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Discovered Pilot Designs in MIMO OFDM System with optimization algorithms

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Abstract

For Wi-Fi and IoT wireless systems, high-capacity wireless communication technology is being applied using MIMO OFDM. Because of virtu0al subcarriers in the practical MIMO OFDM system, the pilot subcarriers cannot be spaced equally. Thus, it is difficult to obtain a good mean square error (MSE) performance of the channel estimate. This paper proposes applicable methods and the newly discovered locations of pilot subcarriers in four transmitted antennas resulting in a good MSE performance with proposed optimization algorithms.

Keywords: *Pilot design, OFDM, MIMO, channel estimate, MSE, pilot subcarriers.*

1. Introduction

Multiple input, multiple input multiple output (MIMO) orthogonal frequency division multiplexing (OFDM) systems have been studied for high data transmission in wireless communications. To guess transmitted data correctly, the channel state information (CSI) is essential [1]. By minimizing the mean square error (MSE) performance of the channel estimate, the CSI can be acquired correctly. To estimate the CSI, comb pilots are utilized. The minimum MSE performance of the channel estimate can be obtained using equally spaced locations and equal powers [2]. Because of virtual subcarriers, however, it is difficult to obtain the mini-mum MSE of the channel estimate [2]. Even though many algorithms [3]-[5] have tried to look for the locations and powers of the pilot subcarriers, resulting in a good MSE channel estimate performance, the methods proposed in [3]-[5] cannot be applied to a practical MIMO OFDM system such as 802.16m [6]. In this paper, applicable pilot designs are proposed for a MIMO OFDM system that has practical parameters and four transmitted antennas, such as in the 802.16m standard [6] with proposed optimization algorithms. This paper's different approach methodology is application of proposed optimization algorithms compared to [8]. The results of this paper are valuable research that can be applicable to the recent wireless network system and Wi-Fi wireless channel, and IoT wireless communications [9] [10].

2. System Model

The system herein is considered a MIMO OFDM system with N_t transmitted antennas and N_r received

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antennas. The received data in the q-th antenna is as follows [2]:

$$\mathrm{Y}^{\mathrm{q}} = \sum_{\mathrm{p}=1}^{\mathrm{N_t}} \mathrm{diag}(\mathrm{X}^{\mathrm{p}})\mathrm{F_L}\mathrm{h}^{\mathrm{q,p}} + \mathrm{N}^{\mathrm{q}} = \mathrm{A}\mathrm{h}^{\mathrm{q}} + \mathrm{N}^{\mathrm{q}} \tag{1}$$

where M represents the number of pilot subcarriers per one transmitted antenna, Y_q is the received data with an M ×1 vector in the q-th antenna, X_p is the transmitted data with an M ×1 vector in the p-th antenna, F_L is an M×1 fast Fourier transform (FFT) matrix, diag() is a diagonal matrix, L indicates the maximum channel length, $h^{q,p}$ is a channel impulse response matrix from the p-th transmitted antenna to the q-th received antenna having an L ×1 matrix, and N_q is a q-th (i.i.d.) Gaussian noise with mean zero and variance σ_{n}^2. Also, $\mathrm{A} = \left[\mathrm{X_D^1 F_L}, \cdots, \mathrm{X_D^{N_t} F_L}\right]$ and $\mathrm{X_D^p} = \mathrm{diag}(\mathrm{X^p})$. The least square (LS) channel estimate is $\hat{\mathrm{h}}^{\mathrm{q}} = \mathrm{A}^{\dagger}\mathrm{Y}^{\mathrm{q}}$, where $\mathrm{A}^{\dagger} = (\mathrm{A}^H\mathrm{A})^{-1}\mathrm{A}^H$, $\mathrm{A}^{\dagger}$is the pseudoinverse of A, and A^H is the hermitian matrix of A. After finishing the LS channel estimate, the MSE performance is

$$\mathrm{MSE} = \frac{1}{\mathrm{LN_t}}\mathrm{E}\left\{\left\|\hat{\mathrm{h}}^{\mathrm{q}} - \mathrm{h}^{\mathrm{q}}\right\|^2\right\} = \frac{\sigma_{\mathrm{n}}^2}{\mathrm{LN_t}} tr\{(\mathrm{A}^H\mathrm{A})^{-1}\} \tag{2}$$

where E{} is an expectation, I_M is $\mathrm{M} \times \mathrm{M}$ identity matrix, $\mathrm{E}\left\{N^{\mathrm{q}} + N^{q^H}\right\} = \sigma_{\mathrm{n}}^2 I_M$; $\|\cdot\|$ is an ℓ_2 norm; $tr\{\}$ is a trace operator, as shown in [5], for gaining the minimum MSE performance of the LS channel estimate; and $\mathrm{A^H A} = \mathcal{P}\mathrm{I_{LN_t}}$ must be satisfied [5], where $\mathcal{P}$ represents the fixed power dedicated for training. Thus, the following is the result of the minimum MSE of the LS channel estimate:

$$(\mathrm{MSE})_{\mathrm{minimum}} = \frac{\sigma_{\mathrm{n}}^2}{\mathcal{P}} \tag{2}$$

In the next section, a method for finding the locations of the pilot subcarriers satisfying (3) is explained for the case in which practical parameters of a MIMO OFDM system are used.

3. Proposed Methodology

In [3], the pilot design is implemented with $\mathcal{K} = 256$, $\mathcal{K}_{\mathrm{u}} = 201$, $\mathrm{L} = 8, 16$, and $\mathcal{V} = \mathcal{K} - \mathcal{K}_{\mathrm{u}} - 1$, where $\mathcal{K}$ means the total number of subcarriers in an OFDM symbol, $\mathcal{K}_{\mathrm{u}}$ and $\mathcal{V}$ represent the numbers of useful and virtual subcarriers, and -1 is for a DC subcarrier in an OFDM symbol. However, the design method is not applicable to the pilot design, which leads to a good MSE channel estimate performance for a practical four-transmitted-antenna OFDM system such as 802.16m [6]. For four transmitted antennas, pilot subcarriers as small as possible have to be used instead. Because more pilot subcarriers are used, lower pilot subcarriers close to the locations of the virtual subcarriers can be utilized. Thus, not only is it important to find the locations and powers of the pilot subcarriers, which result in a good MSE performance of the channel estimate, but it is also important to utilize a lower number of pilot subcarriers for multiple transmitted antennas. In this section, the proposed methods using practical parameters in four transmitted antennas are explained. With the results in III.1, the pilot design is implemented for those in III.2 and III.3

3.1 $\mathcal{K} = 512,\ \mathcal{K}_u = 432,\ L = CP = 32$

The locations of pilot subcarriers are obtained by solving the following [4]:

$$\begin{cases} \min F = \sum_{p=1}^{N_t}(P_m^p)(P_m^p - |P_m^p|) \\ \mathcal{B}^p P^p = b, \qquad P^p \geq \mathbf{0}_{M^p \times 1} \end{cases} \tag{3}$$

where $P_m^p = [P_1^p, \cdots, P_{M^p}^p]^T$ is an $M^p \times 1$ vector, P_m^p is the pilot power for the subcarrier κ_m^p, and M^p is the number of pilot subcarriers for the p-th transmitted antenna. Also, $\mathcal{B}^p = [\nu_1^p, \cdots, \nu_{M^p}^p]^T$has a $2(L-1) \times M^p$ vector, $\nu_m^p = [\cos\frac{2\pi}{\mathcal{K}}\kappa_m^p, \cdots, \cos\frac{2\pi(L-1)}{\mathcal{K}}, \sin\frac{2\pi}{\mathcal{K}}\kappa_m^p, \cdots, \sin\frac{2\pi(L-1)}{\mathcal{K}}\kappa_m^p, 1]^T$, and $b = [0, \cdots, 0, \mathcal{P}]$ is a $2(L-1)$ vector, $[]^T$ is a matrix transpose. In order to solve (4), the following steps are proposed [3], [7]. Steps 1), 2) are modified schemes from [3], and step 3) is an additional step.

1) P_p is defined by a sdpvar command. Set the initial values close to zero with an assign command. Then, set the nonlinear function with a sdpfun command. Then, solve (4) with constraints by solvesdp and sdpsettings commands.
2) Find the locations of subcarriers having positive values.
3) Eliminate one of each two subcarriers at an interval of one subcarrier.

According to the tutorials in [7], set P_p, which is an object for solving (4), with a sdpvar command. In step 2), the applicable optimizing tool is automatically selected with a sdpsettings command. After finishing step 2), 38 subcarriers are left. If step 3) is performed, the output leads to 32 subcarriers. Thus, the obtained locations of the subcarriers and equal powers result in an MSE performance of the channel estimate that is comparable to the minimum MSE [2]. From the sec-ond to fourth transmitted antennas, the locations of the subcarriers and powers are gained using Pilot Design with Seed Sequence (PDSS)1. With PDSS1, good MSE channel estimate performance can be achieved as shown in Figure. 1. In PDSS1, the pilot elements in the p-th antenna are $\mathcal{M}^p = \{\mathcal{U}_1^p, \mathcal{U}_2^p\}$, $\mathcal{M}^{p+1} = \{\mathcal{U}_1^{p+1}, \mathcal{U}_2^{p+1}\}$, $\mathcal{U}_1^p = \{\kappa_1, \cdots, \kappa_m\}$, $\mathcal{U}_1^{p+1} = \mathcal{U}_1^p - 1$, $\mathcal{U}_2^p = \{\kappa_{m+1}, \cdots, \kappa_n\}$, and $\mathcal{U}_2^{p+1} = \mathcal{U}_2^p + 1$, where κ_n represents the location of the n-th pilot subcarrier, $\kappa_m = \frac{\mathcal{K}_u}{2}$, and $\kappa_{m+1} = \frac{\mathcal{K}_u}{2} + \mathcal{V}$. The pilot subcarriers used in the first antenna, $p = 1$, are the seed sequence for obtaining the pilot subcarriers from the second to fourth antennas. Also, the pilot elements have equal powers when the total power is fixed power $\mathcal{P}$.

3.2 $\mathcal{K} = 512$, $\mathcal{K}_u = 432$, $L = CP = 64$

With $M_{u\_1} = \mathcal{M}^1$ in the first antenna of Table 1, find the middle locations, M_{u_mid}, of M_{u_1}. That is, $M_{u\_1} = \{a_1, a_2, \cdots\}$, $M_{u\_mid} = \mathrm{round}\{\frac{(a_1+a_2)}{2}, \ldots\}$, and $M_{u\_2} = M_{u\_1} + M_{u\_mid}$. Then, add one of the values of a middle location between the end of the subcarriers in an OFDM symbol and the element at the end of M_{u_2}. With one OFDM symbol, the elements in Table 2 are utilized for the first and second antennas. With another OFDM symbol, the elements are used for the third and fourth antennas. The simulation results are shown in Figure 1.

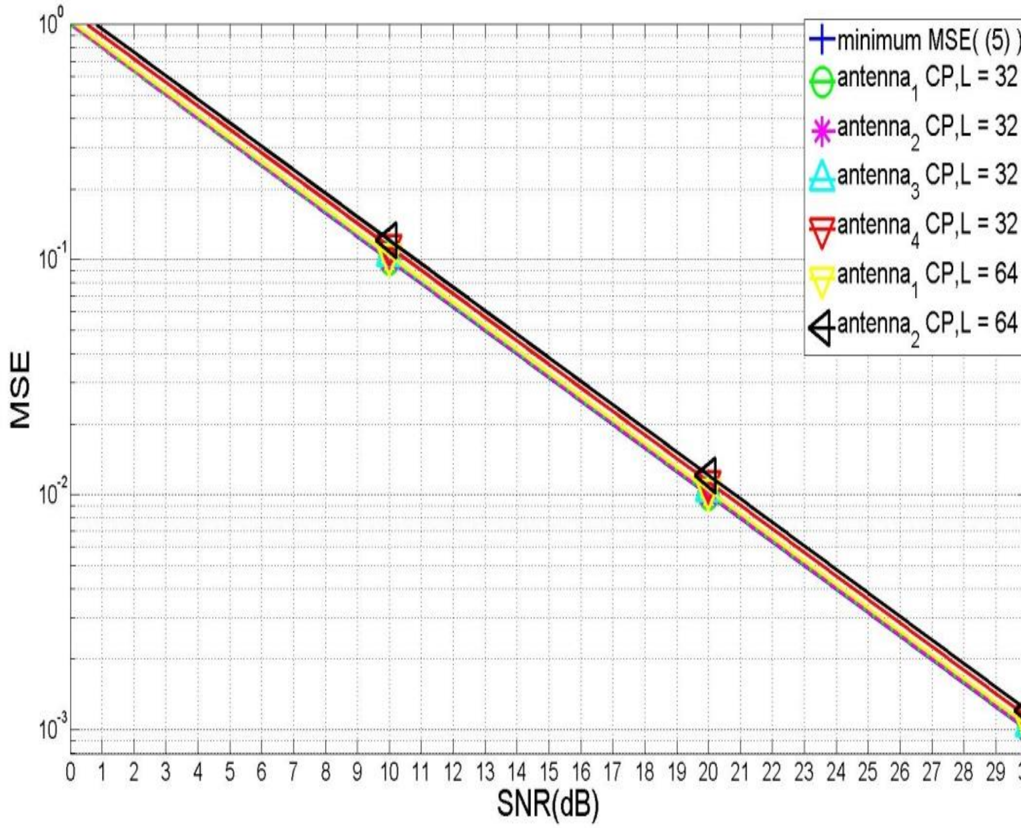

Figure 1. $\mathcal{K} = 512,\ \mathcal{K}_u = 432,\ \mathrm{L} = \mathrm{CP} = 32, 64,\ \mathcal{V} = 79$

= minimum MSE (ideal minimum MSE),
= second antenna, in case of CP = 32, L = 32,
= fourth antenna, in case of CP = 32, L = 32,
= second antenna, in case of CP = 64, L = 64

= first antenna, in case of CP = 32, L = 32
= third antenna, in case of CP = 32, L = 32
= first antenna, in case of CP = 64, L = 64

TABLE I. The Discovered locations of the pilot subcarriers, $\mathcal{K} = 512,\ \mathcal{K}_u = 432,\ \mathrm{L} = \mathrm{CP} = 32,\ \mathcal{V} = 79$

$\mathcal{M}_\ell$	8, 24, 40, 55, 71, 87, 102, 118, 133,148, 163, 178, 191, 203, 212, 216
$\mathcal{M}_r$	296, 300, 309, 321, 335, 349, 364, 379, 394, 410, 425, 441, 457, 473, 488, 504
$\mathcal{M}^1$(first antenna) = $\mathcal{M}_\ell, \mathcal{M}_r$, $\mathcal{M}^2$(second antenna) = $\mathcal{M}_\ell - 1, \mathcal{M}_r + 1$ $\mathcal{M}^3$(third antenna) = $\mathcal{M}_\ell - 2, \mathcal{M}_r + 2$, $\mathcal{M}^4$(fourth antenna) = $\mathcal{M}_\ell - 3, \mathcal{M}_r + 3$	
$\boldsymbol{\mathcal{M}_\ell, \mathcal{M}_r}$: the pilot elements which indicate the locations of the pilot subcarriers, $\boldsymbol{\mathcal{M}^1}$: the pilot elements in the first transmitted antenna, $\boldsymbol{\mathcal{M}^2}$: the pilot elements in the second transmitted antenna, $\boldsymbol{\mathcal{M}^3}$: the pilot elements in the third transmitted antenna, $\boldsymbol{\mathcal{M}^4}$: the pilot elements in the fourth transmitted antenna	

TABLE II. The Discovered locations of the pilot subcarriers for $\mathcal{K} = 512,\ \mathcal{K}_u = 432,\ L = CP = 64,\ \mathcal{V} = 79$

$\mathcal{M}_\ell$	4,8,16,24,32,40,48,55,63,71,79,87,95,102,110,118,126,133,141,148,156,163,170,178, 185,191,197,203,208,212,216
$\mathcal{M}_r$	296,300,305,309,315,321,328,335,342,349,357,364,372,379,387,394,402,410,418,426,434,441,449,457,465,473,481,488,496,504
$\mathcal{M}^1$(first antenna) = $\mathcal{M}_\ell, \mathcal{M}_r$, $\mathcal{M}^2$(second antenna) = $\mathcal{M}_\ell - 1, \mathcal{M}_r + 1$	
$\boldsymbol{\mathcal{M}_\ell, \mathcal{M}_r}$: the pilot elements which indicate the locations of the pilot subcarriers, $\boldsymbol{\mathcal{M}^1}$: the pilot elements in the first transmitted antenna, $\boldsymbol{\mathcal{M}^2}$: the pilot elements in the second transmitted antenna	

3.3 $\mathcal{K} = 1024,\ \mathcal{K}_u = 864,\ L = CP = 64$

The following is the proposed method [8]:

1) $\mathcal{S}_1$, the pilot locations of the subcarriers in the first antenna with that in Table 1, are used. Then, $\mathcal{S}_2 = \mathcal{S}_1 \times 2$.
2) Search the middle elements of the $\mathcal{S}_1$ elements, that is, $\mathcal{S}^1 = \{\mathfrak{a}_1, \mathfrak{a}_2, \cdots\}$, $\mathcal{S}^1_{\text{mid}} = \text{round}\left\{\frac{\mathfrak{a}_1+\mathfrak{a}_2}{2}, \cdots\right\}$, and remove one subcarrier in the area of the virtual subcarriers and add each middle element between the elements loca ted at the end of the subcarriers of an OFDM symbol and the elements located at the end of the $\mathcal{S}^1$elements. Also, $\mathcal{S}_{\text{mid}}$ is acquired with 30 elements.
3) $\mathcal{S}_1 = \{\mathcal{S}_1, \mathcal{S}_{\text{mid}}\}$. The number of elements is 64. Then, find the middle elements $\mathcal{S}^2_{\text{mid}} = \{\mathcal{S}_1, \mathcal{S}_{\text{mid}}\}$ of the $\mathcal{S}_2$ elements as in (b), and eliminate the one subcarrier in the area of the virtual subcarriers.
4) Add the elements of (c) to $\mathcal{S}_2$, and the pilot element $\mathcal{S}_3$ is obtained. Then, eliminate some subcarriers.

In step (d), eliminating some specific subcarriers is important to obtain a good MSE channel estimate performance in four transmitted antennas with an OFDM symbol. First, remove each one subcarrier of the subcarriers at intervals of two subcarriers, and four subcarriers except for two elements near the edge of the area of the virtual subcarriers. The number of pilot subcarriers is then 116. Second, eliminate one subcarrier from each pair of subcarriers except for two elements near the edge of the area of the virtual subcarriers. Finally, the number of pilot subcarriers is equal to 60 in Table 3. And PDSS1 is used for the second, third, and fourth transmitted antennas. The simulation results are shown in Figure 2.

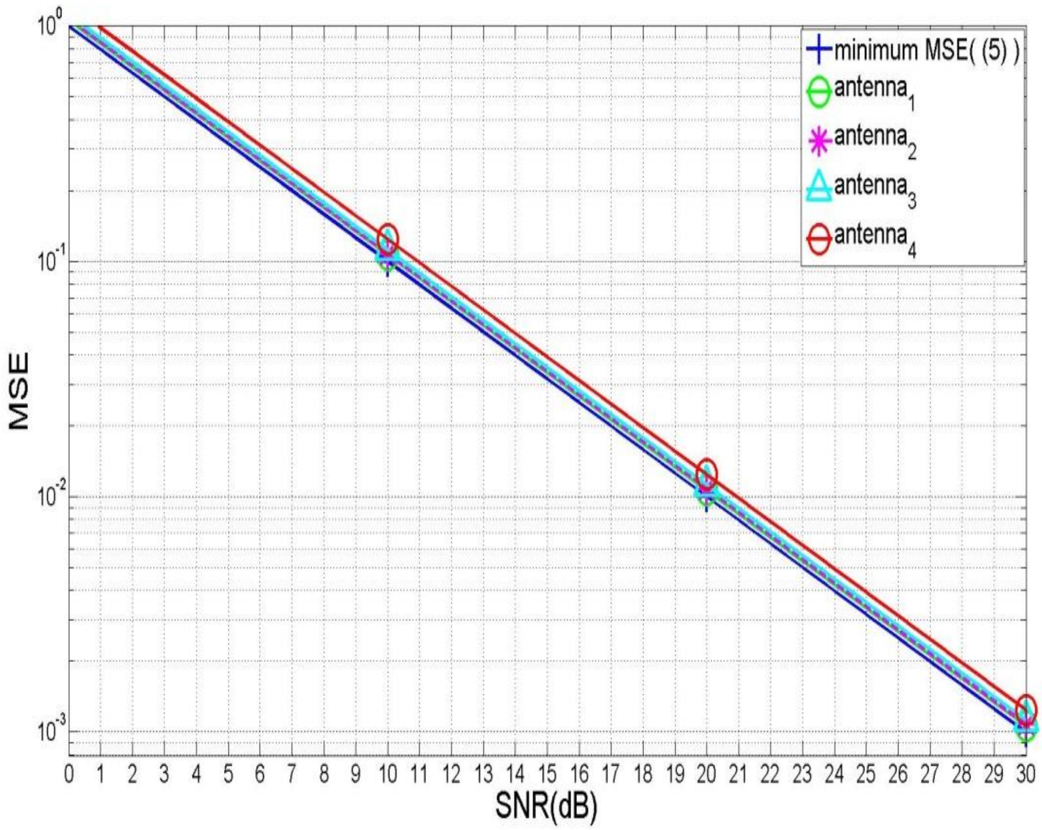

Figure 2. $\mathcal{K} = 1024,\ \mathcal{K}_{\mathrm{u}} = 864,\ \mathrm{L} = \mathrm{CP} = 64,\ \mathcal{V} = 159$

= Minimum MSE (ideal minimum MSE),
= first antenna, in case of CP = 64, L = 64
= second antenna, in case of CP = 64, L = 64,
= third antenna, in case of CP = 64, L = 64
= fourth antenna, in case of CP = 64, L = 64

TABLE III. The Discovered locations of the pilot subcarriers for $\mathcal{K} = 1024,\ \mathcal{K}_u = 864,\ L = CP = 64,\ \mathcal{V} = 159$

$\mathcal{M}_\ell$	8, 24, 40, 56, 72, 88, 103, 118, 134, 150, 166, 182, 197, 212, 228, 244, 259, 274, 289, 304, 319, 334, 349, 363, 376, 388, 400, 415, 428, 432
$\mathcal{M}_\mathrm{r}$	592, 596, 605, 624, 636, 649, 663, 677, 691, 706, 721, 736, 751, 766, 781, 796, 812, 828, 843, 858, 874, 890, 906, 922, 938, 954, 969, 984, 1000, 1016
$\mathcal{M}^1$(first antenna) $= \mathcal{M}_\ell, \mathcal{M}_\mathrm{r}$, $\mathcal{M}^2$(second antenna) = $\mathcal{M}_\ell - 1, \mathcal{M}_\mathrm{r} + 1$ $\mathcal{M}^3$(third antenna) = $\mathcal{M}_\ell - 2, \mathcal{M}_\mathrm{r}$ +2, $\mathcal{M}^4$(fourth antenna) = $\mathcal{M}_\ell - 3, \mathcal{M}_\mathrm{r} + 3$	

$\boldsymbol{\mathcal{M}_\ell, \mathcal{M}_\mathrm{r}}$: the pilot elements which indicate the locations of the pilot subcarriers, $\boldsymbol{\mathcal{M}^1}$: the pilot elements in the first transmitted antenna, $\boldsymbol{\mathcal{M}^2}$: the pilot elements in the second transmitted antenna, $\boldsymbol{\mathcal{M}^3}$: the pilot elements in the third transmitted antenna, $\mathcal{M}^4$: the pilot elements in the fourth transmitted antenna

4. Conclusion

Studies on MIMO OFDM system are useful in wireless communication technology field. In a practical technology, the system has the problem of optimizing the channel estimate performance. With the proposed methodology, the results can lead to a good MSE channel estimate performance. The simulation results demonstrate that the pilot design is applicable and optimized, and that its methods are expected to significantly improve the performance of the MIMO OFDM system. And the newly discovered results are valuable to mobile or IoT system such as Wi-Fi channels & multiple antenna communications because of applicable & optimized & lightweight benefits. The design method and proposed pilot subcarriers locations should be emphasized on the system explained in this paper.

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