Research Paper
J. Astron. Space Sci. 40(1), 29-33 (2023)
https://doi.org/10.5140/JASS.2023.40.1.29

JASS Journal of Astronomy and Space Sciences

Constant Acceleration in Fractal Structures with Fractal Dimension D = 2

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An unexplained acceleration on the order of 10^{-8} cm s^{-2}, which is close to cH, where c is the speed of light and H is the Hubble constant, is detected in gravitationally bound systems of different scales, from the solar system to clusters of galaxies. We found that any test body located inside a fractal structure with fractal dimension D = 2 experiences acceleration of the same order and confirmed the previous work that photons propagating through this structure decrease the frequency owing to gravitational redshift. The acceleration can be directed against the movement of the test body. The fractal distribution of the matter should be at scales of at least hundreds of megaparsecs to a few gigaparsecs for the existence of this acceleration.

Keywords: stars, cosmology, fractals—cosmology, galaxy distribution—astrophysical processes, non-standard theories of gravity—cosmology, Hubble constant

1. INTRODUCTION

The unexplained acceleration of the order of cH~$7\cdot10^{-8}$ cm s^{-2}, where c is the speed of light and H is the Hubble constant, is now observed for many objects and systems at various scales. The acceleration discussed has been termed as universal acceleration by many researchers.

The first use of similar acceleration was possibly the modified theory of Newtonian dynamics, which explained the flat rotational curves of spiral galaxies, proposed by Milgrom (1983a, b, c). Later, the similar scale deviations from Newtonian law of gravity were found in solar system for two Pioneer spacecrafts by Anderson et al. (2002) and several spacecraft flyby maneuvers near the Earth by Anderson et al. (2008).

The observations of stellar radial velocity dispersion in galactic globular clusters allow the detection of the unusual motion of stars in the outer regions of these stellar systems at the radii where the acceleration produced by globular clusters becomes comparable with the universal acceleration of 10^{-8} cm s^{-2}. The investigation of this effect was published, for example, by Scarpa et al. (2011).

Similar uncertainty, namely the flat dependence of stellar velocity dispersion on radius, was also found for elliptical galaxies (see Chae et al. 2020 as an example). The investigations of observed correlations between dynamical and baryonic masses in galaxies with different sizes and morphologies [radial acceleration relation (RAR)] were published by McGaugh et al. (2016), Chan & Del Popolo (2020), etc.

This short overview shows that the unknown acceleration

Received 01 FEB 2023 **Revised** 24 FEB 2023 **Accepted** 28 FEB 2023
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http://janss.kr pISSN: 2093-5587 eISSN: 2093-1409

is an observational fact; it is observed at scales ranging from several radii of our planet to those of a typical galaxy. If these accelerations are explained by one physical scenario, the scenario should affect moving bodies at all the above-mentioned scales, implying the possible scenario to be simple and universal. Many studies, starting from Milgrom (1983a, b, c), have discussed modifications of Newtonian gravitation. In this work, however, we have tried to indicate that the standard Newtonian law of gravity, together with some special distribution of matter (fractal distribution), might be sufficient to explain the observed excess accelerations.

In particular, we showed that the fractal structure with fractal dimension D = 2 exhibits an acceleration of the order of cH for all objects within or near the structure at scales of at least some hundreds of megaparsecs to a few gigaparsecs. In addition, we also attempted to construct the simplest model for acceleration in parallel with the simplified calculation of the gravitational redshift published by Baryshev (1981, 1994) and Baryshev et al. (1994).

2. ACCELERATION AND GRAVITATIONAL REDSHIFT IN STRUCTURES WITH FRACTAL DIMENSION D = 2

We determined the acceleration of the test body inside the fractal structure and showed the calculations of acceleration and gravitational redshift for the test body and photon, respectively. For the photon's gravitational redshift, we used the simplification of results published by Baryshev (1981, 1994) and Baryshev et al. (1994). The acceleration calculations are presented herein for the first time.

The test body is located near any massive body (for example, a fractal structure) of mass M. The acceleration experienced by radius R can be expressed as

$$\mathbf{a} = \frac{\mathbf{GM}}{\mathbf{R}^2} \tag{1}$$

A photon leaving the colossal body experiences a gravitational frequency shift

$$\mathbf{z} = \frac{\Delta \mathbf{v}}{\mathbf{v}} = \frac{\mathbf{GM}}{\mathbf{Rc}^2} \tag{2}$$

For a fractal structure with dimension D, the relationship between its mass and radius is

$$\mathbf{M}(\mathbf{R}) = \frac{4\pi}{3}\mathbf{r}_0^3\rho\left(\frac{\mathbf{R}}{\mathbf{r}_0}\right)^{\mathbf{D}}, \tag{3}$$

where ρ is the average density of the minimum sphere of radius r_0 where the fractal structure begins. This dependence of mass on radius in fractal structures was discussed by Wertz (1971), Baryshev (1981), and Pietronero (1987). According to Baryshev (1994)

$$\mathbf{r_0} = 10\,\mathrm{kpc}, \quad \boldsymbol{\rho} = 5.2 \cdot 10^{-24}\,\mathrm{g\,cm^{-3}} \tag{4}$$

It defines the mass near 3 × 10^{11} solar masses inside the sphere with 10 kpc radii, which is the mass and size of a typical galaxy. Assuming

$$\mathbf{C_f} = \frac{4\pi}{3}\mathbf{r}_0^3\rho\frac{1}{\mathbf{r}_0^{\mathbf{D}}} \tag{5}$$

or, using Eq. (4), for the case D = 2

$$\mathbf{C_f} = \frac{4\pi}{3}\mathbf{r}_0\rho = 0.6721\,\mathrm{g\,cm^{-2}} \tag{6}$$

that is

$$\mathbf{M(R)} = \mathbf{C_f R^2} \tag{7}$$

Thus, the acceleration of the test body and the gravitational photon frequency shift for a fractal of dimension D = 2 are

$$\mathbf{a}(\mathbf{R}) = \mathbf{GC_f} \tag{8}$$

$$\mathbf{z}(\mathbf{R}) = \frac{1}{\mathbf{c}^2}\mathbf{GC_f R} = \frac{H}{c}\mathbf{R}, \tag{9}$$

where the Hubble constant $\mathrm{H} = \frac{1}{\mathrm{c}}\mathbf{GC_f}$

Thus, the acceleration is constant and the photon frequency shift depends linearly on the radius. The linearity of gravitational redshift for a fractal dimension D = 2 was mentioned by Baryshev (1981, 1994) and Baryshev et al. (1994). The coefficient of proportionality in Eq. (9) can be compared to the Hubble constant. It is equal to

$$\frac{\mathbf{G}}{\mathbf{c^2}}\mathbf{C_f} = 7.426 \cdot 10^{-29}\,\mathrm{cm\,g^{-1}} \cdot \mathbf{C_f} \tag{10}$$

Using Eq. (4) and Eq. (6), this coefficient can be calculated as follows:

$$\frac{G}{c^2}C_f = 4.991 \cdot 10^{-29}\,\mathrm{cm}^{-1}$$

then, the Hubble constant

$$H = \frac{1}{c}GC_f = 46.2\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{11}$$

and the acceleration

$$\mathbf{a} = \mathbf{GC_f} = 4.49 \cdot 10^{-8}\,\mathrm{cm\,s^{-2}} \tag{12}$$

The average density of a fractal (in particular, a fractal of D = 2) structure is accepted to decrease afar any point. Therefore, the test body moves from a point with a higher to lower gravitational potential. Therefore, naturally, the acceleration of a test body will be directed against its movement, and similarly, a photon emitted in any direction will experience a redshift.

The situation should also be valid for any test body existing near or inside a fractal structure with a sufficiently large radius for the validity of Eq. (3). The scale of the fractal structure is not comparable to that of our solar system or galaxy; rather, it is at least several hundred megaparsec or up to a few gigaparsecs. Assuming the homogeneity of the universe at this scale, we can suppose that any moving test body can be located near or inside fractal structures.

Currently, indicating that the above-discussed acceleration by a fractal structure could be detected only in hyperbolic orbits, when a spacecraft changes its direction leaving the gravitational field of the Earth or Solar system, is necessary. The elliptical orbits in the solar system were not influenced by acceleration. An explanation of this phenomenon is beyond the scope of the present study.

Thus, the acceleration of a test body located near or inside a massive structure with fractal dimension D = 2 and the linear gravitational redshift of a photon near or inside this structure can be calculated. Shan et al. (2021) reviewed the recent determinations of the Hubble constant. Most recent determinations are in the range of 66–73 km s^{-1} Mpc^{-1}.

A better fit can be achieved by changing the initial conditions Eq. (4) or using a more realistic model. To change the value of Hubble constant calculated by Eq. (11), from 46.2 to 69 km s^{-1} Mpc^{-1}, changing the density in Eq. (4) from $\rho = 5.2 \cdot 10^{-24}$ to $7.8 \cdot 10^{-24}$ g cm^{-3} or the radius from r_0 = 10 to 14.9 kpc is sufficient. The corresponding value of acceleration in Eq. (12) is increased to $a = 6.70 \cdot 10^{-8}$ cm s^{-2}. Note that the calculated values of the Hubble constant Eq. (11) and acceleration Eq. (12) are linearly dependent on the accepted values of the density and radius in Eq. (4).

3. DISCUSSION AND CONCLUSION

Baryshev (1994) and Baryshev et al. (1994), using the same conditions Eq. (4) but a more sophisticated model, calculated the value of Hubble constant as H = 68.6 and 69 km s^{-1} Mpc^{-1}, respectively. These values approximate the values of the Hubble constant determined by many other authors in recent decades, although the large-scale distribution of matter in the universe is still an open question.

As discussed so far, do fractal structures with dimension D = 2 actually exist at several hundred megaparsec to a few gigaparsec scales? What is the observed distribution of matter on these scales?

In accordance with Jones et al. (2004), where some investigations of fractal dimension were reviewed, a fractal structure with dimensions close to D = 2 was detected in relatively small regions of the universe. Deeper surveys seemed to allow a possible increase in D with a transition to a homogeneous distribution of matter (D = 3).

Scrimgeour et al. (2012) used the WiggleZ survey for galaxies with $z \leq 1$. In this research a fractal distribution with fractal dimension below D = 2.97 in the range of 80–300 h^{-1} Mpc was excluded. This result is based on redshift measurements of more than 200 thousand galaxies by the WiggleZ survey.

Raikov et al. (2014) found D = 2.69 using a set of 822 extragalactic supernovae; Shirokov et al. (2017) found D = 2.55 ± 0.06 for the distribution of 384 gamma-ray bursts on scales of 2–6 gigaparsec. Raikov et al. (2014) and Shirokov et al. (2017) used a pairwise separation method (Raikov & Orlov 2011) to determine the fractal dimension. This method allows for the use of a significantly lower number of objects to obtain reliable results.

Teles et al. (2021) found the opposite trend, that is, a decrease in the fractal dimension at high redshifts. The fractal dimension appeared to be equal to D = 1.63 ± 0.20 for $z \leq 1$ and D = 0.52 ±0 .29 for $1 \leq z \leq 6$. A reduced UltraVISTA survey subsample containing 166,566 galaxies was used in this study.

Teles et al. (2022) extended the previous data sample by adding 750 k new galaxies with measured redshifts using SPLASH and COSMOS2015 catalogues. A decrease in the fractal dimension for a redshift $z > 1$ was confirmed. For $z < 1$ the fractal dimension was found to be D = 1.00 ± 0.12 for the SPLASH galaxies and D = 1.39 ± 0.19 COSMOS2015. For galaxies with $1 \leq z \leq 4$ the values $D = 0.83^{+0.36}_{-0.37}$ and $D = 0.54^{+0.27}_{-0.26}$ were obtained for SPLASH and COSMOS2015, respectively.

Notably, all the above-mentioned researchers tried to determine the distribution of visible matter. The distribution of the dark matter remains an open question. All the cited investigations were based on different methods, possibly leading to different results. Obviously, the fractal nature of the distant universe requires more attention.

The acceleration and gravitational redshift due to fractal structures with dimensions close to D = 2, even if they exist only in relatively small regions, not in the entire observed universe, should still significantly influence the global distributions of redshift and accelerations from our perspective of the universe.

Our simple model of fractal dimension D = 2, or with dimensions close to this value, shows that some accelerations and redshifts are observable because of the large-scale fractal structure of matter distribution. More sophisticated models are necessary in the future. That is why we used here only Newtonian approximation, while relativistic effects were not considered at all because constructing the simple fractal model of a matter distribution seems necessary.

All future theories, such as the next version of lambda cold dark matter (ΛCDM), or the theories listed by Lópes-Corredoira & Marmet (2022), should include these effects of fractal nature to construct a realistic picture of the universe.

ACKNOWLEDGMENTS

AD was funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP14972694).

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REFERENCES

Anderson JD, Campbell JK, Ekelund JE, Ellis J, Jordan JF, Anomalous orbital-energy changes observed during spacecraft flybys of Earth, Phys. Rev. Lett. 100, 091102 (2008). https://doi.org/10.1103/PhysRevLett.100.091102

Anderson JD, Laing PA, Lau EL, Liu AS, Nieto MM, et al., Study of the anomalous acceleration of Pioneer 10 and 11, Phys. Rev. D 65, 082004 (2002). https://doi.org/10.1103/PhysRevD.65.082004

Baryshev YV, Hierarchical structure of metagalaxy: problem review, Bull. Spec. Astrophys. Obs. 14, 24-43 (1981).

Baryshev YV, On the fractal nature of the large-scale structure of the universe, Astron. Astrophys. Trans. 5, 15-23 (1994). https://doi.org/10.1080/10556799408245848

Baryshev YV, Labini FS, Montuori M, Pietronero L, Facts and ideas in modern cosmology, Vistas Astron. 38, 419-500 (1994). https://doi.org/10.1016/0083-6656(94)90013-2

Chae KH, Bernardi M, Sánchez HD, Sheth RK, On the presence of a universal acceleration scale in elliptical galaxies, Astrophys. J. Lett. 903, L31 (2020). https://doi.org/10.3847/2041-8213/abc2d3

Chan MH, Del Popolo A, The radial acceleration relation in galaxy clusters, Mon. Not. R. Astron. Soc. 492, 5865-5869 (2020). https://doi.org/10.1093/mnras/staa225

Jones BJT, Martínez VJ, Saar E, Trimble V, Scaling laws in the distribution of galaxies, Rev. Mod. Phys. 76, 1211 (2004). https://doi.org/10.1103/RevModPhys.76.1211

López-Corredoira M, Marmet L, Alternative ideas in cosmology, Int. J. Mod. Phys. D 31, 2230014 (2022). https://doi.org/10.1142/S0218271822300142

McGaugh SS, Lelli F, Schombert JM, Radial acceleration relation in rotationally supported galaxies, Phys. Rev. Lett. 117, 201101 (2016). https://doi.org/10.1103/PhysRevLett.117.201101

Milgrom M, A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis, Astrophys. J. 270, 365-370 (1983a). https://doi.org/10.1086/161130

Milgrom M, A modification of the Newtonian dynamics: implications for galaxies, Astrophys. J. 270, 371-383 (1983b). https://doi.org/10.1086/161131

Milgrom M, A modification of the Newtonian dynamics: implications for galaxy systems, Astrophys. J. 270, 384-389 (1983c). https://doi.org/10.1086/161132

Pietronero L, The fractal structure of the universe: correlations of galaxies and clusters and the average mass density, Phys. A. Stat. Mech. Appl. 144, 257-284 (1987). https://doi.org/10.1016/0378-4371(87)90191-9

Raikov AA, Orlov VV, Method of pairwise separations and its astronomical applications, Mon. Not. R. Astron. Soc. 418, 2558-2564 (2011). https://doi.org/10.1111/j.1365-2966.2011.19645.x

Raikov AA, Orlov VV, Gerasim RV, Determination of the fractal dimensionality of large-scale structure with type Ia supernovae by the method of pairwise distances, Astrophysics 57, 287-295 (2014). https://doi.org/10.1007/s10511-014-9334-9

Scarpa R, Marconi G, Carraro G, Falomo R, Villanova S, Testing Newtonian gravity with distant globular clusters: NGC 1851 and NGC 1904, Astron. Astrophys. 525, A148 (2011). https://doi.org/10.1051/0004-6361/201014462

Scrimgeour MI, Davis T, Blake C, James JB, Poole GB, et al., The WiggleZ dark energy survey: the transition to large-scale cosmic homogeneity, Mon. Not. R. Astron. Soc. 425, 116-134 (2012). https://doi.org/10.1111/j.1365-2966.2012.21402.x

Shan P, Lemos P, Lahav O, A buyer's guide to the Hubble constant, Astron. Astrophys. Rev. 29, A9 (2021). https://doi.org/10.1007/s00159-021-00137-4

Shirokov SI, Raikov AA, Baryshev YV, Spatial distribution of gamma-ray burst sources, Astrophysics 60, 484-496 (2017). https://doi.org/10.1007/s10511-017-9500-y

Teles S, Lopes AR, Ribeiro MB, Fractal analysis of the UltraVISTA galaxy survey, Phys. Lett. B 813, 136034 (2021). https://doi.org/10.1016/j.physletb.2020.136034

Teles S, Lopes AR, Ribeiro MB, Galaxy distributions as fractal systems, Eur. Phys. J. C 82, 896 (2022). https://doi.org/10.1140/epjc/s10052-022-10866-0

Wertz JR, A Newtonian big-bang hierarchical cosmological model, Astrophys. J. 164, 227 (1971). https://doi.org/10.1086/150834