J. Appl. & Pure Math. Vol. 5(2023), No. 1 - 2, pp. 109 - 119
https://doi.org/10.23091/japm.2023.109

A STUDY ON THE TELEOLOGY OF MATHEMATICS EDUCATION IN THE LIGHT OF KANT'S EPISTEMOLOGY

CHUNGHYUN YU

Abstract. As for the practical purpose of mathematics education, the extrinsic purpose is emphasized. As an alternative to this, a discussion on mathematics education as a character education is urgently requested. It can be said that the main purpose of learning mathematics is to have a form of life that values the form and structure of mathematics. The epistemological basis of such an idea can be seen as based on Kant's philosophy. Kant's epistemology can provide one answer to the question of the intrinsic purpose of mathematics education.

AMS Mathematics Subject Classification : 97D99.
Key words and phrases : Kant's epistemology, purpose of mathematics education, intrinsic purpose, character education.

1. Introduction

The voices of criticism about what education is for today and who it is for are rising. Although alternatives and countermeasures are being put forward for this criticism, the core of the criticism is that the current education is focused on entrance exam education and is neglecting character education(see [20]). These criticisms generally seem to be getting widespread appeal. Even in mathematics education, such criticism cannot be avoided.

Perhaps one of the most common questions students ask math teachers is, " Why should I study mathematics?" One of the most common answers is probably that in the reality of entrance exam education, you should study mathematics now in order to get good grades and pass the college entrance exam. However, while this answer at first glance seems to have resolved the question, yet again follow-up questions are posed again, such as why do you want to go to college? It is also answered that mathematics, which is not desperately needed in real life but is difficult, is an important subject that must be learned in order to reach the goal of living comfortably while enjoying social stability and economic

Received January 13, 2023. Revised February 27, 2023. Accepted March 23, 2023.

prosperity. These answers are doubts about mathematics education itself, and it can be said that they come from interpreting the value of mathematics subject in light of external purposes, especially one's own purpose of gaining social wealth and honor.

Then, what are the goals and objectives pursued by the current mathematics education? Looking at the purpose of mathematics education presented in the Korean curriculum, the personal aspect of character cultivation and job preparation and the social aspect of realizing the ideal of national development and humanity's common prosperity are presented together. For this purpose, the goal of mathematics education for each class also pursues the healthy and balanced growth of the body and mind through mathematics study, but the external purpose is also emphasized(see [13]). According to Moon-gyu Lim, Japan also emphasizes the application of mathematics to daily life in mathematics education and values interest and pleasure in mathematics education, but pursues the same purpose of mathematics education as Korea(see [17]). In terms of the purpose of mathematics education, Western curriculum emphasizes extrinsic and practical purposes, while intrinsic objectives are not sufficiently discussed(see [12]). So this phenomenon can be said to be a global trend . According to Seongmo Jang, in the study of Greek philosophy from the Pythagoreans to Plato in the West and the Western educational tradition of liberal education from then until modern times, the inherent purpose of education, including mathematics, was emphasized(see [20]).

As such, until the modern age, mathematics as a science was not a subject taught mainly for practical purposes. Of course, this aspect cannot be completely ruled out, but more than that has been considered more important in relation to mathematics education. Mathematics was a traditional liberal arts subject in the West and was an important subject as a character education as it belonged to the seven free subjects. It is necessary to find possible ways to enhance the value of mathematics education as a character education in the mathematical educational reality where such external purposes take precedence. It is clear that the practical aspect of the purpose has existed in mathematics education, and that the instrumental role also has an important function. However, it is also an undeniable fact that mathematicians have valued the study of mathematics itself and pursued it while maintaining its internal consistency since the history of mathematics education has progressed due to this. Although the current global trend seems to emphasize the external purpose and practical aspect, the value of the intrinsic purpose of mathematics education, which has been traditionally pursued behind it, should not be neglected.

Kant's philosophy can also provide an epistemological explanation for the purpose of mathematics education. Since Kant's epistemology is the purpose of mathematics education rather than the teaching method or student-centered initiative that often forms the surface layer of mathematics education, it can provide an opportunity to reflect on the current mathematics education from a more in-depth perspective. Regarding the form of life, one possible alternative can be found. It can be said that the advantage of learning mathematics is not that it helps to solve the problems encountered in life, but that it is a form of life that values the form and structure of knowledge through the form of mathematics. . This is to understand in what sense the form of life experienced through the form or structure of knowledge of mathematics is valuable in relation to the purpose of mathematics education. Ever since mathematics began in ancient Greece, questions and explorations of reality have continued. Beyond the world of sensory experience with the pursuit of truth, the search for reality as a transcendental assumption led to Kant's epistemology, insight into the nature of mathematical knowledge, and introduction of it to metaphysical questions to establish a new position. In that the idea of the structure of knowledge or the form of life is based on Kant's epistemology, Kant's epistemology can give implications for the purpose of mathematics education. Reflecting on current mathematics education in light of Kant's epistemology can seem very different from the reason for receiving mathematics education from an external point of view. It is necessary to discuss the intrinsic purpose of mathematics education in depth. For this purpose, it is meaningful to consider the teleology of mathematics education in Kant's epistemology.

2. Theoretical Background of Kant's epistemology

Kant's epistemology is very comprehensive and extensive, spanning the realms of nature, human beings, and religion. It can be said that it is impossible to summarize his epistemology in one or two concepts. However, despite these limitations, the basis of Kant's epistemology can be said to be an epistemological methodology derived from mathematical insight. It is not only the basis of Kant's philosophy, but also the beginning of understanding the whole structure of Kant's thought.

Kant's epistemology originated from a mathematical methodology in which the cognizing subject puts what he thought before into the object through reason, and examines whether or not what he thought rationally was justified based on the result. It is to think of laws and orders that can explain empirical facts unifiedly through reason. These laws and orders are expressed in mathematics in the form of universal propositions. However, Kant does not just stay in the empirical facts as in natural science, but goes beyond that and constructs a law that should explain the same kind of facts in a unified way. And by putting it back into nature and examining its legitimacy, the methodology was expanded to epistemology. If a law is wrong, it may be corrected and led to the discovery of a more certain law. In this way, Kant's epistemology is deeply related not only to exploring the way natural phenomena exist, but also to further exploring their existence.

According to S. E. Stumpf, Kant raises two problems. The first problem is that, as the mechanistic method of the natural sciences is applied to the realm of all realities, including human beings, there is a danger that the moral realm of freedom and reality will be incorporated into the mechanistic universe of the natural sciences. The second problem is that Kant asks how to explain truth, that is, mathematical knowledge, which is an a priori synthetic judgment, how to justify mathematical knowledge, which is its fundamental principle, and what makes mathematical knowledge possible(see [21]). For Kant, these two problems are closely related. the fundamental problem of Kant's philosophy is how to reconcile the two contradictory interpretations of man and nature: the claim that all events are products of necessity and the different claim that there is freedom in human action. Kant insights that mathematical knowledge is similar in principle to metaphysical knowledge, and therefore the justification or explanation of mathematical thought is equally applicable to the justification or explanation of metaphysical thought concerning freedom and reality(see [23]).

According to Kant, all knowledge begins with intuition. Experiential intuition is related to the impression formed by the a priori form triggering the sensibility, and formally it is related to the transcendental form of intuition added to the impression, that is, space and time. The intellect thinks about the phenomenon as an object, and by forming a relationship between the phenomena, the intellect forms an orderly law. Kant calls this relation a law of nature. Natural laws are no different from the essential epistemological laws of the understanding in that the natural order is presented according to rules grounded in the a priori nature of the understanding itself. This does not mean that the understanding derives universal laws from nature beyond experience, but that the understanding describes universal laws in nature. Kant not only establishes cognition, but goes further and precisely defines the limits of cognition. Perception is limited only to what is given to intuition, that is, phenomena, but human reason is not satisfied with experience. Reason always has as an a priori condition something given beyond experience, that is, an idea. Kant's philosophy tried to harmonize academic and metaphysical demands by contemplating the possibility behind it while establishing the certainty of mathematical cognition. It is a well-known fact that Kant's philosophy has been called the equivalent of the Copernican turn in the history of philosophy. Kant's philosophy, which recognizes the purpose and reality at the root of natural laws and is based on mathematical recognition, emphasizes the harmony and unity of mathematical perception and ideology(see [3]). It is a well-known fact that Kant's philosophy has been called the equivalent of the Copernican turn in the history of philosophy(see [7]).

3. Discussion on Kant's Epistemology and Mathematics

Empiricism and rationalism could not fully explain mathematical knowledge. Kant's epistemology focuses on revealing the nature of mathematical knowledge that can be regarded as certain knowledge, that is, the nature of knowledge obtained from mathematics(see [1, 6]). Kant presents mathematics as a model of synthetic a priori judgment. Kant's representative book, Critique of Pure Reason, is Kant's representative work written to deal with Kant's main question, 'How is synthetic a priori judgment possible?' As Kant said, "Form without content is empty, and content without form is blind," mathematical knowledge is established by the unity of content and form. It is necessary to examine the process of cognition suggested by Kant in the Critique of Pure Reason.

First, Kant establishes the transcendental form of knowledge. The transcendental forms of cognition are space and time, which are forms of intuition, and categories, which are forms of understanding, and ideas, which are forms of reason. Pure reason no longer contains anything other than an a priori form of cognition, and its activity consists in synthesis. In the form of intuition, experience is synthesized into perception, that is, empirical intuition. This is called the synthesis of perceptions. However, cognition cannot be established by empirical intuition itself. The understanding already contains certain forms of thought before all experience, and such forms are called categories. When the understanding applies the categories to the forms of intuition, space and time, it can form synthetic a priori judgments. This transcendental synthetic judgment expresses the form and relationship of the object and the condition of the object's existence. Kant presents mathematics as a model of synthetic a priori judgment. According to Kant, cognition is divided into empirical cognition based on experience and transcendental cognition independent of experience. If empirical cognition cannot have necessity and universality, knowledge derived from it cannot be called true knowledge. Unlike empirical cognition, transcendental cognition has inevitability in its nature.

On the other hand, Kant divided judgment into analytic judgment and synthetic judgment. Analytical judgment can have necessity but cannot expand cognition. On the other hand, synthetic judgment can expand cognition in that it adds a concept that is not included in the subject concept to the predicate concept. Kant's idea is that true academic knowledge must be a priori synthetic judgment in that strict certainty is required and the knowledge must be gradually increased. A transcendental synthetic judgment is a synthetic judgment in which the concept of the predicate is not included in the concept of the subject, and at the same time it is an a priori judgment that is logically unrelated to the judgment describing the sensory experience. Kant is capable of synthetic transcendental judgments, and presents mathematics as a typical example of synthetic transcendental judgments. According to Kant, all mathematical knowledge is an a priori synthetic judgment. Since mathematics is conceptually a priori judgment, the certainty and universality of recognition are guaranteed, and at the same time, it is possible to expand and grow mathematical knowledge because it is a comprehensive judgment. Arithmetic and geometrical knowledge are presented as examples of this a priori synthetic judgment.

If mathematical knowledge is an empirical judgment, then mathematics becomes nothing more than unreliable. Mathematics is necessarily a priori synthetic judgment. ant explains that it is a priori that makes mathematics possible as a solid science. Transcendentality is a property due to the transcendental form of time and space. A priori form is at the root of all mathematical knowledge.

Geometry is based on the transcendental form of space, and arithmetic is based on the transcendental form of time. Such space and time are the basis for the a priori nature of mathematics(see [5]). This tells us that mathematics is possible as a transcendental synthetic judgment due to the transcendental form of space and time. The transcendental form of time and space is called transcendental intuition in that it is the source of mathematical cognition and the transcendental condition of perception. The existence of transcendental intuition is an indispensable condition for the establishment of a transcendental synthetic judgment, and if transcendental intuition is possible, it is inevitable to think that the a priori form is already contained in the given object. Human perception is intuition only through the transcendental form of space and time, and human intuitive representation is established by the transcendental form of space and time. Therefore, Kant's argument is that mathematical knowledge is also based on an a priori form in that it is an a priori synthetic judgment.

According to Kant, space and time, the epistemological sources of mathematical knowledge, have empirical reality. The fact that space and time have empirical reality can be interpreted as meaning that space and time participate as essential elements in empirical cognition. This is a basic premise of Kant's epistemology and is already included in Kant's statement that intuition must presuppose the existence of an object. For example, in terms of empirical reality, to say that geometry takes space as its object means that geometry deals with various shapes as a space conceptually defined by the understanding, not space as a fundamental representation(see [5]). From the point of view of empirical reality, the way mathematics proceeds should be interpreted as meaning that it is not only the conceptual cognition of the understanding but also the combination of perception and understanding. Mathematical reasoning also requires 'intuition', not strictly logical reasoning for Kant. Intuition here means transcendental intuition, not empirical intuition(see [10]). In Kant's view, the necessity and composability of all mathematical principles are based on the necessity of a priori intuition of space and time(see [19]). In the end, a transcendental synthetic proposition is possible only when the object is given a priori in intuition. Mathematics, as an a priori synthetic judgment, relies on a certain 'interpretation system' that allows mathematical objects to be grasped as more than their original concepts(see[24]). However, Kant uses the term 'pure intuition' to express his position that this system of interpretation is logically impossible to grasp completely in that it has transcendent ideality in its nature.

4. Discussion on the purpose of mathematics education based on Kant's epistemology

Kant believed that mathematics not only exists as pure mathematics but also applies to nature. According to Kant, mathematics occupies an important part in educating human beings(see[22]). We will discuss the implications for the purpose of mathematics education based on Kant's epistemology.

Kant's epistemology explains cognition by dividing it into three levels: perception, intellect, and reason. Kant's three levels of cognition are all composed of a combination of content and form, and as these three represent levels, the cognition of the upper level includes the meaning that the cognition of the lower level takes the result of cognition as its content. At the perceptual level of cognition, time and space produce intuition by applying form to the content of sensory matter. Here content can be said to be a property of an object as sensory matter, but time and space are subjective conditions of perception.

At the level of the understanding, concepts are produced by applying the form of categories to the content of intuition, and the concepts are used to judge concrete phenomena. Here, intuition is the content of the understanding obtained as a result of applying the subjective conditions of perception, such as time and space, to sensory matter, whereas category is the subjective form of the understanding that must be logically assumed to produce the result of concept. In order for a mathematical concept to be established, it is formed through primary integration through the form of intuition, but it can never become a concept unless it is reintegrated through categories. In other words, the concept of number and counting should be regarded as belonging to the subjective condition of cognition, and it can be explained that it is made into a unified concept by the subjective condition(see[15]). The rational level of cognition produces some result by applying the form of the idea to the content of the concept. According to Kant, reason is related to the concept and judgment of the understanding (B363)(see[8, 9]). Reason is defined as the highest cognitive ability that gives the highest unity to the various cognitions of the understanding and at the same time imposes upon itself problems that transcend one's abilities (B355)(see[8, 9]).

Then, how does the idea relate to mathematical concepts? It can be seen that the relation of ideas to concepts is already implied by the application of time and space to sensory matter and the application of categories to intuition. Time, space, and categories belong before empirical cognition, and they have transcendental characteristics. That it has transcendental characteristics means that it excludes all distinctions, including the distinction between subject and object, which are established along with cognition. Nevertheless, if all cognition is established through it, it must be regarded as containing everything without exception. In this way, if time, space, and categories are logical assumptions to explain perception-level cognition and comprehension-level cognition, ideology is also a logical assumption that must be considered to exist prior to empirical cognition to explain rational-level cognition. As such, all three levels of cognition in Kant's explanation are composed of 'content' and 'form'. Here, form is something that cannot be said in principle, in which all distinctions, including mind and object, are excluded, and content exists by distinction between mind and object or between objects(see[16]). Form and content are not separated in fact, but are connected in a conceptually distinct relationship.

However, the three levels of cognition never occur separately, and therefore the forms at the three levels also point to the same entity in their referents. To the extent that reason is at the highest level of the stage of knowledge, its form represents the form or subjective condition of all stages. The reason why Kant divided cognition into three levels and presented the form corresponding to each level is to illustrate how the form specifically works that universally valid cognition of the world, that is, mathematics is a science that has entered the safe path of science. Therefore, an idea can be said to be an a priori idea insofar as it is a logical assumption to explain the establishment of a mathematical concept. According to Shin Chun-ho, if ideology takes on transcendental ideology, then ideology exists in a way that is neither the same nor different from the way things exist and is capable of recognizing the existence of things rather than the existence of things themselves. It can be said to create a 'form of cognition'(see[2]). Therefore, in order to theoretically explain the establishment of a mathematical concept, it contains an idea as a logical assumption, which means that an idea is related to a mathematical concept.

What can we say about the purpose of mathematics education according to Kant's epistemology? In this way, the ideology is a logical assumption that must be considered that mathematics exists in order to enter the safe path of learning. The ideology that is fixed as a logical assumption in mathematics is not just a logical assumption, but a standard that guides our lives and an ideal of life(see[4]). To realize the ideology as the standard of life and the ideal of life, there is no other way than through mathematical concepts as it can never reveal itself unless it is through the medium of expressing itself(see[14]). Therefore, the significance of Kant's epistemology in relation to mathematics education is that it explains mathematics as an ideology. Concepts that make up mathematics take phenomena and explain them. Because of this, any explanation of the phenomenon is impossible in itself unless the ideology is logically assumed, and the concepts constituting mathematics are established from the ideology.

According to Kant's argument, teaching mathematics is not just to give information about mathematics to the student who learns it, but to bring about a change in the student's existence. In other words, mathematical knowledge is established with the idea of principle as a logical assumption. As such, when teaching mathematical knowledge, it must be seen that not only the knowledge about the subject is conveyed, but also something else beyond the knowledge(see[18, 25]). Therefore, when teaching mathematics, mathematics teachers and students must see that they are exchanging not only mathematical knowledge but also ideology, which is the logical assumption of that knowledge. In mathematical concepts, ideas are included as logical assumptions. As such, it should be seen that ideology not only constructs mathematical concepts by being interested in objects, but also causes changes in existence by looking back at the mind itself, that is, pure subjectivity. In other words, it must be seen that the change of existence by the mind looking back at the mind itself is accomplished simultaneously with the mind's interest in the object and the creation of knowledge about the object. Therefore, it can be said that the purpose of mathematics education based on Kant's epistemology is to construct mathematical concepts of students learning mathematics and to cause changes in existence at the same time.

5. Conclusions

The question of why mathematics should be learned and taught in mathematics education is also the most fundamental question in mathematics education. Kant's theory of epistemology asserts that there is a reality that cannot be grasped by the understanding. Kant, who converted to subjective of transcendental intuition in order to acquire the necessity and universality of mathematical cognition, is evaluated to have synthesized the empiricism and rationalist explanations of the time. And Kant's explanation of mathematical concepts and ideologies can be seen as having a Platonic idea about mathematical objects. For Kant, mathematics occupies an important position related to ideology. The connection between mathematics and ideology is revealed in a special way for Kant.

In the nature of time and space, categories and ideas as logical assumptions, these three levels of form of cognition are inseparable. It can only be said that it is an inevitable step taken by Kant to understand cognition analytically. The reason why Kant divided cognition into three levels is to explain that both objective objects and subjective conditions are necessary for cognition to be established, and that, nevertheless, the subjective condition is more fundamental among the two elements. In order to theoretically explain the establishment of a mathematical concept, it must be seen that it contains an idea as a logical assumption. In mathematics education, ideology, on the one hand, does the work of constructing the concept, and at the same time, on the other hand, it does the work of leading the concept to orient itself. Therefore, the idea is expressed as a mathematical concept, and the mathematical concept realizes the idea. However, from the standpoint of ideology, as long as mathematics is a means of ideology, its realization can never be completed. If so, it is not that one-time learning of mathematics can realize the mentality behind it, but it is gradually being realized through endless learning. Mathematical knowledge with ideology as a logical assumption is always in the process of change, and in principle, something that cannot be expressed in a complete form is expressed as a mathematical concept. Therefore, ideology realizes itself by relating to mathematical concepts, and learning mathematical concepts is the path to realizing ideologies. Mathematical concepts not only have a status as expedients, but furthermore, ideologies are established by demonstrating their capabilities.

Kant's epistemology could provide an alternative answer to the question of the purpose of mathematics education. The purpose of mathematics education based on Kant's philosophy can be an alternative that can revive the idea of holistic education presented in the mathematics curriculum, and Kant's epistemology can be said to be a philosophical basis for mathematics education as character education.

Conflicts of Interest: The author declares that there is no conflict of interests regarding the publication of this paper.

References

1. Carl J. Posy, *Kants philosophy of mathematics: modern essays*, Kluwer Academic Publishers, edited Dordrecht, Boston, 1992.
2. Chun-ho Shin, *Kant's philosophy as an educational theory; commentary on the critique of judgment*, Seoul National University Doctoral Dissertation, 2005.
3. Chung-hyeon Yu, *Kant's transcendental philosophy and mathematics education*, Seoul National University Doctoral Dissertation, 2008.
4. Dae-hee Han, *The meaning of mathematics education in Froebel's thought of human education*, Research on Mathematics Education **10** (2000), 100-111.
5. E. Carson, *Kant on intuition in geometry*, Canadian Journal of Philosophy **27** (1997), 489-512.
6. E. Carson and R. Huber, *Intuition and the axiomatic method*, Springer, New York, 2006.
7. F. Copleston, *History of Philosophy*, Vol 6, Newman Press, 1961.
8. I. Kant, *Critique of pure reason*, translated by P. Guyer and A. Wood, The Cambridge Edition of the works of Immanuel Kant, Cambridge, 1998.
9. I. Kant, *Critique of pure reason*, translated by J.M.D. Meiklejohn, 2009. eBooks@Adelaide. http://ebooks.adelaide.edu.au/k/kant/immanuel/k16p/
10. Ja-gyeong Han, *Kant and Transcendental Philosophy: What is Human?*, Seogwangsa, Seoul, 1992.
11. Jae-hee Choi, *Kant's Critique of Pure Reason*, Parkyoungsa, Seoul, 1983.
12. Jae-Hoon Lim, *A Study on Plato's Philosophy of Mathematics Education*, Seoul National University Doctoral Dissertation, 1998.
13. Jae-hoon Lim, Kyung-hwa Lee, Dae-hee Han, *Inherent Purpose of Mathematics Education*, Proceedings of the 2003 Summer Math Education Research Conference **1** (2003), 225-242.
14. Jae-moon Park, *Bruner's Structure of Knowledge in the Light of Structuralist Epistemology*, Seoul National University Doctoral Dissertation, 1981
15. J. Dewey, J.A. Mclellan, *The Psychology of Number and its Application to Methods of Teaching Arithmetic*, D. Appleton and company, New York, 1895.
16. J. Piaget. *Psychology and epistemology: towards a theory of knowledge*, (A. Denoël, Rosin, Trans.). Penguin Books, New York, 1977.
17. Moon-gyo Lim, *A Study on the Goals of Elementary School Mathematics in Korea and Japan: Focusing on the Revised Curriculum and Commentary at the End of the 20th Century*, Journal of the Korean Society for Elementary Mathematics Education **9** (2005), 111-135.
18. Oh-Nam Kwon, Ji-Hyun Park, Jeong-Suk Park, *Case of Mathematics Class Model for Creative Character Education*, Education of Mathematics **50** (2011), 403-428.
19. S.R. Campbell, *Constructivism and the limits of reason: revisiting the Kantian problematic*, Studies in Philosophy and Education **21** (2002), 421-445.
20. Seongmo Jang, *Curriculum Education and Cultivation of Mind*, The Journal of Mathematics Education **1** (2004), 1-13.
21. S.E. Stumpf, *A History of Philophy*, McGraw-Hill. Inc, New York, 1975.
22. Tae-pyeong Im , *Kant's Philosophy of Education*, Academic History, Seoul, 1983.

23. Yang-ja Kim, *Comparative Study on the Moral Theory of Kant and Dewey*, Ewha Womans University Graduate School Master's Thesis, 1990.
24. Wi-seong Kim, *Kant's a priori deduction and Popper's falsifiability*, Kyungpook National University graduate school doctoral dissertation, 1983.
25. Young-bae Park, *A Study on the Constructivist Development of Mathematics Teaching-Learning*, Seoul National University Graduate School Doctoral Dissertation, 1996.

ChungHyun Yu received Ph.D. at Seoul National University. His research interests are mathematics education and special functions.

Department of Mathematics Education, Hannam University, Daejeon 34430, Korea.
e-mail: profyu@hnu.kr