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Assessing the Impact of Network Effects on Brand Choice in the Growth Market: A Multi-Brand Diffusion Model

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Abstract

This study investigates network effects to measure how strongly the early adopters affect the brand choice of the potential consumer. By using the Gumbel-Hougaard (GH) copula, this study checks the magnitude of network effects varied from country to country. To consider consumer heterogeneity and network effects in the growth market, this study proposes the multi-brand Gamma/Shifted-Gompertz (m-G/SG) model based on the GH copula. Out of eighteen Western European cellular phone market data and South Korea smartphone data sets, the m-G/SG model provides an improvement in the estimation accuracy over the Libai, Muller, and Peres model. The results show that network effects enhance (i) the polarization of brand choice probabilities as time elapses; (ii) the dominance of the more preferred and the earlier entered brand; and (iii) the deceleration of category-level diffusion. Potential followers can analyze their relationship with earlier entrants through the m-G/SG model and also establish an optimal market entry strategy.

Keywords*: New Product, Competition, Heterogeneity, Network Effects*

1. INTRODUCTION

Studies have long addressed models that predict the demand for new products. The recent technological advances shorten product life cycles. To keep up with the rapid changes in the market, manufacturers invest more money into research and development (R&D). Hence, it is of crucial interest for marketing managers to formulate investment decisions or marketing activities to investigate the diffusion of new products. Entry of brands accelerates the diffusion of new products [1]; it advances the time to adopt new products. If so, how long time to adopt the entry of brands can reduce? Further, how can the entry of a new brand affect consumer's adoption behavior? There are two kinds of unobservable factors that can be captured by network effects in the brand choice, and consumer heterogeneity in the purchase incidence.

To investigate competition in the growth category, the brand choice based on consumer preference is combined with the diffusion model [2]. However, they independently predict the demand of a category without consideration of the consumer preference. To reflect the brand-level demand and the category-level demand interdependently, [3] suggest the diffusion model for multiple brands that shows the within-, and cross-brand

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influences. To estimate the within-, and cross-brand influences, the Libai, Muller, and Peres (LMP) model assumes that the within-brand influences on all brands q_i*'s* are the same; $q_i = q$ for all i. Hence, the LMP model has a limitation to capture the effect of the entry of a new brand. In the estimation, the LMP model considers categories including two brands only; it may be inefficient to analyze a category including three or more brands. To overcome the issue from the number of brands, [4] propose a multi-brand model regarding the decision process as a two-stage process. Some consumers may make a decision whether to adopt a category ahead of making up their mind which brand to adopt. On the other hand, for the other consumers in the same category, the time to adopt the category can coincide with the time to choose a brand. However, [4] infer the overall market at the aggregate level; they exclude the possibility that two kinds of consumers can co-exist in a single category. Hence, the individual-level approach will be required to reflect this possibility.

Table 1. Brand-level diffusion models

Authors		Descriptions	Implications
[2]	Lee, et al. (2006)	An application of the conjoint analysis to a brand-level diffusion.	The result shows that TV type that will dominate in the market depends on price.
[3]	Libai, Muller, And Peres (2009)	The two types of interpersonal communication with adopters i) of that brand, and ii) of competing brands.	The “dual pattern”; i) a fast takeoff for a follower, and ii) an interaction-based advantage for the earlier entrant.
[4]	Krishnan, Seetharaman, and Vakratsas (2012)	Three types of the interpersonal communication (IPC) among previous adopters; i) brand-to-brand IPC, ii) brand-to-category IPC, and iii) category-to-category IPC.	The adoption process is composed of a two-stage process; the adoption to the category is ahead of the choice of the brand.
[5]	Krishnan, Bass, and Kumar (2000)	The framework to explore the impact of a late entrant on the diffusion of the category, and of the earlier entrants.	The entry of the late entrant can i) increase the size of market potential, ii) hasten the speed of diffusion.
[6]	Landsman and Givon (2010)	The adoption process composed of two consecutive sub-processes: service consideration and brand choice.	There are two types of non-adopters i) who decided to consider the adoption, and, ii) who decided not to consider.
[7]	Tan et al. (2023)	A multi-generation diffusion model under a duopoly.	The simulation outcomes yield several key observations: i) Influence of Pricing on Revenue ii) Differential Brand Competitiveness iii) Impact of Quality Upgrade on Time-to-Market Decision.
[8]	Guseo, and Mortarino (2011)	A Competition and Regime Change Diachronic (CRCD) model.	The entry of the late entrants induces i) the delay of the earlier entrants, and ii) an expansion of the whole category.

A summary in the extant literature is in Table 1. In the extant literature, the brand-level diffusion is divided by brand choice and category adoption. However, both of them can be correlated; brand choice can depend on category adoption, and vice versa. The entry of a new brand can influence not only on the probability of choosing an existing brand, but also on the time to adopt the category. To capture them simultaneously, this study uses a competing risk approach. Based on that, this study proposes a multi-brand diffusion model; it reflects two decision-making processes – brand choice, and purchase timing – in a single framework.

2. THEORY

"Although some insights into the brand choice process derive from behavioral studies, diffusion modeling can combine a brand choice and an individual-level decision and estimate their relative importance at each stage [9]." To adopt new high-tech products that included to high involvement products, consumers take care of the buying decision process. According to the buying decision, consumers pass through evaluation of alternatives before reaching purchase decision [10]. In the stage of evaluation of alternatives, they choose a brand based on preference for brands, however their adoptions can be postponed due to various reasons. (e.g., perceived risk) To investigate brand-level data, this study considers the purchase of a brand as a two-stage process – choosing a brand in the category and then adopting the category- rather a one-stage model [11].

2.1. The Multi-Brand Choice Model

To model brand choice, this study considers not only the kind of brand, but also the time to adopt. This study follows three stages; firstly focuses on the marginal distributions, secondly derives the joint survival function, and finally evaluates the brand-level survival function.

Let time t_1 be the elapsed time from the entry time of the first brand. Let τ_i be time of i-th brand entry; $0 = \tau_1 \leq \tau_2 \leq \cdots \leq \tau_{B(t_1)}$ where $B(t_1)$ is the number of brands in the market at time t_1. Then, t_i can be defined as the elapsed time from the time at which *i*-th brand is introduced to the time t_1; $t_i = t_1 - \tau_i$. The latent random variable $T_{1,i}$ is the potential time elapsed from the entry time of the *i*-th brand to adopt the brand. The preference for the *i*-th brand can be captured by the positive scale parameter b_i $(i = 1, \dots, B(t_1))$. Since preferences are consistent [12], this study assumes that b_i is constant (memory-less) across individual customers; a brand-specific time invariant parameter. To specify the incidence of choosing a brand, a binary variable can be modeled by an exponential distribution for the time of incidence [13]. Hence, this study assumes that $T_{1,i}$ follows an exponential distribution with a constant parameter b_i. Then the net hazard for *i*-th brand, and the marginal survivor function for $T_{1,i}$ are defined as $h_i(t_i)$, and $S_i(t_i)$, respectively.

$$S_i(t_i) = P\left(T_{1,i} > t_i\right) = e^{-b_i t_i}. \tag{1}$$

From equation (1), the larger b_i is, the stronger the preference, and then the faster the adoption.

Due to the entry of a new brand, the potential time to adopt the existing brands can decrease or remain as they are. For example, it is unexpected that the entry of Android shortens a consumer's potential time to adopt iOS, but lengthens her potential time to adopt Windows Mobile. Since $T_{1,1}, \dots, T_{1,i-1}$ can be shifted in the same direction due to the entry of the *i*-th brand, this study assumes that there is the non-negative correlation among the potential times to adopt brands. In order to establish the joint distribution of the latent times to adopt, $S(t_{1,} \dots, t_{B(t_1)})$, this study uses a copula; it describes the interdependence among time-varying variables but differs from a correlation in that the latter describes the dependence among time-invariant variables [14]. This study applies the Gumbel-Hougaard (GH) copula to account for the positive (or no) correlation among $T_{1,i}$'s. The GH copula serves a dual function; i) it connects the joint survival function to its marginal survival functions in a manner that is completely analogous to the way a copula connects the joint distribution function to its margins, and ii) it allows that the initial times at which multiple brands are launched into a growing market may not coincide.

$$P\left(T_{1,1} > t_1, \dots, T_{1,B(t_1)} > t_{B(t_1)}\right) = C_\gamma\left(u_1, \dots, u_{B(t_1)}\right),$$

where $u_i \in [0,1]$. C_γ is the GH copula:

$$= \exp\{-[\textstyle\sum_{i=1}^{B(t_1)} (b_i t_{1,i})^\gamma]^{1/\gamma}\}. \tag{2}$$

From equation (2), the dependence parameter γ is ranged from 1 to ∞. Ultimately, this study is interested in the effect of entry of a follower on the time to adopt the category. To investigate how long time it takes to adopt, and which brand is chosen, this study suggests a competing risk. Since high-tech products are generally adopted at most once per consumer, this study assumes that i) a consumer adopts only one brand in which the potential time to be chosen is shortest, and ii) there is no substitution because competing alternatives are in the same generation (no difference in the level of technology). Let the random variable T_1 be the potential time elapsed from the entry time of a category to adopt any brand in the category.

$$T_1 = \min(T_{1,1} + \tau_1, \dots, T_{1,i} + \tau_i, \dots, T_{1,B(t_1)} + \tau_{B(t_1)}). \tag{3}$$

From equation (3), this study can derive the shortest time to adopt, as well as a brand index which tells us which of the $T_{1,i}{}'\text{s}$ is chosen, and the chosen brand can be observed by $\underset{i}{\operatorname{argmin}}(T_{1,1} + \tau_1, \dots, T_{1,i} + \tau_i, \dots, T_{1,B(t_1)} + \tau_{B(t_1)})$. The time *t* at which the category is introduced is equal to the entry time of the first brand in the category. Hence, the time *t* elapsed from the introduction of the category is equal to t_1. This study investigates how the entry of other brands affects the joint survival function.

$$\boldsymbol{S}(t) = P(T_1 > t),$$

from equation (3),

$$= P\big(T_{1,1} > t_1, \dots, T_{1,B(t_1)} > t_{B(t_1)}\big),$$

from equation (2),

$$= \exp\{-[\textstyle\sum_{i=1}^{B(t_1)} (b_i t_i)^\gamma]^{1/\gamma}\},$$

since t_i is equal to $t - \tau_i$ and $B(t_1) = B(t)$,

$$= \exp(-\{\textstyle\sum_{i=1}^{B(t)} [b_i(t - \tau_i)]^\gamma\}^{1/\gamma}). \tag{4}$$

From equation (4), the survivor function of the time to choose any brand $\boldsymbol{S}(t)$ can be derived such as [15].

2.2. The Multi-Brand Diffusion Model at the Individual-Level

For the adoption of non-durable goods, brand choice and purchase timing can occur simultaneously. However, brand choice and purchase timing of durable goods may not coincide. Although an individual already choose a brand, she may hesitate to adopt the durable good yet due to perceived risk; the amount of perceived risk varies with the amount of money at stake, the amount of attribute uncertainty, and the amount of consumer self-confidence [10]. Since T_1 is based on the homogeneous population, this study assumes that the time to adopt a category can be mediated by an unobserved heterogeneity; the latent random variable T_2 denotes the elapsed time at which an individual consumer adopts the category conditional on the intrinsic tendency to adopt late η. Since a perceived risk at the purchase timing process is unrelated to the entry of a new brand, this study can assume that the intrinsic tendency to adopt late affects not to T_1, but to T_2. To capture the category-level diffusion, [16] suggest the Gompertz process as a classical S-shape growth curve. Hence, this study assumes the corresponding product adoption process as the Gompertz distribution. Because the Gompertz distribution is a reverted Gumbel distribution, this study assumes that T_2 follows the Gumbel distribution with parameter η and $\{\sum_{i=1}^{B(t)} [b_i(t - \tau_i)]^\gamma\}^{1/\gamma}$; the Gumbel distribution for T_2 is as follows:

$$P(T_2 \le t \mid \eta) = \exp(-\eta e^{-\{\sum_{i=1}^{B(t)} [b_i(t-\tau_i)]^\gamma\}^{1/\gamma}}). \tag{5}$$

From equation (5), a positive parameter η is a continuous variable that varies across the population within the category; the parameter η is at the individual level. The higher η, the higher the expected adoption time is [17]. Let the observable random variable T be the elapsed time at which an individual consumer adopts the chosen brand conditional on η. For the consumers whose η is low, time to adopt a brand T will be the equal to the time to choose the brand T_1. For the other consumers who show high η, adopting a brand is delayed from choosing the brand; the time to adopt a brand T will be followed by T_1, and will be the equal to the time to adopt the category conditional on η, T_2; T is defined as the largest of two independent random variables T_1 and T_2:

$$T = \max(T_1, T_2). \tag{6}$$

From equation (6), both T_1 and T_2 are times but only the largest value T is observed because the time to adopt can be observed after choosing a brand and overcoming the tendency to adopt late. The shift can be due to the availability (or lack thereof) of the new product. To reflect the probability that an individual consumer adopts the chosen brand, the shifted-Gompertz (SG) model [18] is considered as a function on the individual-level.

$$F(t \mid \eta) = P(T \leq t \mid \eta)$$

From equation (6),

$$= P(\max(T_1, T_2) \leq t \mid \eta)$$

Since T_1 and T_2 are independent,

$$= P(T_1 \leq t)P(T_2 \leq t \mid \eta)$$

From equations (4) and (5),

$$= [1 - \exp(-\{\textstyle\sum_{i=1}^{B(t)}[b_i(t-\tau_i)]^\gamma\}^{1/\gamma})]\exp(-\eta e^{-\{\sum_{i=1}^{B(t)}[b_i(t-\tau_i)]^\gamma\}^{1/\gamma}}), \tag{7}$$

From equation (7), $F(t \mid \eta)$ is the SG model for the chosen brand at time t with shape parameter η ($\eta > 0$).

2.3. The Multi-Brand Diffusion Model at the Aggregate-Level

This study assumes that η varies according to a gamma distribution with shape parameter α and scale parameter β across consumers:

$$k(\eta) = [1/(\beta^\alpha\Gamma(\alpha))]\eta^{\alpha-1}e^{-\eta/\beta},\ \eta > 0,\ \alpha, \beta > 0. \tag{8}$$

From equation (8), the parameter α captures the degree of consumer heterogeneity: as α approaches zero, consumers become more heterogeneous.

$$F(t) = \int_0^\infty F(t \mid \eta)k(\eta)\,d\eta$$

$$= [1 - \exp(-\{\textstyle\sum_{i=1}^{B(t)}[b_i(t-\tau_i)]^\gamma\}^{1/\gamma})]/[1 + \beta\exp(-\{\sum_{i=1}^{B(t)}[b_i(t-\tau_i)]^\gamma\}^{1/\gamma})]^\alpha. \tag{9}$$

From equation (9), this study addresses the cumulative density function (cdf) of multi-brand Gamma/Shifted-Gompertz (m-G/SG) model to derive a closed-form equation at the aggregate level.

$$F(t) = \{1 - \exp[-\textstyle\sum_{i=1}^{B(t)} b_i(t-\tau_i)]\}/\{1 + \beta\exp[-\sum_{i=1}^{B(t)} b_i(t-\tau_i)]\}^\alpha \tag{10}$$

From equation (10), the m-G/SG model reduces to the constrained model when γ is equal to 1.

In addition, when α is equal to 1 with the simultaneous entry of brands, the equation (10), the constrained m-G/SG model coincides with the model proposed by [5]. Because the interaction depends on the length of time, the adoption times of brands are correlated. To check the effect of the entry of the followers, this study checks the interaction among the times to adopt brands; the choice probability of *i*-th brand in the category at time *t*.

$$f(t) = \{\textstyle\sum_{i=1}^{B(t)} b_i[b_i(t-\tau_i)]^{\gamma-1}\}\{\sum_{i=1}^{B(t)}[b_i(t-\tau_i)]^\gamma\}^{(1/\gamma)-1}\exp(-\{\sum_{i=1}^{B(t)}[b_i(t-\tau_i)]^\gamma\}^{1/\gamma})$$

$$[1 + \beta\exp(-\{\textstyle\sum_{i=1}^{B(t)}[b_i(t-\tau_i)]^\gamma\}^{1/\gamma})]^{-\alpha-1}[1 + \alpha\beta + \beta\exp(-\{\sum_{i=1}^{B(t)}[b_i(t-\tau_i)]^\gamma\}^{1/\gamma})(1-\alpha)]. \tag{11}$$

Equation (11) represents the probability density function (pdf) of the m-G/SG model.

Let $f_i(t)$ be the pdf of the m-G/SG model for *i*-th brand at the aggregate level.

$$f_i(t)/f(t) = b_i[b_i(t-\tau_i)]^{\gamma-1}/\textstyle\sum_{j=1}^{B(t)} b_j[b_j(t-\tau_j)]^{\gamma-1},\ t > \tau_i. \tag{12}$$

From equations (11) and (12),

$$f_i(t) = b_i[b_i(t-\tau_i)]^{\gamma-1}\{\sum_{j=1}^{B(t)}[b_j(t-\tau_j)]^{\gamma}\}^{(1/\gamma)-1}\exp(-\{\sum_{j=1}^{B(t)}[b_j(t-\tau_j)]^{\gamma}\}^{1/\gamma})$$

$$[1+\beta\exp(-\{\sum_{j=1}^{B(t)}[b_j(t-\tau_j)]^{\gamma}\}^{1/\gamma})]^{-\alpha-1}[1+\alpha\beta+\beta\exp(-\{\sum_{j=1}^{B(t)}[b_j(t-\tau_j)]^{\gamma}\}^{1/\gamma})(1-\alpha)]. \quad (13)$$

From equation (13), the choice probability for the *i*-th brand conditioned upon buying the category at time *t* at the aggregate level can be measured. Let $F_i(t)$ be the cdf of the m-G/SG model for *i*-th brand at the aggregate level. From the equation (13),

$$F_i(t) = \int_0^t f_i(s)\mathrm{ds}, \quad (14)$$

where $F(t) = \sum_i F_i(t)$. Equation (14) represents the market share of *i*-th brand at time *t*.

2.4. Network Effects

Network effects mean that the increase in a consumer's utility from a product when the number of other users of that product increases [19]. From the definition, network effects corresponded by a particular brand are positively proportional to the adoptions of the brand; the network size. Network effects can occur due to cellular operators when they charge the discounted fees for within than for cross operator calls. The mobile phone users whose contacts all use the same cellular operator pay low fares for the calls. Hence, communications in cellular phone market are characterized as direct network effects [20]. Moreover, installed-base users are somewhat tied to the incumbent brand. It creates a bias against the new brand [21].

To extract the network size of a particular brand, this study controls the interaction among brands. The network size of the *i*-th brand $\widetilde{N}_i(t)$ is proportional to the marginal cdf $\tilde{F}_i(t)$ [15].

$$G(x) = (1-\exp(-x))/(1+\beta\exp(-x))^{\alpha}. \quad (15)$$

From equation (15), G is an invertible function that maps [0,∞] onto [0,1].

$$\tilde{F}_i(t) = \{1-\exp[-b_i(t-\tau_i)]\}/\{1+\beta\exp[-b_i(t-\tau_i)]\}^{\alpha} = G\big(b_i(t-\tau_i)\big), \quad (16)$$

From equation (16), the marginal cdf $\tilde{F}_i(t)$ can be expressed as the function G. Preference as measured by b_i and the time elapsed since launch, $t-\tau_i$, play important roles in measuring the network size.

$$G^{-1}\left(\tilde{F}_i(t)\right) = b_i(t-\tau_i). \quad (17)$$

From equation (17), $b_i(t-\tau_i)$ is equal to the quantile function of the network size of the i-th brand $G^{-1}(\tilde{F}_i(t))$. Hence, the network size corresponded with a particular brand is positively proportional to $b_i(t-\tau_i)$. [22] has mentioned the existence of path dependence - earlier adopters' decisions can influence on the decisions of later adopters (e.g., consumers' choices of video-recorder formats). An early dominance of earlier entrants might give rise to the failure of subsequent more preferred followers. Network effects are at the origin of path dependence. This study suggests that network effects for the *i*-th brand can be expressed as $[b_i(t-\tau_i)]^{\gamma-1}$, where network effects are positively proportional to the network size, and the dependence parameter γ represents the degree of path dependence. Network effects $[b_i(t-\tau_i)]^{\gamma-1}$ serve as an improvement of preference for the *i*-th brand. Recency in the market has to be compensated by a relatively higher preference because of a short run premium to the earlier entrants as captured by $t-\tau_1$. Regardless of the degree of path dependence γ, the ratio of $(t-\tau_i)^{\gamma-1}$ to $(t-\tau_j)^{\gamma-1}$ converges to 1 as *t* approaches ∞. The more preferred brand can enjoy more network effects in the long run because there is no premium from the earlier entrance.

3. EXPERIMENTS

3.1. Data

Table 2. Description of the Data Set

This table describes that the entry time of each operator for individual country. The fourth entrant is omitted.

Data set/ Country	Observed period	First entrant		Second entrant		Third entrant	
		Operator	Entry Time	Operator	Entry Time	Operator	Entry Time
Cellular phone							
Austria	'93 - '98	PTV	Dec. '93	Maxmobil	Oct. '96		
Belgium	'94 - '98	Belgacom	Jan. '94	Mobistar	Aug. '96		
Denmark	'92 - '98	TeleDenmark	July '92	Sonefon	July '92	Telia	Jan. '98
Finland	'92 - '98	Sonera	July '92	Radiolinja	July '92	Finnet G	Feb. '98
France	'92 - '98	FT	July '92	SFR	Dec. '92	Bouygues	May '96
Germany	'92 - '98	Mannesmann	June '92	T-Mobil	July '92	E-Plus	May '94
Greece	'93 - '98	Panafon	July '93	Stet-Hellas	July '93	Cismote	May '98
Iceland	'94 - '98	PTT	Aug. '94	TAL	May '98		
Ireland	'93 - '98	Eircell	July '93	Eset DigiP	Mar '97		
Italy	'92 - '98	Telecom Italia	Oct. '92	Omnitel	Oct. '95		
Luxemburg	'93 - '98	PTT	July '93	Tango	Aug '98		
Holland	'94 - '98	PTT Telecom	July '94	Libertel	Sept. '95		
Norway	'93 - '98	Telenor	May '93	Netcom	Sept. '93		
Portugal	'92 - '98	Telecel	Oct. '92	TMN	Dec. '92	Optimus	Sep. '98
Spain	'95 - '98	Telefonia M	July '95	Spain Airtel	Oct. '95		
Sweden	'92 - '98	Comviq	Sep. '92	Telia	Nov. '92	Nordic T	Sep. '93
Suiss	'93 - '98	Swisscom (900)	Mar '93	(1800)	Mar '95		
UK	'92 - '98	Vodafone	July '92	One-2-One	Sep. '93	Cellent	Jan. '94
Smartphone							
S. Korea[a]	'07 - '11	Windows Mobile	Nov. '09	iOS	Dec. '09	Android	Jan. '10

[a] July 2007 is the announcement date of iOS in the U. S.

This study uses two data sets (Table 2). First, the Western European cellular phone market, which were included in two or more service providers are under the competition structure. To ignore the generation change, this study restricted the data sets to a single generation - Global System for Mobile Communications (GSM). The publisher of the Global Mobile newsletter is Informa in the UK that provided the Western European cellular phone data sets. Second, this study uses a smartphone as a category to investigate the diffusion of high-technology products. The smartphone category is divided into sub-categories (Android, iOS, Windows Mobile) based on mobile OS. To explore adoptions of the smartphone including a particular mobile OS, the mobile OS-level smartphone subscription data in South Korea is used. To ignore the repeat purchase, an observation period of the data set is 2 years – the usual stipulated time period – from October 2009 to October 2011. In addition, there is not a big difference in price among competing smartphones within this period. The Windows Mobile by Microsoft is a market early mover in the time origin of the smartphone category (December 2008). The iOS followed on December 2009. Lastly, Android followed after a month later. ZDNet Korea (http://www.zdnet.co.kr) provided the data in South Korea.

3.2. Estimation

This study assumes that a maximum of one individual-level adoption occurs in a given period. Let m be the number of eventual adopters. Since the m-G/SG model is based on a competing risk, this study uses the maximum likelihood estimation (MLE) [15].

$$L = \prod_{j=1}^{m} \{ \prod_{i=1}^{B(t_j)} [f_{ji}(t_j)/S_j(t_j)]^{\delta_{ji}} \} S_j(t_j), \tag{18}$$

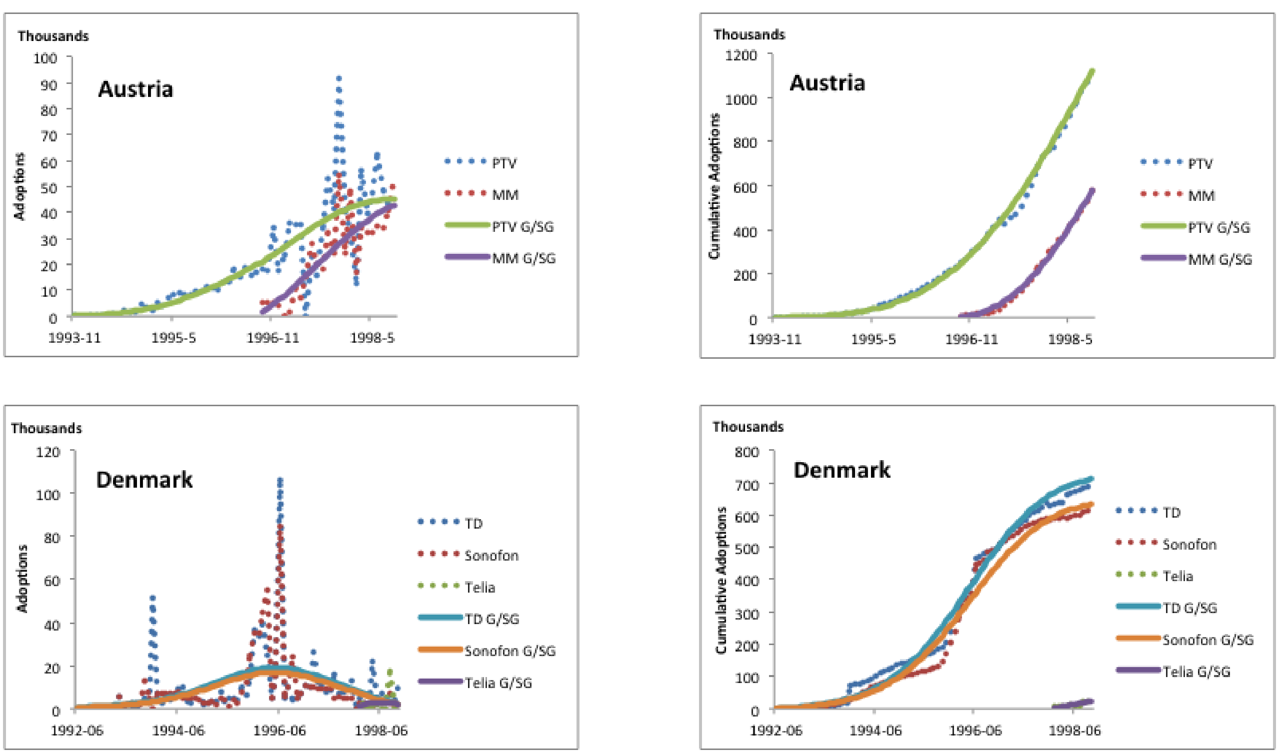
where t_j is the observation time for consumer j, δ_{ji} is an indicator for consumer j whether to adopt the i-th brand (1 if adopt, 0 if not), $f_{ji}(t_j)$ is the choice probability for consumer j to adopt the i-th brand; $f_{ji}(t_j) = f_i(t_j | \eta)$, and $S_j(t_j)$ is the category-level survivor function for consumer j; $S_j(t_j) = 1 - F(t_j | \eta)$. When m adopters are observed once, equation (18) is the likelihood. Because the equation (18) is based on the non-fixed number of potential adopters, this study estimates the aggregate-level likelihood using the maximum likelihood [23]. Let M, and c be the population size, and the probability of eventually adopting the product, respectively; $m = cM$.

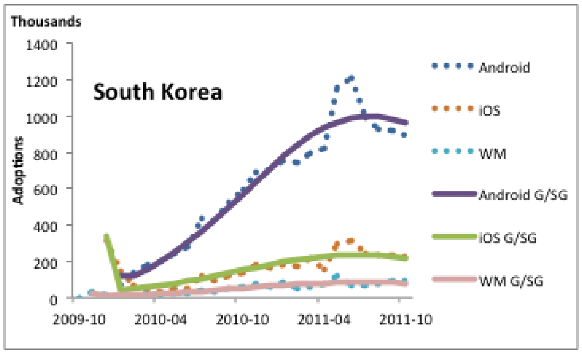
$$L = \prod_{t=1}^{T} \prod_{i=1}^{B(t)} (cf_i(t))^{x_i(t)} (1 - cF(t))^{M - \sum_{t=1}^{T} \sum_{i=1}^{B(t)} x_i(t)}. \tag{19}$$

where T, and $x_i(t)$ are the total length of the observation period, and the sales of the i-th brand in the time interval $(t-1, t]$, respectively. Since the given data is right-censored and at the aggregate level, the likelihood can be expressed as equation (19). This study maximizes the log-likelihood ln L numerically with respect to the parameters c, γ, β and b_i (i = 1, 2, .., $B(T)$).

4. RESULTS AND DISCUSSION

4.1. Empirical Results





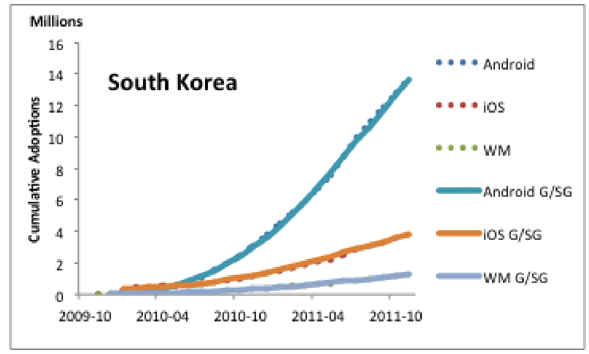


Figure 1. Actual versus Fitted Brand-Level Diffusion Curves

Note. Each panel depicts the monthly number of adoptions on the left-hand side and the cumulative number of adoptions on the right-hand side. MM in the upper panels denotes Maxmobil, TD in the middle panels is TeleDenmark, and WM in the lower panels is Windows Mobile. Mobilix, the fourth entering brand in Denmark, has been omitted.

Table 3. MLE of the Parameters: Constrained versus Unconstrained m-G/SG model

This table describes the estimates for individual country. The estimates of b_3 and b_4 are omitted.

Country/ Operator	Constrained version (γ = 1)					Unconstrained version (γ ≥ 1)						M (* 10^5)
	b_1	b_2	α	β	c	b_1	b_2	α	β	c	γ	
Cellular phone												
Austria	0.02	0.01	151	0.04	1.00	0.03	0.04	202	0.03	0.62	1.96	82
Belgium	0.23	0.11	0.17	1E06	0.31	0.05	0.03	1.00	33.3	0.33	1.00	104
Denmark	0.05	0.05	1.07	88.6	0.26	0.07	0.07	1.02	111	0.26	1.78	54
Finland	0.03	0.02	3.03	8.38	1.00	0.03	0.02	3.02	8.48	1.00	1.49	52
France	0.05	0.04	0.85	1E03	0.33	0.06	0.05	0.84	1E03	0.30	1.54	589
Germany	0.04	0.03	0.51	1E04	1.00	0.04	0.04	0.55	7E03	0.97	1.66	823
Greece	0.05	0.03	0.61	4E03	1.00	0.05	0.04	0.62	3E03	1.00	1.36	111
Iceland	0.06	0.04	0.98	21.3	0.40	0.06	0.04	0.98	21.3	0.40	1.00	3
Ireland	0.04	0.03	947	0.01	0.20	0.04	0.05	984	0.01	0.19	1.23	41
Italy	0.04	0.02	4.20	11.0	0.51	0.04	0.04	9.96	2.87	0.53	1.66	576
Luxemburg	0.11	1.26	0.43	916	0.25	0.11	1.26	0.43	916	0.25	1.00	5
Holland	0.10	0.06	0.41	1E05	1.00	0.10	0.06	0.41	1E05	1.00	1.02	163
Norway	0.02	0.01	55.6	0.09	0.73	0.02	0.01	55.6	0.09	0.73	1.00	46
Portugal	0.08	0.08	0.41	5E05	0.50	0.08	0.08	0.41	5E05	0.50	1.00	105
Spain	0.03	0.02	6E03	0.00	0.23	0.03	0.02	6E03	0.00	0.23	1.00	431
Sweden	0.01	0.01	679	0.01	0.72	0.01	0.01	686	0.01	0.72	1.03	90
Suiss	0.10	0.00	0.86	914	0.34	0.10	0.00	0.86	914	0.34	1.00	73
U. K.	0.01	0.00	452	0.01	0.73	0.01	0.00	452	0.01	0.73	1.00	585
Smartphone												
S. Korea	0.01	0.03	1.32	54.3	0.62	0.02	0.03	2.64	8.99	0.70	1.38	486

The Western European cellular phone market The parameter estimates and the comparison between estimated and actual sales are reported in Table 3 and figure 1 (the upper). To control network effects, the constrained model $(\gamma = 1)$ assumes that there is no network effect. The estimated $\gamma - 1$ is less than 1 for the unconstrained m-G/SG model. ($1 \leq \gamma \leq 1.955$) Since any magnitude of network effects (≤ 0.955) is less than a magnitude of the ratio of preferences, the ratio of preference for a brand plays a much more important role in choice probability than the difference in network size. This result is consistent with [19]. For countries that there is no network effect; $\gamma = 1$, the estimated preference for brand i in the constrained model is the same to that in the unconstrained model. In those countries, the majority of market share over the given period of time is occupied by the most preferred brand. For countries that there is network effect; $\gamma > 1$, the estimated preference for brand i in the constrained model depends on network effects for brand i. In the constrained model, the estimated preference for an individual brand can be skewed because of network effects.

For Austria data set, the estimated γ (1.955) is the most among countries. The ratio of preference for 1st brand to preference for 2nd brand in the constrained model is 1.594. However, the ratio of preference for 1st brand to preference for 2nd brand in the unconstrained model is 0.675; preference for 2nd brand is higher than preference for 1st brand. Network effects serve as a product enhancement, in particular an incumbent firm's. For countries that there is network effect, an incumbent firm pretends to be more preferred brand. Hence, the estimated preference for an individual brand in the constrained model is distorted because preference is convoluted with network effects.

No matter how high preference is, it is impossible for a follower to enter the category in the mature stage. For Denmark data set, the ratio of preference for 1st brand to that for 3rd brand in the unconstrained model is 0.431 (2.143 for the constrained model). However, the ratio of cumulative adoption of 1st brand to that of 3rd brand is 27.92 because of the late entry of 3rd brand.

The South Korea smartphone market Given the gap between the international announcement (July, 2007) and the time at which iPhone was launched into the South Korean market (December, 2009), there is the potential demand of iPhone – the device of iOS. Since the m-G/SG model reflects not only preferences for an individual brand, but also the gap of between the international announcement and the domestic entry time, this study analyzes a fluctuation on the early stage. The parameter estimates and the comparison between estimated and actual sales are reported in Table 3 and figure 1 (the below). The existence of network effects ($\gamma = 1.38$) demonstrates high usage of applications only for a particular mobile OS. In South Korea, network effects are advantages of not the market innovator (Windows Mobile), but the more preferred follower (Android) because of a rather small ratio of preference for 1st brand to that for 3rd brand (0.148); Android entered into the South Korea market with a significantly higher preference. Moreover, the entry of Android is early enough to catch-up the market share. In general, an incumbent firm hardly loses the lead in the market when there are network effects. However, the more preference and the early entry for Android are enough to take the lead in South Korea. Once Android has become the market leader, network effects rather work to the advantage of Android, and the gap among market shares has been expanded. However, the South Korea smartphone market is not the market with a winner-take-all outcome because the estimated network effects ($\gamma = 1.38$) is not enough of a monopoly to Android.

4.2. Comparison between the m-G/SG model and the LMP model

Table 4. Comparing Models: Multivariate G/SG Versus Libai, Muller and Peres' (2009) Model

This table compares Multivariate G/SG (m-G/SG) model and Libai, Muller and Peres' (LMP) Model for individual country. The number of parameters varies between 6 and 8 for the unconstrained m-G/SG and between 5 and 7 for the LMP and constrained m-G/SG models depending on the number of brands. When the constrained model cannot be rejected against the unconstrained version, we compare it to the LMP model.

Data set/ Country	m-G/SG model (* 10^6)					LMP model (* 10^6)			Superior model
	LL* ($\gamma \geq 1$)	LL ($\gamma = 1$)	LRT[a]	BIC[b]	AIC[c]	LL	BIC	AIC	

Cellular phone									
Austria	-11.2	-11.3	20.8	22.5	22.5	-11.2	22.5	22.5	m-G/SG
Belgium	-11.0	-11.0	21.8	21.9	21.9	11.0	21.9	21.9	LMP
Denmark	-9.9	-9.9	17.8	19.7	19.7	9.9	19.7	19.7	m-G/SG
Finland	-14.3	-14.3	13.0	28.6	28.6	-14.3	28.7	28.7	m-G/SG
France	-68.9	-68.9	20.6	137.8	137.8	-69.0	137.9	137.9	m-G/SG
Germany	-94.9	-94.9	21.5	189.8	189.8	-95.0	190.0	190.0	m-G/SG
Greece	-13.1	-13.1	16.5	26.1	26.1	-13.1	26.2	26.2	m-G/SG
Iceland	-0.4	-0.4	0.0	0.9	0.9	-0.4	0.9	0.9	LMP
Ireland	-3.7	-3.7	15.5	7.5	7.5	-3.7	7.5	7.5	m-G/SG
Italy	-93.5	-93.6	22.6	187.0	187.0	-94.6	189.6	189.2	m-G/SG
Luxemburg	-0.7	-0.7	0.0	1.4	1.4	-0.7	1.4	1.4	LMP
Netherlands	-19.8	-19.8	6.9	39.6	39.6	-19.8	39.6	39.6	m-G/SG
Norway	-11.2	-11.2	0.0	22.3	22.3	-11.2	22.4	22.4	m-G/SG
Portugal	-17.2	-17.2	0.0	34.4	34.4	-18.5	37.1	37.1	m-G/SG
Spain	-38.0	-38.0	0.0	75.9	75.9	-38.1	76.1	76.1	m-G/SG
Sweden	-20.8	-20.8	4.8	41.7	41.7	-24.0	48.0	48.0	m-G/SG
Switzerland	-9.4	-9.4	0.0	18.9	18.9	-9.4	18.9	18.9	LMP
United Kingdom	-76.3	-76.3	0.0	152.7	152.7	-76.5	152.9	152.9	m-G/SG
Smartphone									
S. Korea	-101.5	-101.6	20.6	203.1	203.1	-101.7	203.4	203.4	m-G/SG

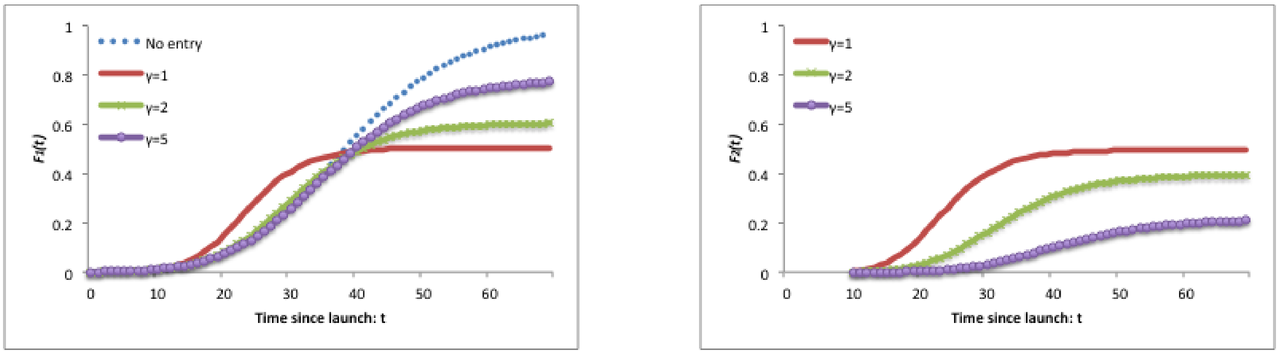
* LL denotes the log-likelihood.
a LRT Likelihood ratio test. The critical value at 95% is equal to 3.84.
b BIC (Bayesian Information Criterion) = -2 * LL + p ln(N), p = the number of parameters and N = sample size.
c AIC (Akaike Information Criterion) = -2 LL + 2p.

To check the validity of the m-G/SG model, this study estimates the data by using the LMP model. Table 4 compares estimations of the m-G/SG model and the LMP model. Since two models require the different number of parameters, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) are used to consider the estimation accuracy. The LMP model provides a better fit than the m-G/SG model in four countries – Luxembourg, Iceland, Switzerland, and Belgium among nineteen countries. There are i) no network effects (The estimated dependence parameter γ in those countries is 1), and ii) only two brands in those four countries. But it is not the sufficient condition for the superiority of LMP model because of Norway and Spain. In the other fifteen countries, the m-G/SG model outperforms the LMP model. For the countries that there are i) network effects, or ii) three or more brands, then the m-G/SG model is superior to the LMP model. For South Korea data, the m-G/SG model shows a higher accuracy than the LMP model in terms of AIC or BIC. For the category including three or more brands, the LMP model cannot be expressed as a closed form. The LMP model is slightly defective when the data is incomplete; actual sales in the initial period are truncated. To analyze three or more brands in the incomplete data, the m-G/SG model is more feasible than the LMP model.

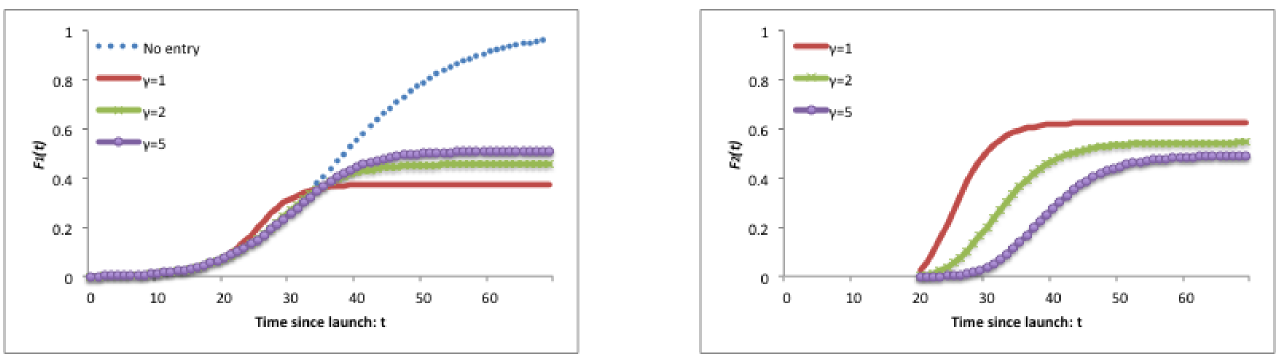
4.3. Discussion

Cumulative penetration depends on the degree of path dependence γ. To investigate the brand-level diffusion, this study analyzes the cumulative penetration of the first brand and that of the second brand. For the reflection of an early entry of the follower with an improvement in preference, three cases are given in figure 2; i) an early entry without an improvement in preference ($b_1 = b_2 = 0.1$ and $\tau_2 = 10$), ii) a late entry with an improvement in preference ($b_1 = 0.1, b_2 = 0.2$ and $\tau_2 = 20$), and iii) an early entry with an improvement in preference ($b_1 = 0.1, b_2 = 0.2$ and $\tau_2 = 10$).

$b_2 = 0.1$ and $\tau_2 = 10$



$b_2 = 0.2$ and $\tau_2 = 20$



$b_2 = 0.2$ and $\tau_2 = 10$

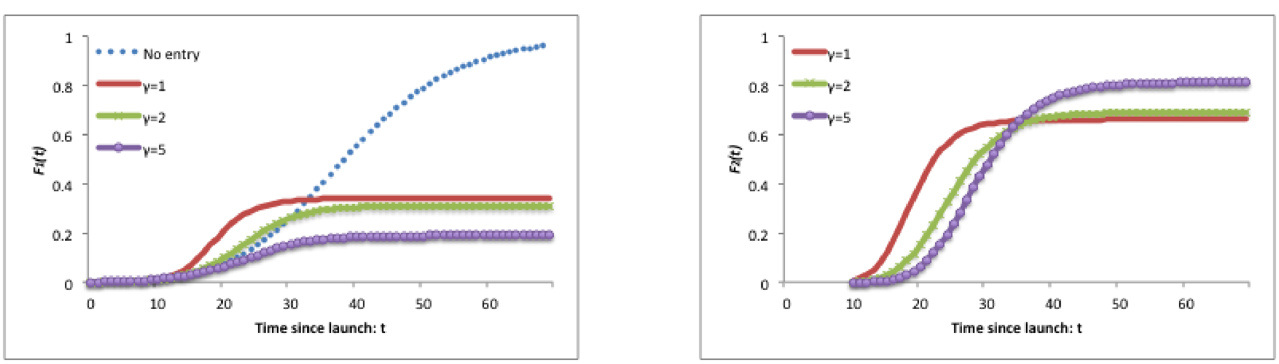


Figure 2. Cumulative Penetration of the First Brand and That of the Second Brand

Note. Cumulative penetration of the time to adopt the first brand in a market and corresponding cumulative penetration for the second brand as a function of the dependence parameter γ, the preferences for the second brand and the time to entry of the second brand. Each panel depicts the cumulative penetration of the first brand on the left-hand side and the cumulative penetration of the second brand on the right-hand side. The preference for the first brand is $b_1 = 0.1$. The product class parameters are $\alpha = 2$ and $\beta = 20$.

For the first case in figure 2, the higher degree of path dependence γ, the slower diffusion for the follower is. In other words, the more path dependent, the more consumers are involved to the earlier entrant. It is due to that the network size of the follower is always smaller than that of the earlier entrants. A follower cannot overcome the disadvantage of a late entry without an improvement in preference. For the second case in figure 2 - the entry of the follower is not early enough even though the follower is more preferred, network effects still work to the advantage of the earlier entrant (e.g., the success of the QWERTY keyboard over the Dvorak keyboard that shows higher performance). When network effects are weak, minority brands such as Apple in a personal computer category can survive [24]. As network effects increase, the predominance of the more preferred and the earlier entered brand is accelerated. Hence, strong network effects enable the brand with larger network size to stay longer as a market leader and result in a strong tendency towards higher market concentration [20]. One of keys to the monopoly power of Microsoft in the operating system (OS) market is strong network effects: The Windows OS and Microsoft Office can appeal to customers more because of many earlier users and the compatibility issue [25]. The cumulative penetration of the follower decreases as network effects increase; the entry of the follower turns out to be good for earlier entrants. Meanwhile, network effects cannot always defend the earlier entrants from the entry of the follower [19]. Network effects can work to the advantage of the follower conditionally. For the third case in figure 2, the cumulative penetration for a follower can overtake that for the follower of which there are no network effects; an early entry with higher preference overcomes network advantages. After this overtake, the cumulative penetration for the follower increases as network effects increase. To make network effects the advantage, therefore a follower should enter the market with higher preference as early as possible. It is consistent with market efficiency [19].

Network effects can significantly influence the growth of a category [26]. Because of the positive interaction among brands, the entry of the follower in the growth stage draws the faster growth of the category.

$$F(t;\gamma=\infty)\leq F(t)\leq F(t;\gamma=1). \qquad (20)$$

Equation (20) shows the speed of category-level diffusion corresponding to the degree of path dependence γ. In general, outcomes from path independent process - no network effects - are most efficient and lead to the highest benefit [27]. Network effects cancel out the positive effect of an entry of a new brand; as γ increases, the category-level diffusion curve shifts to the right side. Network effects may slow the rate of adoptions of a new product as adopters wait for sufficient others (threshold) to adopt. The impact of network effects on category-level adoption is always negative but approaching zero [28]. As network effects increase, the speed of category-level diffusion decreases. Since compatibility among brands is limited, convergence to the brand with largest network size should be expected. If network effects excessively increase, the market has a tendency towards a natural monopoly [28]; the category is governed by “First choice”. Network effects may result in the negative effect to the category diffusion [26]. And also, this result is consistent with the chilling effects of network effects [29].

5. CONCLUSION

Why do consumers hesitate to buy a new product in spite of the positive effect due to entries of new brands? There are two kinds of negative effects to time to adopt; the first one depends on the interaction among times to adopt competing brands, and the latter one depends on the individual consumer’s tendency to postpone adoption. “Interpersonal communication is not necessarily needed for network externalities to work [9].” This research suggests an alternative model to overcome the limitations of the earlier studies. In order to incorporate network effects, competition among brands, and individual heterogeneity, this study suggests the m-G/SG model. Based on the estimation of the degree of path dependence, this study can check whether network effects can play an important role and how strong network effects are in a given category. When brands are path dependent, the m-G/SG model really comes into its own. This study provides the theoretical basis to construct a market strategy and contribute that the degree of path dependence can capture the strength of network effects. The proposed model has significant managerial implications. Marketers have been interested in network effects as the tool to investigate the social communication. A strategic decision from the m-G/SG model could be applied to analyze the relationship with incumbent firms. [19] show that it may be better to be better than to be first. According to the results in our analyses, however, the preference does not always dominate the entry of order. It is better to be better than to be first conditional on "early catch-up".

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