Vol. 8, No. 6, December 2024, pp. 659-663

Current Optics and Photonics

Regular Paper

Nonlinear Effects of a Laser Phase-conjugate Wave and Synchronization Transmission of a Signal

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(Received September 19, 2024 : revised October 31, 2024 : accepted November 4, 2024)

In this paper, a simple and fast synchronization-transmission technology is designed to realize the synchronization transmission of a laser phase-conjugate wave to a target signal. First, the nonlinear effects of a laser phase-conjugate wave are analyzed. On this basis, a unique synchronization-transmission technology is designed. Finally, the effectiveness of the synchronization-transmission technology is verified by numerical calculation. This technology does not need to calculate the Lyapunov exponent of the system, nor to design the Lyapunov function. Moreover, the technology has no limitation on the selection of the target signal.

Keywords : Nonlinear effect, Phase conjugate wave, Synchronization transmission
OCIS codes : (190.3100) Instabilities and chaos; (190.4380) Nonlinear optics, four-wave mixing; (190.4410) Nonlinear optics, parametric processes; (190.4420) Nonlinear optics, transverse effects in; (190.5040) Phase conjugation

I. INTRODUCTION

A laser phase-conjugation wave is a new light-wave field generated by four-wave mixing, three-wave mixing, or stimulated scattering in nonlinear optical media. Because the phase-conjugate wave can compensate for the wave-front distortion of a laser system and yield high-brightness laser output, it shows attractive application prospects in optical communication, image processing, real-time information processing, and many other areas [1–5]. In particular, the use of laser systems such as the phase-conjugated wave has a unique role in remote communication, and in synchronization transmission and conversion of a relay signal. Therefore, the related research in this field has become a hot spot. People have done fruitful research and designed many effective synchronization-transmission technologies, including the master stability function criterion [6], adaptive control [7, 8], pinning technique [9], impulsive control [10], the open-loop and closed-loop technique [11], etc. There are many types of synchronization transmission, such as complete synchronization, cluster synchronization, lag synchronization, and finite-time synchronization [12–16].

Although there are all kinds of transmission technologies mentioned above for the synchronization transmission of a signal, the criteria for realizing synchronization transmission are nothing more than calculating the Lyapunov exponent of the system and designing the Lyapunov function. However, for nonlinear systems such as a phase-conjugate wave, the calculation of the Lyapunov exponent and the design of the Lyapunov function are very complicated, which is not conducive to practical application. Therefore, it is necessary to improve the original synchronization-transmission technologies and propose new, practical, and effective synchronization-transmission schemes.

Based on the above discussion, we design a simple and fast synchronization-transmission technology to realize the synchronization transmission of a laser phase-conjugate wave to a target signal. The features of this technology are

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pISSN: 2508-7266 / eISSN: 2508-7274
https://doi.org/10.3807/COPP.2024.8.6.659

that one does not need to calculate the Lyapunov exponent of the system, nor to design the Lyapunov function. The only work needed is to determine the range of adjustable parameters. Moreover, the technology has no limitation on the selection of the target signal.

This work is presented as follows: In Section 2, we analyze the nonlinear effects of a laser phase-conjugate wave. Section 3 gives the synchronization-transmission criterion. At last, conclusions are summarized.

II. NONLINEAR EFFECTS OF A LASER PHASE-CONJUGATE WAVE

The structure of the phase-conjugate resonator is shown in Fig. 1. It is composed of a normal mirror (NM), a phase-conjugate mirror (PCM), and a lossless Kerr medium that can achieve multilevel scattering as a phase-conjugate medium.

In Fig. 1, A_1 and A_2 are two pump beams in the process of four-wave mixing. The detection light field is A_4, and its phase-conjugate wave is A_3. For the lossless Kerr medium of the phase-conjugate mirror, the Raman-Nath approximation is adopted, and the scattered lights in the zeroth-order and first-order approximations are, respectively [17]:

$$A_3 = J_0(2\left|A_2 A_4^*\right|)e^{i[|A_2|^2+|A_1|^2+|A_4|^2]}, \tag{1}$$

$$A_3 = iJ_1(2\left|A_2 A_4^*\right|)\frac{A_2 A_4^*}{\left|A_2 A_4^*\right|}e^{i[|A_2|^2+|A_1|^2+|A_4|^2]}, \tag{2}$$

where J_0 and J_1 are the zeroth-order and first-order Bessel functions, respectively. Based on Eqs. (1) and (2), an iterative mapping can be constructed for the detection light field A_4. If the phase of the probe field A_4 is $\theta(n)$ and the intensity is $I(n) = |A_4(t + n\tau)|^2$, then the iterative mapping can be obtained for 0^{th}-order reflection:

$$I(n+1) = RI_1 J_0^2\{2[I_2 I(n)]^{1/2}\}, \tag{3}$$

$$\theta(n+1) = [2kL + \theta_1 + I_1 + I_2 + I(n)] \bmod 2\pi. \tag{4}$$

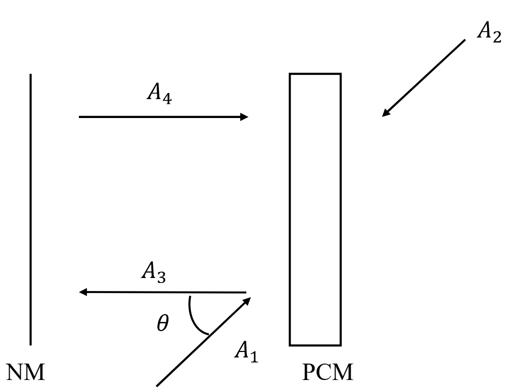

FIG. 1. Structure of a phase-conjugate resonator.

For first-order reflection, the form of the mapping is:

$$I(n+1) = RI_1 J_1^2\{2[I_2 I(n)]^{1/2}\}, \tag{5}$$

$$\theta(n+1) = [2kL + \theta_1 + \theta_2 + I_1 + I_2 + I(n) - \theta(n)] \bmod 2\pi, \tag{6}$$

where L is the cavity length, R is the reflectivity of the ordinary mirror, $2kL$ is the phase shift of the light in the cavity, I_1 and I_2 are the intensities of the two pumps, θ_1 and θ_2 are the phases of the two pumps, and n is the iteration step. Here we only consider the light intensity, refer to $x(n)$, and let $x(n) = 4I_2 I(n)$, $M = 4RI_2 I_1$; Then Eqs. (3) and (5) can be expressed as

$$x(n+1) = MJ_{0/1}^2[x^{1/2}(n)]. \tag{7}$$

If we consider the threshold effect or switching behavior, we can get the mapping with two parameters:

$$x(n+1) = MJ_{0/1}^2\{[x(n) - B]^{1/2}\}, \tag{8}$$

where B is the threshold parameter.

We use numerical simulation to show the nonlinear effects of a phase-conjugated wave. First, the evolution of the state variable with the threshold parameter B in the zeroth-order Bessel function is given when the pump-light intensity $M = 10$, as shown in Fig. 2. It can be seen that the system begins in a single-period state, and through period-doubling bifurcation enters the two-period and four-period states near the threshold-parameter values $B = -2.35$ and $B = -0.35$, respectively. The period merger occurs near $B = 2.3$. When $B = 2.65$ the two-period curves jump, and then these nonlinear effects continue to appear.

When the pump intensity $M = 15$, the nonlinear effects of the laser phase-conjugate wave are more abundant. Figure 3 shows that the system can not only exhibit nonlinear effects such as period-doubling bifurcation and period merging, but also exhibits chaotic behavior with random-

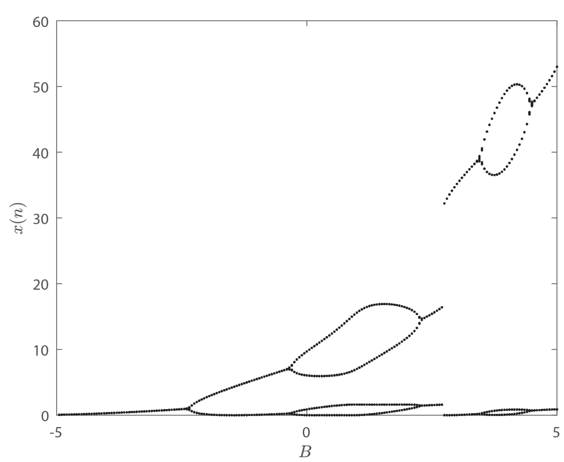

FIG. 2. Evolution of state variables with threshold parameter B ($M = 10$).

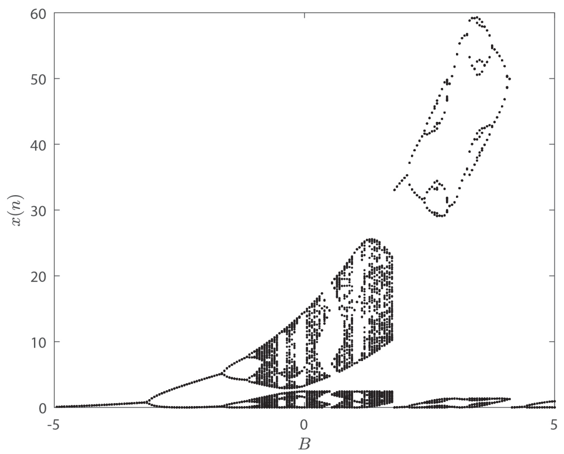

FIG. 3. Evolution of state variables with threshold parameter B (M = 15).

ness when the threshold parameter B is between −1 and 1.7. When the pump intensity M takes on other values, these nonlinear effects will also appear.

III. MECHANISM OF SIGNAL SYNCHRONIZATION TRANSMISSION

We use the laser phase-conjugate wave to transmit the target signal synchronously. The transmission system is

$$x(n+1) = MJ_0^2\{[x(n)-B]^{1/2}\} + \varphi(n), \tag{9}$$

where $\varphi(n)$ is the control input.

Suppose the target signal is sent by

$$s(n+1) = f[s(n)]. \tag{10}$$

If the error between the state variable of the transmission system and the target signal is defined as $e(n) = x(n) - s(n)$, then the following relation can be obtained:

$$e(n+1) = MJ_0^2\{[x(n)-B]^{1/2}\} - f[s(n)] + \varphi(n). \tag{11}$$

When $\lim_{n\to\infty}|x(n)-s(n)| \to 0$, the laser phase-conjugate wave can transmit the target signal synchronously. The above formula is equivalent to the inequality $|e(n+1)| < |e(n)|$. Obviously, the inequality can be decomposed into the following two expressions.

$$e(n)[e(n+1) - e(n)] < 0, \tag{12}$$

and

$$e(n)[e(n+1) + e(n)] > 0. \tag{13}$$

We design a control input

$$\varphi(n) = -MJ_0^2\{[x(n)-B]^{1/2}\} + f[s(n)] + \delta e(n), \tag{14}$$

where δ is the adjustable parameters.

By substituting Eqs. (11) and (14) into (12), the following relation can be deduced:

$$\begin{aligned} &e(n)[e(n+1)-e(n)] \\ &= e(n)\{MJ_0^2[(x(n)-B)^{1/2}] - f[s(n)] + \varphi(n) - e(n)\} \\ &= (\delta-1)e^2(n) < 0. \end{aligned} \tag{15}$$

It can be seen that the parameter δ must satisfy $\delta < 1$.

Similarly, by substituting Eqs. (11) and (14) into (13), the following relation can be deduced.

$$\begin{aligned} &e(n)[e(n+1)+e(n)] \\ &= e(n)\{MJ_0^2[(x(n)-B)^{1/2}] - f[s(n)] + \varphi(n) + e(n)\} \\ &= (\delta+1)e^2(n) > 0. \end{aligned} \tag{16}$$

It can be seen that the parameter δ must satisfy $\delta > -1$.

Since the synchronization transmission between the laser phase-conjugate wave and the target signal must satisfy Eqs. (12) and (13), the range of the adjustable parameters. δ is determined:

$$-1 < \delta < 1. \tag{17}$$

We use numerical simulation to verify the effectiveness of the above synchronization-transmission technology. The target signal is taken arbitrarily, and here we take the Gibbs laser model [18],

$$s(n+1) = A\sin^2(s(n) - s_b), \tag{18}$$

where A and s_b are the parameters, with $A = 3$ and $s_b = 0.85\pi$.

The signal-transmission system is described by Eq. (9), the control input is described by Eq. (14), and the adjustable parameters δ is arbitrarily taken to be 0.8, within the value range determined. The error between the state variable of the transmission system and the target signal is shown in Fig. 4. It can be seen that as long as the value of δ is within the determined range, the error tends to zero steadily after a short time evolution, which means that as a transmission system, the laser phase-conjugate wave has effectively transmitted the target signal.

To demonstrate that this technology has no limitation on the selection of target signal, we choose the laser phase-conjugate wave to replace the Gibbs laser model as the target signal:

$$s(n+1) = MJ_0^2\{[s(n)-B]^{1/2}\}. \tag{19}$$

We repeat the above simulation process with all system parameters unchanged, and the adjustable parameters δ arbitrarily taken to be −0.6, within the value range determined. From Fig. 5, it can be observed that the error still tends to zero steadily, which means that the stability of signal synchronization transmission is still quite ideal.

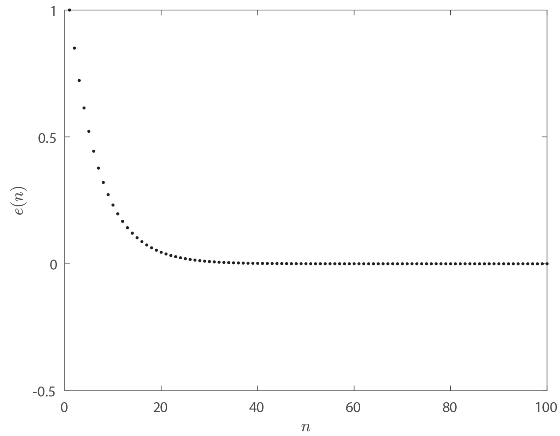

FIG. 4. Error between the state variable of the transmission system and the target signal ($\delta = 0.8$).

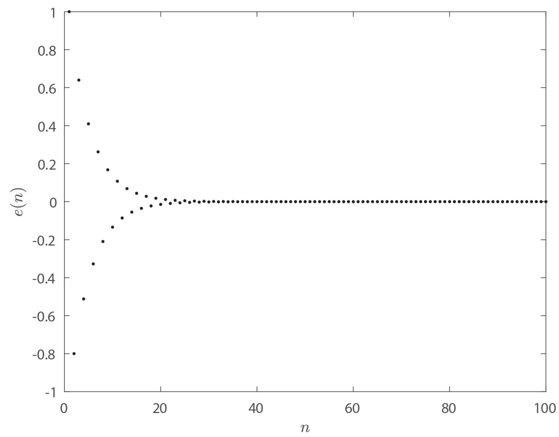

FIG. 5. Error between the state variable of the transmission.

IV. CONCLUSIONS

We have analyzed the nonlinear effects of a laser phase-conjugate wave and completed synchronization transmission of the laser phase-conjugate wave to the target signal. The results show that the laser phase-conjugate wave can exhibit not only period-doubling bifurcation and period merging, but also chaotic behavior with randomness. Furthermore, as a transmission system the laser phase-conjugate wave can effectively transmit the target signal, as long as the value of the adjustable parameters is within the determined range.

FUNDING

This research was supported by the National Natural Science Foundation of China (Grant No. 11747318), and the Science and Technology Innovation Team of Dalian University of Science and Technology, China (Grant No. KYTD202202).

DISCLOSURES

The authors declare no conflicts of interest.

DATA AVAILABILITY

The authors state that the data of the current study will be made available on reasonable request.

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