the minimum and the maximum Plithogenic fuzzy cardinalities taken over all minimal Plithogenic vertex dominating sets of G_P [6].

Lemma 2.1. *If a Plithogenic product fuzzy graph $G_P = (V_P, E_P)$ is complete with $P_{VW}(V_i) = c$ (constant) for every $V_i \in V_P$, then its minimal Plithogenic vertex dominating sets are singleton sets containing every element of V_P* [6].

Lemma 2.2. *If $G_P = (V_P, E_P)$ is a Plithogenic product fuzzy graph with distinct P-weights of P-vertices, then every Plithogenic vertex dominating set of G_P contains $V_j \in V_P$ such that $P_{VW}(V_j)$ is the maximum* [6].

3. Main Results

Theorem 3.1. *For any incomplete Plithogenic product fuzzy graph with distinct P-weights of P-vertices, the Plithogenic vertex domination number of its Plithogenic complement is always greater than or equal to that of its perfect Plithogenic complement.*

Proof. Let $G_P = (V_P, E_P)$ be an incomplete $PPFG$ where $V_P = \{V_1, V_2, \ldots, V_n\}$ and $E_P = \{E_1, E_2, \ldots, E_m\}$ are Plithogenic fuzzy sets of P-vertices and P-edges respectively characterized by k attributes. Consider $\overline{G_P} = (\overline{V_P}, \overline{E_P})$ and $P\overline{G_P} = (\overline{V_{PP}}, \overline{E_{PP}})$ to be its Plithogenic complement and perfect Plithogenic complement respectively. Suppose that for every $V_i \in V_P$ in G_P, $P_{VW}(V_i)$ is distinct. This implies that for every $V_j \in \overline{V_P}$ in $\overline{G_P}$, and the corresponding $V_r \in \overline{V_{PP}}$ in $P\overline{G_P}$, $P_{VW}(V_j) = P_{VW}(V_r)$, where $1 \leq i, j, r \leq n$ and $i = j = r$.
Case (i) When the minimum $PVDSs$ of $\overline{G_P}$ and $P\overline{G_P}$ are singleton sets. Since the P-vertex with the maximum P-weight is the same both in $\overline{G_P}$ and $P\overline{G_P}$, and it is the only element that belongs to the minimum $PVDSs$ of both $\overline{G_P}$ and $P\overline{G_P}$ by Lemma 2.2, $\gamma_v(\overline{G_P}) = \gamma_v(P\overline{G_P})$.
Case (ii) When the minimum $PVDS$ of $\overline{G_P}$ consists of more than one element. Since $P\overline{G_P}$ is a complete $PPFG$ and the minimum $PVDS$ of $P\overline{G_P}$ is a singleton set containing only the P-vertex with the maximum P-weight by Lemma 2.2., and the minimum $PVDS$ of $\overline{G_P}$ consists of more than one element including the P-vertex with the maximum P-weight, $\gamma_v(\overline{G_P}) > \gamma_v(P\overline{G_P})$. □

Proposition 3.2. *In a Plithogenic product fuzzy graph G_P, if for every $V_i \in \overline{G_P}$, $P_{VW}(V_i)$ is the same, then $\gamma_v(P\overline{G_P}) = c$.*

Proof. Let G_P, $\overline{G_P}$ and $P\overline{G_P}$ be a $PPFG$, its Plithogenic complement and perfect Plithogenic complement respectively. Given that for every $V_i \in \overline{G_P}$, $P_{VW}(V_i)$ is the same. This implies $P_{VW}(V_j)$ is also the same, where V_j is the corresponding P-vertex in $P\overline{G_P}$.
Case (i) When $\overline{G_P}$ is complete and for every $V_i \in \overline{G_P}$, $P_{VW}(V_i)$ is the same, it implies that since $P\overline{G_P}$ is complete, every P-vertex is adjacent to the rest of the P-vertices in $P\overline{G_P}$, and any one P-vertex dominates $P\overline{G_P}$. Therefore, $\gamma_v(P\overline{G_P}) = c$, where c is the P-weight of any P-vertex in $P\overline{G_P}$.
Case (ii) When $\overline{G_P}$ is incomplete and for every $V_i \in \overline{G_P}$, $P_{VW}(V_i) = c$, it implies that since $P\overline{G_P}$ is complete, every P-vertex is adjacent to the rest of the P-vertices in $P\overline{G_P}$, and any one P-vertex dominates $P\overline{G_P}$. Therefore, $\gamma_v(P\overline{G_P}) = c$, where c is the P-weight of any P-vertex in $P\overline{G_P}$. □

Definition 3.3. Any two Plithogenic product fuzzy graphs are said to be crisp isomorphic, if they have the same number of P-vertices, P-edges and also the same edge connectivity.

Definition 3.4. Any two crisp isomorphic Plithogenic product fuzzy graphs G_{P_1} and G_{P_2} are said to be Plithogenic product fuzzy isomorphic if the number of attribute values is the same in both.

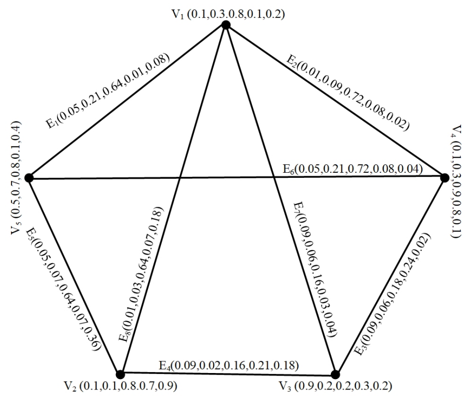

FIGURE 1. G_{P_1}

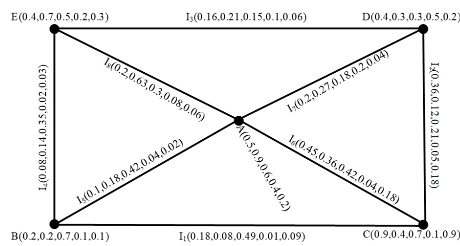

FIGURE 2. G_{P_2}

Example 3.5. Consider Plithogenic product fuzzy graphs G_{P_1} and G_{P_2} as shown in Figures 1 and 2 respectively.Though their structures are different, they are crisp isomorphic Plithogenic product fuzzy graphs, since the number of P-vertices and their adjacencies are the same in both G_{P_1} and G_{P_2}. The corresponding P-vertices in G_{P_1} and G_{P_2} are as follows: $V_1 = A$; $V_2 = B$; $V_3 = C$; $V_4 = D$ and $V_5 = E$. Moreover, since both G_{P_1} and G_{P_2} have 5 attribute values each, they are also Plithogenic product fuzzy isomorphic graphs.

Lemma 3.6. *If the Plithogenic vertex domination numbers of a complete Plithogenic product fuzzy graph and its Plithogenic complement are distinct, then the Plithogenic vertex domination number of its perfect Plithogenic complement equals that of Plithogenic complement.*

Proof. Let $G_P = (V_P, E_P)$ be a complete *PPFG*. Consider $\overline{G_P} = (\overline{V_P}, \overline{E_P})$ and $P\overline{G_P} = (\overline{V_{PP}}, \overline{E_{PP}})$ to be its Plithogenic complement and perfect Plithogenic complement respectively. Obviously $\overline{G_P}$ and P$\overline{G_P}$ are also complete *PPFGs*. Let $\gamma_v(G_P)$, $\gamma_v(\overline{G_P})$ and $\gamma_v(P\overline{G_P})$ be $PVDNs$ of G_P, $\overline{G_P}$ and $P\overline{G_P}$ respectively. Suppose that $\gamma_v(G_P) \neq \gamma_v(\overline{G_P})$. i.e., $V_i \in G_P$ such that $P_{VW}(V_i)$ is the maximum dominates G_P implies $\gamma_v(G_P) = P_{VW}(V_i)$, and $V_j \in G_P$ such that

$P_{VW}(V_j)$ is the minimum, implies that for the corresponding $V_r \in \overline{G_P}$, $P_{VW}(V_r)$ is the maximum in $\overline{G_P}$, and V_r dominates $\overline{G_P}$. Therefore $\gamma_v(\overline{G_P}) = P_{VW}(V_r)$ and vice versa. Since $\overline{G_P}$ and $P\overline{G_P}$ are Plithogenic product fuzzy isomorphic with the same P-weights, for $V_s \in P\overline{G_P}$ corresponding to $V_r \in \overline{G_P}$, $P_{VW}(V_s) = P_{VW}(V_r)$, and the singleton set containing V_s dominates $P\overline{G_P}$. This implies $\gamma_v(P\overline{G_P})$=$P_{VW}(V_s)$. Therefore, $\gamma_v(\overline{G_P}) = \gamma_v(P\overline{G_P})$. □

Theorem 3.7. *For a Plithogenic product fuzzy graph, G_P if $\gamma_v(G_P)$ and $\gamma_v(\overline{G_P})$ are not equal, then either $\gamma_v(P\overline{G_P}) = \gamma_v(\overline{G_P})$ or $\gamma_v(P\overline{G_P}) \neq \gamma_v(\overline{G_P})$.*

Proof. Let G_P, $\overline{G_P}$ and $P\overline{G_P}$ be Plithogenic product fuzzy graph, its Plithogenic complement and perfect Plithogenic complement respectively. Suppose that $\gamma_v(G_P) \neq \gamma_v(\overline{G_P})$.
Case (i) When G_P is a complete *PPFG*, then obviously both $\overline{G_P}$ and $P\overline{G_P}$ are also complete. By Lemma 3.6, $\gamma_v(P\overline{G_P}) = \gamma_v(\overline{G_P})$.
Case (ii) When G_P is an incomplete *PPFG*, then $\overline{G_P}$ is also incomplete, but $P\overline{G_P}$ is complete. Suppose the minimum *PVDS* of $\overline{G_P}$ and $P\overline{G_P}$ are singleton sets, then by Theorem 3.1, $\gamma_v(P\overline{G_P}) = \gamma_v(\overline{G_P})$. Otherwise, $\gamma_v(P\overline{G_P}) < \gamma_v(\overline{G_P})$, i.e., $\gamma_v(P\overline{G_P}) \neq \gamma_v(\overline{G_P})$. □

Remark 3.1. For a complete Plithogenic product fuzzy graph G_P where every attribute value of P-vertices in G_P is θ,
(a) $\gamma_v(G_P) < \gamma_v(\overline{G_P}) = \gamma_v(P\overline{G_P})$ when $0 \leq \theta < 0.5$
(b) $\gamma_v(G_P) = \gamma_v(\overline{G_P}) = \gamma_v(P\overline{G_P})$ when $\theta = 0.5$
(c) $\gamma_v(G_P) > \gamma_v(\overline{G_P}) = \gamma_v(P\overline{G_P})$ when $0.5 < \theta \leq 1$

Remark 3.2. For an incomplete Plithogenic product fuzzy graph G_P, when every attribute value of P-vertices in G_P is θ,
(a) $\gamma_v(G_P) < \gamma_v(\overline{G_P}) \geq \gamma_v(P\overline{G_P})$ when $0 \leq \theta < 0.5$
(b) $\gamma_v(G_P) = \gamma_v(\overline{G_P}) \geq \gamma_v(P\overline{G_P})$ when $\theta = 0.5$
(c) $\gamma_v(G_P) > \gamma_v(\overline{G_P}) \geq \gamma_v(P\overline{G_P})$ when $0.5 < \theta \leq 1$

Theorem 3.8. *The Plithogenic vertex domination number of the union of any two Plithogenic product fuzzy isomorphic graphs is always greater than or equal to the average of the Plithogenic vertex domination numbers of the corresponding graphs.*

Proof. Let $G_{P_1} = (V_{P_1}, E_{P_1})$ and $G_{P_2} = (V_{P_2}, E_{P_2})$ be two Plithogenic product fuzzy isomorphic graphs with k attribute values each. Consider $\gamma_v(G_{P_1}) = x$, $\gamma_v(G_{P_2}) = y$, and the union of G_{P_1} and G_{P_2} as $G_P = (V_P, E_P)$. To prove that $\gamma_v(G_{P_1} \cup G_{P_2}) \geq (x + y)/2$, the following cases are considered.
Case (i) Suppose for every $V_i \in V_{P_1}$ and the corresponding $V_j \in V_{P_2}$ such that $P_{VW}(V_i) = P_{VW}(V_j)$ and for every $a_i \in V_i$ and the corresponding $b_i \in V_j$ implies that $a_i = b_i$ where $a_i, b_i \in [0, 1]$ and $1 \leq i \leq k$, then $\gamma_v(G_{P_1}) = \gamma_v(G_{P_2})$, i.e., $x = y$. Therefore, $\gamma_v(G_{P_1} \cup G_{P_2}) = (x + y)/2$ since $G_{P_1} \cup G_{P_2} = G_{P_1} = G_{P_2}$.
Case (ii) Suppose for every $V_i \in V_{P_1}$ and the corresponding $V_j \in V_{P_2}$ such

that $P_{VW}(V_i) \neq P_{VW}(V_j)$, then $\gamma_v(G_{P_1}) \neq \gamma_v(G_{P_2})$, i.e., $x \neq y$. This implies that since in $G_{P_1} \cup G_{P_2}$ for every $V_r \in V_P$, $P_{VW}(V_r) = \sum_{i=1}^{k} max(a_i, b_i)$ where $a_i \in V_i$ and $b_i \in V_j$; $a_i, b_i \in [0,1]$; $1 \leq i \leq k$ and V_i and V_j are the corresponding common P-vertices of $V_r \in G_P$ in G_{P_1} and G_{P_2} respectively, it follows that $\gamma_v(G_{P_1} \cup G_{P_2}) > 1/2\{\gamma_v(G_{P_1}) + \gamma_v(G_{P_2})\}$, i.e., $\gamma_v(G_{P_1} \cup G_{P_2}) > (x+y)/2$.
Case (iii) Suppose there exist P-vertices in G_{P_1} and G_{P_2} such that for any $V_i \in V_{P_1}$ and the corresponding $V_j \in V_{P_2}$, either $P_{VW}(V_i) = P_{VW}(V_j)$ or $P_{VW}(V_i) \neq P_{VW}(V_j)$. Since $P_{VW}(V_r) = \sum_{i=1}^{k} max(a_i, b_i)$ where $a_i \in V_i$ and $b_i \in V_j$; $a_i, b_i \in [0,1]$; $1 \leq i \leq k$ and V_i and V_j are the corresponding common P-vertices of $V_r \in G_P$ in G_{P_1} and G_{P_2} respectively, it follows that $\gamma_v(G_{P_1} \cup G_{P_2}) > 1/2\{\gamma_v(G_{P_1}) + \gamma_v(G_{P_2})\}$, i.e., $\gamma_v(G_{P_1} \cup G_{P_2}) > (x+y)/2$.

□

Proposition 3.9. *The Plithogenic vertex domination number of the union of any two Plithogenic product fuzzy graphs is greater than the average of the Plithogenic vertex domination numbers of the corresponding graphs.*

Proof. Let $G_{P_1} = (V_{P_1}, E_{P_1})$ and $G_{P_2} = (V_{P_2}, E_{P_2})$ be two Plithogenic product fuzzy graphs with k attribute values each such that $V_{P_1} \cap V_{P_2} \neq \emptyset$. Consider $\gamma_v(G_{P_1}) = x$, $\gamma_v(G_{P_2}) = y$ and $\gamma_v(G_{P_1} \cup G_{P_2})$ to be the $PVDNs$ of G_{P_1}, G_{P_2} and $G_{P_1} \cup G_{P_2}$ respectively. For any $V_r \in G_{P_1} \cup G_{P_2}$, any of the following holds true: (i) $P_{VW}(V_r) = \sum_{i=1}^{k} max(a_i, b_i)$ where $a_i \in V_i$ and $b_i \in V_j$; $a_i, b_i \in [0,1]$; $1 \leq i \leq k$ and V_i and V_j are the corresponding common P-vertices of $V_r \in G_{P_1} \cup G_{P_2}$ in G_{P_1} and G_{P_2} respectively, (ii) $P_{VW}(V_r) = P_{VW}(V_i)$ when $V_r = V_i$ and V_i is the corresponding P-vertex of $V_r \in G_{P_1} \cup G_{P_2}$ in G_{P_1}, and (iii) $P_{VW}(V_r) = P_{VW}(V_j)$ when $V_r = V_j$ and V_j is the corresponding P-vertex of $V_r \in G_{P_1} \cup G_{P_2}$ in G_{P_2}. Therefore, it follows that $\gamma_v(G_{P_1} \cup G_{P_2}) > 1/2\{\gamma_v(G_{P_1}) + \gamma_v(G_{P_2})\}$, i.e., $\gamma_v(G_{P_1} \cup G_{P_2}) > (x+y)/2$. □

Remark 3.3. When $V_{P_1} \cap V_{P_2} = \emptyset$, Plithogenic vertex domination doesn't exist, since $G_{P_1} \cup G_{P_2}$ results in a disconnected Plithogenic product fuzzy graph.

Remark 3.4. The Plithogenic vertex domination number of the union of any two Plithogenic product fuzzy graphs is always less than or equal to the sum of the Plithogenic vertex domination numbers of the corresponding graphs.

Theorem 3.10. *For any two Plithogenic product fuzzy isomorphic graphs,*
$\gamma_v(G_{P_1} \cap G_{P_2}) \leq min\{\gamma_v(G_{P_1}), \gamma_v(G_{P_2})\}$.

Proof. Let $G_{P_1} = (V_{P_1}, E_{P_2})$ and $G_{P_2} = (V_{P_2}, E_{P_2})$ be any two Plithogenic product fuzzy isomorphic graphs with k attribute values each. Consider $\gamma_v(G_{P_1}) = x$ and $\gamma_v(G_{P_2}) = y$ and the intersection of G_{P_1} and G_{P_2} as $G_P = (V_P, E_P)$.
Case (i) When for every $V_i \in V_{P_1}$ and the corresponding $V_j \in V_{P_2}$ such that $P_{VW}(V_i) = P_{VW}(V_j)$ where for every $a_i \in V_i$ and the corresponding $b_i \in V_j$,

$a_i = b_i$; $a_i, b_i \in [0,1]$ and $1 \leq i \leq k$, then $\gamma_v(G_{P_1}) = \gamma_v(G_{P_2})$. i.e., $x = y$. Therefore, $\gamma_v(G_{P_1} \cap G_{P_2}) = min(x,y)$ since $G_{P_1} \cap G_{P_2} = G_{P_1} = G_{P_1}$.

Case (ii) When there exists for every $V_i \in V_{P_1}$ and the corresponding $V_j \in V_{P_2}$, either $P_{VW}(V_i) = P_{VW}(V_j)$ or $P_{VW}(V_i) \neq P_{VW}(V_j)$, then $P_{VW}(V_r) = \sum_{i=1}^{k} min(a_i, b_i)$ where $a_i \in V_i$ and $b_i \in V_j$ such that $a_i, b_i \in [0,1]$; $1 \leq i \leq k$ and V_i and V_j are the corresponding common P-vertices of $V_r \in G_P$ in G_{P_1} and G_{P_2} respectively, it follows that $\gamma_v(G_{P_1} \cap G_{P_2}) < min\{\gamma_v(G_{P_1}), \gamma_v(G_{P_2})\}$, i.e., $\gamma_v(G_{P_1} \cap G_{P_2}) < min(x,y)$.

Case (iii) When for any $V_i \in V_{P_1}$ and the corresponding $V_j \in V_{P_2}$, $P_{VW}(V_i) \neq P_{VW}(V_j)$, then $\gamma_v(G_{P_1}) \neq \gamma_v(G_{P_2})$. i.e., $x \neq y$. This implies that since in G_P for any $V_r \in V_P$, $P_{VW}(V_r) = \sum_{i=1}^{k} min(a_i, b_i)$ where $a_i \in V_i$ and $b_i \in V_j$ such that $a_i, b_i \in [0,1]$; $1 \leq i \leq k$ and V_i and V_j are the corresponding common P-vertices of $V_r \in G_P$ in G_{P_1} and G_{P_2} respectively, it follows that $\gamma_v(G_{P_1} \cap G_{P_2}) < min\{\gamma_v(G_{P_1}), \gamma_v(G_{P_2})\}$, i.e., $\gamma_v(G_{P_1} \cap G_{P_2}) < min(x,y)$. □

Remark 3.5. Plithogenic vertex domination number of the intersection of any two Plithogenic product fuzzy isomorphic graphs is always greater than the absolute difference between Plithogenic vertex domination numbers of the corresponding graphs.

4. Conclusion

The analysis of *PVDN* in the basic operations on *PPFGs* is a mathematical tool for better perception and optimization of the solution to complemented and merged *PPFGs*. Since perfect Plithogenic complement of a *PPFG* is always a complete *PPFG*, the *P*-vertex with the maximum *P*-weight dominates the rest of the *P*-vertices, and the corresponding *P*-weight becomes its *PVDN*. Whenever the minimum *PVDSs* of Plithogenic complement and perfect Plithogenic complement of a *PPFG* are singleton sets, their *PVDNs* are equal, since *P*-vertices belonging to these sets have the same *P*-weights. Otherwise, *PVDN* of Plithogenic complement is greater than that of its perfect Plithogenic complement. Since the maximum of the attribute values of the common *P*-vertices in the respective *PPFGs* is allotted to the corresponding *P*-vertex in their union, the *PVDN* of the union is greater than or equal to the average and less than or equal to the sum of the *PVDNs* of the concerned graphs. On the contrary since the minimum of the attribute values of the common *P*-vertices in the Plithogenic product fuzzy isomorphic graphs is assigned to the corresponding *P*-vertex in their intersection, the *PVDN* of the intersection is always less than or equal to the minimum of *PVDNs* of the corresponding graphs and greater than the absolute difference between them. Likewise, further research to analyze the variations and relations of Plithogenic edge domination numbers in operations on *PPFGs* is to be carried out in the future.

Conflicts of interest : The authors declare that they have no conflict of interest.

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