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COMPLETELY V-REGULAR ALGEBRA ON SEMIRING AND ITS APPLICATION IN EDGE DETECTION

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Abstract. In this paper, Completely V-Regular on semiring is defined and used to derive new theorems with some of its properties. This paper also illustrates V-Regular algebra and Completely V-Regular Algebra with examples and properties. By extending completely V-Regular to fuzzy, a new concept, fuzzy V-Regular is brought out and fuzzy completely V-Regular algebra is introduced too. It is also developed by defining the ideals of Completely V -Regular Algebra and fuzzy completely V-Regular algebra. Finally, this fuzzy algebra concept is applied in image processing to detect edges. This V-Regular Algebra is novel in the research area.

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1. Introduction

Von Neumann regular ring was introduced by Von Neumann[1] in 1936, under the name of regular rings. It is explained that as for any a in a ring R, there exists x in R such that $axa = a$. In 1934[2] Vandiver introduced an algebraic system; the system he constructed was ring-like rather than a ring. Vandiver called this system a Semiring, which is defined as a system (S, +, $\cdot$) is said to be a semiring if it is closed and associative under addition and multiplication. In 1951, [3] S. Bourne introduced a regular on ring R as, for any a in a semiring R, there exist x, y in R such that $a + axa = aya$. In 1996, Adhikari and Sen came out with k-regular Semiring which is defined as, for any a in a semiring R, there exists x in R such that $a + axa = axa$ [4]. Later completely regular semirings were studied and developed by Rick Schumann in 2013 [5]. Followed by the aforementioned study the completely V-Regular was developed. The concept of fuzzy sets is

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introduced by Zadeh. In the algebraic structures like semigroups, groups, rings, modules, vector spaces and topologies, these ideas have been applied in ([6], [7]). In 1966, the concept of fuzzy sets to BCK -algebras is introduced by Imai and Iseki[8], also which is applied by Xi in 1991[9].

Computer vision is concerned with the development of systems that can interpret the content of natural scenes. Computer vision systems start with detective work and finding some options within the input image. One of the main advantages of feature extraction is that it significantly reduces the amount of data required to represent an image in order to understand its content. To advance the fuzzy algebra theory in our research, we applied the fuzzy idea to image processing. In this paper, Completely V-Regular is developed by deriving some of its properties from the comparision with completely V-Regular with Von Neumann regular, Borne regular and k-regular on semirings ([10], [11]). In the extension, V-Regular algebra and Completely V-Regular algebra are introduced and analyzed with examples. From these algebras, fuzzy V-Regular algebra and fuzzy completely V-Regular algebra are introduced. Some results also proved. By referring to [12] and [13], Fuzzy completely V-Regular ideal is also demonstrated with examples. Finally, having gone through ([14], [15], [16], [17], [18], [19], [20]), this concept is applied in the image processing to detect the edges by using the prpposed Accelerated Segment Test (FVRFAST) edge detector method. This paper is divided into five different parts each discussing one aspect.

2. Basic Concepts

Here, some basic definitions have been discussed.

Definition 2.1. *k-inverse*

Let S be a semiring and $a \in S$. Then $x \in S$ is called a k-inverse of a if

$$a + axa = axa \tag{1}$$

$$x + xax = xax \tag{2}$$

Definition 2.2. *k- semifields*

An additive idempotent commutative semiring S is a k-semifield if for $a \in S$ and $b \in S_1$, where S_1 is the set of all non zero element of S, there are x_1, x_2, y_1 and $y_2 \in S$ such that

$$a + x_1 b = x_2 b. \tag{3}$$

$$a + by_1 = by_2 \tag{4}$$

3. Completely V-Regular Semirings

The definition of completely V-Regular and some of its results have been discussed in this section.

Definition 3.1. *Completely V-Regular semiring*

Let $(S, +, \cdot)$ be a semiring and $a \in S$. Then a is called completely V-Regular if *there exists an element x satisfying*

$$a = a + x + a \tag{5}$$

$$a + x = x + a \tag{6}$$

$$a(a + x) = (a + x) \tag{7}$$

$$a \bullet x \bullet a = a \tag{8}$$

Furthermore, S is called completely V-Regular if every element of S is completely V-Regular.

Example 3.2. *On $S = \{a,\ b,\ c\}$ the binary operations are defined in tables 1 and 2.*

This system $(S,\ +,\bullet)$ is a completely V-Regular semiring, it is an example

TABLE 1. The operator +.

$+$	a	b	c
a	a	b	c
b	b	b	c
c	c	c	b

TABLE 2. The operator $\bullet$

$\bullet$	a	b	c
a	a	a	a
b	a	b	b
c	a	b	c

with the least order.

Theorem 3.3. *In a semi ring $(S,\ +,\ \bullet)$ every completely V-Regular is Bourne regular.*

Proof. Let $(S, +, \bullet)$ be semi ring and let S be a completely V-Regular. Then for all $a \in S$ there exists $x \in S$ such that (i) $a = a + x + a$, (ii) $a + x = x + a$, (iii) $a(a + x) = (a + x)$, (iv) $axa = a$. Let $y \in S$. Then $aya \in S$. Adding aya on both sides of (iv), we get

$$\begin{aligned} a + aya &= axa + aya \\ a + aya &= a(x + y)a \\ a + aya &= aza \quad \textit{where} \quad z = x + y \in S \end{aligned}$$

Hence for all $a \in S$ there are $y, z \in S$ such that $a + aya = aza$. □

Theorem 3.4. *Every additive idempotent completely V-Regular is k-regular*

Proof. Let $(S, +, \bullet)$ be an additive idempotent completely V-Regular semiring. Then for all $a \in S$ there exists an element $u \in S$ such that (i) $a = a + u + a$, (ii) $a + u = u + a$, (iii) $a(a + u) = (a + u)$, (iv) $aua = a$. From (iv), we get

$$\begin{aligned} aua &= a \\ a + aua &= a + a \\ a + aua &= a \quad (additive \quad idempotent) \\ a + aua &= aua \quad (By \quad iv) \end{aligned}$$

Hence $(S, +, \bullet)$ is k-regular. □

Theorem 3.5. *Every commutative idempotent k-regular in which cancellation law holds, is completely V-Regular semiring.*

Proof. Let $(S, +, \bullet)$ be a commutative idempotent k-regular semiring. Then for $a \in S$ there exists an element $x \in S$ such that $a + axa = a$. Clearly $axa \in S$.

Now $a + axa = axa \Rightarrow a + axa = axa + axa$(idempotent)

$$\begin{aligned} \Longrightarrow a &= axa \quad (By \quad R.C.L) & (9) \\ \Longrightarrow a &= aaxaa \quad (idempotent) \\ \Longrightarrow a &= a(axa)a & (10) \end{aligned}$$

Now

$$\begin{aligned} a &= a + a + a \quad (idempotent) \\ \Longrightarrow a &= a + axa + a \quad (by (9)) & (11) \end{aligned}$$

Since S is commutative

$$\begin{aligned} a + axa &= axa + a & (12) \\ \Longrightarrow a(a + axa) &= aa + aaxa \\ &= a + axa \quad (idempotent) & (13) \end{aligned}$$

From (10),(11),(12) and (13), for all $a \in S$, there exists $axa \in S$ such that the conditions of Completely V-Regular Semiring are satisfied. Hence S is completely V-Regular semiring. □

Theorem 3.6. *Every completely V-Regular element has a k-inverse.*

Proof. If $a \in S$ is k-regular then there is $x \in S$ such that $a = axa$. Then

$$\begin{aligned} a + axa &= axa & (14) \\ a + axa + axaxa &= axa + axaxa \\ a + ax(a + axa) &= ax(a + axa) \\ a + a(xax)a &= a(xax)a & (15) \end{aligned}$$

Also $x, a \in S \Longrightarrow xax \in S$, and since S is semiring $xax + xaxax = xaxax$ (pre and post multiplying by x in (14))

$$\Longrightarrow xax + xaxax + xaxaxax = xaxax + xaxaxax$$

$$\Longrightarrow xax + xax(a + axa)x = xax(a + axa)x$$
$$\Longrightarrow xax + (xax)a(xax) = (xax)a(xax)$$

Hence xax is a k-inverse of a in S. Thus every completely V-Regular element has a k-inverse. □

Theorem 3.7. *Let S be an idempotent k-semifield in which cancellation law holds. Then S is completely V-Regular.*

Proof. Let S be a k-semifield and $a \in S_1$. Then there are $t, x \in S$ such that $a + ta = ta$ and $t + ax = ax$. These two implies that $a + ta + axa = ta + axa$. Thus $a + (t + ax)a = (t + ax)a$ which implies that $a + axa = axa$. Hence S is k-regular. Then S is completely V-Regular. □

4. V-Regular Algebra and Fuzzy V-Regular Algebra on Semirings

In this section, V-Regular algebra and fuzzy V-Regular algebra are introduced on semirings and examples are given. Some of it's properties have been discussed. Also fuzzy V-Regular ideal is introduced and explained.

Definition 4.1. *V-Regular algebra*

Let (S, +, •) be a semiring. Then S is called V-Regular algebra if there exists an element x in S satisfying

$$0 = 0 + x + 0 \tag{16}$$
$$0 = 0 \bullet x \bullet 0 \tag{17}$$
$$a + x = x + a \quad \forall a \in S \tag{18}$$

Example 4.2. *On $S = \{0, 1, 2\}$ the binary operations are defined in tables 3 and 4.*

From the tables, for every element of S, $2 \in S$ satisfying all the conditions (16),

TABLE 3. The operator +.

+	0	1	2
0	0	1	0
1	1	1	0
2	0	0	1

TABLE 4. The operator •.

•	0	1	2
0	0	0	0
1	0	1	1
2	0	1	0

(17) *and* (18). *Then S is a V-Regular algebra.*

Definition 4.3. ***Subalgebra of V-Regular algebra***

Let (S, +, •) be a V-Regular algebra. Let U be a subset of S. Then U is said to be a subalgebra of S if $a \in S$, $x \in U \Rightarrow ax \in U$.

Definition 4.4. ***V-Regular ideal***

Let (S, +, •) be a V-Regular algebra. Let U be a subset of S. Then U is said to be a V-Regular left ideal if (i) $0 \in U$, *(ii)* $a \in S$, $x \in U \Rightarrow a+x, ax \in U$. *Again U is said to be a V-Regular right ideal if (i)* $0 \in U$, *(ii)* $a \in S$, $x \in U \Rightarrow x+a, xa \in U$. *Then U is said to be a V-Regular ideal if U is both V-Regular left and right ideals.*

Definition 4.5. ***Fuzzy V-Regular***

Let S be a V-Regular algebra. Then a fuzzy set μ *is said to be fuzzy V-Regular if for any x in S* $\mu(x) \geq min\{\mu(a+x), \mu(a)\}$ *for all* $a \in S$.

Example 4.6. *On* $S = \{0, 1, 2\}$ *the binary operations are defined in tables 5 and 6, for every element of S,* $2 \in S$ *satisfying all the conditions* (16), (17) *and* (18). *Then S is*

Table 5. The operator +.

+	0	1	2
0	0	1	0
1	1	1	1
2	0	1	1

Table 6. The operator •.

•	0	1	2
0	0	0	0
1	0	1	1
2	0	1	0

a V-Regular algebra. Define μ *as* $\mu(0) = 0.4$, $\mu(1) = 0.3$, $\mu(2) = 0.5$. *Then it is easy to verify that* μ *is a fuzzy V-Regular algebra.*

Definition 4.7. ***Fuzzy Subalgebra of V-Regular***

Let S be a V-Regular algebra. Then a fuzzy set μ *is said to be fuzzy Subalgebra of S if for any x in S* $\mu(a \bullet x) \geq min\{\mu(a), \mu(x)\}$ *for all a in S.*

Definition 4.8. ***Fuzzy V-Regular ideal***

A fuzzy ideal μ *of a V-Regular algebra S is said to be a fuzzy V-Regular left ideal of S if* $\mu(x) \geq min\{\mu(x + a), \mu(x \bullet a), \mu(a)\}$ *for all x,* $a \in S$ *and* μ *is said to be a fuzzy V-Regular right ideal of S if* $\mu(x) \geq min\{max\{\mu(a + x), \mu(a \bullet x), \mu(a)\}$ *for all x,* $a \in S$. *Then* μ *is fuzzy V-Regular ideal of S if* $\mu(x) \geq min\{max\{\mu(x + a), \mu(a + x), \mu(a \bullet x), \mu(x \bullet a), \mu(a)\}$ *for all x,* $a \in S$.

Proposition 4.9. *If ν and λ are fuzzy V-Regular subalgebras of a V-Regular algebra S, then so is $\nu\cup\lambda$.*

Proof. Let $s, t\in A$. Then

$$\begin{aligned}
\nu\cup\lambda(s\bullet t) &\geq min\{\nu(s\bullet t),\lambda(s\bullet t)\}\\
&\geq (min\{\nu(s),\lambda(s)\})\bullet(min\{\nu(t),\lambda(t)\})\\
&= (\nu\cup\lambda)(s)\bullet(\nu\cup\lambda)(t)\\
&= min\{(\nu\cup\lambda)(s)),(\nu\cup\lambda)(t)\}
\end{aligned}$$

Hence $\nu\cup\lambda$ is a fuzzy V-Regular subalgebra of a V-Regular algebra S. □

Proposition 4.10. *If λ and ν are fuzzy V-Regular subalgebras of a V-regular algebra S, then $\lambda\times\nu$ is fuzzy V-Regular subalgebra of a V-Regular algebra $S\times S$.*

Proof. For any $s_1, s_2, t_1, t_2\in S$,

$$\begin{aligned}
(\lambda\times\nu)((s_1,t_1)\bullet(s_2,t_2)) &\geq min\{(\lambda\times\nu)(s_1\bullet s_2),(\lambda\times\nu)(t_1\bullet t_2)\}\\
&= min\{\lambda(s_1\bullet s_2)\times\nu(s_1\bullet s_2),\\
&\quad \lambda(t_1\bullet t_2)\times\nu(t_1\bullet t_2)\}\\
&= \{(min\{\lambda(s_1),\nu(s_1)\})\bullet(min\{\lambda(s_2),\nu(s_2)\})\}\\
&\quad \times\{(min\{\lambda(t_1),\nu(t_1)\})\bullet(min\{\lambda(t_2),\nu(t_2)\})\}\\
&= min\{(\lambda(s_1)\times\nu(t_1)),(\lambda(s_2)\times\nu(t_2))\}\\
&= min\{(\lambda\times\nu)(s_1,t_1),(\lambda\times\nu)(s_2,t_2)\}
\end{aligned}$$

Hence $\lambda\times\nu$ is fuzzy V-Regular subalgebra of a V-Regular algebra $S\times S$. □

Proposition 4.11 ([12])**.** *A fuzzy subset ν of a V-Regular algebra S is a fuzzy left (right) V-Regular ideal of S if and only if each nonempty level subset of ν is a left (right) V-Regular ideal of S.*

Proof. Assume that μ is a fuzzy V-Regular ideal of S. Suppose that $a\in\mu_t$ and $x\in S$, and $(a+x)\in\mu_t$ and $ax\in\mu_t$. Then $\mu(a)\geq t, \mu((a+x))\geq t$ and $\mu(ax)\geq t$, and hence $\{\mu(a+x),\mu(ax)\}\geq t$. Since μ is a fuzzy V-Regular ideal of S, $\mu(x)\geq min\{\mu(a+x),\mu(ax),\mu(a)\}$, i.e., $x\in\mu_t$. Hence μ_t is a V-Regular ideal of S.

Conversely, assume μ_t is a V-Regular ideal of S, for any $t\in[0,1]$ with $\mu_t\neq\emptyset$. For any $x, a\in S$, let $\mu(a)=t_1$, $\mu(a+x)=t_2$, $\mu(ax))=t_3$ ($t_i\in[0,1]$). If we let $t=min\{max\{t_2,t_3\},t_1\}$, then $a\in\mu_t$ and $a+x\in\mu_t$ and $ax\in\mu_t$. Since μ_t is a V-Regular ideal of S, we have $x\in\mu_t$, i.e., $\mu(x)\geq min\{max\{\mu(a+x),\mu(\text{a x}),\mu(a)\}$, so that μ is a fuzzy V-Regular ideal of S. □

Proposition 4.12. *An onto homomorphic preimage of a fuzzy V-Regular left (or right) ideal is a fuzzy V-Regular left (or right) ideal.*

Proof. Let $h : S \rightarrow S^1$ be an onto homomorphism. Let σ be a fuzzy completely V-Regular left ideal and let μ be the preimage of ν under h. Then for any $x, a \in S$,

$$\begin{aligned} \mu(x) &\geq \sigma(h(x)) \\ &\geq min\{\sigma(h(x) + h(a)), \sigma(h(a)h(x)), \sigma(h(a))\} \\ &= min\{\sigma(h(x + a)), \sigma(h(a \bullet x)), \sigma(h(a))\} \\ &= min\{\mu((x + a)), \mu(a \bullet x), \mu(a)\} \end{aligned}$$

Thus μ is a fuzzy completely V-Regular left (or right) ideal of S. □

5. Completely V-Regular Algebra and Fuzzy Completely V-Regular Algebra on Semirings

Definition 5.1. *Completely V-Regular algebra*

Let (S, +, •) be a semiring. Then S is said to be a V- Regular algebra if there exists an $x \in S$ satisfying

$$0 = 0 + x + 0 \quad and \quad 1 = 1 + x + 1 \tag{19}$$

$$0 = 0 \bullet x \bullet 0 \quad and \quad 1 = 1 \bullet x \bullet 1 \tag{20}$$

$$a + x = x + a, \quad \forall \ a \in S \tag{21}$$

$$a \bullet (a + x) = (a + x), \forall \ a \in S \tag{22}$$

Example 5.2. *From the example 4.6, for every element of S, 2 ∈ S satisfying all the conditions 19,20,21 and 22, then S is a completely V-Regular algebra.*

Definition 5.3. *Subalgebra of completely V-Regular algebra*

Let (S, +, •) be a completely V-Regular algebra. Let U be a subset of S. Then U is said to be a subalgebra of S if a∈ S, x ∈ U ⇒a •(a + x) ∈ S.

Definition 5.4. *completely V-Regular ideal*

Let (S, +, •) be a completely V-Regular algebra. Let U be a subset of S. Then U is said to be a completely V-Regular left ideal if (i) 0, 1 ∈ U, (ii) a ∈S, x ∈U ⇒a•(a + x) ∈ U. U is said to be Completely V-Regular right ideal if (i) 0, 1∈ U, (ii) a∈S, x ∈U ⇒a•(x + a)∈ U. U is Completely V-Regular ideal if U is both completely V-Regular left and right ideal.

Definition 5.5. *Fuzzy completely V-Regular*

Let S be a completely V-Regular algebra. Then a fuzzy set μ is said to be fuzzy completely V-Regular of S if for any x in S μ(x) ≥ min{μ(a + x), μ(a •x), μ(a)} for all a in S.

Example 5.6. *By the example 5.2, it is very clear that μ is a fuzzy completely V-Regular algebra.*

Definition 5.7. *Fuzzy Subalgebra of completely V-Regular*

Let S be a completely V-Regular algebra. Then a fuzzy set μ is said to be fuzzy Subalgebra of S if for any x in S μ(a•(a + x)) ≥ min{μ(a),μ(a + x)} for all a in S.

Definition 5.8. *Fuzzy completely V-Regular ideal*

A fuzzy set μ of a completely V-Regular algebra S is said to be a fuzzy completely V-Regular left ideal of S if $\mu(x) \geq min\{max\{\mu(a\bullet(x + a)), \mu(a)\}$ for all x, $a\in S$ and μ is said to be a fuzzy completely V-Regular right ideal of R if $\mu(x) \geq min\{max\{\mu(a\bullet(a + x)), \mu(a)\}$ for all x, $a\in S$. Then μ is said to be a fuzzy completely V-Regular ideal of S if $\mu(x) \geq min\{max\{\mu(a\bullet(x + a)), \mu(a\bullet(a + x))\}, \mu(a)\}$ for all x, $a \in S$.

Proposition 5.9. *Let μ be a fuzzy subset of S. Then μ is a fuzzy completely V-Regular ideal of S if and only if, for any $t\in[0,1]$ such that $\mu_t \neq \emptyset$, μ_t is a completely V-Regular ideal of S, where $\mu_t = \{x\in S| \ \mu(x) \geq t\}$, which is called a level subset of μ.*

Proof. Assume that μ is a fuzzy completely V-Regular ideal of S. Suppose that $a\in \mu_t$ and $x\in S$, and $a\bullet(a+x)\in\mu_t$or $a\bullet(x+a)\in \mu_t$. Then $\mu(a) \geq t, \mu(a\bullet(a+x)) \geq t$ and $\mu(a\bullet(x+a)) \geq t$, and hence $\max\{\mu(a\bullet(a+x)), \mu(a\bullet(x+a))\} \geq t$. Since μ is a fuzzy completely V-Regular ideal of S, $\mu(x) \geq \min\{\max\{\mu(a+x), \mu(x+a)\}, \mu(a)\}$, $i.e., x\in\mu_t$. Hence μ_t is a completely V-Regular ideal of S.

Conversely, assume μ_t is a completely V-Regular ideal of S, for any $t\in[0,1]$ with $\mu_t\neq\emptyset$. For any $x, a\in S$, let $\mu(a) = t_1$, $\mu(a\bullet(x+a)) = t_2$, $\mu(a\bullet(a+x)) = t_3(t_i\in[0,1])$. If we let $t = min\{max\{t_2, t_3\}, t_1\}$, then $a\in\mu_t$ and $a\bullet(a+x)\in\mu_t$ and $a\bullet(x+a)\in\mu_t$. Since μ_t is a completely V-Regular ideal of S, we have $x\in\mu_t$, i.e., $\mu(x) \geq min\{max\{\mu(a\bullet(x+a)), \mu(a\bullet(a+x))\}, \mu(a)\}$, so that μ is a fuzzy completely V-Regular ideal of S. □

Proposition 5.10. *An onto homomorphic pre image of a fuzzy completely V-Regular left (or right) ideal is a fuzzy completely V-Regular left (or right) ideal.*

Proof. Let $g : S\to S^1$ be an onto homomorphism. Let σ be a fuzzy completely V-Regular left ideal and let μ be the preimage of ν under g. Then for any $x, a\in S$,

$$\begin{aligned} \mu(x) &\geq \sigma(g(x)) \\ &\geq min\{max\{\sigma[g(a)\bullet(g(x)+g(a))], \sigma[g(a)\bullet(g(a)+g(x))]\}, \sigma(g(a))\} \\ &= min\{max\{\sigma[g(a)\bullet(g(x+a))], \sigma[g(a)\bullet(g(a+x))]\}, \sigma(g(a))\} \end{aligned} \quad (23)$$

Thus μ is a fuzzy completely V-Regular left (or right) ideal of S. □

6. Completely V-Regular fuzzy algebra in Image Processing

In recent years, the field of computer vision has experienced significant growth. To obtain the appropriate purpose of a feature using similarity matching, a template window is used. This window is moved element by element over a larger search window around a computable corresponding purpose, and the similarity between the two regions is measured at each position. The position of the most effective match is determined by the maximum or minimum value of the resulting measurements. Normalized cross correlation is a well-known technique for determining the degree of similarity between two regions.

6.1. Application of V-Regular rules in Edge Detection. Edge detection is an essential device in image processing, particularly for function detection and extraction, with the intention of figuring out factors in a virtual photograph in which the brightness of the image adjustments abruptly, and detecting discontinuities. Edge detection reduces the quantity of statistics in an image whilst keeping the structural functions for similarly image processing. Edge in a grayscale photograph is a neighborhood function that separates areas inside a community in which the grayscale is greater or much less uniform and has special values on both aspect of the threshold. It is tough to locate edges in a loud image due to the fact each the threshold and the noise include excessive frequency content, ensuing in a blurred and distorted image.

Here the addition rule is used as the input key Δ_x and the multiplication rule is used as the other input key Δ_y. The Fuzzy V-Regular Features from Accelerated Segment Test (FVRFAST) edge detector is a suitable edge detector, which extracts the edge of the image. The working rules of FVRFAST corner detection are

{(i-1, j) & (i, j-1) & (i, j) & (i, j+1)&(i+1, j)& (i+1, j+1)} are white and {(i-1,j-1) & (i-1, j+1) & (i+1, j-1)} are black, which are denoted as Δ_x (Figure 1)

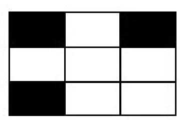


FIGURE 1. Δ_x.

{(i-1, j-1) &(i-1, j) & (i-1, j+1) &(i, j-1) & (i+1, j-1) &(i+1, j+1)} are black and {(i,j) & (i, j+1) & (i+1, j)} are white, which are denoted as Δ_y (Figure 2)

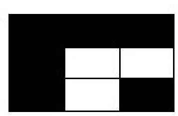


FIGURE 2. Δ_y.

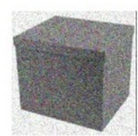
In the proposed F edge detection, a detailed configuration space is considered in order to provide a more efficient solution, instead of only considering a restricted configuration space. The F edge detector works as follows:

Algorithm 6.1. (1) *Using a good edge detector such as Canny, extract the edge contours from the input image.*

(2) *Fill in any small gaps in the edge contours. Mark the gap as a T-corner when it forms a T=1.96* $\widehat{\sigma}_s$ *-junction.*
(3) *Calculate the curvature of the edge contours at a large scale.*
(4) *The edge points are defined as the mini - maxi principles (Fuzzy Concept) of absolute curvature that are greater than a certain threshold value.*
(5) *To improve localization, track the corners through multiple lower scales.*

6.2. Experimental Study. The F edge detection algorithm was implemented in MATLAB Tool. Using the input keys Δx and Δy as specified in the algorithm, the performance of the F edge is evaluated with images of various file types. The processing for detection of edges in images summarized in Figure 3.

Original image



original image (with noise-salt & pepper)

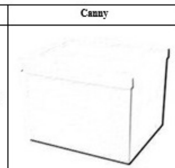
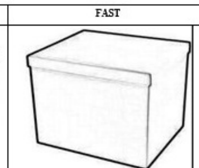
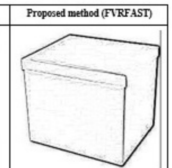
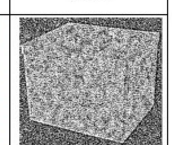
File Type	Canny	FAST	Proposed method (FVRFAST)
Without Noise			
Processing Time (in Sec's)	0.143	0.137	0.122
With Noise (Salt & Pepper)			
Processing Time	0.152	0.144	0.127

Figure 3. Edge detection with and without noise

7. Conclusions

The paper has discussed elaborately the properties on completely V-Regular on Semirings. This paper also has thrust upon the development of Fuzzy from Completely V-Regular on semirings. This idea has been applied to the edge detection in image processing. It may be applied to technical and electrical components to monitor their functions more effectively. The research may be elaborated in future with findings of more applications and detailed discussion.

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Data availability : Not applicable

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