

Research Paper

## Advancing an Automated Algorithm for Estimating Rebar Quantities in Columns

Rachmawati, Titi Sari Nurul<sup>1</sup> · Widjaja, Daniel Darma<sup>1</sup> · Kim, Sunkuk<sup>2\*</sup>

<sup>1</sup>Doctoral Student, Department of Architectural Engineering, Kyung Hee University, Giheung-Gu, Yongin, 17104, Korea

<sup>2</sup>Professor, Department of Architectural Engineering, Kyung Hee University, Giheung-Gu, Yongin, 17104, Korea

\*Corresponding author

Kim, Sunkuk

Tel : 82-2-1201-2922

E-mail : [kimsuk@khu.ac.kr](mailto:kimsuk@khu.ac.kr)

Received : June 4, 2023

Revised : July 11, 2023

Accepted : July 24, 2023

### ABSTRACT

Manual estimation of rebar quantities by contractors often yields discrepancies between projected and actual amounts used in the construction phase, leading to cost inaccuracies and potential logistical challenges. To address these issues, there is a clear need for a method that allows precise estimation of rebar quantities during the design phase. Such a method would enhance contractor competitiveness during project bids, promote accurate cost calculations, and avert superfluous rebar purchases on-site. Given that columns are the primary structural components in reinforced concrete(RC) buildings and necessitate considerable amounts of rebar, this study focuses on creating an automated algorithm for estimating column rebar quantities. An analysis of the accurate quantities obtained via the study and those derived from manual estimation reveals a discrepancy of 0.346 tons or 2.056%. This comparison affirms the proposed algorithm's efficacy in eliminating errors from overestimation or underestimation of rebar quantities. The practical implications of this study are significant for construction companies as it fosters efficient and precise estimation of rebar quantities, ensuring compliance with related specifications and governing regulations.

Keywords : estimation, algorithm, column rebar, precise quantity, rebar usage

## 1. Introduction

Quantity takeoff(QTO) is an essential task in the construction project. It involves the measurement and enumeration of building components[1]. In the preconstruction stage, QTO is used to approximate the project's expenses and create a bill of quantities(BOQ) for project bidding and tendering. In the construction stage, QTO is used to generate schedule planning and material procurement[2]. Consequently, the accuracy of quantity from QTO is considerably important as it influences the reliability of cost calculation and project planning[3].

Traditionally, QTO has relied on manual techniques utilizing 2D drawing and human interpretation[4]. Quantity estimators examine a set of drawings and apply specific measurement rules to determine the quantities of each building element. In cases where the drawings lack adequate information due to the project being in the schematic of development stages, knowledge, expertise, and experience are necessary to deduce the missing details. Consequently, manual QTO is a time-consuming and error-prone procedure, and the outcomes may vary among quantity estimators[5].

Quantity surveyors roughly estimate the quantities of rebars by employing a standard method of measurement. Unlike the actual rebar quantity calculation in the construction stage, which consider rebar information, specifications, building codes, or



This is an Open Access article distributed under the terms of the Creative Commons Attribution Non-Commercial License (<http://creativecommons.org/licenses/by-nc/4.0/>) which permits unrestricted non-commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

regulations such as lap splice length, lap splice position, anchorage and hook length, and bending deduction, the manual quantity estimation generally does not consider this information[6]. To minimize such a significant difference between estimated quantity and actual quantity, this study introduced automatic quantity estimation which can generate precise quantity even from the construction planning stage. In this study, an automated algorithm for estimating rebar quantities in columns is developed. The column was chosen because it is one of the main structural members of RC buildings and requires an excessive amount of rebar.

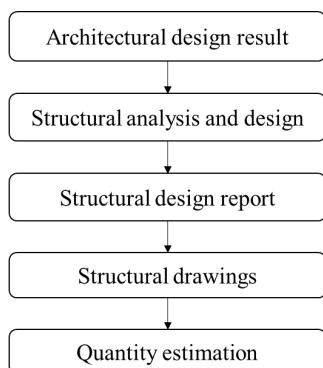
This study follows the subsequent procedures to prove the effectiveness of the proposed algorithm: (1) develop the proposed algorithm, (2) validate the result by using case studies where the estimated quantity obtained from manual quantity estimation is compared with the precise quantity obtained from the proposed algorithm, and (3) analyze and discuss the results.

The paper is organized as follows. Section 2 presents preliminary studies that explore manual quantity estimation and column rebar characteristics. Section 3 presents the concept and development of an automated quantity estimation algorithm specifically for column main rebar. Section 4 presents case studies to verify the effectiveness of the proposed algorithm. In the end, Section 5 concludes the results and sets future studies.

## 2. Preliminary studies

### 2.1 Manual Quantity Estimation

Typically, once an architectural design is finished, structural engineers carry out structural analysis and design. Once the structural analysis and design are completed, a structural design report and structural drawing are prepared[7]. Following that, quantity estimators estimate the rebar quantity based on structural drawings and specifications. The result is further processed into bill of quantity which is used for initial rebar cost estimation that will help contractors to win the tenders. If contractors win the tender, contractors continue to generate shop drawings and recalculate the quantity for construction. The process is summarized in Figure 1.



**Figure 1.** Workflow for manual quantity estimation

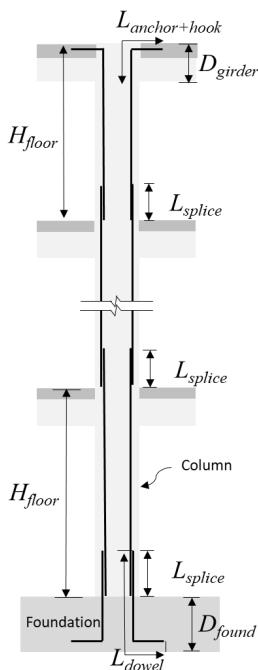
The manual quantity estimation commonly follows the standard method of measurement which provides a set of standards to measure and quantify construction works. Examples of the standard are the New Rules for Measurement(NRM) developed

by RICS[8] and Civil Engineering SMM 4(CESMM) developed by ICE[9]. In the case of rebar, the general steps for quantifying rebar manually are as follows: (1) identifying each structural member associated with rebar on the structural drawings, (2) understanding rebar details including bar sizes, types, shapes, bar spacing, lap lengths, etc., (3) measure the lengths considering rebar details, and (4) calculate the quantities by applying rebar unit weight[8,9].

Some studies[1,4,5] showed that manual quantity estimation based on these standards may generate errors and be time-consuming. First, the procedure implies that this method depends on structural drawing. There may be human errors in the process of preparing structural drawings and structural design reports which cause miscalculations in quantity estimation. Furthermore, significant time and effort are required to make the structural drawings. Second, SMM guidelines do not clearly state the measurement method of rebar quantity in detail, resulting in open interpretation to quantity estimators which may differ between persons due to differences in experience and justification. Therefore, this study proposed to develop an algorithm to precisely calculate the rebar quantity by directly using the structural analysis and design which is the master data for automated quantity estimation of column rebar, thereby the potency of error or loss of information in the structural drawing can be avoided.

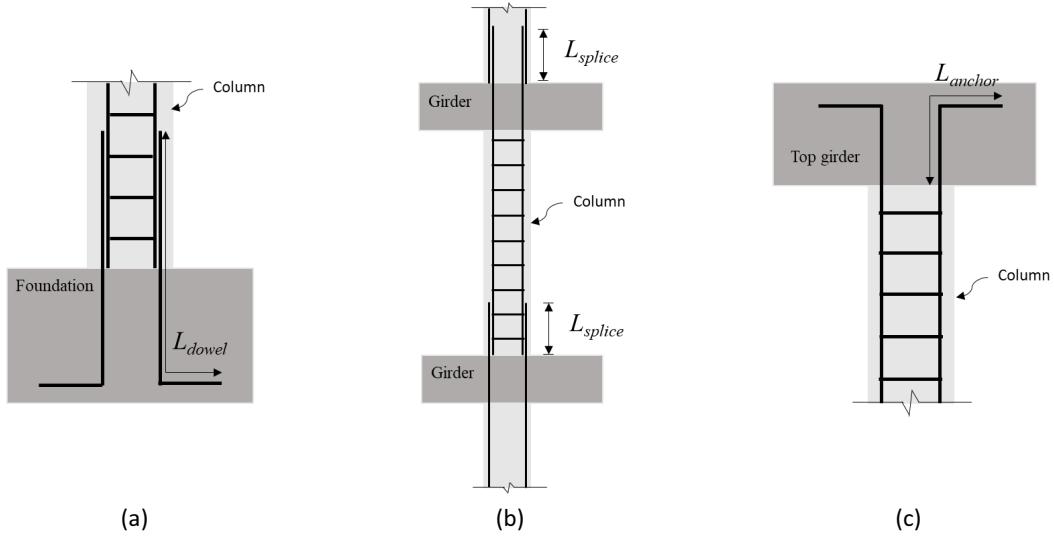
## 2.2 Column Rebar Characteristics

Columns are the main parts of the structural frame which are defined as vertical load-bearing members supporting axial compressive loads[10]. Column transfers the total loads from beams to the foundation and provides stability to the structure. The main rebars of continuous columns are composed of dowel bars that connect foundations and columns, longitudinal rebars that are repeatedly connected by lap splices on each floor, and rebars that are anchored to the top beam, as shown in Figure 2. These main bars are confined with transverse reinforcement.



**Figure 2.** Typical column rebar arrangement

In general, the diameter of the main bar of columns in high-rise buildings and skyscrapers is at least 20mm and 32mm or greater, respectively[11]. The height of a typical floor is intended to remain uniform, although the height of specialized floors, such as mezzanine floors, may differ depending on their specific purpose. A detailed column rebar arrangement is shown in Figure 3. Figure 3(a) shows a dowel bar anchored to the foundation. The length anchored to the foundation is calculated by the tensile anchorage length and the length of the 90-degree standard hook. Figure 3(b) shows the lap splice in the bottom position of each floor and Figure 3(c) shows the column main bar anchored to the top beam.



**Figure 3.** Column rebar detailing by position

As depicted in Figure 2, the lap splice length of the column rebar is calculated using building codes such as ACI and BS[12,13]. Based on BS, Equations (1) through (3) are used to calculate the compression lap splice length ( $l_{lap\_c}$ ) and tension lap splice length ( $l_{lap\_t}$ ) considering the development length of rebar ( $l_d$ ). Where  $f_y$  is yield stress of rebar(MPa),  $d_b$  is the diameter of rebar(mm),  $\gamma_m$  is partial safety factor(1.4),  $\beta$  is a coefficient dependent of tension rebar(0.5) or compression rebar(0.63), and  $f_{cu}$  is the ultimate compressive strength of concrete(MPa).

$$l_d = \frac{f_y d_b}{\gamma_m 4\beta \sqrt{f_{cu}}} \quad (1)$$

$$l_{lap\_c} = 1.25 l_d \quad (2)$$

$$l_{lap\_t} = 1.4 l_d \quad (3)$$

As depicted in Figure 3(a), the dowel bar is anchored to the column using a 90-degree hook and connected to the first floor through a lap splice. It is known that dowel bar rests on foundation rebar. In general, foundation rebar consists of two layers transverse to x-axis and y-axis. Therefore, dowel bar length is calculated considering the depth of the foundation( $D_f$ ),

foundation concrete cover( $Cc_f$ ), rebar diameter of foundation( $d_{b\_f}$ ), lap splice length( $L_{splice}$ ), and hook length( $L_{hook}$ ), as presented in Equation (4).

$$L_{dowel} = (D_f - Cc_f - 2 \times d_{b\_f}) + L_{splice} + L_{hook} \quad (4)$$

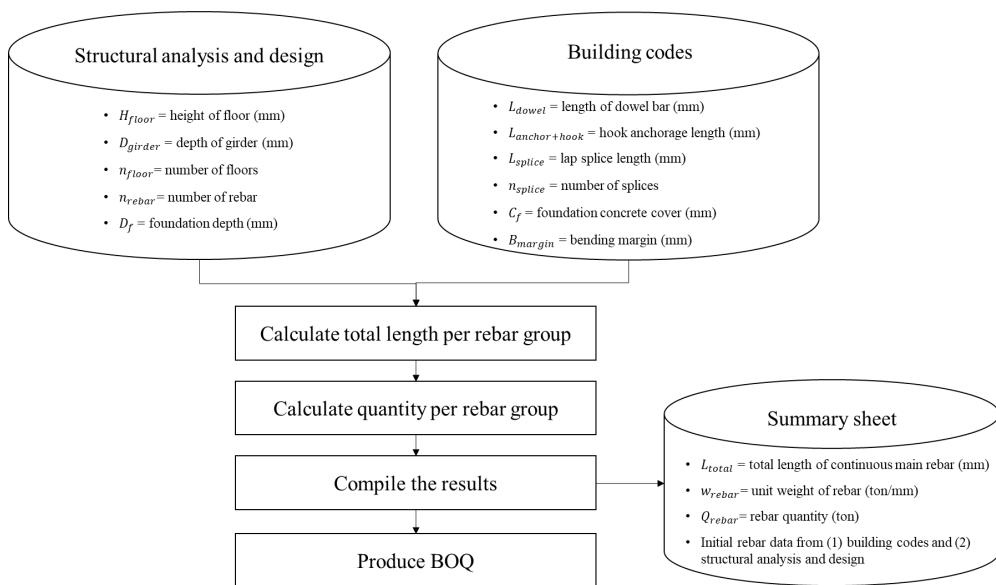
Finally, the dowel bar as shown in Figure 3(a), and the rebar that is anchored to the roof floor as shown in Figure 3(c) are bent. Therefore, when the total length of these rebars is calculated, the bend deduction from the steel bar shape must be considered. Equation (5) presents the bend deduction for a 90-degree bent following BS[14] where  $R$  is the bending radius and  $d_b$  is the rebar diameter.

$$B_{margin} = 0.43R + 1.2d_b \quad (5)$$

### 3. Automated algorithm for estimating rebar quantities in columns

Preliminary studies[1,4,5] have shown that manual quantity estimation may generate errors and be time-consuming. Hence, automatic rebar estimation for column rebar is needed. The proposed algorithm automates the quantity take-off using mathematical algorithms. The mathematical algorithm uses structural analysis and design information and follows rebar specifications, codes, and regulations. The working process of the proposed algorithm is shown in Figure 4. The algorithm can be defined in a sequential manner to effectively attain the desired objective as follows:

1. Obtain the column and rebar information from the structural analysis and design process. The collected data is then supplemented with relevant information derived from rebar specifications and building codes.
2. Calculate the total length of rebar per rebar group.



**Figure 4.** Workflow for automated column rebar quantity estimation algorithm

3. Calculate the rebar quantity per rebar group.
4. Compile the generated results in a summary sheet that will serve as a basis for the creation of a rebar bill of quantity. The summary sheet comprises the initial data from the structural analysis and design process, building codes, the total length, and the total quantity of rebar.

First, the column and rebar information from structural design is obtained. In general, rebar information such as diameter, number of, and spacing of rebar are obtained. In addition, column information such as the height of the floor, the number of floors, the foundation depth, and the depth of the top girder is obtained. These data are complemented by calculation results based on rebar specifications and building codes, including hook anchorage length, dowel bar length, number of lap splices, lap splice length, bending margin, and foundation concrete cover.

After all necessary data are acquired, the working process continues to calculate the total length of the main rebar. The total length equation of column main rebar that stretches from the foundation to the top girder is shown in Equation (6) as referred to Figure 2 which incorporates the height of the floor, number of floors, depth of girder, length of dowel, length of anchorage, lap length, and number of lap splice.

$$L_{total} = \sum H_{floor} - D_{girder} + L_{dowel} + L_{anchor+hook} + \sum L_{splice} - \sum B_{margin} \quad (6)$$

Where  $L_{total}$  is the total length of continuous main rebar(mm),  $H_{floor}$  is the height of floor(mm),  $n_{floor}$  is the number of floors,  $D_{girder}$  is the depth of girder(mm),  $L_{dowel}$  is the length of dowel bar(mm),  $L_{anchor+hook}$  is the hook anchorage length(mm),  $L_{splice}$  is the lap splice length(mm),  $n_{splice}$  is the number of splices, and  $B_{margin}$  is the bending deduction.

In the next step, the quantity of the main rebar ( $Q_{rebar}$ ) is calculated by multiplying the total length of continuous main rebar ( $L_{total}$ ) with the unit weight of rebar ( $w_{rebar}$ ), and the number of rebar ( $n_{rebar}$ ), as shown in Equation (7). By this step, precise rebar quantity is obtained.

$$Q_{rebar} = \sum_{n=1}^N (L_{total} \times w_{rebar} \times n_{rebar}) \quad (7)$$

The proposed algorithm offers several advantages. First, the algorithm can directly use the structural design information as master data without the need for structural drawing. The algorithm implementation is not only saving time from generating structural drawing but also avoiding the error or loss of information that may occur in the structural drawing. Second, contractors can calculate rebar quantity quickly and precisely while satisfying the codes related to rebars such as anchorage length, hook length, lap splice length, etc. Third, the result of this proposed algorithm can be further used to optimize rebar usage and achieve near-zero rebar-cutting waste. Fourth, the obtained rebar information can be further processed to retrieve the bar bending schedule, rebar cutting list, and BOQ of rebar. Fifth, the result of this proposed algorithm can be further used to develop BIM-based drawing automation and assist rebar cutting and bending, installation, and documentation for progress payment.

## 4. Validation of the algorithm

One continuous column which is built from the foundation to the roof floor of the RC building was used to verify the proposed algorithm. The building consists of 22 floors in total(20 floors above ground and 2 basement floors). The column height per floor varies following the floor height. The standard floor height is 3800mm while the shortest and tallest height floor is 3700mm and 6000mm, respectively. The foundation depth and girder depth are 600mm and 700mm, respectively. The detailed data regarding the column study case is summarized in Table 1.

**Table 1.** Column and column rebar data

Description	Contents
Foundation depth( $D_f$ )	600mm
Foundation concrete cover( $C_f$ )	50mm
Basement level(B2-B1) height	8300mm
Upper ground level(F1-Roof) height	87,400mm
Total floor height( $\sum H_{floor}$ )	95,700mm
Girder depth( $D_{girder}$ )	700mm
Rebar diameter( $d$ )	UHD600 D29
Concrete strength( $f_{ck}$ )	B2-F20: 35MPa
Girder depth( $D_{girder}$ )	700mm
Lap splice length( $L_{splice}$ )	1500mm
Anchorage length( $L_{anchor}$ )	1050mm
90-degree hook length( $L_{hook}$ )	350mm
Dowel bar length( $L_{dowel}$ )	2350mm
Bending deduction( $B_{margin}$ )	79mm

Along the column itself, there are several rebar layout arrangements from the B2 up to the roof floor due to the decreased dimension of columns and the number of rebars on the higher floors. For example, there are 42 rebar from B2 to B1 while there are only 14 rebar from F13 to F20. Figure 5 depicts the rebar layout arrangement per section.

From this layout arrangement, it can be concluded that there are rebars stretch from the foundation to the roof floor and other rebars stretch from the foundation to some specific point in the column. The proposed algorithm of this study utilizes that rebar layout by grouping the rebar with the same length. Therefore, column rebar is further categorized based on the same length. For example, rebars that are stretching from the foundation to the roof floor are grouped and named 1<sup>st</sup> rebar group. While rebars that are stretching from the foundation to F13 are grouped and named the 2<sup>nd</sup> rebar layer. This grouping was performed repeatedly until seven rebar groups were obtained, as illustrated in Figure 6.

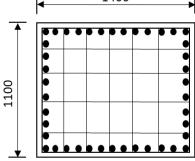
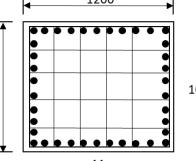
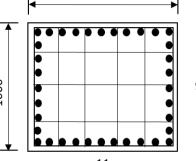
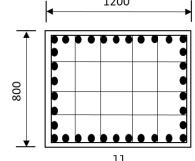
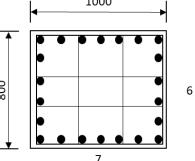
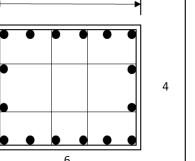
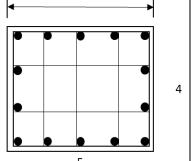
Floors	B2-B1	F1	F2-F3
C14	 1400 1100 13	 1200 1100 11	 1200 1000 11
Concrete strength (fck)	35	35	35
Dimension	1400 x 1100	1200 x 1100	1200 x 1000
Reinforcement	42 – UHD29	38 – UHD29	36 – UHD29
Hoops	Both ends Center	HD10 @300 HD10 @150	HD10 @150 HD10 @300
F4-F6	F7-F8	F9-F12	F13-F20
 1200 800 11	 1000 800 7	 1000 800 6	 1000 800 5
35	35	35	24
1200 x 800	1000 x 800	1000 x 800	1000 x 800
34 – UHD29	22 – UHD29	16 – UHD29	14 – UHD29
HD10 @150	HD10 @150	HD10 @150	HD10 @150
HD10 @300	HD10 @300	HD10 @300	HD10 @300

Figure 5. Section-wise rebar arrangement

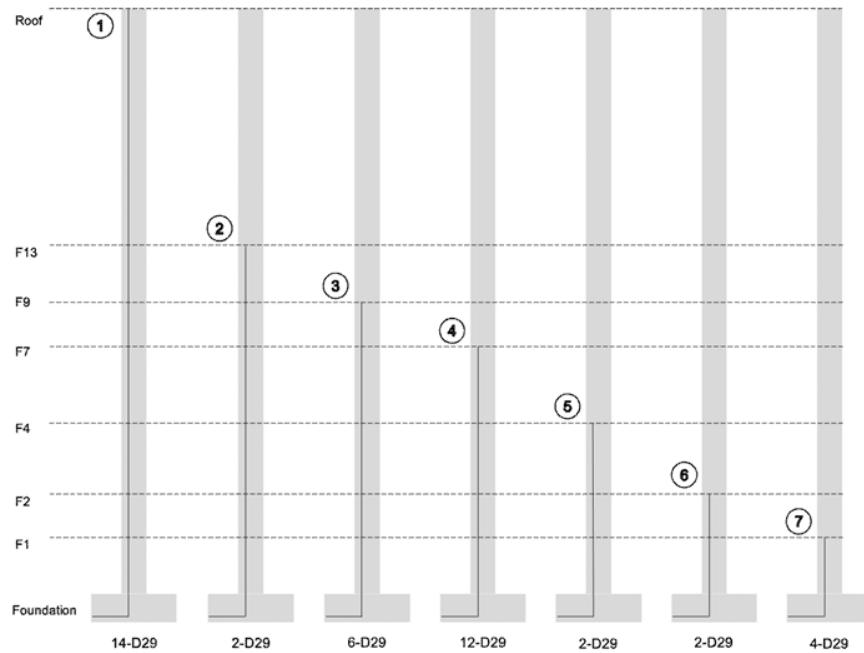


Figure 6. Grouping of rebars with similar length

As all necessary data to calculate the rebar total length was obtained, the total rebar length was calculated as shown in Equation (6). Take the 1<sup>st</sup> rebar group(foundation to the roof level) as an example. By entering all the data, a total length of 130.25m for the 1<sup>st</sup> rebar group was obtained.

$$L_{total} = \sum H_{floor} - D_{girder} + L_{dowel} + L_{anchor} + \sum L_{splice} - B_{margin} \quad (8)$$

$$L_{total} = 95.7 - 0.7 + 2.35 + 1.4 + (1.5 \times 21) - (79 \times 2) = 130.092m$$

After calculating the total length for all rebar groups, rebar quantity was calculated by multiplying the total length of continuous main rebar( $L_{total}$ ) with the unit weight of rebar( $w_d$ ), and the number of rebars( $n_{rebar}$ ), as shown in Equation (7). For example, using the unit weight of rebar D29 is  $5.185 \times 10^{-6}$  ton/mm, and the 1<sup>st</sup> rebar group has 14 rebars( $n_{rebar}$ ) in total, the total quantity for the 1<sup>st</sup> rebar group was 9.443 tons.

$$Q_{rebar} = \sum_{n=1}^N (L_{total} \times w_{rebar} \times n_{rebar}) \quad (9)$$

$$Q_{rebar} = 130,092 \times 5.18 \times 10^{-6} \times 14 = 9.443 \text{ tons}$$

Finally, the calculation result for all rebar groups is summarized as shown in Table 2. The overall precise required quantity for the column main rebar is 16.485 tons.

**Table 2.** Summary of the calculation results

Rebar group	Total length(mm)	Number of rebars	Precise required quantity(ton)
1 <sup>st</sup>	130,092	14	9.443
2 <sup>nd</sup>	85,871	2	0.890
3 <sup>rd</sup>	64,671	6	2.012
4 <sup>th</sup>	54,071	12	3.364
5 <sup>th</sup>	32,371	2	0.336
6 <sup>th</sup>	18,171	2	0.188
7 <sup>th</sup>	12,071	4	0.250
Total			16.485

In the last step, the precise required quantity was compared with the estimated quantity obtained from manual quantity estimation. A comparison of the rebar quantities generated by the manual estimation and the proposed algorithm is depicted in Table 3. The quantity of rebar in manual estimation was obtained based on expert judgments.

**Table 3.** Comparison of generated rebar quantities

Description	Manual estimation(ton)	Proposed algorithm(ton)	Difference(%)
	(1)	(2)	$\frac{(1-2)}{(1)} * 100\%$
Main rebars	16.831	16.485	2.056

The estimated quantity and precise quantity of column main rebar are 16.831 tons and 16.485 tons, respectively. The difference is 0.346 tons, equal to 2.056%. According to the results, there was an overestimation of the required rebar quantity. Such overestimated estimation can make the contractor less competitive and potentially fail to secure the contract. Conversely, the underestimated estimation may compel the contractors to spend extra expenses as they need to procure more rebar than initially estimated, leading to reduced profits or even losses.

Furthermore, an overestimation condition in construction projects can result in adverse outcomes for contractors, including the loss of bids, financial losses, and wastage of resources invested in the bidding process. In addition, such a condition can harm the contractor's reputation and credibility, leading to the loss of future project opportunities. Even if the contractor successfully secures the contract, the construction may yield a high waste generation, resulting in several serious problems. On the other hand, underestimation can result in lower productivity, project delays, and lower quality of the project due to insufficient material supply. In severe cases, the situation may push the contractor into financial distress and bankruptcy. Therefore, the proposed algorithm offers an effective solution to mitigate the negative consequences associated with both conditions, benefiting the contractor.

The disparity in quantities arises from the potential errors made by humans during manual QTOs, leading to both overestimation and underestimation. In contrast, the proposed algorithm is a computer-based system that eradicates the potency of errors by directly incorporating rebar information from structural analysis and design. However, it is important to note that both quantities represent the required quantity that needs to be incorporated with stock length or special length to obtain the necessary cutting pattern and purchased quantity.

The rebar information obtained from this study can be further used to build a 3D model in BIM. Consequently, the bar bending schedule, rebar cutting list, and BOQ of rebar can be automatically retrieved from BIM. The automation can significantly assist contractors in planning rebar cutting and bending, installation, and documentation for progress payment efficiently. Furthermore, the rebar information can be employed to generate rebar cutting patterns using special length or stock length to optimize rebar usage, thereby achieving near-zero rebar cutting waste.

## 5. Conclusion

Existing quantity estimation relies on a manual process that is prone to human error and can only generate an estimated quantity. In this regard, this study proposed an automated algorithm for quantity estimation to obtain the precise quantity of column rebar. The algorithm uses structural design information with other necessary information obtained by following rebar specifications and building codes. The estimated quantity and precise quantity of column main rebar are 16.831 tons and 16.485 tons, respectively. The difference is 0.346 tons, which is equal to 2.056%. The disparity in results from the possibility of human error during manual QTOs, which can result in both overestimation and underestimation. By contrast, the proposed

algorithm is a computerized system that eliminates the possibility of errors by directly incorporating rebar information from structural analysis and design.

Despite the proposed algorithm's ability to precisely estimate the rebar quantities, the prevailing and well-established take-off quantity notion facilitates each construction company in formulating its methods for rebar estimation, making the practical application becomes quite challenging. Recognizing the existence of diverse methods across the companies, a greater effort is needed to promote the benefits and advantages of the proposed algorithm within the industry. The algorithm presented in this study should be developed in the future as computerized software that automatically performs total length and quantity calculations using a summary sheet of column main rebar. Not only that, but the result can also be further used to automatically generate BBS, RCL, BOQ, and shop drawings. For future studies, the proposed algorithm can be further used to develop BIM-based drawing automation and assist rebar cutting and bending, installation, and documentation for progress payment. Besides the rebar automation, the proposed algorithm can be further used as a bridge to develop a rebar-cutting waste minimization algorithm. The construction sites continue to generate a high amount of cutting waste due to manual estimation, procurement, and installation problems. Hence, the precise estimation algorithm can mitigate the potential of errors made during the procurement and installation process. The practical application of this study's findings offers significant potential for construction companies, as it facilitates efficient and precise estimation of rebar quantities while ensuring adherence to related specifications and regulatory codes.

## Funding

This work was supported by the National Research Foundation of Korea(NRF) grants funded by the Korean government (MOE)(No. 2022R1A2C2005276).

## ORCID

Titi Sari Nurul Rachmawati,  <http://orcid.org/0000-0002-8190-9077>

Daniel Darma Widjaja,  <http://orcid.org/0009-0003-5077-1284>

Sunkuk Kim,  <http://orcid.org/0000-0002-7350-4483>

## References

1. Holm L, Schaufelberger JE, Griffin D, Cole T. Construction cost estimating: Process and practices. 1st ed. London (UK): Pearson; 2005. 354 p.
2. Khosakitchalert C, Yabuki N, Fukuda T. Improving the accuracy of BIM-based quantity takeoff for compound elements. Automation in Construction. 2019 Oct;106:102891. <https://doi.org/10.1016/j.autcon.2019.102891>
3. Aram S, Eastman C, Sacks R. A knowledge-based framework for quantity takeoff and cost Estimation in the AEC industry using BIM. The 31st International Symposium on Automation and Robotics in Construction and Mining; 2014 Jul 9-11; Sydney, Australia. Australia: The International Association for Automation and Robotics in Construction; 2014. P. 434-42. <https://doi.org/10.22260/ISARC2014/0058>

4. Monteiro A, Martins JP. A survey on modeling guidelines for quantity takeoff-oriented BIM-based design. *Automation in Construction*. 2013 Nov;34:238-53. <https://doi.org/10.1016/j.autcon.2013.05.005>
5. Sacks R, Eastman C, Lee G, Teicholz P. *BIM Handbook: A guide to building information modeling for owners, designers, engineers, contractors, and facility managers*. 3rd ed. Hoboken (NJ): Wiley; 2018. 688 p.
6. Kim D, Lim C, Liu Y, Kim S. Automatic estimation system of building frames with integrated structural design information (AutoES). *Iranian Journal of Science and Technology, Transactions of Civil Engineering*. 2020 Sep;44:1145-57. <https://doi.org/10.1007/s40996-019-00308-5>
7. Halpin DW, Senior BA. *Construction management*. 4th ed. Hoboken (NJ): Wiley; 2011. 445 p.
8. Northen A, Benge D. *NRM 2: Detailed Measurement for Building Works*. 2nd ed. London (UK): Royal Institution of Chartered Surveyors (RICS); 2021. 348 p.
9. Civil Engineers I. *CESMM4: civil engineering standard method of measurement*. 4th ed. London (UK): Thomas Telford Ice; 2012. 111 p.
10. Samarakkody DI, Thambiratnam DP, Chan THT, Moragaspitiya. Differential axial shortening and its effects in high rise buildings with composite concrete filled tube columns. *Construction and Building Materials*. 2017 Jul;143:659-72. <https://doi.org/10.1016/j.conbuildmat.2016.11.091>
11. Murcia-Delso J, Stavridis A, Shing B. Modeling The Bond-Slip Behavior of Confined Large Diameter Reinforcing Bars. III ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering; 2011 May 25-28; Corfu, Greece. Athens (Greece): National Technical University of Athens; 2011. p. 1-14.
12. American Concrete Institute 318-14. *Building code requirements for structural concrete*. Farmington Hills (MI): American Concrete Institute; 2014. 519 p.
13. British Standard Institute. *Structural use of concrete - part 1: code of practice for design and construction*. Londun (UK): British Standard Institute; 1997. 150 p.
14. BS 8666:2020. *Scheduling, dimensioning, cutting and bending of steel reinforcement for concrete - Specification* Londun (UK): British Standard Institute; 2020. 32 p.