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UTILIZING COUPLING STRATEGY TO GENERATE A NEW SIMPLE 7D HYPERCHAOTIC SYSTEM AND ITS CIRCUIT APPLICATION

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Abstract. By utilizing coupling the strategy in the 5D Sprott B system, a new no equilibrium 7D hyperchaotic system is introduced. Despite the proposed system being simple with twelve-term, including solely two cross product nonlinearities, it displays extremely rich dynamical features such as hidden attractors and the dissipative and conservative nature. Besides, this system has largest Kaplan-Yorke dimension compared with to the work available in the literature. The dynamical properties are fully investigated via Matlab 2021 software from several aspects of phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, offset boosting and so on. Moreover, the corresponding circuit is done through Multisim 14.2 software and preform to verify the new 7D system. The numerical simulations wit carryout via both software are agreement which indicates the efficiency of the proposed system.

1. Introduction

The n-D dynamical systems are divided into low-dimension and high-dimension under $n = 3$ and $n \geq 4$, respectively. Currently, generating high-dimensional systems that possess multiple positive Lyapunov exponentials with the fewest number of terms and largest Kaplan-Yorke dimension poses a challenging task due to several factors. The first challenge is to ensure that these systems adhere to the simplicity criteria established by researcher Sprott [46]. The second obstacle involves systems containing numerous terms, which tend to be challenging and more complex to implement in potential applications compared to systems with only a few terms, such as chaos control, chaos synchronization [15, 18, 19, 23, 26, 47], encryption [5, 13], and optimization [37]. Especially the subject of electronic circuits, which witnessed increasing interest from many researchers due to the importance of testing the efficiency and effectiveness of the proposed systems [20, 25, 27, 35, 36, 39, 48–50, 52, 53].

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The reported 7-D systems with different attractors, total number of terms, and largest Kaplan-Yorke dimension are categorized in Table 1. The reported various dimensional 5D/6D systems with a total number of terms are listed in Table 2. Mostly 7-D systems are more than 19-term with a dissipative nature [7, 11, 16, 22, 28, 30, 62, 63, 65]. Only very few 7-D systems are introduced with the fewest possible number of terms with a conservative nature. It is clear via Table 1, that 7-D conservative systems with the least number of terms with higher Kaplan-Yorke dimension are not available in the literature. In addition, it was noticed from Table 2 show that most dynamical systems for 5D [1, 42, 44, 51, 54, 58, 61, 64] and 6D [3, 4, 6, 8–11, 21, 38, 59, 60] were composed of equal or more than 12-term with dissipative nature and not find any simple conservative 7D hyperchaotic system consist of 12-term solely. This discussion on the literature motivated us to explore a new seven dimension hyperchaotic system with different features. The features of the proposed system are:

• This system having four positive Lyapunov exponents so it classify as hyperchaotic system,

• It is consider as a simple system according to Sprott criterion due to it composed of merely 12-term,

• It has the largest Kaplan-Yorke dimension compared with other available traditional systems. Therefore, it becomes more complexity,

• The new system has dissipative and conservative nature,

• Electronic circuit for this system is implementation and both phase portraits are agreement through software: Matlab 2021 and Multisim 14.2 which indicated the efficiency of the proposed system.

Table 1. Reported 7D systems with total number of terms and largest Kaplan-Yorke dimension.

Reference	Attractors	No. of positive LEs	Total of terms	Nature of system	Dky
2020 [11]	...	$n-2$	21	Dissipative	6.1281
2012 [30]	Self-excited	$n-4$	19	Dissipative	5.0828
2014 [28]	...	$n-5$	21	Dissipative	2.0599
2018 [16]	...	$n-2$	50	Dissipative	6.7318
2018 [62]	Self-excited	$n-2$	18	Dissipative	6.1486
2019 [63]	Self-excited	$n-5$	18	Dissipative	5.2777
2021 [65]	...	$n-4$	23	Dissipative	4.0322
2021 [7]	Self-excited	$n-3$	23	Dissipative	5.0638
2023 [22]	...	$n-4$	19	Dissipative	5.7791
This work	**Hidden**	$n-3$	**12**	**Dissipative/ Conservative**	**6.8744/ 7.0000**

2. A new 7D hyperchaotic Sprott B system

Many works dealt with Sprott systems to generate new higher dimensional systems due to their simplicity [17, 24, 31, 56]. In 2016, Ojoniyi and Njah [34] generated 5D system from the well-known 3D Sprott B system by utilizing a

TABLE 2. Reported various dimensional 5D/6D systems with total number of terms.

Dimension system	Reference	Total of terms	No. of Nonlinear	Nature of system
5D	2014 [54]	12	3	Dissipative
5D	2013 [58]	12	3	Dissipative
5D	2017 [64]	15	4	Dissipative
5D	2018 [42]	13	3	Dissipative
5D	2019 [51]	13	4	Dissipative
5D	2019 [61]	12	2	Dissipative
5D	2020 [1]	13	6	Dissipative
5D	2023 [44]	13	5	Dissipative
6D	2020 [11]	17	2	Dissipative
6D	2015 [59]	14	3	Dissipative
6D	2020 [38]	15	2	Dissipative
6D	2020 [60]	14	3	Dissipative
6D	2021 [3]	17	2	Dissipative
6D	2022 [10]	13	3	Dissipative
6D	2022 [8]	12	4	Dissipative
6D	2022 [6]	17	3	Dissipative
6D	2023 [9]	12	4	Dissipative
6D	2023 [4]	17	3	Dissipative
6D	2024 [21]	12	4	Dissipative
7D	**This work**	**12**	2	**Dissipative/ Conservative**

linear state feedback control strategy which expressed as

$$\begin{cases} \frac{dx_1}{dt} = x_2x_3 - x_5, \\ \frac{dx_2}{dt} = x_1 - x_2 - x_4, \\ \frac{dx_3}{dt} = 1 - x_1x_2, \\ \frac{dx_4}{dt} = ax_1 + x_2, \\ \frac{dx_5}{dt} = x_1, \end{cases} \tag{1}$$

x_i, $i = 1, 2, \ldots, 5$ are the five independent state variables, a is the real constant control parameter. This system is composed of ten terms, including two cross product nonlinearities with one constant, and exhibits hyperchaotic behavior at $a = 99.8$, and the corresponding Lyapunov exponents are $LE_1 = 0.7810, LE_2 = 0.0232, LE_3 = -\mathbf{0.0006} \approx 0, LE_4 = -0.0482, LE_5 = -1.7558$ [34]. Based on the 5D Sprott B system and coupling strategy, by adding a 2D system into the 5D system, a new 7D hyperchaotic system has been modeled as

$$\begin{cases} \frac{dx_1}{dt} = x_2x_3 - x_5, \\ \frac{dx_2}{dt} = x_1 - x_2 - x_4, \\ \frac{dx_3}{dt} = 1 - x_1x_2, \\ \frac{dx_4}{dt} = ax_1 + x_2, \\ \frac{dx_5}{dt} = x_1, \\ \frac{dx_6}{dt} = bx_6, \\ \frac{dx_7}{dt} = cx_7, \end{cases} \tag{2}$$

x_i, $i = 1, 2, \ldots, 7$ are state variables, b and c are both coupling parameters and $b, c \neq 0$, whereas a is the control parameter. Obviously, this system is simple, with twelve terms only. In general, the system can be written in vector form as

$$\dot{X} = f(x_i) = [x_2x_3 - x_5, x_1 - x_2 - x_4, 1 - x_1x_2, ax_1 + x_2, x_1, bx_6, cx_7]^T. \tag{3}$$

3. Dynamical analysis

The dynamic characteristics that hold significant importance for the proposed system will be seek herein.

Theorem 3.1. *The proposed system without equilibrium points.*

Proof. Set all $\frac{dx_i}{dt} = 0$ in system (2) and solving the system equations

$$\begin{cases} x_2x_3 - x_5 = 0, \\ x_1 - x_2 - x_4 = 0, \\ 1 - x_1x_2 = 0, \\ ax_1 + x_2 = 0, \\ x_1 = 0, \\ bx_6 = 0, \\ cx_7 = 0. \end{cases} \tag{4}$$

The fifth equation of system (4) shows that $x_1 = 0$, which contradicts the third equation where 1 is equal to 0. As a result, the proposed system does not have any equilibrium points and falls under the category of hidden attractors. □

3.1. Nature of the proposed system

The divergence or Trace of the matrix of system (3) is computed as

$$Tr(f(x_i)) = \nabla \cdot f(x_i) = divf(x_i) = \sum_{i=1}^{7} \frac{\partial \dot{x}_i}{\partial x_i} = -1 + b + c. \tag{5}$$

From Eq. (5) and relying on both coupling parameters, it is feasible to determine whether the nature of the proposed system is either dissipative or conservative through the relationship (6)

$$Nature = \begin{cases} \text{dissipative; if } c < 1 - b, \\ \text{conservative; if } c = 1 - b, \\ \text{unbonded; if } c > 1 - b. \end{cases} \tag{6}$$

3.2. Phase portraits and hyperchaotic behavior

Analytical, the new system can exhibit two distinct natures, namely dissipative and conservative, under the parameters $(a, b, c) = (0.1, 0.9, 0.01)$ and $(a, b, c) = (0.1, 0.9, 0.1)$, respectively upon the divergence of the system as defined in Eqs. (7) and (8), respectively. The precision and consistency of the analytical findings were verified through numerical simulation employing the Wolf algorithm [34]. Figure 1 portrays the hyperchaotic attractors of the proposed system, employing initial conditions $x_i(0) = (0.1, 0.1, 0.3, 0.3, 0.2, 0.1, 0.1)$. Eqs. (7) and (8) provides the Lyapunov exponents, their sum, and the (Dky) corresponding to each nature. The Lyapunov exponents are also illustrated in Figure 2.

$$\frac{\begin{cases} LE_1 = 0.9000 \\ LE_2 = 0.0840 \\ LE_3 = 0.0100 \\ LE_4 = 0.0072 \\ LE_5 = \mathbf{0.0000} \\ LE_6 = -0.3745 \\ LE_7 = -0.7167 \end{cases}}{\left(\underbrace{\sum_{i=1}^{7} LE_i = -0.09}_{\text{Dissipative}}\right)}, \ D_{KY} = 6.8744, \tag{7}$$

$$\frac{\begin{cases} LE_1 = 0.9000 \\ LE_2 = 0.1000 \\ LE_3 = 0.0840 \\ LE_4 = 0.0072 \\ LE_5 = \mathbf{0.0000} \\ LE_6 = -0.3745 \\ LE_7 = -0.7167 \end{cases}}{\left(\underbrace{\sum_{i=1}^{7} LE_i = -0.00}_{\text{Conservative}}\right)}, \ D_{KY} = 7.000. \tag{8}$$

System (3) has hyperchaotic attractors with four positive Lyapunov exponents $(+veLE_s)$, i.e., realized $(n-3) + veLE_s$ and the largest Lyapunov exponent (new 7D system) = 0.9000 is greater than the original system (LE_1) (original 5D Sprott B system) = 0.7810, which refers to the new system is highly complex compared to the system (1).

Remark 1.

- Lyapunov exponents are ordered from largest to smallest [57],
- The largest (first) Lyapunov exponent must exceed the limit of 0.001, i.e., $LE_1 > 0.001$ [32],
- Zero Lyapunov exponent is necessary for finding exponent [45],
- If the sign of the sum Lyapunov exponents is negative (equal to zero), then the nature of the system is dissipative (conservative),
- If the sign of the sum Lyapunov exponents is positive, then the system is unbonded,
- The sum of Lyapunov exponents should be almost equal to divergence, i.e., $\sum_{i=1}^{7} LE_i = div f(x_i)$ [43],
- A system with a higher Kaplan-Yorke dimension indicates complexity,

• Kaplan-Yorke dimension for the conservative (dissipative) system must be an integer (non-integer),

• In most applications, a system that has multiple positive Lyapunov exponents is considered more significant.

Remark 2. By implementing a coupling control strategy and exploring the influence of parameters on the proposed system, we have obtained the following findings:

• The first and third Lyapunov exponents in Eq. (8) are the same values of the coupling parameters $b = 0.9$, $c = 0.01$, respectively, i.e., $LE_1 = b$, $LE_3 = c$.

• Due to the above finding, if select b, c are negative signs, then $LE_1 = -b$, $LE_3 = -c$. Thus, the system has hyperchaotic attractors with two positive Lyapunov exponents and satisfied $(n - 5) + veLE$.

• If the 2D system is added to the 5D system as $\dot{x_6} = -bx_6$, $\dot{x_7} = -cx_7$, then the negative sign has no effect on the exponents.

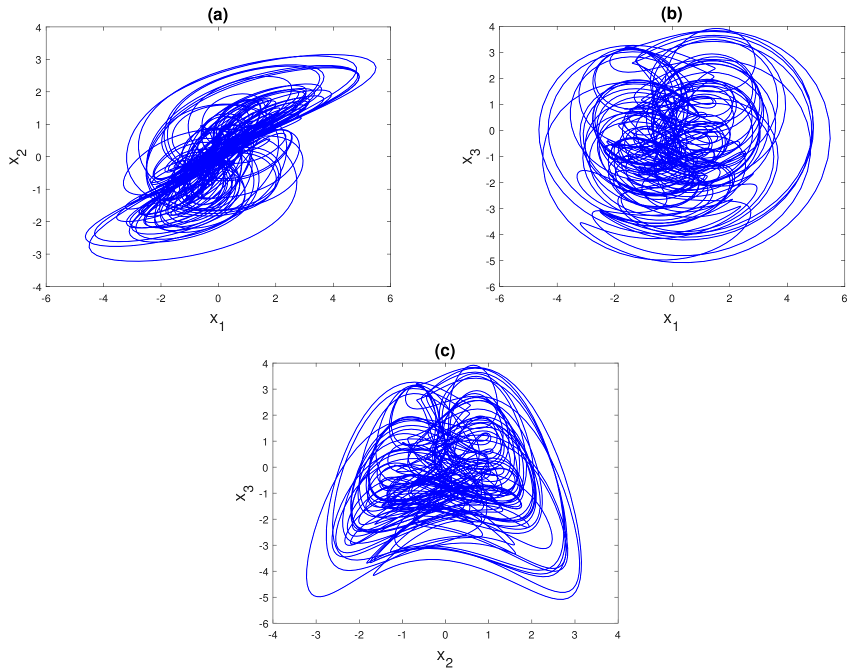

FIGURE 1. Phase portraits of a system (2) in the planes: (a) $x_1 - x_2$, (b) $x_1 - x_3$, (c) $x_2 - x_3$.

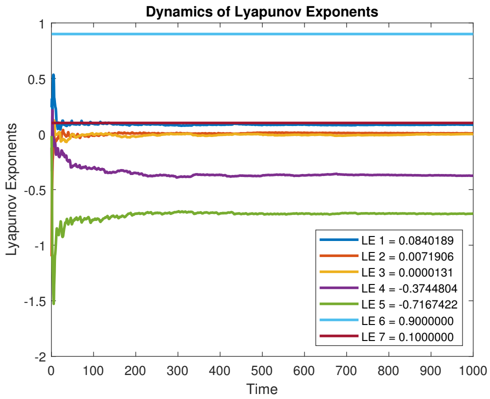

FIGURE 2. LEs of the proposed system.

4. Offset boosting

Offset boosting control has several applications in hyperchaotic systems [29, 33]; it is very useful to convert a unipolar signals into a bipolar signals and vice versa. Clearly, the state variable x_3 solely appears in the first equation of the proposed system, so it is simplicity to control and variable x_3 can be offset boosting by introducing feedback d and the corresponding offset boosting system as:

$$\begin{cases} \frac{dx_1}{dt} = x_2(x_3 + d) - x_5, \\ \frac{dx_2}{dt} = x_1 - x_2 - x_4, \\ \frac{dx_3}{dt} = 1 - x_1x_2, \\ \frac{dx_4}{dt} = ax_1 + x_2, \\ \frac{dx_5}{dt} = x_1, \\ \frac{dx_6}{dt} = bx_6, \\ \frac{dx_7}{dt} = cx_7, \end{cases} \tag{9}$$

in which d offset boosting controller. Under the parameters $(a, b, c) = (0.1, 0.9, 0.01)$ and initial conditions $(0.1, 0.1, 0.3, 0.3, 0.2, 0.1, 0.1)$, the phase portraits of the system (2) in $x_1 - x_3$ plane, $x_2 - x_3$ plane with different offset boosting control d, which are depicted in Figure 3(a), (b), respectively.

5. Circuit implementation

To investigate the efficiency and effectiveness of the proposed system and to obtain their attractors by another approach, a circuit design is utilized through Multisim 14.2 software. This circuit design is comprised of fundamental components such as amplifiers (TL082CD), supply voltage ($\pm 16V$) capacitors, resistors, and multipliers (AD633), which are illustrated in Figure 4. Typically,

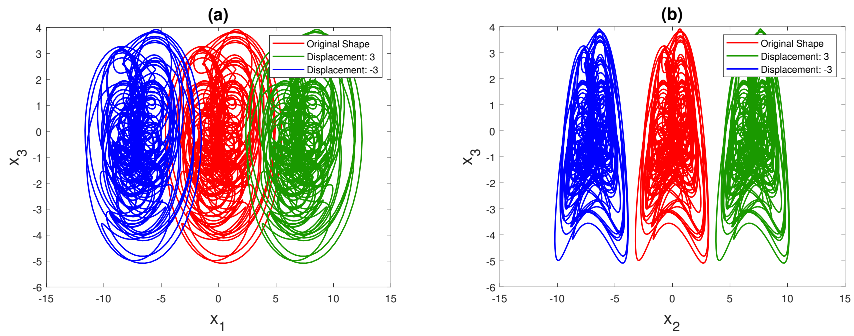

FIGURE 3. Phase portraits of the system (2) with various offset boosting control d and different planes: (a) $x_1 - x_3$, (b) $x_2 - x_3$ for $d = 7$ (red), $d = 0$ (green) and $d = -7$ (blue).

two factors are used: amplitude scale factor and time scale factor. Some studies incorporate both of these factors [2, 12, 29, 66], while others find the time scale factor alone sufficient [14, 55, 67]. Meanwhile, certain works do not consider either of these factors [40, 41].

Theorem 5.1. *An electronic circuit for the proposed system is implemented under the typical parameters as $a = 0.1$, $b = 0.9$ and $c = 0.01$, and the factor of time scale transform $\tau = \tau_0 t$, ($\tau_0 = 1000$).*

Proof. The new system can be rewritten as

$$\left\{\begin{aligned} \frac{dx_1}{dt} &= -1000(-x_2)x_3 - 1000x_5,\\ \frac{dx_2}{dt} &= -1000(-x_1) - 1000x_2 - 1000x_4,\\ \frac{dx_3}{dt} &= -1000(-1) - 1000x_1x_2,\\ \frac{dx_4}{dt} &= -100(-x_1) - 1000(-x_2),\\ \frac{dx_5}{dt} &= -1000(-x_1),\\ \frac{dx_6}{dt} &= -900(-x_6),\\ \frac{dx_7}{dt} &= -10(-x_7). \end{aligned}\right. \tag{10}$$

Based on Kirchhoffs law, the circuit equations can be formulated for system (10) as

$$\left\{\begin{aligned} \frac{dx_1}{dt} &= -\frac{1}{R_1C_1}(-x_2)x_3 - \frac{1}{R_2C_1}x_5,\\ \frac{dx_2}{dt} &= -\frac{1}{R_3C_2}(-x_1) - \frac{1}{R_4C_2}x_2 - \frac{1}{R_5C_2}x_4,\\ \frac{dx_3}{dt} &- \frac{1}{R_6C_3}(-V_1) - \frac{1}{10R_7C_3}x_1x_2,\\ \frac{dx_4}{dt} &= -\frac{1}{R_8C_4}(-x_1) - \frac{1}{R_9C_4}(-x_2),\\ \frac{dx_5}{dt} &= -\frac{1}{R_{10}C_5}(-x_1),\\ \frac{dx_6}{dt} &= \frac{1}{R_{11}C_6}(-x_6),\\ \frac{dx_7}{dt} &= -\frac{1}{R_{12}C_7}(-x_7). \end{aligned}\right. \tag{11}$$

In which each capacitor $\forall C_i = 10nF$, $i = 1, 2, \dots, 7$. By comparing the coefficients in Eqs. (10) and (11), the values of the resistors (R_i) can be determined as

$$
\left\{
\begin{array}{lll}
\frac{1}{R_1C_1} = 1000 & \Longrightarrow & R_1 = 100k\Omega, \\
\frac{1}{R_2C_1} = 1000 & \Longrightarrow & R_2 = 100k\Omega, \\
\frac{1}{R_3C_2} = 1000 & \Longrightarrow & R_3 = 100k\Omega, \\
\frac{1}{R_4C_2} = 1000 & \Longrightarrow & R_4 = 100k\Omega, \\
\frac{1}{R_5C_2} = 1000 & \Longrightarrow & R_5 = 100k\Omega, \\
\frac{1}{R_6C_3} = 1000 & \Longrightarrow & R_6 = 100k\Omega, \\
\frac{1}{10R_7C_3} = 1000 & \Longrightarrow & R_7 = 10k\Omega, \\
\frac{1}{R_8C_4} = 100 & \Longrightarrow & R_8 = 1M\Omega, \\
\frac{1}{R_9C_4} = 1000 & \Longrightarrow & R_9 = 100k\Omega, \\
\frac{1}{R_{10}C_5} = 1000 & \Longrightarrow & R_{10} = 100k\Omega, \\
\frac{1}{R_{11}C_6} = 900 & \Longrightarrow & R_{11} = 111.111K\Omega, \\
\frac{1}{R_{12}C_7} = 10 & \Longrightarrow & R_{12} = 10M\Omega.
\end{array}
\right. \tag{12}
$$

Figure 5 shows the simulation results of Multisim 14.2 of an electronic circuit, which presents hyperchaotic attractors of the system (2) in different planes. These results are identical to the results of Matlab simulation as depicted in Figure 1, which indicated the circuit feasibility of the proposed system, whereas the simulation result is captured via digital oscilloscope (Tektronix) in plane $x_2 - x_3$ supports the validity and accuracy of results. $\square$

6. Time series

In the context of dynamical systems, time series data capture the behavior or evolution of a system over time. Time series data in a chaotic/hyperchaotic systems are mostly employed to examine and study the behavior of dynamic systems. By investigating the points of sequential data, scientists can distinguish irregularities, periodicities, and trends. This diagnosis helps in identifying the dynamical system, forecasting future states, and making suitable decisions. Many observations can be listed from several sources, such as simulations, real-world phenomena, or physical experiments. In addition, the analysis of time series can be employed to examine the state variables for the new model x_i, $i = 1, 2, \dots, 7$ with respect to time as in Figure 7.

7. Discussion

Through the comparison made between the original model (5D Sprott B system), and the new 7D system, and highlight the various characteristics, including the number of terms, Trace of the matrix, nature of the system, largest Lyapunov exponent (first exponent) and number of $+veLE_s$ which is listed in Table 3.

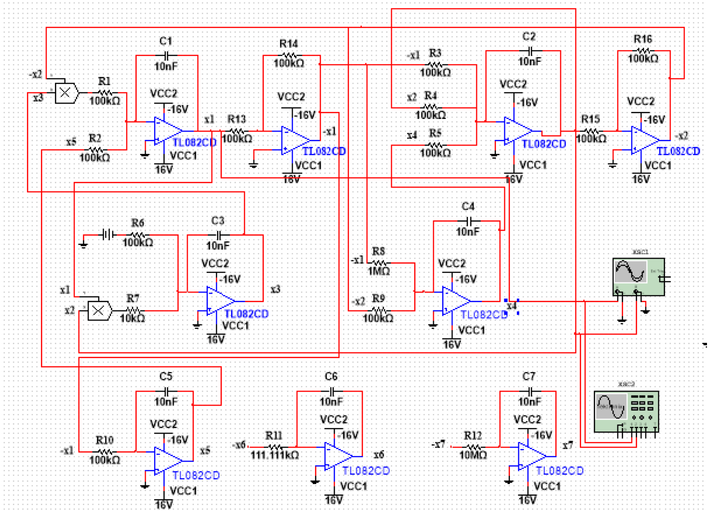

FIGURE 4. Electronic circuit of the new system using Multisim 14.2 software.

TABLE 3. A comparison between the original 5D Sprott B system and the proposed 7D system.

The details	5D Sprott B system (Original system)	New 7D system(Proposed system)
No. of terms	10-term	12-term
$\nabla . f(x_i)$	-1	$-1+b+c$
Nature of system	Dissipative	**Dissipative/ Conservative**
LE_1	0.7810	**0.9000**
No. of positive LEs	2	**4**

8. Conclusions

A new 7D hyperchaotic system without equilibrium is introduced by applying a coupling strategy to the 5D Sprott B system. Despite its simplicity, with twelve terms including two cross product nonlinearities, the system exhibits a wide range of dynamic features, such as hidden attractors and a combination of dissipative and conservative behaviors. Furthermore, compared to existing literature, this system possesses the largest Kaplan-Yorke dimension. Extensive investigations of the system's dynamic properties are conducted using Matlab 2021 software, including phase portraits, LE_i, D_ky, and offset boosting. To validate the proposed 7D system, an electronic circuit is designed using Multisim 14.2 software and performs as expected. The numerical simulations carried

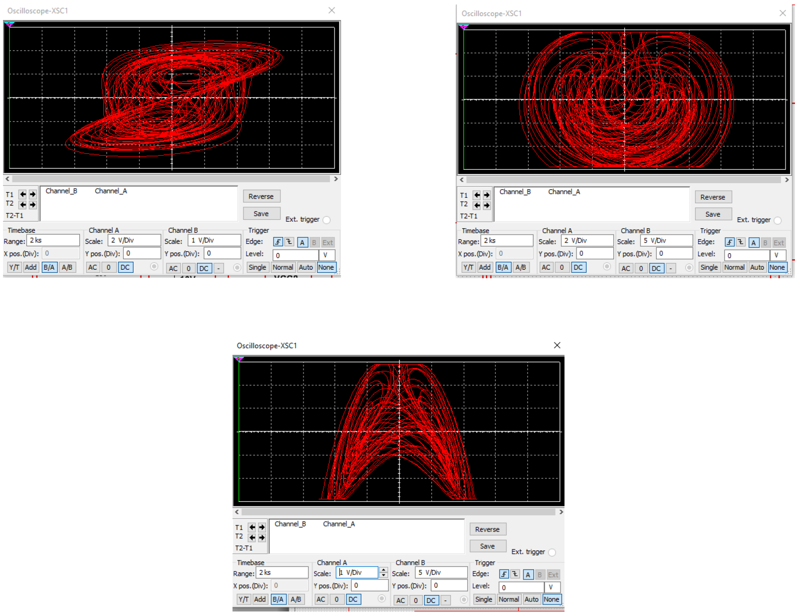

FIGURE 5. Electronic circuit of the new system uses Multisim 14.2 software.

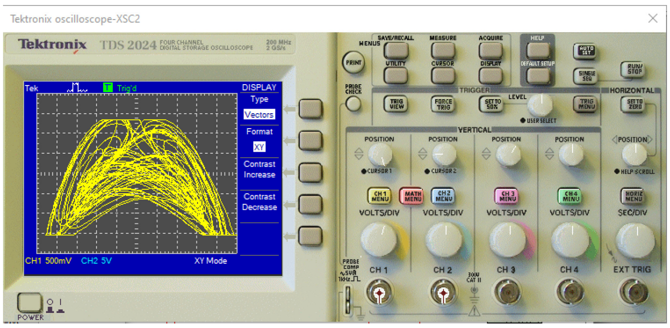

FIGURE 6. Simulation results were captured utilizing NI Multisim 14.2 software via digital oscilloscope (Tektronix) in plane $x_2 - x_3$.

out using both software platforms agree, demonstrating the feasibility of the new system.

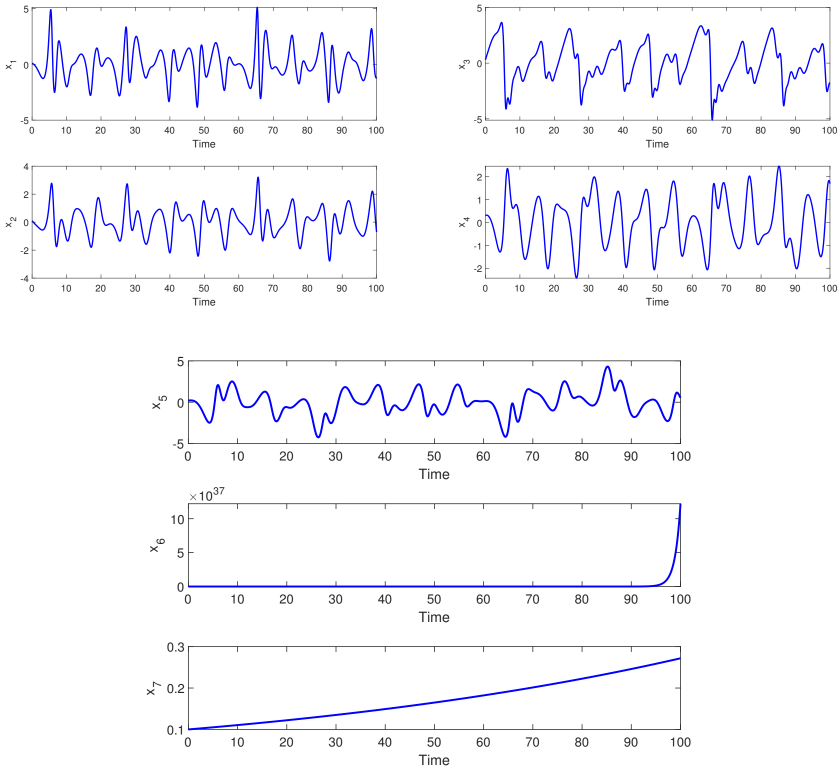

Figure 7. Analysis of time series of state variables x_i, $i = 1, 2, \ldots, 7$ for the new 7D system.

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