Nuclear Engineering and Technology 55 (2023) 324–329

Contents lists available at ScienceDirect

Nuclear Engineering and Technology

journal homepage: www.elsevier.com/locate/net

Original Article

Using the Monte Carlo method to solve the half-space and slab albedo problems with Inönü and Anlı-Güngör strongly anisotropic scattering functions

Bahram R. Maleki [a, b]

[a] Department of Nuclear Engineering, Hacettepe University, Beytepe, Ankara, 06800, Turkey
[b] Nuclear Engineering Department, Sinop University, Sinop, 57000, Turkey

ARTICLE INFO

Article history:
Received 21 August 2022
Received in revised form
21 September 2022
Accepted 22 September 2022
Available online 27 September 2022

Keywords:
Strongly anisotropic scattering
Monte Carlo
Transmission factor
Albedo
Variance reduction

ABSTRACT

Different types of deterministic solution methods were used to solve neutron transport equations corresponding to half-space and slab albedo problems. In these types of solution methods, in addition to the error of the numerical solutions, the obtained results contain truncation and discretization errors. In the present work, a non-analog Monte Carlo method is provided to simulate the half-space and slab albedo problems with Inönü and Anlı-Güngör strongly anisotropic scattering functions. For each scattering function, the sampling method of the direction of the scattered neutrons is presented. The effects of different beams with different angular dependencies and the effects of different scattering parameters on the reflection probability are investigated using the developed Monte Carlo method. The validity of the Monte Carlo method is also confirmed through the comparison with the published data.

1. Introduction

The neutron motion and its interactions with nuclei of a medium are described by the neutron transport equations. Except for some simple cases such as a steady-state neutron transport problem in a purely absorbing one-dimensional medium, the neutron transport equations cannot be solved analytically. Monte Carlo (stochastic) and deterministic methods are extensively used to solve the neutron transport equations. In deterministic methods, the angular dependency is approximated either by spherical harmonic or by discrete ordinate methods; the energy, space, and time variables are also discretized. In these types of solution methods, in addition to the error of the numerical solutions, the obtained results contain truncation and discretization errors as well. In contrast, in the Monte Carlo simulation methods without using different types of approximations, the neutronic behavior of the system is simulated by using the corresponding Probability Distribution Functions (*PDF*) in which the neutron's motion, interaction type, the elapsed time during the interaction, energies of the generated particles and groups of the generated precursors are sampled randomly. In these types of problems, because of using random numbers the simulation results always include uncertainty. To reduce the uncertainty and to obtain more precise results, different types of variance reduction techniques are taken into consideration. The deterministic methods are faster but fall short in addressing the nuclear systems with complex geometries, strong anisotropy of neutron scattering, and complicated neutron energy spectrums [1–6].

The one-speed, steady-state, source-free, and azimuthally integrated neutron transport equation for a one-dimensional and non-multiplying homogeneous medium with strongly anisotropic scattering is expressed as follows:

$$\mu \frac{\partial \psi(z,\mu)}{\partial z} + \Sigma_t\ \psi(z,\mu) = \int_{-1}^{1} \Sigma_s(\mu' \to \mu)\ \psi(z,\mu')\ d\mu' \quad (1)$$

where Σ_t is the neutron total macroscopic cross section, μ' and μ represent the direction cosine of the neutron velocity with z-axis before and after the interaction, respectively, $\psi(z,\ \mu)$ is the azimuthally integrated angular flux, and $\Sigma_s(\mu' \to \mu)$ is the scattering kernel [2,6–9].

The integral of the scattering kernel gives us the total scattering cross section which is denoted by Σ_{s_0}.

E-mail address: bmaleki@sinop.edu.tr.

https://doi.org/10.1016/j.net.2022.09.024
1738-5733/

$$\int_{-1}^{1} \Sigma_s(\mu' \rightarrow \mu)\ d\mu' = \Sigma_{s_0} \tag{2}$$

For the given direction cosine before the scattering event (*i.e.* μ'), the neutron direction cosine after scattering (*i.e.* μ) can be sampled using the probability distribution function of:

$$Pdf(\mu' \rightarrow \mu) = Pdf(\mu) = \frac{\Sigma_s(\mu' \rightarrow \mu)}{\Sigma_{s_0}} \tag{3}$$

where $\int_{-1}^{1} Pdf(\mu' \rightarrow \mu)\ d\mu\ = 1.0$.

The ratio of the total scattering cross section to the total cross section is denoted by c and is known as average number of secondary neutrons per collision [10,11]. For the total cross section of equal of 1.0 cm^{-1} and using the given definitions the Eq.1 is rewritten as follows:

$$\mu \frac{\partial \psi(z,\mu)}{\partial z} + \psi(z,\mu) = c \int_{-1}^{1} Pdf(\mu' \rightarrow \mu)\ \psi(z,\mu')\ d\mu' \tag{4}$$

The equation above also known as transport equation in term of optical thickness, in such a condition z represents the optical thickness and the total cross section can take any value.

Different types of deterministic solution methods were presented to solve the neutron transport equations with strongly anisotropic scattering in spherical, half-space, and both reflected and bare slab geometries [7–9,12–20]. The effects of the strongly anisotropic scatterings on the spectrum of the time-eigenvalues for one-speed neutron transport in the spherical geometry were investigated by Sahni and Sjöstrand. The critical size of both reflected and bare slab reactors with strongly anisotropic scatterings was studied by using the P_N, modified U_N, ultraspherical polynomial $P_N^{(\lambda)}$ and T_N approximation methods [7,8,13,14]. The corresponding transport equation for the monoenergetic slab problem with strongly anisotropic scatterings was solved using both the F_N method and the variational technique and used to calculate the albedo and transmission factor [15,17]. Besides, for different types of anisotropic scatterings, the calculation of the half-space albedo problem was performed by using both F_N and modified F_N approximation methods [18,20]. The Singular Value Decomposition (SVD) method [21,22] was used to solve the transport equations of a half-space problems with Anlı-Güngör strongly anisotropic scattering kernel [23]. The critical thickness problem in the reflected system with tetra-anisotropic scattering was solved by Koklu and Ozer [28]. Rashidian Maleki presented a Monte Carlo method to solve the slab albedo problem with linearly anisotropic scattering, in which by sampling the scattering cosine and then rotating the coordinate systems [24] the direction of the scattered neutron was calculated.

In this study, a pure Monte Carlo simulation method is developed to solve both slab albedo and half-space albedo problems with Inönü and Anlı-Güngör scattering function. To confirm the validity of the Monte Carlo simulation, the simulation results are compared with the results of different deterministic solution methods.

2. Half-space and slab albedo problems

In half-space problem, an angular dependent neutron beam of the form $\psi(0,\mu) = \mu^P\ (P = 0, 1, 2, ..)$ incidents from the vacuum to a non-multiplying half-space $z \geq 0$ [18,19,23]. The reflection probability of the system (albedo) is obtained subsequent to the solution of the corresponding transport equation. Several scattering functions have been assumed and proposed to investigate the half-space albedo problem, where we only consider two more complicated scattering functions:

The first one is the Inönü scattering function, which is expressed as a combination of the linearly anisotropic scattering, forward scattering, and backward scattering [18,25]:

$$\Sigma_s(\mu' \rightarrow \mu) = \Sigma_{s_0}\left(\frac{a}{2}(1 + 3\ f_1\ \mu'\ \mu) + b\ \delta(\mu' - \mu) + d\ \delta(\mu' + \mu)\right) \tag{5}$$

where f_1 is a constant and can take both positive and negative values. a,b and d are the positive constants where $a + b + d = 1$.

The second one is the Anlı-Güngör scattering function [23]:

$$\Sigma_s(\mu' \rightarrow \mu) = \frac{\Sigma_{s_0}}{2}\left(1 + t\ P_1(\mu')\ P_1(\mu) + t^2\ P_2(\mu')\ P_2(\mu)\right) \tag{6}$$

where t is known as scattering parameter and takes values between − 1.0 and 1.0. $P_1(\mu) = \mu$ and $P_2(\mu) = \frac{1}{2}(3\mu^2 - 1)$ are the first and second order Legendre polynomials, respectively.

In the case of slab albedo problem, a non-multiplying slab medium of a thickness of $2a$ cm (extended from $- -a$ to a) is subjected to a monoenergetic neutron beam from the left side. There is a vacuum boundary condition on the right side as well. The aim of this problem is to calculate the albedo and transmission factor by solving the neutron transport equation. In this case, Inönü scattering function is assumed as a combination of the isotropic, forward, and backward scattering terms [16,17]:

$$\Sigma_s(\mu' \rightarrow \mu) = \Sigma_{s_0}\left(\frac{1-2k}{2} + (k+\gamma)\ \delta(\mu' - \mu) + (k-\gamma)\ \delta(\mu' + \mu)\right) \tag{7}$$

where k and γ are the constant values.

3. Monte Carlo algorithm

3.1. Particle tracking

To start the Monte Carlo simulation, the N_n number of incident neutrons of the weight of $w_0 = 1$ which enter into the system at $z_0 = z_{in}$ cm is taken into account. For the incident neutrons with an angular flux of $\psi(0,\mu) = \mu^P$, the incoming partial current of incident neutrons becomes equal to $1/(P+2)$. Hereby, using the $(P+2)\ \mu^{P+1}$ function as the probability distribution function, the cosine of the polar angle of the incident neutron (denoted by μ') direction is sampled as follows:

$$\mu' = \xi^{\frac{1}{P+2}} \tag{8}$$

where ξ represents a uniformly distributed random number between zero and one.

Since we deal with a one-dimensional problem, the azimuth angle is taken as free variable (system is rotationally invariant). Therefore, sampling the direction cosine is sufficient to simulate the neutron direction.

Due to the simulation time-cost, it is impossible to use a large number of histories in the Monte Carlo simulations. Killing a neutron due to leakage imposes a variance on the simulation results. To obtain more precise result with a small number of histories (e.g. 10^5 or 10^6 neutrons), the forced collision variance reduction technique is implemented during the simulation. In this variance reduction technique, first of all, the minimum distance to surface in the neutron direction (d_s) is calculated, then $e^{-\Sigma_t d_s}$ fraction of the neutron weight is allowed to leak from the system and the remaining weight $\left(w_{int} = (1 - e^{-\Sigma_t d_s})\ w_0\right)$ is forced to do a collision after traveling a path length of $d(\in[0, d_s])$. This method causes neutron to live longer and subsequently have more chance to score, that is, the forced collision technique increases sampling of collisions in specified regions.

$$d = -\frac{1}{\Sigma_t} ln\left[1 - \xi(1 - e^{-\Sigma_t d_s})\right] \tag{9}$$

The new position of the interacted neutron is calculated as:

$$z = z_0 + d \times \mu' \tag{10}$$

Killing a neutron due to absorption also imposes an additional variance on the simulation results. To minimize this imposed variance, implicit capture variance reduction technique is implemented as well. Therefore, Σ_a/Σ_t fraction of the $w_{\rm int}$ is killed due to absorption and the remaining weight undergoes a scattering event. Since, we deal with one-speed neutrons, the neutron energy after the scattering does not change. Therefore, only the neutron direction changes and should be sampled. For a neutron with an initial direction cosine of μ', the scattered neutron direction for different scattering functions is sampled as follows:

1. The Inönü scattering function: In this case, the probability distribution function to sample the direction cosine of the scattered neutron is in the form of:

$$Pdf(\mu) = \left(\frac{a}{2}(1 + 3\ f_1\ \mu'\ \mu) + b\ \delta(\mu' - \mu) + d\ \delta(\mu' + \mu)\right) \tag{11}$$

To sample the μ parameter, a random number ξ is generated. According to the generated random number magnitude and using the $a + b + d = 1$ condition, there are three different possibilities:

(a) If $\xi \le a$, the $Pdf(\mu)$ is taken as follows:

$$Pdf(\mu) = \frac{1}{2}(1 + 3\ f_1\ \mu'\ \mu) \tag{12}$$

By calculating the corresponding cumulative distribution function and using the inverse transform method the μ is the solution of the quadratic equation below:

$$(3f_1\mu')\mu^2 + 2\mu + (2 - 3f_1\mu' - 4\xi) = 0 \tag{13}$$

For different values of $\mu' \in [-1, 1]$ and $\xi \in [0, 1]$, one of the obtained values for μ always take a value between -1 and 1. It also should be noted that, for the cases with $f_1 = 0$, the μ is easily sampled as $2\xi - 1$.

(b) Else if $\xi > a$ and $\xi \le (a + b)$, the $Pdf(\mu)$ is taken equal to $\delta(\mu' - \mu)$. Subsequently μ becomes equal to μ', that is, neutron does not change its direction.
(c) Otherwise, $Pdf(\mu)$ is taken equal to $\delta(\mu' + \mu)$, and μ becomes equal to $-\mu'$, that is, the neutron's new direction is directly opposite to its old direction.

2. Anlı-Güngör scattering function: In this case, the probability distribution function takes the form below:

$$Pdf(\mu) = \frac{1}{2}\left(1 + t\ P_1(\mu')\ P_1(\mu) + t^2\ P_2(\mu')\ P_2(\mu)\right) \tag{14}$$

For the given t and μ' values, the μ is obtained by solving the following cubic equation:

$$\mu^3 + \frac{2\mu'}{t\left(3\mu'^2 - 1\right)}\mu^2 + \frac{1 - \frac{t^2}{4}\left(3\mu'^2 - 1\right)}{\frac{t^2}{4}\left(3\mu'^2 - 1\right)}\mu + \frac{1 - 2\xi - \frac{t\mu'}{2}}{\frac{t^2}{4}\left(3\mu'^2 - 1\right)} = 0.0 \tag{15}$$

The solution method of equation above was presented by Press et al. and Bose [27,29]. For different values of μ', t and ξ, one of the obtained values for μ always take a physically meaningfull value between -1 and 1.

To track the transport of the scattered neutron, z_0, w_0 and μ' values are updated to z, $(\Sigma_s/\Sigma_t)w_{int}$ and μ respectively.

Figure Of Merit (FOM) is defined as $1/(R^2T)$ where R and T represent relative error and computation time, respectively. FOM is used as a criterion in the performance analysis of Monte Carlo simulations, and the aim is to maximize the FOM value. Monitoring the particles with negligible weights takes a long simulation time. This, in turn, decreases the FOM. To overcome this undesired problem the Russian-Roulette method is taken into account. In the Russian-Roulette method, threshold and survival weights which are denoted by w_{rr} and w_{sur}, respectively, are selected in terms of the average weight of the incident neutrons. If the weight of the scattered neutron be less than the threshold weight ($w_0 < w_{rr}$) a uniformly distributed random number is chosen. If this random number be less than the $P_{sur}(= w_0/w_{sur})$ the particle with the new weight of $w_0(= w_{sur})$ is survived; otherwise, the neutron is killed and transport of the other neutrons is simulated. In this manuscript, the threshold and survival weights are set to 0.25 $w_{av_{inc}}$ and 0.50 $w_{av_{inc}}$, where the $w_{av_{inc}}$ denotes the average weight of the incident neutrons and is equal to unity.

The transport of each neutron is monitored until it is killed by the Russian-Roulette method.

3.2. Tallying

Reflection probability (Albedo) is denoted by α. The transmission factor for a slab problem is also denoted by τ. These quantities are tallied as follows [26].

$$\alpha = \frac{\sum_{i=1}^{N_{L_l}} w_{0_i} e^{-\Sigma_t d_{s_i}}}{N_n \times 1.0} \tag{16}$$

$$\tau = \frac{\sum_{j=1}^{N_{L_r}} w_{0_j} e^{-\Sigma_t d_{s_j}}}{N_n \times 1.0} \tag{17}$$

where i is the number of leakage events that occur at the left boundary, j also represents the number of leakage events that occur at the right boundary of the slab media, w_{0_i} and w_{0_j} represent the weight of neutrons leaking from the left and right boundaries, respectively.

3.3. Error estimation

For each incident neutron of the weight of unity, the total outgoing partial current at the left side gives us the albedo per incident neutron (α_i). Since the N_n number of incident neutrons is used in the Monte Carlo simulation, the estimated albedo can be expressed as follows.

$$\alpha = \frac{\sum_{i=1}^{N_n} \alpha_i}{N_n \times 1.0} = \frac{\alpha_1 + \alpha_2 + \cdots + \alpha_{N_n}}{N_n} = \bar{\alpha_i} \tag{18}$$

Table 1
Comparison of the albedo values obtained from Monte Carlo (MC) and $F_{N=7}$ deterministic method for P=0.0, d=0.0 and c=0.8.

f_1	Method	$b = 0.20$	$b = 0.40$	$b = 0.60$	$b = 0.80$
0.10	MC	$0.288231 \pm 2.92588E-4$	$0.244203 \pm 2.88785E-4$	$0.188536 \pm 2.74615E-4$	$0.112967 \pm 2.32996E-4$
	$F_{N=7}$	0.288155	0.244357	0.188624	0.113273
0.20	MC	$0.269151 \pm 2.90613E-4$	$0.226812 \pm 2.85155E-4$	$0.173496 \pm 2.68082E-4$	$0.101764 \pm 2.23188E-4$
	$F_{N=7}$	0.269071	0.226496	0.173011	0.102171
0.30	MC	$0.247879 \pm 2.87109E-4$	$0.207040 \pm 2.78580E-4$	$0.156097 \pm 2.58821E-4$	$0.090371 \pm 2.12856E-4$
	$F_{N=7}$	0.247976	0.206959	0.156188	0.0904984

It is evident that the estimated albedo is equal to the mean of the calculated albedos per incident neutron. Therefore, the corresponding error for the estimated system albedo becomes equal to the standard error of $\bar{\alpha}_i'$s, and is denoted by $\sigma_{\bar{\alpha}_i}$:

$$\sigma_{\bar{\alpha}_i} = \frac{\sqrt{\frac{1}{N_n-1}\sum_{i=1}^{N_n}\left(\alpha_i - \bar{\alpha}_i\right)^2}}{\sqrt{N_n}} \tag{19}$$

In this paper, the confidence interval is taken as $\alpha \pm (1 \times \sigma_{\bar{\alpha}_i})$, that is, with a probability of 68% the true albedo value is in this confidence interval. To estimate the corresponding error for the transmission factor a similar procedure is followed.

4. Results and discussions

In this section, the validity of the proposed Monte Carlo simulation is tested on different benchmark problems given in the literature. In this manuscript in order to make our results comparable with the results given in the literature, the total cross section is taken equal to 1 cm^{-1} [16,18,20,23]. In addition, the mean number of secondary neutrons per collision (c) is used as an input parameter to specify the required cross sections. It is worth noting that, the Monte Carlo simulation codes are written in FORTRAN 90, and in each simulation, the $1E+6$ number of incident neutrons is taken into consideration.

4.1. Case1: Half-space problem

The reflection probability of the half-space media, for different scattering functions, is calculated as followings:

4.1.1. İnönü strongly anisotropic scattering function

In this test case, for different input parameters, the resulted albedo values for a half-space problem are compared with the results of $F_{N=7}$ [19] deterministic method. In Table 1, the value of the d parameter is taken equal to zero, that is, the contribution of the backward scattering is decreased. It is seen that, for a fixed value of f_1, by increasing the b value the contribution of the forward scattering increases, and subsequently, the albedo value goes down. Also, for constant b values, any increase in the f_1 value causes the contribution of the forward scattering to increase, and then the albedo declines. It is clearly seen that, the results of $F_{N=7}$ method and Monte Carlo simulation method are in good agreement with each other.

The albedo values for the cases that b value is taken equal to zero are presented in Table 2. It is seen that for a fixed value of f_1, by increasing the d value, the backward scattering contribution is increased and then the albedo value goes up. As seen in Table, any increase in the f_1 value causes to decrease in the albedo.

4.1.2. Anlı-Güngör strongly anisotropic scattering functions

In this test case, the effect of change of scattering parameter and c value on the half-space reflection probability is investigated using the developed Monte Carlo method and presented in Tables 3 and 4. The results obtained are compared with the results of the Singular Value Decomposition (SVD) method [23].

As seen in the tables, for a constant c value, by increasing the scattering parameter the albedo goes down. This, in turn, is because of increasing the forward scattering contribution for more positive t values. It is also seen that the results of both methods are close to each other.

Table 2
The albedo values for P=0.0, b=0.0 and. c=0.8.

f_1	Method	$d = 0.20$	$d = 0.40$	$d = 0.60$	$d = 0.80$
0.10	MC	$0.365589 \pm 2.87871E-4$	$0.399820 \pm 2.80651E-4$	$0.431729 \pm 2.71089E-4$	$0.463167 \pm 2.58614E-4$
	$F_{N=7}$	0.360123	0.392586	0.423530	0.456254
0.20	MC	$0.352436 \pm 2.89626E-4$	$0.391777 \pm 2.83173E-4$	$0.427338 \pm 2.73446E-4$	$0.461188 \pm 2.59736E-4$
	$F_{N=7}$	0.341954	0.376424	0.410137	0.447274
0.30	MC	$0.339320 \pm 2.91670E-4$	$0.383869 \pm 2.85667E-4$	$0.422895 \pm 2.75265E-4$	$0.459447 \pm 2.60657E-4$
	$F_{N=7}$	0.321804	0.358635	0.395585	0.437754

Table 3
The albedo values for varying c and $-1.0 \le t \le -0.2$.

c	Method	-1.0	-0.60	-0.20
0.70	MC	$0.305108 \pm 2.27063E-4$	$0.286054 \pm 2.28276E-4$	$0.2667385 \pm 2.28754E-4$
	SVD	0.305346	0.286501	0.266829
0.80	MC	$0.393433 \pm 2.56004E-4$	$0.373899 \pm 2.59265E-4$	$0.353391 \pm 2.61832E-4$
	SVD	0.393338	0.373840	0.352979
0.90	MC	$0.526991 \pm 2.77537E-4$	$0.509055 \pm 2.83959E-4$	$0.489324 \pm 2.9036E-4$
	SVD	0.527150	0.509010	0.488967

Table 4
: The albedo values for varying c and $0.20 \leq t \leq 1.0$.

		t		
C	*Method*	0.20	0.60	1.0
0.70	*MC*	$0.2461012 \pm 2.27844E-4$	$0.2235172 \pm 2.25564E-4$	$0.1992413 \pm 2.21695E-4$
	SVD	0.245910	0.223414	0.199098
0.80	*MC*	0.330298 ± 2.63484	$0.305277 \pm 2.63564E-4$	$0.276995 \pm 2.62415E-4$
	SVD	0.330196	0.304949	0.276682
0.90	*MC*	$0.466033 \pm 2.95979E-4$	$0.439994 \pm 3.01066E-4$	$0.409927 \pm 3.05038E-4$
	SVD	0.466338	0.440319	0.409863

Table 5
The albedo for different degrees of P with $2a = 1.0\ cm$ and $c = 0.8$.

k	γ	$P = 0.0$		$P = 2.0$		$P = 5.0$		$P = 10.0$	
		MC	*Exact*	*MC*	*Exact*	*MC*	*Exact*	*MC*	*Exact*
0.0	0.0	0.280410	0.2801	0.247585	0.2471	0.231613	0.2316	0.222872	0.2228
0.375000	$-k$	0.397687	0.3940	0.370811	0.3669	0.356787	0.3529	0.348625	0.3444
	$-k/2$	0.348933	0.3454	0.321174	0.3175	0.306840	0.3034	0.298783	0.2951
	0.0	0.288221	0.2844	0.260666	0.2567	0.246814	0.2432	0.2394139	0.2354
	$k/2$	0.208068	0.2052	0.182684	0.1798	0.170841	0.1681	0.164432	0.1617
	$(3k)/4$	0.157438	0.1557	0.134499	0.1331	0.124768	0.1233	0.119567	0.1180
	k	0.096975	0.0970	0.079540	0.0793	0.072190	0.0722	0.068784	0.0685
0.446428	k	0.417683	0.4184	0.393444	0.3943	0.379947	0.3809	0.371657	0.3726
	$-k/2$	0.362818	0.3637	0.337675	0.3383	0.323876	0.3247	0.315509	0.3164
	0.0	0.291438	0.2921	0.265618	0.2666	0.252461	0.2535	0.244496	0.2458
	$k/2$	0.193095	0.1934	0.170235	0.1709	0.159475	0.1602	0.153444	0.1542
	$(3k)/4$	0.127760	0.1278	0.109750	0.1097	0.101360	0.1017	0.097090	0.0974
	k	0.045327	0.0450	0.035887	0.0358	0.032516	0.0323	0.030625	0.0306

4.2. Case2: Slab albedo problem

In this test case, using the scattering model presented in Eq.7, for incident neutrons with different inlet currents, the reflection and transmission probabilities are estimated by employing the proposed Monte Carlo method. Moreover, the effects of different types of scattering are also investigated. The albedo and transmission factor for different types of inlet currents (different P values) with $2a = 1.0\ cm$ and $c = 0.8$ are presented in Tables 5 and 6, respectively. The Monte Carlo results are compared with the results of an Exact named deterministic method [16,17]. As seen in the Tables, for the fixed values of k and γ, by increasing the order of P a decrease of albedo and an increase of the transmission factor is observed. Also, it is clearly seen that for cases with a constant k value, by increasing the γ value the forward scattering contribution to overall scattering goes up, then albedo and transmission factor experience a decrease and an increase, respectively.

It is also seen that the results of both methods are comparable with each other. It is important to note that the incident neutron is absorbed by the medium's nuclei with a probability of $1 - \alpha - \tau$.

5. Conclusion

It is a well-known fact that, except for the simple cases that a purely absorbing media containing one-speed neutrons, the neutron transport equation can not be solved analytically. To get informed about the reaction rate and other required parameters the angular flux which is the solution of the transport equation is required. Therefore, the transport equation is solved and simulated using the Monte Carlo and deterministic methods. Half-space and slab albedo problems have been investigated using different deterministic solution methods. In this study, the author presents a Monte Carlo algorithm to simulate these problems with two different strongly anisotropic scattering functions. The incident neutron's direction is sampled using the concept of inlet current. Moreover, the scattered neutron's direction is also sampled using

Table 6
The Transmission factor for different degrees of P with $2a = 1.0\ cm$ and $c = 0.8$.

k	γ	$P = 0.0$		$P = 2.0$		$P = 5.0$		$P = 10.0$	
		MC	*Exact*	*MC*	*Exact*	*MC*	*Exact*	*MC*	*Exact*
0.0	0.0	0.416090	0.4163	0.471847	0.4721	0.500662	0.5008	0.517363	0.5176
0.375000	$-k$	0.320227	0.3214	0.381554	0.3824	0.413011	0.4138	0.431205	0.4321
	$-k/2$	0.364947	0.3657	0.429880	0.4305	0.462698	0.4628	0.480839	0.4813
	0.0	0.421186	0.4221	0.489053	0.4903	0.522316	0.5230	0.540377	0.5413
	$k/2$	0.496034	0.4967	0.566155	0.5667	0.597720	0.5986	0.615632	0.6160
	$(3k)/4$	0.543589	0.5441	0.613523	0.6137	0.644009	0.6443	0.660525	0.6607
	k	0.600962	0.6010	0.668354	0.6684	0.696989	0.6968	0.711747	0.7117
0.446428	$-k$	0.309045	0.3090	0.372139	0.3722	0.404183	0.4041	0.422968	0.4226
	$-k/2$	0.358496	0.3584	0.426424	0.4262	0.459447	0.4593	0.478881	0.4781
	0.0	0.424179	0.4241	0.496495	0.4859	0.530024	0.5295	0.548821	0.5481
	$k/2$	0.516289	0.5159	0.589615	0.5896	0.622363	0.6219	0.639309	0.6391
	$(3k)/4$	0.577698	0.5774	0.649495	0.6497	0.679970	0.6799	0.695684	0.6956
	k	0.655270	0.6555	0.722588	0.7225	0.748849	0.7487	0.762213	0.7620

the corresponding scattering function. It is observed that the results of Monte Carlo and deterministic methods are in good agreement.

It is well-known fact that the Monte Carlo methods are widely used in the simulation of the systems with complex geometry and with complex energy and angular dependence of neutron behavior. However, deterministic methods may fall short in addressing such complex systems. Therefore, as future works, the presented method in this study can be extended to simulate the more complex nuclear systems with strongly anisotropic scattering behaviors.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

The author thanks Professor Mehmet Tombakoglu for his valuable suggestions and helpful discussions.

References

[1] J.J. Duderstadt, L.J. Hamilton, Nuclear Reactor Analysis, Wiley, 1976.
[2] W. Stacey, Nuclear reactor physics, comp. rev. and enl 2 (2007).
[3] M. Hanuš, Mathematical Modeling of Neutron Transport, PhD thesis, PhD thesis, University of West Bohemia, 2014.
[4] Y. Wu, Neutronics of Advanced Nuclear Systems, Springer, 2019.
[5] A.J. Hoffman, A Time-dependent Method of Characteristics Formulation with Time Derivative Propagation, PhD thesis, University of Michigan, 2013.
[6] A.K. Prinja, E.W. Larsen, General principles of neutron transport, in: Handbook of Nuclear Engineering, Springer, 2010, 427–542.
[7] A. Yılmazer, M. Tombakoglu, Unified treatment of the p (λ) n approximation to solve the reflected slab criticality problem with strong anisotropy, Kerntechnik 73 (5-6) (2008) 260–268.
[8] A. Yilmazer, Solution of one-speed neutron transport equation for strongly anisotropic scattering by tn approximation: slab criticality problem, Annals of Nuclear Energy 34 (9) (2007) 743–751.
[9] D. Sahni, N. Sjöstrand, Time eigenvalue spectrum for one-speed neutron transport in spheres with strong forward-backward scattering, Annals of Nuclear Energy 26 (4) (1999) 347–358.
[10] J. Wang, W.R. Martin, B.S. Collins, Code verification of mpact using ganapol critical rod benchmark, in: EPJ Web of Conferences, 247, EDP Sciences, 2021, 10027.
[11] N.G. Sjoestrand, Accurate critical slab calculations for various degrees of anisotropy and for different reflection coefficients, Kerntechnik 75 (1-2) (2010) 58–59.
[12] C. Tezcan, Third form of the transport equation for extremely anisotropic scattering kernel, Journal of quantitative spectroscopy and radiative transfer 55 (1) (1996) 33–40.
[13] C. Yildiz, Variation of the critical slab thickness with the degree of strongly anisotropic scattering in one-speed neutron transport theory, Annals of Nuclear Energy 25 (8) (1998) 529–540.
[14] H. Öztürk, The effect of strongly anisotropic scattering on the critical size of a slab in one-speed neutron transport theory: modified un method, Annals of nuclear energy 65 (2014) 24–29.
[15] C. Yildiz, Variation of the albedo and the transmission factor with forward and backward scattering in neutron transport theory-the fn method, Annals of Nuclear Energy 27 (9) (2000) 831–840.
[16] C. Yildiz, Calculation of the albedo and transmission in a slab with a specified isotropic scattering using the fn method, Journal of Quantitative Spectroscopy and Radiative Transfer 70 (1) (2001) 37–45.
[17] A.A. Hendi, A. Elghazaly, The solution of the neutron transport equation in a slab with anisotropic scattering, Journal of Quantitative Spectroscopy and Radiative Transfer 84 (3) (2004) 339–347.
[18] R. Tureci, Half-space albedo problem for İnönü, linear and quadratic anisotropic scattering, Nuclear Engineering and Technology 52 (4) (2020) 700–707.
[19] M. Koçmen, A. Teğmen, D. Türeci, M. Güleçyüz, R. Türeci, Albedo and constant source problems for extremely anisotropic scattering, Kerntechnik 78 (3) (2013) 245–249.
[20] A. Kaşkaş, C. Tezcan, The fn method for anisotropic scattering in neutron transport theory: the half-space problems, Journal of Quantitative Spectroscopy and Radiative Transfer 55 (1) (1996) 41–46.
[21] R.G. McClarren, Calculating time eigenvalues of the neutron transport equation with dynamic mode decomposition, Nuclear Science and Engineering 193 (8) (2019) 854–867.
[22] D.C. Sahni, R.G. Tureci, A.Z. Bozkir, Partial range completeness of case eigenfunctions and numerical solution of singular integral equations of particle transport problems, Journal of Computational and Theoretical Transport 49 (7) (2020) 349–367.
[23] A.Z. Bozkır, R.G. Türeci, D.C. Sahni, Half-space albedo problem for the anlıgüngör scattering function, Kerntechnik 87 (2) (2022) 237–248.
[24] A.F. Bielajew, Fundamentals of the Monte Carlo Method for Neutral and Charged Particle Transport, 1, The University of Michigan, 2001.
[25] E. İnönü, A theorem on anisotropic scattering, Transport theory and statistical physics 3 (2-3) (1973) 137–146.
[26] B. Rashidian Maleki, A non-analog Monte Carlo simulation method for slab albedo problem with linear-anisotropic scattering, ALKÜ Fen Bilimleri Dergisi 3 (1) (2021) 1–13.
[27] S.K. Bose, Numerical Methods of Mathematics Implemented in Fortran, Springer, 2019.
[28] H. Koklu, O. Ozer, Critical thickness problem for tetra-anisotropic scattering in the reflected reactor system, Pramana 95 (4) (2021) 1–11.
[29] W.H. Press, S.A. Teukolsky, W.T. Vetterling, B.P. Flannery, Numerical Recipes in Fortran 77, i, 2003.