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RESEARCH ARTICLE

Analysis of topics in elementary pre-service teachers' writings on constructivist mathematics lessons using topic modeling

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Abstract

This study explores the professional development of pre-service mathematics teachers by analyzing their written work, with a focus on how their perspectives evolved over the course of a semester. Writing plays a critical role in mathematics education, fostering reflective practice, improving communication, and deepening conceptual understanding. Using latent Dirichlet allocation (LDA), a topic modeling technique, this research examined a collection of writings from pre-service teachers enrolled in a constructivist-focused mathematics education course. The analysis identified key topics that emerged, changes in the prominence of these topics over time, and unique characteristics distinguishing the writing of pre-service mathematics teachers from their peers in other disciplines. The findings revealed that writing effectively captured the shifting priorities of pre-service teachers, such as their emphasis on teacher roles and learner-centered instruction. Variability in topic focus throughout the semester may reflect the influence of a changing educational context, particularly in remote or hybrid learning settings. These dynamic shifts underscore the interplay between the pre-service teachers' growing understanding and their evolving learning environments. This study contributes to the expanding literature on pre-service teacher development by demonstrating how machine learning techniques can complement traditional qualitative methods, offering deeper insights into their professional growth. The results suggest that integrating writing analysis into teacher education programs can enhance support for reflective practice and adaptive teaching strategies.

Keywords: pre-service mathematics teachers, writing in mathematics education, topic modeling, latent Dirichlet allocation

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I. INTRODUCTION

Writing provides a unique means of uncovering students' conceptual understanding, problem-solving strategies, and critical thinking processes in learning mathematics (Kim & Choi-Koh, 2023). Often overlooked in a field dominated by symbols and calculations, writing creates opportunities for students to reflect on their work, helping them solidify their thinking and refine their understanding of mathematical ideas (National Council of Teachers of Mathematics [NCTM], 2000). In knowledge-generative environments for learning mathematics (Hwang, 2024), writing plays a crucial role by enabling students to express their thoughts, construct knowledge, and engage deeply with mathematical concepts (Casa et al., 2020). Writing is both a process and a product of mathematical thinking, allowing students to use their language to think mathematically while incorporating multiple modes of representation to articulate and communicate their understanding effectively.

Effective writing, however, requires a range of skills and considerations, including content, context, process, rhetorical elements, and linguistic features. Ultimately, writing is a complex process that demands attention to detail and careful consideration of the audience to communicate effectively (Kitajroonchai et al., 2022). This complexity underscores the value of writing in fostering both mathematical understanding and broader communication skills. For students from linguistically diverse backgrounds, writing in mathematics presents additional challenges, such as navigating multiple linguistic systems (Moschkovich, 2007). These challenges highlight the importance of fostering writing skills in mathematics education to help students convey their ideas clearly and adapt to the subject's language demands. Writing, as a tool for both self-expression and conceptual understanding, is essential for advancing learning and bridging gaps in mathematical communication across diverse learning contexts.

Beyond students' writing to learn mathematics, writing also plays a significant role in mathematics education as a reflective and analytical tool for teachers. In teacher education, writing allows pre-service teachers to document their evolving perspectives, analyze teaching practices, and construct a deeper understanding of pedagogical concepts (Chamberlin, 2009). Writing in this context is not merely a communication tool; it is a process through which pre-service teachers engage with theoretical and practical knowledge to teach mathematics, reflect on their experiences, and articulate their professional growth. By selecting their own topics based on their understanding, pre-service teachers carefully choose expressions that reflect both their knowledge and their intended audience. This process goes beyond showcasing their writing skills; it reveals what they consider important and the messages they aim to deliver. Writing, therefore, becomes a window into their priorities, values, and growing expertise as educators.

This study focuses on the writings of pre-service teachers as a lens to explore their evolving views on teaching and learning mathematics. By examining these writings, the study aims to uncover key topics, analyze how their focus changes over time, and identify distinct features of pre-service mathematics teachers' approaches to writing compared to those of their peers in other disciplines. Through this analysis, the study seeks to illuminate

the role of writing as a critical lens to examine their professional development in mathematics education.

As the role of writing in mathematics education evolves, modern tools like machine learning present new opportunities to analyze and enhance these practices. While writing has been widely explored using various theoretical frameworks in education, the use of data-driven approaches, particularly topic modeling, to analyze pre-service teacher writings remains relatively new. In mathematics education specifically, there is a notable gap in research that examines pre-service teachers' perspectives through their writing.

Furthermore, most topic modeling research in mathematics education has been conducted in English, with little attention given to Korean texts. This gap adds another layer of complexity to understanding diverse educational contexts. This study seeks to address these gaps by analyzing pre-service teachers' writings in Korean, offering a perspective on their learning experiences and professional development. By examining the content and evolution of their writings over a semester, this research aims to provide meaningful insights into how future mathematics teachers conceptualize their learning and refine their teaching philosophies.

This study sought to answer the following research questions:

- What topics emerged in pre-service teachers' writing?
- What are the changes in the number of writings for each topic throughout the semester?
- Are there specific characteristics that distinguish the writings of pre-service mathematics teachers from those of other majors?

II. LITERATURE REVIEW

Writing is a crucial element in the professional development of mathematics teachers, functioning as a tool for self-reflection, pedagogical enhancement, and fostering greater student engagement. Reflective writing, in particular, provides a structured method for teachers to critically evaluate their instructional practices, leading to more effective teaching strategies and improved student outcomes. For example, Ngololo and Kanandjebo (2021) described writing which encourages teachers to examine their decision-making processes and carefully design classroom activities, aligning their practices with modern educational goals.

Through writing, teachers often have the opportunity to re-evaluate and even transform their beliefs about mathematics teaching and learning. Research involving middle-school educators has shown that writing tasks helps teachers articulate their learning experiences and instructional philosophies, resulting in a deeper understanding of effective teaching methods (Chamberlin, 2009). The extent to which educators engage in reflective thinking through writing can vary significantly. Studies (e.g., Aghakhani et al., 2023) have highlighted differences in the levels of reflection among teachers, emphasizing the importance of developing higher-order reflective skills to enhance instructional effectiveness.

In teacher education programs, writing tasks can play a central role in preparing

pre-service teachers for the complexities of the classroom. Research analyzing the writing of prospective mathematics teachers revealed its value in fostering critical thinking and adaptability—skills essential for addressing the diverse needs of students (Soares & Santos, 2022). Reflective journaling allows pre-service teachers to document their evolving perspectives, analyze teaching experiences, and develop self-awareness, which collectively support their professional growth (Zarestky & Bigler, 2021).

Beyond reflective writing, mathematics teachers engage in various other forms of writing that contribute to their professional development. These include writing lesson plans, designing curriculum materials, and drafting academic papers or reports. Such practices not only require teachers to articulate their instructional objectives and design effective learning experiences but also enhance their ability to communicate complex mathematical ideas clearly. For example, Pugalee (2004) found that teachers who regularly wrote about their instructional strategies gained a deeper understanding of both mathematical content and pedagogy, which translated into more effective teaching. Collaborative writing efforts, such as co-authoring research articles or contributing to educational publications, further enable teachers to share best practices and engage with the broader educational community (Taylor et al., 2024).

While reflective and professional writing has received considerable attention, there is a notable gap in research examining the development of pre-service teachers through their writing. Current studies often analyze reflective practices or teaching strategies in isolation but fail to investigate how pre-service teachers' professional growth unfolds over time through their written work. By analyzing writings through the semesters, future research could uncover valuable insights into how pre-service teachers' priorities, understandings, and approaches to teaching evolve throughout their training.

Recent advancements in machine learning provide new opportunities to address such gaps. One promising approach is topic modeling with latent Dirichlet allocation (LDA), a hierarchical Bayesian model that identifies patterns and clusters of topics within large text datasets (Blei et al., 2003). LDA operates as an unsupervised learning technique that models documents as mixtures of latent topics, each characterized by distributions of words. This technique allows for the repeated testing of topics across documents, making it possible to identify trends and shifts in focus over time. While machine learning primarily emphasizes prediction, as opposed to the inferential focus of traditional statistical methods, tools like LDA can complement existing approaches by identifying patterns in pre-service teachers' writings and revealing their evolving priorities and professional development trajectories. Combining such advanced methodologies with traditional reflective practices could open new avenues for understanding and supporting teacher development in mathematics education.

III. RESEARCH BACKGROUND

The data analyzed in this study was collected in the course designed to help pre-service teachers experience teaching methods based on constructivist philosophy at a

university specializing in elementary teacher education in South Korea. It is crucial that these future teachers experience how constructivist principles are applied in practice. This course aimed to equip them with the necessary skills to incorporate constructivism into their teaching when they become in-service teachers. The specific course design would revolve around implementing these constructivist teaching strategies in a practical, engaging way.

Course Design

We collected writing samples from elementary pre-service teachers enrolled in a mathematics education course open to students from all majors. The first two weeks of the course were dedicated to video lectures, which provided foundational knowledge and prepared students for the report submissions that began in the third week. These initial weeks were essential as they gave students the time needed to write substantial and well-structured reports. In the first week, the focus was on a comprehensive orientation. The orientation covered an introduction to basic teaching materials, guidance on using the university's online library for academic research, and instructions on utilizing resources like "Research Information Sharing Service (RISS)" to search for relevant papers. Students were also given detailed guidance on report writing, including specific formatting instructions such as using A4 size, setting appropriate margins, selecting font size, adding titles, and ensuring they included their student ID and name.

Following this orientation, students watched the video lesson (Kamii, 1989) based on constructivist teaching principles. A key feature of this video was the absence of traditional teacher-centered instructional practices, making it a clear example of how constructivist principles could be applied in the classroom. After viewing the video, an hour-long explanation was provided, breaking down and analyzing the different scenes to highlight the application of constructivist methods.

In the second week, the video lecture shifted focus to the 2022 revised curriculum, particularly the key aspects emphasized in this curriculum, such as inclusivity, learner-centered approaches, individualized learning, and the integration of content relevant to students' lives. These concepts were crucial for future teachers, as they would need to implement them in their own classrooms. Without this foundational understanding, there was a risk that their teaching methods might default to traditional practices.

For this course, students began their learning process by preparing ahead of time through reading the designated materials. After watching the Week 1 video lecture, they were instructed to start reading the primary texts selected by the instructor: *Students' Enjoyable Personalized Mathematics Classroom and Students' Enjoyable Mathematics Classroom*. These books are the product of the instructor's direct application of constructivist principles in an elementary school over the course of a year, focusing on the number and operations units. The most notable features of these texts are the absence of specific learning objectives, the simultaneous learning of various knowledge forms, and the emphasis on student-driven knowledge construction. Additionally, commentary is provided throughout the book, helping readers understand the rationale behind the teaching methods.

Course Writing Assessment

Each week, the instructor communicated the assigned readings through the course syllabus, and students were encouraged to select topics of interest from the readings. To deepen their understanding, students would also read additional papers or books that could offer insights into their chosen topics. From a constructivist perspective, each student has a different cognitive framework (Piaget, 1952), meaning that while they may be reading the same core material, they will likely be interested in different topics and bring different interpretations to the table. Even if two students choose the same paper, their reports will reflect different perspectives. If students interpret the core text based solely on personal experience, only superficial abstractions might occur. However, by reflecting on the material with the help of academic papers or books, students engage in a deeper learning process, essentially "standing on the shoulders of giants."

Students then wrote reports based on the topic they selected every week. A basic report involved identifying a topic from the core text, finding a related paper, and summarizing it. A more advanced report took various forms, one of which involved analyzing the core text according to the research questions found in the academic papers. For example, if the core text author mentioned creativity, a student might analyze the text through the subdomains of creativity, such as fluency and flexibility, after reading a relevant paper on the topic. Students were required to cite the papers they used, and as the semester progressed, many students cited multiple references in their reports. These reports were submitted via the learning management system (LMS) 24 hours before the lecture for twelve weeks (Weeks 3-14). Despite the challenge, nearly all students completed their reports each week, showing growing commitment and achievement.

In writing their reports, students were not expected to simply summarize the core text. True knowledge is not found in the text itself but in how the reader constructs that knowledge. The individualized understanding that each student develops becomes the core content of the course. Even though the students are reading the same material, their interpretations differ, and this variety in understanding is the essence of the course. The primary focus of the class discussions is to understand each student’s unique perspective.

Role of the Instructor

The instructor printed and read the reports submitted via the LMS, assessing whether they reflected the core text, referenced papers or books, and conveyed the student's original ideas. The instructor also looked for ideas worthy of discussion in class. While the goal was to find at least one discussion-worthy idea per report, some reports had none, while others had several. Occasionally, the instructor would search and read a paper referenced by a student. Reading the reports of an entire class of 25 to 28 pre-service teachers took more than an hour and a half. Although this was a laborious task for both students and the instructor, it was a rewarding process because it supported the students’ intellectual journeys and allowed the instructor to guide them.

This task provides pre-service teachers with authentic experience of learner autonomy and individualized lessons. At the heart of this approach is the idea that learners should set their own learning goals. As mentioned earlier, after reading the reports, the

instructor would facilitate discussions during the lecture based on the ideas presented by individual students. As they saw their ideas become the subject of class discussions, students were able to develop their understanding at their own pace, setting their learning goals independently.

Classroom Discussion

At the start of each class, the instructor would begin discussions by addressing the ideas in the report of the first student on the attendance list. In subsequent weeks, the instructor rotated through the list to ensure that all students had an equal chance to share their work. After presenting a student's idea, the instructor either explained or elaborated on it, then asked others to share their thoughts. For example, some theorists argue that constructivist teaching should have no predefined learning objectives (Bruner, 1961; Kirschner et al., 2006; Papert, 1980). However, most students, based on their high school experiences, had never encountered a lesson without clear learning objectives. Therefore, they often expressed negative opinions early in the semester. Later on, students who were initially uninterested in a topic would select it for their reports, leading to follow-up discussions. Through these informal discussions and the knowledge gained from papers and core texts, students continuously adapted and developed their cognitive frameworks, incorporating new ideas through processes of assimilation, accommodation, and equilibrium.

These discussions exposed students to various perspectives. For example, constructivist theorists typically advocate for constructivist approaches, while proponents of objective epistemology tend to critique them. Since these positions often oppose one another, students were constantly engaging in metacognitive reflection as they formed their own viewpoints.

IV. RESEARCH METHODS

Data Descriptions

The data for this study were collected from elementary pre-service teachers enrolled at a national university of education located in a metropolitan area of South Korea over two semesters. The participants are 260 elementary pre-service teachers, who were categorized based on their major fields of study. The distribution of participants by major is as follows: 29 pre-service teachers were Science majors, 62 were Korean Language majors, 30 were Social Studies majors, 81 were Mathematics majors, and 58 were Ethics majors. All participants completed weekly writing tasks, resulting in a total of 2,780 pieces of writing in Korean being collected during the study period.

The overall design of this study is shown in Figure 1, and all data analysis was conducted using Python 3.

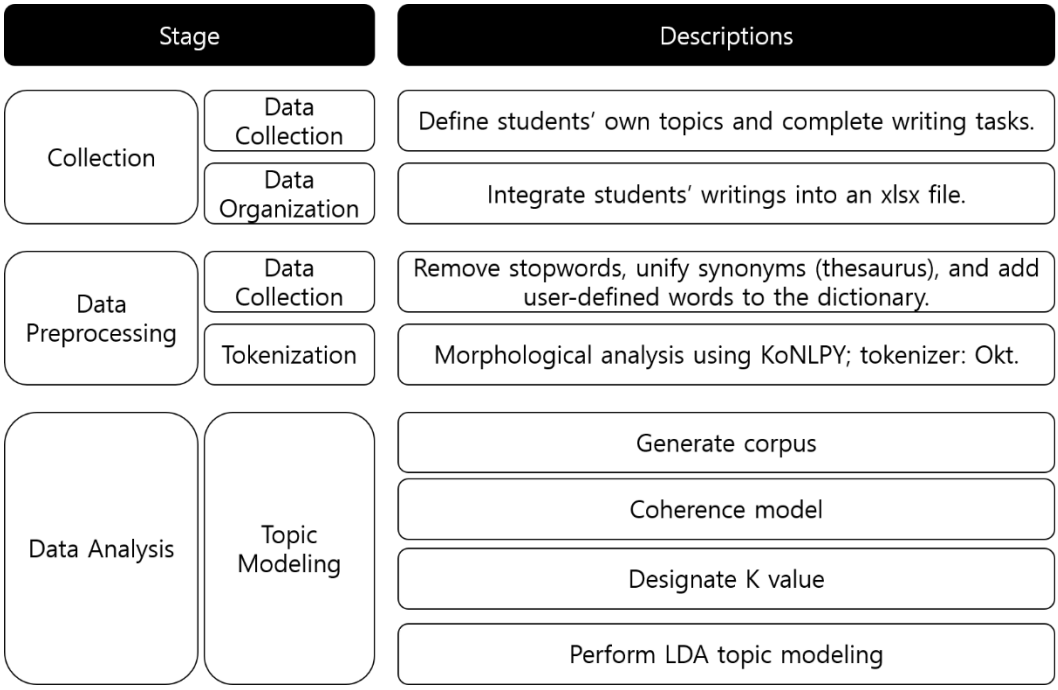

Figure 1. Research design

Data Analysis

The data analysis process began with a thorough pre-processing phase, where all collected texts were tokenized into individual words, focusing specifically on nouns for the analysis. Nouns carry key meanings in the collected writing tasks and play an important role in identifying topics or concepts (Kang et al., 2013). Through this, meaningful information within the text can be effectively selected. Moreover, by focusing on nouns for data analysis, unnecessary words that serve grammatical functions can be removed, improving the efficiency of the analysis. Given that the texts were in Korean, this process involved several critical steps to ensure the accuracy and relevance of the data for topic modeling.

Pre-processing and Tokenization

Firstly, stop words, which are common words that typically do not carry significant meaning in the analysis, were removed. This step was crucial for eliminating noise from the data. Additionally, words that had different forms but the same meaning were standardized into a single term to maintain consistency across the dataset. For instance, words used interchangeably to represent the same concept were unified under one specific term.

In the context of natural language processing, one-letter tokens are often classified as stop words and removed. However, certain single-character tokens like "수" (meaning "number" or "water" depending on context) were retained due to their significance in the context of mathematical and educational texts. To accommodate these nuances, a custom

dictionary was created. This dictionary included key terms and domain-specific terminology relevant to mathematics education, ensuring that important concepts were preserved during tokenization.

Table 1. Most frequent terms in a custom user dictionary

Term	Frequency	Term	Frequency
Mathematics class (수학 수업)	1,860	Mathematics classroom (수학 교실)	457
Feedback (피드백)	1,095	Mathematical concept (수학적 개념)	360
Mathematical thinking (수학적 사고)	673	Problem-solving ability (문제해결 능력)	46
Learning ability (학습 능력)	1,355	Problem-solving (문제해결)	694
Open-ended task (개방형 과제)	2,065	Social knowledge (사회적 지식)	128
Learning environment (학습 환경)	245	Learning goal (학습 목표)	476
Idea (아이디어)	2,210	Individual differences (개인차)	727
Peer learner (동료 학습자)	499	Reasoning ability (추론능력)	891

The tokenization process was performed using the KoNLPy library with the Okt morphological analyzer, which is well-suited for handling Korean text. The Okt analyzer was employed to extract nouns, as these are typically the core components for understanding the context within sentences. For data analysis, a custom user dictionary was added to the Okt morphological analyzer. Morphological analysis was performed repeatedly to identify proper nouns that were not initially extracted as nouns, and these words were manually registered into the user dictionary. Furthermore, specific terms that are compound words, such as "구성주의" (constructivism), which could be mistakenly split into "구성" (construction) and "주의" (ism), were treated as single tokens to maintain their intended meaning. Using Python, the custom user dictionary added to the Okt morphological analyzer is seen in Table 1, and the added words were designated as proper nouns for the analysis. A total of 206 user-defined words were added to the custom dictionary for analysis.

Latent Dirichlet Allocation and Model Specification

Topic modeling is a technique used to identify patterns in unstructured text and

extract latent themes. It probabilistically clusters words with similar meanings, allowing the inference of specific topics within the text. In this study, LDA was employed as the primary statistical method for topic modeling following the pre-processing stage. LDA is a probabilistic approach that describes the distribution of topics across a dataset based on the words occurring in the texts. The objective was to uncover the underlying themes present in the pre-service teachers' writings. Through this method, we aimed to identify and interpret the latent topics that might be addressed within the data.

To determine the optimal number of topics, coherence scores were evaluated across models with varying numbers of topics. Coherence scores measure the degree of semantic similarity between the words in a topic, helping to identify the most meaningful and distinct topics. In the LDA model, specifying the appropriate number of topics (k) is crucial for model evaluation and optimized topic modeling. Typically, the number of topics is determined based on the coherence value, which assesses the consistency of a topic. A higher coherence value indicates greater semantic consistency within a topic. In this study, the parameter k was set to range from 2 to 10 topics, and the models were executed accordingly. After careful evaluation, the model with eight topics yielded the highest coherence score, indicating that it was the most suitable model for this dataset, as shown in Figure 2.

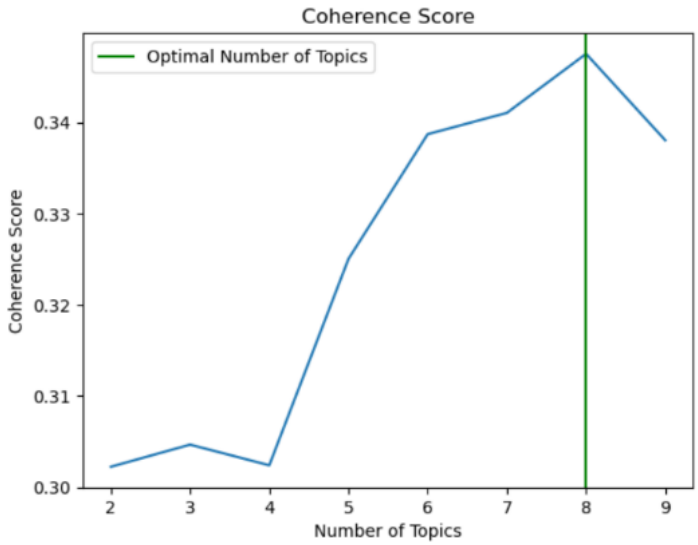

Figure 2. The optimal number of topics based on coherence scores

In LDA topic modeling, adjusting the λ value allows for the identification of both frequently occurring words within each topic and words that distinguish one topic from another. When the λ value is closer to 1, the words with the highest frequency within that topic are primarily displayed, while a λ value closer to 0 highlights words that are more distinctive across topics. This adjustment makes it possible to more clearly discern similarities and differences between topics. The lambda value, which controls the balance between word relevance and frequency within a topic, was optimized to values of 1 and 0.6

to enhance interpretability.

To accurately label each topic, the writings associated with the highest probability terms were closely analyzed. This thorough examination ensured that the assigned labels were not only statistically coherent but also contextually relevant to the educational themes reflected in the pre-service teachers' writings. The process of naming each topic was guided by the combination of the terms identified and their respective relevance within the context of the dataset.

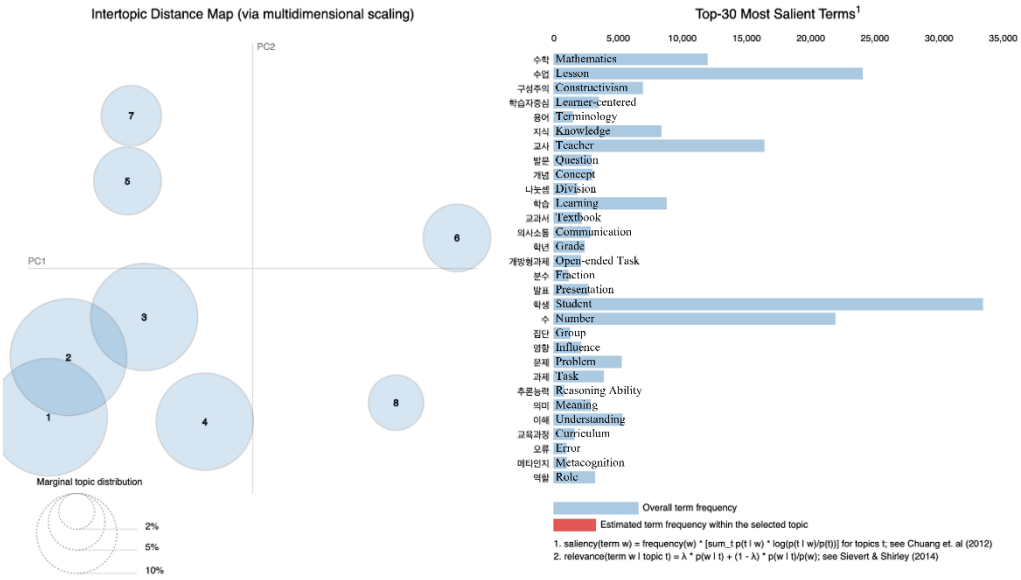

Figure 3. Intertopic distance map and top-30 most salient terms

On the left side of Figure 3, the intertopic distance map visualizes the relationships and distances between the topics identified by the model. This map employs principal components (PC1 and PC2) for dimensionality reduction, enabling the topics to be represented in a two-dimensional space. Each circle in the map represents a topic, with the size of the circle indicating the prevalence of that topic within the corpus. The size of each circle corresponds to the frequency of words assigned to that topic, meaning that a larger circle reflects a greater proportion of the topic within the overall corpus. Topics that are closer together on the map are more similar in terms of the words they include, while topics that are farther apart are more distinct. The small inset at the bottom left shows the overall distribution of topics, with percentages reflecting the relative frequency of each topic.

On the right side of Figure 3, the top-30 Most Salient Terms bar chart displays the thirty most significant terms across the entire dataset. This chart highlights both the overall term frequency (shown in blue) and the estimated term frequency within the most relevant topic (shown in red; see Figures 4 and 5). Salience, in this context, refers to how frequent and relevant a term is to one or more topics. The chart provides insight into which terms are central to the dataset. For instance, the term "학생" (student) appears frequently and is

highly salient across multiple topics, indicating its importance in the corpus.

V. RESULTS

Latent Topics

The topics identified and labeled from the analysis of the students' writings are outlined in Table 2, which includes eight topics along with the words that characterize each topic at lambda values of 1 and 0.6. The topic labels are presented in Korean, reflecting the original language of all the writings. It is important to note that caution was exercised in naming the identified topics due to the nature of the unsupervised machine learning approach used (Henson & Roberts, 2006). Since LDA is an exploratory analysis intended to uncover latent topics, the naming of each topic required careful consideration of all terms with lambda values of 1 and 0.6. The lambda value of 0.6 was specifically used to optimize the results, following established guidelines (Sivert & Shirley, 2014). Specific results with λ=0.6 for Topics 2 and 7 – related to learner-centered instruction – are showed in Figures 4 and 5 respectively.

Table 2. Eight topics with characteristic words (λ=1 and 0.6)

Characteristic words when λ=1	Characteristic words when λ=0.6
1. Teacher-Student Interaction in the Classroom (수업에서의 교사와 학생의 상호작용)	
student (학생), teacher (교사), class (수업), number (수), presentation (발표), thinking (사고), learner (학습자), participation (참여), communication (의사소통), question (질문)	student (학생), teacher (교사), class (수업), number (수), presentation (발표), question (질문), participation (참여), communication (의사소통), thinking (사고), opinion (의견)
2. The Role of the Teacher in Learner-Centered Instruction (학습자 중심 수업에서 교사의 역할)	
class (수업), teacher (교사), student (학생), constructivism (구성주의), number (수), knowledge (지식), construction (구성), role (역할), learning (학습), education (교육)	class (수업), teacher (교사), constructivism (구성주의), student (학생), role (역할), knowledge (지식), construction (구성), learner-centered (학습자중심), education (교육), number (수)
3. Instruction Considering Individual Differences and Implementing Open-Ended Tasks (학생의 개인차를 고려한 수업과 개방형 과제 사용)	
student (학생), mathematics (수학), class (수업), problem (문제), task (과제), open-ended task (개방형과제), learning (학습),	student (학생), task (과제), problem (문제), open-ended task (개방형과제), number (수), class (수업), mathematics (수학), level (수준),

solution (해결), level (수준)	solution (해결), learning ability (학습능력)
4. Construction of Knowledge (지식의 구성)	
student (학생), mathematics (수학), knowledge (지식), questioning (발문), process (과정), thinking (사고), construction (구성), teacher (교사), learning (학습)	knowledge (지식), questioning (발문), metacognition (메타인지), number (수), student (학생), process (과정), mathematics (수학), thinking (사고), construction (구성), cognition (인지)
5. Impact of Instruction on Learning (수업이 학습에 미치는 영향)	
class (수업), learning (학습), student (학생), mathematics (수학), number (수), group (집단), communication (의사소통), activity (활동), assessment (평가), game (게임)	group (집단), learning (학습), class (수업), game (게임), assessment (평가), mathematics (수학), communication (의사소통), student (학생), discussion (토의), activity (활동)
6. Learning of Arithmetic Operations and the Elementary Mathematics Curriculum (사칙연산의 학습과 초등수학 교육과정)	
number (수), mathematics (수학), division (나눗셈), student (학생), grade (학년), concept (개념), textbook (교과서), fraction (분수), curriculum (교육과정), content (내용)	division (나눗셈), fraction (분수), grade (학년), textbook (교과서), concept (개념), multiplication (곱셈), addition (덧셈), curriculum (교육과정), mathematics (수학), calculation (계산)
7. Learner-Centered Instruction (학습자 중심 수업)	
class (수업), student (학생), constructivism (구성주의), learner-centered (학습자중심), number (수), reasoning ability (추론능력), impact (영향), equal sign (등호), result (결과), research paper (논문)	reasoning ability (추론능력), equal sign (등호), learner-centered (학습자중심), constructivism (구성주의), class (수업), impact (영향), academic achievement (학업성취도), experiment (실험), group (집단), student (학생)
8. Use of Mathematical Terminology (수학 용어 사용)	
mathematics (수학), terminology (용어), student (학생), number (수), error (오류), understanding (이해), concept (개념), learning (학습), storytelling (스토리텔링)	terminology (용어), mathematics (수학), storytelling (스토리텔링), error (오류), symbol (기호), definition (정의), failure (실패), figure (도형), meaning (의미), teaching tools (교구)

Additionally, we analyzed pre-service teachers' writings with high probabilities for each topic to ensure accurate labeling and comprehensive coverage of the corresponding themes. For instance, Student B addressed Topic 2 – the role of the teacher in learner-centered instruction – emphasizing the teacher's role from a constructivist perspective. According to Student B, "the teacher acts as a practitioner of 'learner-centered instructional activities,' faithfully adhering to the constructivist assumptions that serve as the foundation of learner-centered teaching principles" (translated). Student B's writing clearly outlines that the teacher's role can be divided into three stages: introduction, where the teacher presents the learning activities and explains the learning process; development, where the learning unfolds; and conclusion, where the learning is summarized (see Appendix for additional examples).

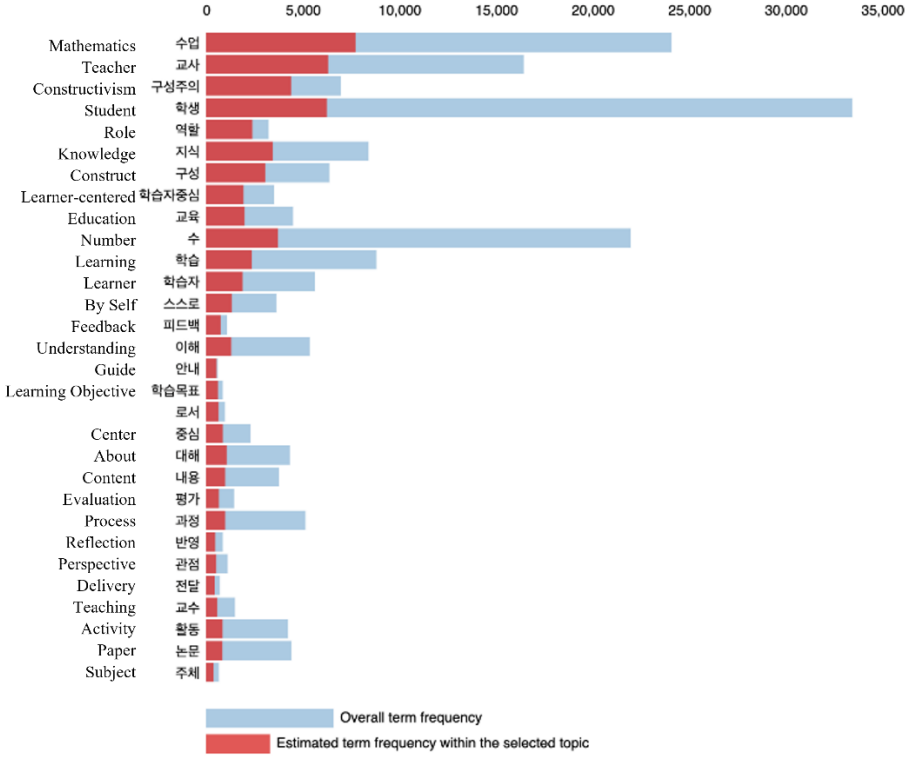

Figure 4. Top-30 most relevant terms for topic 2 with $\lambda = 0.6$

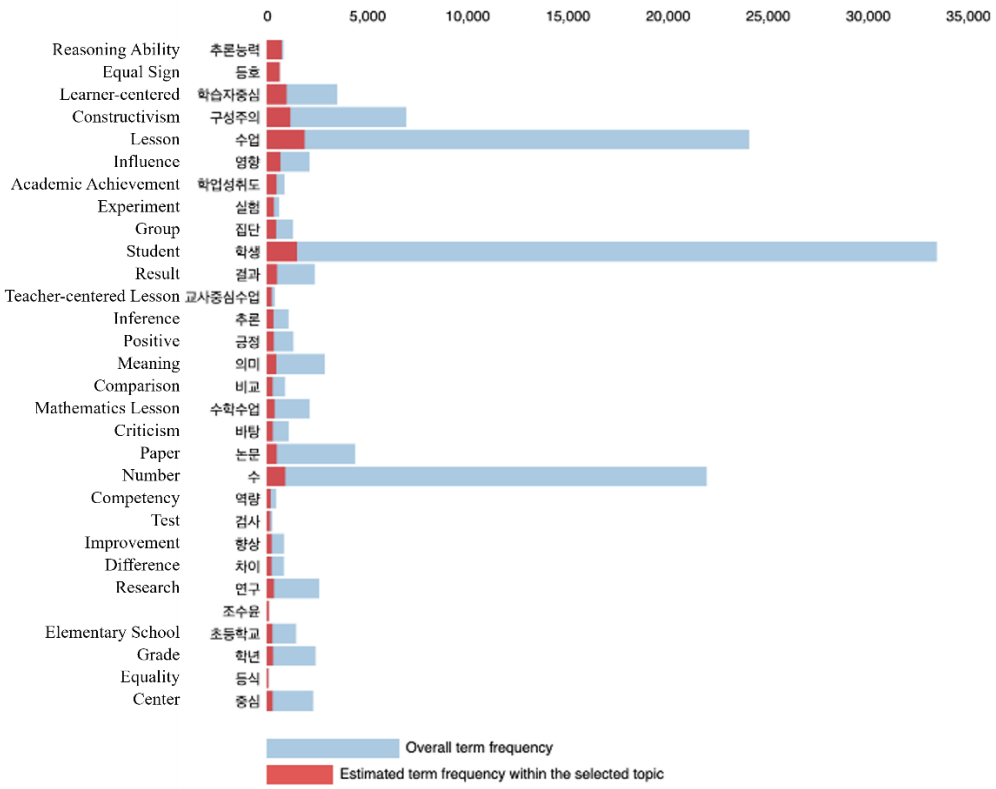

Figure 5. Top-30 most relevant terms for topic 7 with $\lambda = 0.6$

Student F provided a strong example of writing on Topic 6, the learning of arithmetic operations and the elementary mathematics curriculum. Student F focuses on division, where the quotient is a fraction, noting that "the concept of division where the dividend is a whole number and the quotient is a fraction is a concept that even lower elementary students can fully understand." The student makes a compelling argument that, although fraction division is formally taught in fifth grade, many studies have shown that younger students can grasp this concept easily. This curiosity led Student F to explore "which teaching methods can make higher-grade content more accessible to younger students," sparking further investigation into instructional methods (see Appendix for additional examples).

Student G's writing on Topic 7 highlights the positive impact of learner-centered instruction on students' academic achievement. The student states, "students who received learner-centered math instruction demonstrated higher achievement not only in the content learned during the experimental treatment but also in content they had not studied, compared to the control group." This shows that learner-centered instruction not only helps students master the material they are currently studying but also enhances their ability to understand content they may not have covered. The student further emphasizes that learner-centered approaches foster deeper learning outcomes.

In Student H's response on Topic 8, use of mathematical terminology, there is a

strong critique of the current state of elementary school textbooks, particularly the lack of definitions for important mathematical terms. The student argues that "in current elementary school textbooks, the definitions of such terms are significantly lacking," which leads to a situation where students learn mathematical concepts without properly understanding the terminology. This is described as an "absurd and unfortunate" reality. To address this, Student H suggests that "textbooks should present terms alongside their definitions" and recommends that time be allocated for explicitly teaching mathematical terminology. This would provide students with the opportunity to fully understand both the concepts and the language used to describe them (see Appendix for additional examples).

Writing Topics through a Semester

Table 3, Figure 6, 7, and 8 presents the distribution of topics identified in the pre-service teachers' writings across different weeks. We do not report the results for Week 9 and Week 15 due to unusually low numbers of submissions, which coincided with midterm and final exams. The total number of writings collected each week remained relatively stable, ranging from 240 to 247, indicating consistent participation and output by the pre-service teachers throughout the study period. Topics 1 (teacher-student interaction in the classroom) and 2 (the role of the teacher in learner-centered instruction) were the most prevalent, reflecting their prominence in the teachers' reflections and assignments. Topic 1 had the highest overall count of 658 writings, followed by Topic 2 with 635 writings. Topics 3 (instruction considering individual differences and implementing open-ended tasks) and 4 (construction of knowledge) were also fairly common, with 527 and 414 writings, respectively. Topics 5 through 8 appeared less frequently, with Topic 8 (use of mathematical terminology) being the least represented at 110 writings. Notably, Topic 7 (learner-centered instruction) showed variable presence, with higher counts in Weeks 12 and 13, indicating a possible shift in focus during the later part of the semester.

Figure 6 shows the number of student writings on Topic 2 and Topic 7 related to learner-centered instruction over a 14-week period. Topic 2 had a high level of interest in the early weeks, peaking in Week 3 with over 100 writings, followed by a gradual decline and fluctuations. In contrast, Topic 7 had consistently fewer writings, with less than 20 per week until Week 11, when interest began to rise, peaking in Week 14. This suggests that Topic 2 garnered more attention early on, while Topic 7 gained momentum later in the course.

Figure 7 shows the number of student writings on Topics 1, 3, and 4 over a 14-week period. Topic 1 shows relatively stable activity, with moderate fluctuations and a slight increase towards the end, maintaining around 60 writings from Week 7 onwards. Topic 3 experiences a sharp peak in Week 4, reaching 100 writings, followed by a significant drop in Week 6 and fluctuating thereafter. Topic 4 starts with moderate interest, peaking in Week 6 at around 90 writings, but then shows a continuous decline, hitting its lowest point by Week 14.

Figure 8 displays the number of student writings on Topics 5, 6, and 8 over a 14-week period. All three topics show relatively low levels of engagement, with the number of writings generally staying below 20 throughout. Topic 5 experiences a gradual increase,

peaking around Week 13, followed by a drop in Week 14. Topic 6 follows a similar trend, with some fluctuations but a noticeable rise between Weeks 5 and 6 and another peak around Week 12. Topic 8 shows the least variation, remaining mostly consistent, with a slight increase towards Week 13 before declining in Week 14.

Table 3. Distribution of topics identified across different weeks

	Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic 6	Topic 7	Topic 8	Total
Week 3	38	107	50	32	4	3	10	2	246
Week 4	28	59	100	31	6	13	9	1	247
Week 5	60	75	22	35	15	20	15	5	247
Week 6	89	43	36	33	21	7	1	14	244
Week 7	58	32	38	80	7	11	10	6	242
Week 8	61	47	62	42	14	11	5	4	246
Week 10	60	40	52	45	15	17	3	11	243
Week 11	66	36	80	26	14	12	8	5	247
Week 12	43	71	28	30	25	10	27	8	242
Week 13	63	37	32	15	25	17	10	27	226
Week 14	63	62	16	31	8	2	41	22	245
Total	658	635	527	414	157	136	143	110	2780

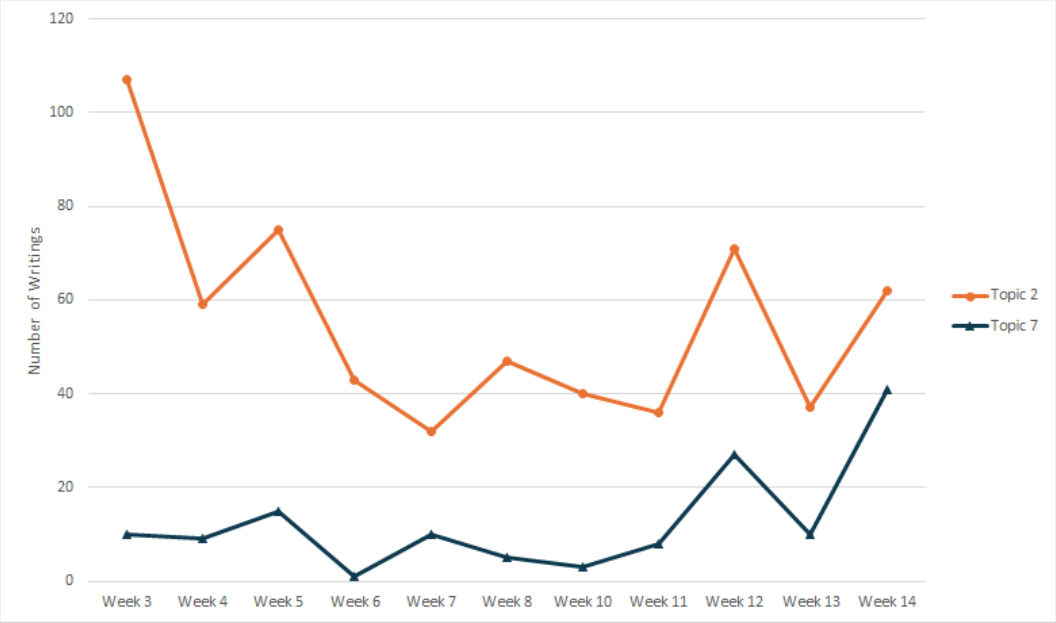

Figure 6. The number of writings covering topics 2 and 7 in writing tasks

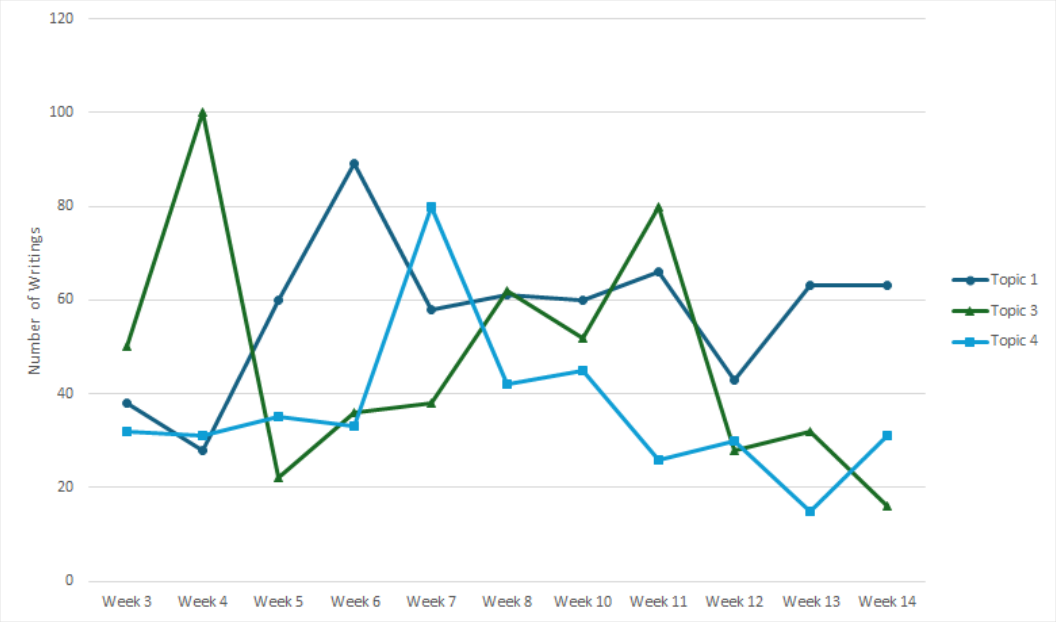

Figure 7. The number of writings covering topics 1, 3, and 4 in writing tasks

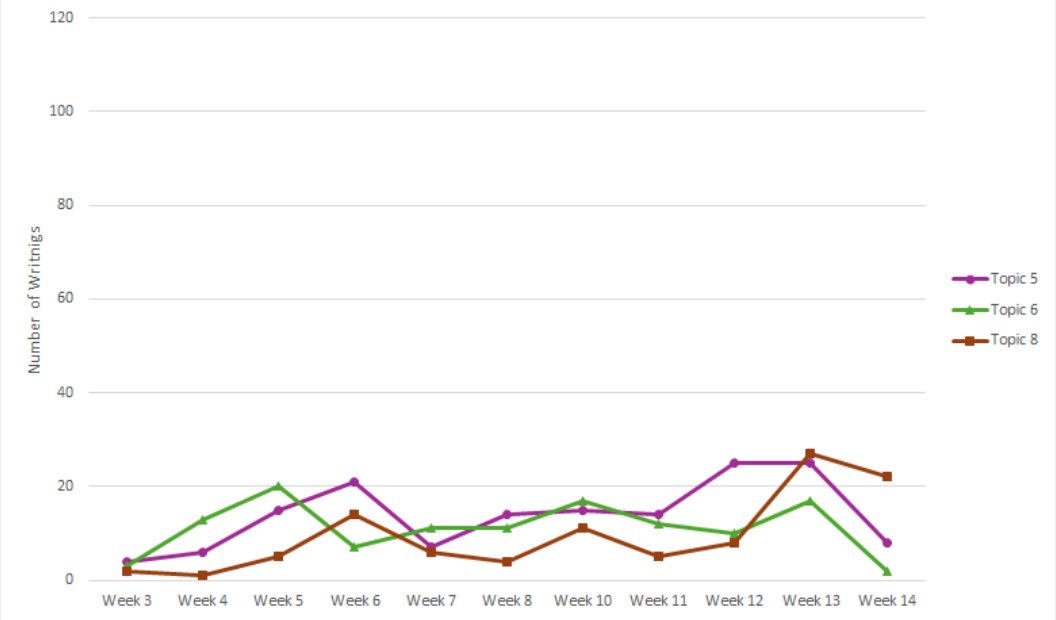

Figure 8. The number of writings covering topics 5, 6, and 8 in the writing tasks

Differences in Writing Topics across Majors of Pre-Service Teachers

Table 4 presents the distribution of topics across different majors of pre-service teachers, highlighting noticeable variations in topic prominence among disciplines. Topics 1 and 2 emerged as the most prominent across all fields, particularly in mathematics and

Korean language, where they accounted for 24–26% of the writings. Social studies majors showed a stronger emphasis on Topic 2, with 31% of their writings focused on this area. In contrast, Topic 7 had the lowest representation across most majors, ranging from 5–6%, indicating a more specialized or less universally relevant focus.

These differences in writing preferences likely reflect the unique characteristics and pedagogical priorities of each subject area. Certain topics may naturally align with the emphasis of specific educational fields, shaping how pre-service teachers approach their writing tasks. These patterns suggest that pre-service teachers' writing is not only influenced by individual perspectives but also by the broader instructional goals and subject-specific emphases of their academic disciplines.

Table 4. Distribution of topics across different majors of pre-service teachers

	Science	**Korean Language**	**Social Studies**	**Mathematics**	**Ethics**
Topic 1	75 (24%)	172 (26%)	46 (16%)	207 (24%)	158 (25%)
Topic 2	62 (20%)	147 (22%)	89 (31%)	209 (24%)	128 (20%)
Topic 3	67 (21%)	127 (19%)	55 (19%)	154 (18%)	124 (19%)
Topic 4	45 (14%)	96 (14%)	41 (14%)	135 (15%)	97 (15%)
Topic 5	20 (6%)	32 (5%)	21 (7%)	49 (6%)	35 (5%)
Topic 6	16 (5%)	34 (5%)	14 (5%)	45 (5%)	27 (4%)
Topic 7	13 (4%)	38 (6%)	14 (5%)	42 (5%)	36 (6%)
Topic 8	17 (5%)	24 (4%)	5 (2%)	32 (4%)	32 (5%)
Total	315 (100%)	670 (100%)	285 (100%)	873 (100%)	637 (100%)

VI. DISCUSSION AND CONCLUSION

The findings of this study highlight several important trends in how pre-service teachers engage with and reflect on various educational topics over time. One of the key observations is the evolving focus of the pre-service teachers as they progressed through the semester. Initially, the pre-service teachers actively selected a wide range of topics, which allowed them to cover diverse areas of interest. This suggests that the freedom to choose topics gave them the opportunity to explore different aspects of teaching and learning, thereby broadening their understanding of these areas. However, as time went on, certain topics became less prominent in their writings. This decline could indicate that, as their understanding deepened, they either felt more confident in those areas or shifted their focus to new challenges and concepts that required further exploration.

Another significant observation is the strong emphasis placed on the role of the

teacher, particularly in topics related to teacher-student interaction and the teacher's role in learner-centered instruction (Topic 1). This suggests that pre-service teachers are primarily focused on understanding their future responsibilities and the impact they will have in the classroom. Interestingly, there appears to be less attention paid to the perspective of students, which could indicate a need for more emphasis on understanding students' needs and learning processes in teacher education programs. The focus on teacher-centered topics might also reflect the nature of mathematics education, where teacher-led instruction is often more prominent.

The pre-service teachers' shifting attention suggests that their experiences and reflections are dynamic, influenced by their growing understanding and the changing context of their learning environment. This dynamic process underscores the importance of allowing pre-service teachers the autonomy to explore various topics and adapt their focus as they progress in their training.

This study has several limitations that warrant consideration. First, the data was derived from a single cohort of pre-service teachers at one institution, which may limit the generalizability of the findings to other contexts. However, the study's design and insights offer a transferable framework that could be adapted by other teacher education programs, particularly those seeking to utilize reflective writing and topic modeling to understand pre-service teacher development. Second, the analysis focused on Korean-language writings, which means the findings may not fully translate to contexts with different linguistic structures or educational practices. Additionally, the decision to analyze only nouns may have excluded some nuanced reflections, as topic modeling, while providing valuable insights into overarching trends, may not fully capture the depth of individual expressions. To address these limitations, incorporating complementary qualitative methods, such as interviews or focus groups, could provide a richer and more comprehensive understanding of pre-service teachers' perspectives.

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