Commun. Korean Math. Soc. **38** (2023), No. 2, pp. 461–468
https://doi.org/10.4134/CKMS.c220109
pISSN: 1225-1763 / eISSN: 2234-3024

ON SHARP GENERAL COEFFICIENT ESTIMATES FOR ϑ-SPIRALLIKE FUNCTIONS

Şahsene Altinkaya and Tuğba Yavuz

Abstract. This paper attempts to investigate a new subfamily $\mathcal{ST}_{\vartheta,\sigma}(\alpha,\beta,\gamma,\mu)$ of spirallike functions endowed with Mittag-Leffler and Wright functions. The paper further investigates sharp coefficient bounds for functions that belong to this class.

1. Introduction

We consider the Mittag-Leffler function

$$E_\alpha(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+1)}$$

and its generalization [14]

$$E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)},\quad (\alpha,\beta,z\in\mathbb{C},\ \Re(\alpha)>0,\Re(\beta)>0)$$

where Γ indicates the Gamma function. Since then, the generalization of the function has been studied by different ways [3, 5, 6, 12].

The Wright function $W_{\alpha,\beta}(z)$ is expressed via a power series

$$W_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)n!}\quad (\alpha>-1,\beta,z\in\mathbb{C}).$$

We know that the series is absolutely convergent. If $\alpha=-1$, then the series is absolutely convergent in the open unit disc $\mathcal{E}=\{z\in\mathbb{C}:|z|<1\}$. If $\alpha>-1$, $W_{\alpha,\beta}$ is an entire function [15]. For more detailed features of the Wright function, we can refer to the series of papers [4, 8–11].

Recently, Bansal and Mehrez [2] defined the function

$$T_{\alpha,\beta}(\gamma;z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)\left[\gamma n!+(1-\gamma)\right]},\quad (0\le\gamma\le1)$$

Received April 22, 2022; Revised June 13, 2022; Accepted June 28, 2022.
2020 *Mathematics Subject Classification.* Primary 30C45, 33E12.
Key words and phrases. Wright function, Mittag-Leffler function, spirallike function, subordination, convolution.

that is, we can examine both Mittag-Leffler and Wright function

$$T_{\alpha,\beta}(0;z) = E_{\alpha,\beta}(z),$$
$$T_{\alpha,\beta}(1;z) = W_{\alpha,\beta}(z).$$

Let $\mathcal{A}$ denote the family of functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ analytic in $\mathcal{E}$ with the normalization $f(0) = 0 = f'(0) - 1$. Further, stand by $\mathcal{S}$ the subfamily of $\mathcal{A}$ involving univalent functions.

For analytic functions f_1 and f_2 in $\mathcal{E}$, we ensure that f_1 is subordinate to f_2, expressed by $f_1 \prec f_2$, for a Schwarz function

$$\mathbf{\Lambda}(z) = \sum_{n=1}^{\infty} \kappa_n z^n \quad \left(\mathbf{\Lambda}(0) = 0, |\mathbf{\Lambda}(z)| < 1\right),$$

analytic in $\mathcal{E}$ such that $f_1(z) = f_2(\mathbf{\Lambda}(z))$ $(z \in \mathcal{E})$. Further, the convolution is stated by

$$f_1(z) * f_2(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n = (f_1 * f_2)(z) \quad (z \in \mathcal{E}),$$

where $f_1(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $f_2(z) = z + \sum_{n=2}^{\infty} b_n z^n$.

Since the function $T_{\alpha,\beta}(\gamma;z)$ is not a member of $\mathcal{A}$, it must be normalized by [1]

$$\begin{aligned}\mathbb{T}_{\alpha,\beta,\gamma}(z) &= \Gamma(\beta) z T_{\alpha,\beta}(\gamma;z) \\ &= z + \textstyle\sum_{n=2}^{\infty} \frac{\Gamma(\beta)}{\Gamma(\alpha(n-1)+\beta)[\gamma(n-1)!+(1-\gamma)]} z^n.\end{aligned}$$

Throughout the paper, we consider the case real-valued α, β with $\alpha > 0, \beta > 0$ and $z \in \mathcal{E}$.

Now, for $f \in \mathcal{A}$, we arrive

$$\begin{aligned}\mathcal{T}_{\alpha,\beta,\gamma} f(z) &= \mathbb{T}_{\alpha,\beta,\gamma}(z) * f(z) \\ &= z + \sum_{n=2}^{\infty} \frac{\Gamma(\beta) a_n}{\Gamma(\alpha(n-1)+\beta)\left[\gamma(n-1)! + (1-\gamma)\right]} z^n.\end{aligned}$$

For $-\infty < t < \infty$ and $\vartheta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, the logarithmic ϑ-spiral curve is expressed by $w = w_0 \exp(-e^{-i\vartheta} t)$, where w_0 is a nonzero complex number. We must mention here that 0-spirals are radial half-lines. An analytic univalent function f is asserted ϑ-spirallike if its range is ϑ-spirallike. The family of ϑ-spirallike functions is indicated by $\mathcal{S}_\vartheta$. We can explain it analytically that a function $f \in \mathcal{A}$ belongs to the family $\mathcal{S}_\vartheta$ if and only if $\Re\left(e^{i\vartheta} \frac{zf'(z)}{f(z)}\right) > 0$ [13]. Libera [7] used this approach to ϑ-spirallike functions of order σ, asserted by $\mathcal{S}_\vartheta(\sigma)$ and satisfies

$$\Re\left(e^{i\vartheta} \frac{zf'(z)}{f(z)}\right) > \sigma \cos\vartheta.$$

Clearly, $\mathcal{S}_\vartheta(\sigma) \subset \mathcal{S}_\vartheta$. Further, the general coefficient bounds for functions in $\mathcal{S}_\vartheta(\sigma)$ was proved:

$$|a_n| \le \prod_{j=0}^{n-2}\left(\frac{\left|2\,(1-\sigma)\,e^{-i\beta}\cos\beta + j\right|}{j+1}\right) \quad \left(n \in \mathbb{N}\backslash\{1\}\,;\ \mathbb{N} := \{1,2,3,\ldots\}\right).$$

This result is sharp.

Definition. A function $f \in \mathcal{A}$ belongs to the family $\mathcal{ST}_{\vartheta,\sigma}(\alpha,\beta,\gamma,\mu)$ if

$$\Re\left(e^{i\vartheta}\frac{z\left(\mathcal{T}_{\alpha,\beta,\gamma}f(z)\right)'}{\mu z\left(\mathcal{T}_{\alpha,\beta,\gamma}f(z)\right)' + (1-\mu)\,\mathcal{T}_{\alpha,\beta,\gamma}f(z)}\right) > \sigma\cos\vartheta,$$

where $|\vartheta| < \frac{\pi}{2},\ 0 \le \mu < 1,\ 0 \le \sigma < 1,\ 0 \le \gamma \le 1$ and $z \in \mathcal{E}$.

2. Main results

In this part, we aim to arrive coefficient conditions and sharp bounds for functions in $\mathcal{ST}_{\vartheta,\sigma}(\alpha,\beta,\gamma,\mu)$.

Theorem 2.1. *Assume $\mathcal{T}_{\alpha,\beta,\gamma}f(z) \neq 0$ for $z \in \mathcal{E}\setminus\{0\}$. Then, f is in $\mathcal{ST}_{\vartheta,\sigma}(\alpha,\beta,\gamma,\mu)$ if and only if*

$$\sum_{n=2}^{\infty}\frac{\Gamma(\beta)\left[(n-1)(1+e^{2i\vartheta})(1-\sigma\mu+i(1-\mu)\tan\vartheta)+2(1-\sigma)e^{2i\vartheta}-(n-1)(1-e^{2i\vartheta})(1-\sigma)\mu\right]a_n z^n}{\Gamma(\alpha(n-1)+\beta)[\gamma(n-1)!+(1-\gamma)]} \neq 0.$$

Proof. Let us put

$$I(z) = \mathcal{T}_{\alpha,\beta,\gamma}f(z) = z + \sum_{n=2}^{\infty} K_n z^n, \quad z \in \mathcal{E},$$

where $K_n = \frac{\Gamma(\beta)a_n}{\Gamma(\alpha(n-1)+\beta)[\gamma(n-1)!+(1-\gamma)]}$ with $K_1 = 1$. Now, consider the function

$$p(z) = \frac{e^{i\vartheta}\sec\vartheta\left(\dfrac{zI'(z)}{\mu zI'(z) + (1-\mu)\,I(z)}\right) - i\tan\vartheta - \sigma}{1-\sigma}$$

is analytic, $p(0) = 1$ and $\Re p(z) > 0$, then $f \in \mathcal{ST}_{\vartheta,\sigma}(\alpha,\beta,\gamma,\mu)$ if and only if

$$p(z) \neq \frac{1-e^{2i\vartheta}}{1+e^{2i\vartheta}}$$

or, equivalently

$$\frac{e^{i\vartheta}\sec\vartheta zI'(z) - (\sigma + i\tan\vartheta)(\mu zI'(z) + (1-\mu)\,I(z))}{(1-\sigma)(\mu zI'(z) + (1-\mu)\,I(z))} \neq \frac{1-e^{2i\vartheta}}{1+e^{2i\vartheta}}.$$

Now, from the series expansion of $I(z)$, we arrive

$$\frac{\sum_{n=1}^{\infty}\left[(n-1)(1-\sigma\mu + i(1-\mu)\tan\vartheta) + (1-\sigma)\right]K_n z^n}{(1-\sigma)\sum_{n=1}^{\infty}(1+(n-1)\mu)K_n z^n} \neq \frac{1-e^{2i\vartheta}}{1+e^{2i\vartheta}},$$

which yields for $z \neq 0$

$$\sum_{n=2}^{\infty} \big[(n-1)(1+e^{2i\vartheta})(1-\sigma\mu+i(1-\mu)\tan\vartheta) +2(1-\sigma)e^{2i\vartheta}-(n-1)(1-e^{2i\vartheta})(1-\sigma)\mu\big] K_n z^n \neq 0. \qquad \square$$

Theorem 2.2. *Assume $\mathcal{T}_{\alpha,\beta,\gamma}f(z) \neq 0$ for $z \in \mathcal{E}\setminus\{0\}$. If f is in $\mathcal{ST}_{\vartheta,\sigma}(\alpha,\beta,\gamma,\mu)$, then*

$$(1) \quad |a_n| \leq \frac{\Gamma(\alpha(n-1)+\beta)[\gamma(n-1)!+(1-\gamma)]}{\Gamma(\beta)(n-1)!(1-\mu)^{n-1}} \prod_{j=0}^{n-2} \big|j(1-\mu)+2(1-\sigma)e^{i\vartheta}\cos\vartheta(1+\mu j)\big|,$$

where $n \in \mathbb{N}\setminus\{0\}$ with $a_1 = 1$. This result is sharp.

Proof. Since $f \in \mathcal{ST}_{\vartheta,\sigma}(\alpha,\beta,\gamma,\mu)$, we can use a Schwarz function $\Lambda(z)$ such that

$$e^{i\vartheta}\sec\vartheta\left(\frac{z\left(\mathcal{T}_{\alpha,\beta,\gamma}f(z)\right)'}{\mu z\left(\mathcal{T}_{\alpha,\beta,\gamma}f(z)\right)'+(1-\mu)\,\mathcal{T}_{\alpha,\beta,\gamma}f(z)}\right)-i\tan\vartheta=\frac{1+(1-2\sigma)\Lambda(z)}{1-\Lambda(z)}.$$

If we put the function $I(z)$, we find

$$\sum_{n=1}^{\infty}\big[ne^{i\vartheta}\sec\vartheta-(1+i\tan\vartheta)(1+(n-1)\mu)\big]K_n z^n = \left(\sum_{n=1}^{\infty}\big[ne^{i\vartheta}\sec\vartheta+(1-2\sigma-i\tan\vartheta)(1+(n-1)\mu)\big]K_n z^n\right)\Lambda(z).$$

Now, for $n \in \mathbb{N}$, we can write

$$(2) \quad \sum_{n=1}^{m}\big[ne^{i\vartheta}\sec\vartheta-(1+i\tan\vartheta)(1+(n-1)\mu)\big]K_n z^n+\sum_{n=m+1}^{\infty}b_n z^n = \left(\sum_{n=1}^{m-1}\big[ne^{i\vartheta}\sec\vartheta+(1-2\sigma-i\tan\vartheta)(1+(n-1)\mu)\big]K_n z^n\right)\Lambda(z).$$

For $m = 2,3,\ldots$, the LHS of (2) is convergent in $\mathcal{E}$. Since $|\Lambda(z)| < 1$, it is appealing to use Parseval's Theorem to get

$$\sum_{n=1}^{m-1}\big|ne^{i\vartheta}\sec\vartheta+(1-2\sigma-i\tan\vartheta)(1+(n-1)\mu)\big|^2|K_n|^2 \geq \sum_{n=2}^{m}\big|ne^{i\vartheta}\sec\vartheta-(1+i\tan\vartheta)(1+(n-1)\mu)\big|^2|K_n|^2$$

or

$$(3) \quad \sum_{n=1}^{m-1}4(1-\sigma)(1+(n-1)\mu)\,(n-\sigma(1+(n-1)\mu)\,|K_n|^2$$

$$\geq \frac{(m-1)^2(1-\mu)^2}{\cos^2\vartheta}\left|K_m\right|^2,$$

where $K_1 = 1$. Now, we claim that

$$|K_n| \leq \frac{1}{(n-1)!\,(1-\mu)^{n-1}}\prod_{j=0}^{n-2}\left|j(1-\mu)+2(1-\sigma)e^{i\vartheta}\cos\vartheta(1+\mu j)\right|. \tag{4}$$

For $n = 2$, we find from (3)

$$|K_2| \leq \frac{2(1-\sigma)\cos\vartheta}{(1-\mu)},$$

which is equivalent to (4). The equation (4) is found for larger n from (3) by the principle of the mathematical induction.

Fix $n, n \geq 3$ and for $k = 2, 3, \ldots, n-1$, assume the equation (1) holds. From (3), we arrive

$$|K_n|^2 \leq \frac{4(1-\sigma)\cos^2\vartheta}{(n-1)^2\,(1-\mu)^2}\left\{1-\sigma+\sum_{k=2}^{n-1}K(k,j,\sigma)\right\}, \tag{5}$$

where

$$K(k,j,\sigma) = \frac{(1+(k-1)\mu)(k-\sigma(k-1)\mu)}{((k-1)!\,(1-\mu)^{k-1})^2}\prod_{j=0}^{k-2}\left|j(1-\mu)+2(1-\sigma)e^{i\vartheta}\cos\vartheta(1+\mu j)\right|^2.$$

Now, we must indicate that the square of RHS of (4) is equal to RHS of (5), that is,

$$\prod_{j=0}^{k-2}\frac{\left|j(1-\mu)+2(1-\sigma)e^{i\vartheta}\cos\vartheta(1+\mu j)\right|^2}{((n-1)!\,(1-\mu)^{n-1})^2} = \frac{4(1-\sigma)\cos^2\vartheta}{(n-1)^2\,(1-\mu)^2}\left\{1-\sigma+\sum_{k=2}^{n-1}K(k,j,\sigma)\right\} \tag{6}$$

for $n = 3, 4, \ldots$. After further calculations, we indicate that (6) is true for $n = 3$ and prove the claim. Assume the equation (6) is valid for all k, $3 < k \leq (n-1)$. From (2) and (5), we find

$$\begin{aligned}|K_n|^2 &\leq \frac{4(1-\sigma)\cos^2\vartheta}{(n-1)^2\,(1-\mu)^2}\left\{1-\sigma+\sum_{k=2}^{n-2}K(k,j,\sigma)+K(n-1,j,\sigma)\right\}\\ &\leq \frac{4(1-\sigma)\cos^2\vartheta}{(n-1)^2\,(1-\mu)^2}\\ &\quad\times\left\{1-\sigma+\sum_{k=2}^{n-2}\frac{(1+(k-1)\mu)(k-\sigma(k-1)\mu)}{((k-1)!\,(1-\mu)^{k-1})^2}\right.\end{aligned}$$

$$\times \prod_{j=0}^{k-2} \left| j(1-\mu) + 2(1-\sigma)e^{i\vartheta}\cos\vartheta(1+\mu j)\right|^2$$

$$+ \frac{(1+(n-2)\mu)(n-1-\sigma(n-2)\mu)}{((n-2)!\,(1-\mu)^{n-2})^2}$$

$$\left.\times \prod_{j=0}^{n-3} \left| j(1-\mu) + 2(1-\sigma)e^{i\vartheta}\cos\vartheta(1+\mu j)\right|^2 \right\}$$

$$= \frac{\prod_{j=0}^{n-3} \left| j(1-\mu) + 2(1-\sigma)e^{i\vartheta}\cos\vartheta(1+\mu j)\right|^2}{((n-2)!\,(1-\mu)^{n-2})^2}$$

$$\left\{ \frac{(n-2)^2}{(n-1)^2} + \frac{4(1-\sigma)\cos^2\vartheta(1+(n-2)\mu)(n-1-\sigma(n-2)\mu)}{(n-1)^2\,(1-\mu)^2} \right\}$$

$$= \frac{\prod_{j=0}^{n-3} \left| j(1-\mu) + 2(1-\sigma)e^{i\vartheta}\cos\vartheta(1+\mu j)\right|^2}{((n-1)!\,(1-\mu)^{n-1})^2}$$

$$\times \left\{ (n-2)^2\,(1-\mu)^2 \right.$$

$$\left. +4(1-\sigma)\cos^2\vartheta(1+(n-2)\mu)(n-1-\sigma(n-2)\mu) \right\},$$

$$|K_n| \le \frac{1}{((n-1)!\,(1-\mu)^{n-1})^2} \prod_{j=0}^{n-2} \left| j(1-\mu) + 2(1-\sigma)e^{i\vartheta}\cos\vartheta(1+\mu j)\right|^2 .$$

Since

$$K_n = \frac{\Gamma(\beta)}{\Gamma(\alpha(n-1)+\beta)\,[\gamma(n-1)! + (1-\gamma)]} a_n \quad (K_1 = 1),$$

we obtain the desired result.

To show the estimate is sharp, we consider following equality

$$\mathcal{T}_{\alpha,\beta,\gamma} f(z) = \frac{z}{(1+Kz)^{\frac{2(\sigma-1)e^{-i\vartheta}\cos\vartheta}{K}}}$$

where $K = (1-\mu) - 2\mu(1-\sigma)e^{-i\vartheta}\cos\vartheta$. □

3. Concluding remarks

In the organized paper, we establish a new family $\mathcal{ST}_{\vartheta,\sigma}(\alpha,\beta,\gamma,\mu)$ of spirallike functions. We derive sharp estimates for functions in this family.

Remark 3.1. If we put $\gamma = 0$, we get the function $E_{\alpha,\beta}(z)$ and establish the following subfamily.

A function $f \in \mathcal{A}$ is in $E_{\vartheta,\sigma}(\alpha,\beta,\mu)$ if

$$\Re\left(e^{i\vartheta}\frac{z\left(\mathcal{E}_{\alpha,\beta,\gamma}f(z)\right)'}{\mu z\left(\mathcal{E}_{\alpha,\beta,\gamma}f(z)\right)'+(1-\mu)\,\mathcal{E}_{\alpha,\beta,\gamma}f(z)}\right)>\sigma\cos\vartheta,$$

$$\left(|\vartheta|<\tfrac{\pi}{2},\ 0\le\sigma<1, 0\le\mu<1,\ z\in\mathcal{E}\right)$$

where $\mathcal{E}_{\alpha,\beta}f(z)=z+\sum_{n=2}^{\infty}\frac{\Gamma(\beta)a_n}{\Gamma(\alpha(n-1)+\beta)}z^n$.

Remark 3.2. If we put $\gamma=1$, we get the Wright function $W_{\alpha,\beta}(z)$ and establish the following subfamily.

A function $f \in \mathcal{A}$ is in $W_{\vartheta,\sigma}(\alpha,\beta,\mu)$ if

$$\Re\left(e^{i\vartheta}\frac{z\left(\mathcal{W}_{\alpha,\beta,\gamma}f(z)\right)'}{\mu z\left(\mathcal{W}_{\alpha,\beta,\gamma}f(z)\right)'+(1-\mu)\,\mathcal{W}_{\alpha,\beta,\gamma}f(z)}\right)>\sigma\cos\vartheta,$$

$$\left(|\vartheta|<\tfrac{\pi}{2},\ 0\le\sigma<1, 0\le\mu<1,\ z\in\mathcal{E}\right)$$

where $\mathcal{W}_{\alpha,\beta}f(z)=z+\sum_{n=2}^{\infty}\frac{\Gamma(\beta)a_n}{\Gamma(\alpha(n-1)+\beta)(n-1)!}z^n$.

The related outcomes of Remarks 3.1 and 3.2 are left, inviting scope for further research.

Finally, the approach presented here can be applied to introduce new subfamilies of spirallike functions involving various special functions.

References

[1] O. Ahuja, S. Kumar, and A. Çetinkaya, *Normalized multivalent functions connected with generalized Mittag-Leffler functions*, Acta Univ. Apulensis Math. Inform. No. 67 (2021), 111–123. https://doi.org/10.17114/j.aua

[2] D. Bansal and K. Mehrez, *On a new class of functions related with Mittag-Leffler and Wright functions and their properties*, Commun. Korean Math. Soc. **35** (2020), no. 4, 1123–1132. https://doi.org/10.4134/CKMS.c200022

[3] G. A. Dorrego and R. A. Cerutti, *The k-Mittag-Leffler function*, Int. J. Contemp. Math. Sci. **7** (2012), no. 13-16, 705–716.

[4] Lj. Gajić and B. Stanković, *Some properties of Wright's function*, Publ. Inst. Math. (Beograd) (N.S.) **20(34)** (1976), 91–98.

[5] K. S. Gehlot, *The p-k Mittag-Leffler function*, Palest. J. Math. **7** (2018), no. 2, 628–632.

[6] P. Humbert, *Quelques résultats relatifs à la fonction de Mittag-Leffler*, C. R. Acad. Sci. Paris **236** (1953), 1467–1468.

[7] R. J. Libera, *Univalent α-spiral functions*, Canadian J. Math. **19** (1967), 449–456. https://doi.org/10.4153/CJM-1967-038-0

[8] Y. Luchko, *The Wright function and its applications*, in Handbook of fractional calculus with applications. Vol. 1, 241–268, De Gruyter, Berlin, 2019.

[9] F. Mainardi and A. Consiglio, *The Wright functions of the second kind in Mathematical Physics*, Mathematics **8** (2020), no. 6, 1–26.

[10] N. Mustafa, *Univalence of certain integral operators involving normalized Wright functions*, Commun. Fac. Sci. Univ. Ank. Sér. A1 Math. Stat. **66** (2017), no. 1, 19–28. https://doi.org/10.1501/Commua1_0000000771

[11] J. K. Prajapat, *Certain geometric properties of the Wright function*, Integral Transforms Spec. Funct. **26** (2015), no. 3, 203–212. https://doi.org/10.1080/10652469.2014.983502

[12] A. K. Shukla and J. C. Prajapati, *On a generalization of Mittag-Leffler function and its properties*, J. Math. Anal. Appl. **336** (2007), no. 2, 797–811. https://doi.org/10.1016/j.jmaa.2007.03.018

[13] L. Spacek, *Contribution a la theorie des fonctions univalentes*, Casopis Pro Pestovani Matematiky a Fysiky **62** (1933), 12–19.

[14] A. Wiman, *Über den Fundamentalsatz in der Teorie der Funktionen* $E^a(x)$, Acta Math. **29** (1905), no. 1, 191–201. https://doi.org/10.1007/BF02403202

[15] E. M. Wright, *On the Coefficients of Power Series Having Exponential Singularities*, J. London Math. Soc. **8** (1933), no. 1, 71–79. https://doi.org/10.1112/jlms/s1-8.1.71

Şahsene Altinkaya
Department of Mathematics
Beykent University
34500 Istanbul, Turkey
Email address: sahsenealtinkaya@beykent.edu.tr

Tuğba Yavuz
Department of Mathematics
Beykent University
34500 Istanbul, Turkey
Email address: tugbayavuz@beykent.edu.tr