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PAIR DIFFERENCE CORDIAL LABELING OF PETERSEN GRAPHS $P(n,k)$

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Abstract. Let $G = (V, E)$ be a (p, q) graph.
Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm \rho\}$ called the set of labels.
Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1. A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph. In this paper we investigate pair difference cordial labeling behaviour of Petersen graphs $P(n,k)$ like $P(n,2), P(n,3), P(n,4)$.

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1. Introduction

In this paper we consider only finite, undirected and simple graphs . The graph labeling was first introduced by Rosa in the name of graceful labeling [24]. Cordial labeling was introduced by Cahit in [4] . Subsequently cordial related labeling was defined and studied in [2,3,5,6,7,8,10,11,22,23]. The concept of pair difference cordial labeling was introduced in [12] and the pair difference cordial labeling behavior of several graphs like path, cycle, star,ladder,wheel,helm, web,fan, umberalla,(n,t)-kite, Mobius ladder, Slanting ladder,Triangular ladder etc have been investigated in [12 - 19]. In this paper we investigate pair difference

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cordial labeling behavior of Petersen graphs $P(n,k)$ like $P(n,2), P(n,3), P(n,4)$.

2. Preliminaries

Definition 2.1. Let $G = (V,E)$ be a (p,q) graph.
Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm1, \pm2, \pm3, \cdots, \pm\rho\}$ called the set of labels.
Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1. A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph.

Definition 2.2. [6]. Let $n \geq 3$ and $1 \leq k \leq \left[\frac{n}{2}\right]$. Then $P(n,k)$ is the graph with $V(P(n,k)) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(P(n,k)) = \{u_iu_{i+1}, u_iv_i, v_iv_{i+k} : 1 \leq i \leq n, \textit{subscripts modulo } n\}$. The graph $P(n,k)$ is called generalized Petersen graph.

For illustration , a pair difference cordial labeling of Petersen graph $P(5,2)$ is given in Figure 1.

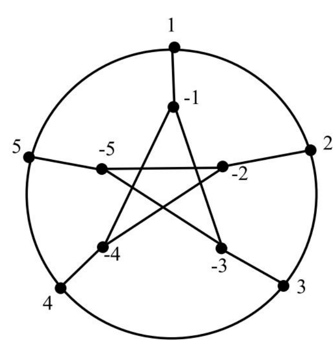

FIGURE 1

3. The Petersen Graph $P(n,2)$

Theorem 3.1. The Petersen graph $P(n,2)$ is pair difference cordial for all even values of $n \geq 4$.

Proof. Let us consider the vertex set and edge set as in Definition 2.2.

Case 1. $n \equiv 0(mod4)$.
Subcase 1. $n = 4$.
A pair difference cordial labeling of Petersen graph $P(4,2)$ is given in Figure 2.

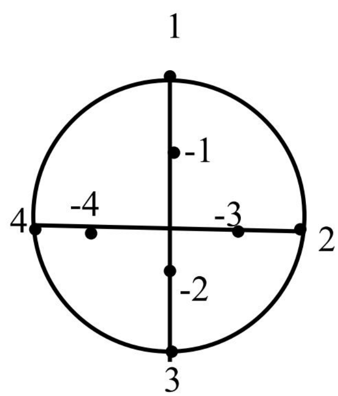

FIGURE 2

Subcase 2. $n > 4$.
Assign the labels $1, 2, 3, \cdots, \frac{3n}{4}$ respectively to the vertices $u_1, u_2, u_3, \cdots, u_{\frac{3n}{4}}$ and assign the labels $\frac{3n+8}{4}, \frac{3n+4}{4}$ respectively to the vertices $u_{\frac{3n+4}{4}}, u_{\frac{3n+8}{4}}$. Next assign the labels $\frac{3n+12}{4}, \frac{3n+16}{4}$ respectively to the vertices $u_{\frac{3n+12}{4}}, u_{\frac{3n+16}{4}}$ and assign the labels $\frac{3n+24}{4}, \frac{3n+20}{4}$ respectively to the vertices $u_{\frac{3n+20}{4}}, u_{\frac{3n+24}{4}}$. Proceeding like this until we reach the vertex u_n.

Now we assign the labels $-1, -3, -5, \cdots, -(n-1)$ to the vertices $v_1, v_3, v_5, \cdots,$ v_{n-1} and assign the labels $-2, -4, -6, \cdots, n$ to the vertices $v_2, v_4, v_6, \cdots, v_n$ respectively.

Case 2. $n \equiv 2(mod4)$.
Subcase 1. $n = 6$.
Assign the labels $1, 2, 3, 4, 5, 6$ to the vertices $u_1, u_2, u_3, u_4, u_5, u_6$ respectively and assign the labels $-1, -2, -3, -4, -5, -6$ respectively to the vertices $v_1, v_2, v_3, v_4, v_5, v_6$.

Subcase 2. $n > 6$.
Assign the labels $1, 2, 3, \cdots, \frac{3n-2}{4}$ respectively to the vertices $u_1, u_2, u_3, \cdots, u_{\frac{3n-2}{4}}$ and assign the labels $\frac{3n+6}{4}, \frac{3n+2}{4}, \frac{3n+10}{4}, \frac{3n+14}{4}$ respectively to the vertices $u_{\frac{3n+2}{4}}$, $u_{\frac{3n+6}{4}}, u_{\frac{3n+10}{4}}, u_{\frac{3n+14}{4}}$. Next assign the labels $\frac{3n+22}{4}, \frac{3n+18}{4}, \frac{3n+26}{4}, \frac{3n+30}{4}$ respectively to the vertices $u_{\frac{3n+18}{4}}, u_{\frac{3n+22}{4}}, u_{\frac{3n+26}{4}}, u_{\frac{3n+30}{4}}$. Proceeding like this until we reach the vertex u_n.

Now we assign the labels $-1, -3, -5, \cdots, -(n-1)$ to the vertices $v_1, v_3, v_5, \cdots$, v_{n-1} and assign the labels $-2, -4, -6, \cdots, -n$ to the vertices $v_2, v_4, v_6, \cdots, v_n$ respectively.

In both cases, obviously $\Delta_{f_1^c} = \Delta_{f_1} = \frac{3n}{2}$.

□

Theorem 3.2. The petersen graph $P(n, 2)$ is pair difference cordial for all odd values of $n \geq 5$.

Proof. We take the vertex set and edge set as in Definition 2.2.

Case 1. $n \equiv 1(mod4)$.

Subcase 1. $n = 5$.
A pair difference cordial labeling of Petersen graph $P(5, 2)$ is given in Figure 1.

Subcase 2. $n > 5$.
Assign the labels $1, 2, 3, \cdots, \frac{3n-3}{4}$ respectively to the vertices $u_1, u_2, u_3, \cdots, u_{\frac{3n-3}{4}}$ and assign the labels $\frac{3n+5}{4}, \frac{3n+1}{4}$ respectively to the vertices $u_{\frac{3n+1}{4}}, u_{\frac{3n+5}{4}}$. Next assign the labels $\frac{3n+9}{4}, \frac{3n+13}{4}$ respectively to the vertices $u_{\frac{3n+9}{4}}, u_{\frac{3n+13}{4}}$ and assign the labels $\frac{3n+21}{4}, \frac{3n+17}{4}, \frac{3n+29}{4}, \frac{3n+25}{4}$ respectively to the vertices $u_{\frac{3n+17}{4}}, u_{\frac{3n+21}{4}}$, $u_{\frac{3n+25}{4}}, u_{\frac{3n+29}{4}}$. Proceeding like this until we reach the vertex u_n.

Now we assign the labels $-1, -3, -5, \cdots, -(\frac{n+1}{2})$ to the vertices $v_1, v_3, v_5, \cdots$, v_n and assign the labels $--(\frac{n+3}{2}), -(\frac{n+5}{2}), -(\frac{n+7}{2}), \cdots, -n$ to the vertices v_2, v_4, $v_6, \cdots, v_{n-1}$ respectively.

Case 2. $n \equiv 3(mod4)$.
Subcase 1. $n = 7$.
Assign the labels $1, 2, 3, 4, 5, 6, 7$ to the vertices $u_1, u_2, u_3, u_4, u_5, u_6, u_7$ respectively and assign the labels $-1, -2, -3, -5, -4, -6, -7$ respectively to the vertices $v_1, v_2, v_3, v_4, v_5, v_6, v_7$.

Subcase 2. $n > 7$.
Assign the labels $1, 2, 3, \cdots, \frac{3n-9}{4}$ respectively to the vertices $u_1, u_2, u_3, \cdots, u_{\frac{3n-9}{4}}$ and assign the labels $\frac{3n-1}{4}, \frac{3n-5}{4}, \frac{3n+3}{4}, \frac{3n+7}{4}$ respectively to the vertices $u_{\frac{3n-5}{4}}$, $u_{\frac{3n-1}{4}}$, $u_{\frac{3n+3}{4}}$, $u_{\frac{3n+7}{4}}$. Next assign the labels $\frac{3n+15}{4}, \frac{3n+11}{4}, \frac{3n+19}{4}, \frac{3n+23}{4}$ respectively to the vertices $u_{\frac{3n+11}{4}}, u_{\frac{3n+15}{4}}, u_{\frac{3n+19}{4}}, u_{\frac{3n+23}{4}}$. Proceeding like this until we reach the vertex u_{n-2}. Now assign the labels $n, n-1$ respectively to the vertices u_{n-1}, u_n.

Now we assign the labels $-1, -3, -5, \cdots, -(\frac{n+1}{2})$ to the vertices $v_1, v_3, v_5, \cdots, v_n$ and assign the labels $-(\frac{n+3}{2})$, $-(\frac{n+5}{2})$, $-(\frac{n+7}{2})$, $\cdots, -n$ to the vertices $v_2, v_4, v_6, \cdots, v_{n-1}$ respectively.

In both cases, clearly $\Delta_{f_1^c} = \frac{3n+1}{2}, \Delta_{f_1} = \frac{3n}{2}$.

□

4. The Petersen Graph $P(n,3)$

Theorem 4.1. If $6 \leq n \leq 15$, then $P(n,3)$ is pair difference cordial.

Proof. Take the vertex set and edge set as in Definition 2.2

First assign the labels $1, 2, 3, \cdots, n$,$6 \leq n \leq 15$ respectively to the vertices $v_i, 1 \leq i \leq n$.
Next assign the labels to the vertices as in Table 1. Table 1 shows that $P(n,3), 6 \leq n \leq 15$ is pair difference cordial.

n	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}	u_{13}	u_{14}	u_{15}
6	-2	-4	-6	-1	-3	-5									
7	-1	-2	-3	-5	-4	-6	-7								
8	-1	-2	-3	-5	-4	-6	-7	-8							
9	-1	-2	-3	-4	-5	-6	-7	-8	-9						
10	-1	-2	-3	-4	-5	-6	-7	-8	-10	-9					
11	-1	-2	-3	-4	-5	-6	-7	-8	-9	-11	-10				
12	-1	-2	-3	-4	-5	-6	-7	-8	-9	-11	-10	-12			
13	-1	-2	-3	-4	-5	-6	-7	-8	-9	-11	-13	-10	-12		
14	-1	-2	-3	-4	-5	-6	-7	-8	-9	-11	-13	-10	-12	-14	
15	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-13	-15	-12	-14

TABLE 1

□

Theorem 4.2. If $n > 15$, then $P(n,3)$ is pair difference cordial.

Proof. Let us consider the vertex set and edge set as in Definition 2.2.

Case 1. $n \equiv 0(mod 3)$.

Assign the labels $1, 2, 3, \cdots, \frac{n}{3}$ respectively to the vertices $u_1, u_4, u_7, \cdots, u_{n-2}$ and assign the labels $\frac{n+3}{3}, \frac{n+6}{3}, \frac{n+9}{3}, \cdots, \frac{2n}{3}$ respectively to the vertices $u_2, u_5, u_8, \cdots, u_{n-1}$. Next assign the labels $\frac{2n+3}{3}, \frac{2n+6}{3}, \frac{2n+9}{3}, \cdots, n$ respectively to the vertices $u_3, u_6, u_9, \cdots, u_n$. Next assign the labels to the vertices $v_i, 1 \leq i \leq n$. There are four cases arises.

Subcase 1. $n \equiv 0(mod 4)$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+8}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+8}{2}}$ and assign the labels $-(\frac{n+12}{2}), -(\frac{n+16}{2}), -(\frac{n+20}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{n+10}{2}}, v_{\frac{n+12}{2}}, v_{\frac{n+14}{2}}, \cdots, v_{\frac{3n+8}{4}}$. Next assign the labels $-(\frac{n+10}{2}), -(\frac{n+14}{2}), -(\frac{n+18}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{3n+12}{4}}, v_{\frac{3n+16}{4}}, v_{\frac{3n+20}{4}}, \cdots, v_n$.

Subcase 2. $n \equiv 1(mod 4)$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+9}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+9}{2}}$ and assign the labels $-(\frac{n+13}{2}), -(\frac{n+17}{2}), -(\frac{n+21}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{n+11}{2}}, v_{\frac{n+13}{2}}, v_{\frac{n+15}{2}}, \cdots, v_{\frac{3n+9}{4}}$. Next assign the labels $-(\frac{n+15}{2}), -(\frac{n+19}{2}), -(\frac{n+23}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{3n+13}{4}}, v_{\frac{3n+17}{4}}, v_{\frac{3n+21}{4}}, \cdots, v_n$.

Subcase 3. $n \equiv 2(mod 4)$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+8}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+8}{2}}$ and assign the labels $-(\frac{n+12}{2}), -(\frac{n+16}{2}), -(\frac{n+20}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{n+10}{2}}, v_{\frac{n+12}{2}}, v_{\frac{n+14}{2}}, \cdots, v_{\frac{3n+6}{4}}$. Next assign the labels $-(\frac{n+10}{2}), -(\frac{n+14}{2}), -(\frac{n+18}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{3n+10}{4}}, v_{\frac{3n+14}{4}}, v_{\frac{3n+18}{4}}, \cdots, v_n$.

Subcase 4. $n \equiv 3(mod 4)$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+9}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+9}{2}}$ and assign the labels $-(\frac{n+13}{2}), -(\frac{n+17}{2}), -(\frac{n+21}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{n+11}{2}}, v_{\frac{n+13}{2}}, v_{\frac{n+15}{2}}, \cdots, v_{\frac{3n+7}{4}}$. Next assign the labels $-(\frac{n+15}{2}), -(\frac{n+19}{2}), -(\frac{n+23}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{3n+11}{4}}, v_{\frac{3n+15}{4}}, v_{\frac{3n+19}{4}}, \cdots, v_n$.

Table 2 shows that $P(n, 3)$ is pair difference cordial for all values of $n \equiv 0(mod 3)$.

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
$n \equiv 0 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1 \pmod 4$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$
$n \equiv 2 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 3 \pmod 4$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$

TABLE 2

Case 2. $n \equiv 1 (mod 3)$.

Assign the labels $1, 2, 3, \cdots, \frac{n+2}{3}$ respectively to the vertices $u_1, u_4, u_7, \cdots, u_n$ and assign the labels $\frac{n+5}{3}, \frac{n+8}{3}, \frac{n+11}{3}, \cdots, \frac{2n+1}{3}$ respectively to the vertices $u_2, u_5, u_8, \cdots, u_{n-1}$. Next assign the labels $\frac{2n+4}{3}, \frac{2n+7}{3}, \frac{2n+10}{3}, \cdots, n$ respectively to the vertices $u_3, u_6, u_9, \cdots, u_{n-2}$. Next assign the labels to the vertices $v_i, 1 \leq i \leq n$. There are four cases arises.

Subcase 1. $n \equiv 0 (mod 4)$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+4}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+4}{2}}$ and assign the labels $-(\frac{n+8}{2})$,$-(\frac{n+12}{2})$, $-(\frac{n+16}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{n+6}{2}}, v_{\frac{n+8}{2}}, v_{\frac{n+10}{2}}, \cdots, v_{\frac{3n+4}{4}}$. Next assign the labels $-(\frac{n+6}{2})$,$-(\frac{n+10}{2})$, $-(\frac{n+14}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{3n+8}{4}}, v_{\frac{3n+12}{4}}, v_{\frac{3n+16}{4}}, \cdots, v_n$.

Subcase 2. $n \equiv 1, 3 (mod 4)$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+5}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+5}{2}}$ and assign the labels $-(\frac{n+9}{2})$,$-(\frac{n+13}{2})$, $-(\frac{n+17}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{n+7}{2}}, v_{\frac{n+9}{2}}, v_{\frac{n+11}{2}}, \cdots, v_{\frac{3n+5}{4}}$. Next assign the labels $-(\frac{n+7}{2})$,$-(\frac{n+11}{2})$, $-(\frac{n+15}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{3n+9}{4}}, v_{\frac{3n+11}{4}}, v_{\frac{3n+15}{4}}, \cdots, v_n$.

Subcase 3. $n \equiv 2 (mod 4)$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+2}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+2}{2}}$ and assign the labels $-(\frac{n+6}{2})$,$-(\frac{n+10}{2})$, $-(\frac{n+14}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{n+4}{2}}, v_{\frac{n+6}{2}}, v_{\frac{n+8}{2}}, \cdots, v_{\frac{3n+2}{4}}$. Next assign the labels $-(\frac{n+4}{2})$,$-(\frac{n+8}{2})$, $-(\frac{n+12}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{3n+6}{4}}, v_{\frac{3n+10}{4}}, v_{\frac{3n+14}{4}}, \cdots, v_n$.

Table 3 shows that $P(n,3)$ is pair difference cordial for all values of $n \equiv 1(mod3)$.

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
$n \equiv 0 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1 \pmod 4$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$
$n \equiv 2 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 3 \pmod 4$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$

Table 3

Case 3. $n \equiv 2(mod3)$.

Assign the labels $1, 2, 3, \cdots, \frac{n+1}{3}$ respectively to the vertices $u_1, u_4, u_7, \cdots, u_{n-1}$ and assign the labels $\frac{n+4}{3}, \frac{n+7}{3}, \frac{n+10}{3}, \cdots, \frac{2n+2}{3}$ respectively to the vertices $u_2, u_5, u_8, \cdots, u_n$. Next assign the labels $\frac{2n+5}{3}, \frac{2n+8}{3}, \frac{2n+11}{3}, \cdots, n$ respectively to the vertices $u_3, u_6, u_9, \cdots, u_n$. Next assign the labels to the vertices $v_i, 1 \leq i \leq n$. There are four cases arises.

Subcase 1. $n \equiv 0(mod4)$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+4}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+4}{2}}$ and assign the labels $-(\frac{n+8}{2})$,$-(\frac{n+12}{2})$, $-(\frac{n+16}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{n+6}{2}}, v_{\frac{n+8}{2}}, v_{\frac{n+10}{2}}, \cdots, v_{\frac{3n+4}{4}}$. Next assign the labels $-(\frac{n+6}{2})$,$-(\frac{n+10}{2})$, $-(\frac{n+14}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{3n+8}{4}}, v_{\frac{3n+12}{4}}, v_{\frac{3n+16}{2}}, \cdots, v_n$.

Subcase 2. $n \equiv 1(mod4)$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+5}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+5}{2}}$ and assign the labels $-(\frac{n+9}{2})$,$-(\frac{n+13}{2})$, $-(\frac{n+17}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{n+7}{2}}, v_{\frac{n+9}{2}}, v_{\frac{n+11}{2}}, \cdots, v_{\frac{3n+5}{4}}$. Next assign the labels $-(\frac{n+7}{2})$,$-(\frac{n+11}{2})$, $-(\frac{n+15}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{3n+9}{4}}, v_{\frac{3n+13}{4}}, v_{\frac{3n+17}{2}}, \cdots, v_n$.

Subcase 3. $n \equiv 2(mod4)$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+4}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+4}{2}}$ and assign the labels $-(\frac{n+8}{2})$,$-(\frac{n+12}{2})$, $-(\frac{n+16}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{n+6}{2}}, v_{\frac{n+8}{2}}, v_{\frac{n+10}{2}}, \cdots, v_{\frac{3n+2}{4}}$. Next assign the labels $-(\frac{n+6}{2})$,$-(\frac{n+10}{2})$, $-(\frac{n+14}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{3n+6}{4}}, v_{\frac{3n+10}{4}}$,

$v_{\frac{3n+14}{2}}, \cdots, v_n$.

Subcase 4. $n \equiv 3(mod4)$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+5}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+5}{2}}$ and assign the labels $-(\frac{n+9}{2})$,$-(\frac{n+11}{2})$, $-(\frac{n+15}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{n+7}{2}}, v_{\frac{n+9}{2}}, v_{\frac{n+11}{2}}, \cdots, v_{\frac{3n+3}{4}}$. Next assign the labels $-(\frac{n+7}{2})$,$-(\frac{n+13}{2})$, $-(\frac{n+17}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{3n+7}{4}}, v_{\frac{3n+11}{4}}, v_{\frac{3n+15}{2}}, \cdots, v_n$.

Table 4 shows that $P(n,3)$ is pair difference cordial for all values of $n \equiv 2(mod3)$.

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
$n \equiv 0 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1 \pmod 4$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$
$n \equiv 2 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 3 \pmod 4$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$

TABLE 4

□

5. The Petersen Graph $P(n,4)$

Theorem 5.1. If $n \geq 8$, then $P(n,4)$ is pair difference cordial.

Proof. We take the vertex set and edge set as in Definition 2.2. There are four cases arises.

Case 1. $n \equiv 0(mod4)$.

Subcase 1. $n = 8$.
Assign the labels $1, 2, 3, 4, 5, 6, 8, 7$ respectively to the vertices $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8$ and assign the labels $-1, -3, -5, -7, -2, -4, -6, -8$ to the vertices $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8$ respectively.

Subcase 2. $n > 8$.
Assign the labels $1, 2, 3, \cdots, \frac{n}{4}$ respectively to the vertices $u_1, u_5, u_9, \cdots, u_{n-3}$ and assign the labels $\frac{n+4}{4}$, $\frac{n+8}{4}$, $\frac{n+12}{4}, \cdots, \frac{n}{2}$ respectively to the vertices $u_2, u_6, u_{10}, \cdots, u_{n-2}$. Next assign the labels $\frac{n+2}{2}$, $\frac{n+4}{2}$, $\frac{n+6}{2}, \cdots, \frac{3n}{4}$ respectively to the vertices $u_3, u_7, u_{11}, \cdots, u_{n-1}$ and assign the labels $\frac{3n+4}{4}$, $\frac{3n+8}{4}$, $\frac{3n+12}{4}, \cdots, n$ respectively to the vertices $u_4, u_8, u_{12}, \cdots, u_n$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+8}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+8}{2}}$ and assign the labels $-(\frac{n+12}{2})$,$-(\frac{n+16}{2})$, $-(\frac{n+20}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{n+10}{2}}, v_{\frac{n+12}{2}}, v_{\frac{n+14}{2}}, \cdots, v_{\frac{3n+8}{4}}$. Next assign the labels $-(\frac{n+10}{2})$,$-(\frac{n+14}{2})$, $-(\frac{n+18}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{3n+12}{4}}, v_{\frac{3n+16}{4}}, v_{\frac{3n+20}{4}}, \cdots, v_n$.

Case 2. $n \equiv 1(mod4)$.

Subcase 1. $n = 9$.
Assign the labels $1, 2, 3, 4, 5, 7, 6, 8, 9$ respectively to the vertices $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9$ and assign the labels $-1, -8, -6, -4, -2, -9, -7, -5, -3$ to the vertices $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9$ respectively.

Subcase 2. $n > 9$.

Assign the labels $1, 2, 3, \cdots, \frac{n+3}{4}$ respectively to the vertices $u_1, u_5, u_9, \cdots, u_n$ and assign the labels $\frac{n+7}{4}, \frac{n+11}{4}, \frac{n+15}{4}, \cdots, \frac{n+1}{2}$ respectively to the vertices $u_4, u_8, u_{12}, \cdots, u_{n-1}$. Next assign the labels $\frac{n+3}{2}, \frac{n+5}{2}, \frac{n+7}{2}, \cdots, \frac{3n+1}{4}$ respectively to the vertices $u_3, u_7, u_{11}, \cdots, u_{n-2}$ and assign the labels $\frac{3n+5}{4}, \frac{3n+9}{4}, \frac{3n+13}{4}, \cdots, n$ respectively to the vertices $u_2, u_6, u_{10}, \cdots, u_{n-3}$.

Assign the labels $-1, -2, -3, \cdots, -(\frac{n+5}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+5}{2}}$ and assign the labels $-(\frac{n+9}{2})$,$-(\frac{n+13}{2})$, $-(\frac{n+17}{2}), \cdots, -(n)$ respectively to the vertices $v_{\frac{n+7}{2}}, v_{\frac{n+9}{2}}, v_{\frac{n+11}{2}}, \cdots, v_{\frac{3n+5}{4}}$. Next assign the labels $-(\frac{n+7}{2})$,$-(\frac{n+11}{2})$, $-(\frac{n+15}{2}), \cdots, -(n-1)$ respectively to the vertices $v_{\frac{3n+9}{4}}, v_{\frac{3n+13}{4}}, v_{\frac{3n+17}{4}}, \cdots, v_n$.

Case 3. $n \equiv 2(mod4)$.

Subcase 1. $n = 10$.
Assign the labels $1, 2, 3, 4, 5, 6, 8, 7, 9, 10$ respectively to the vertices $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}$ and assign the labels $-1, -6, -4, -9, -2, -7, -5, -10, -3, -8$ to the vertices $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}$ respectively.

Subcase 2. $n > 10$.

Assign the labels $1, 2, 3, \cdots, \frac{n+2}{4}$ respectively to the vertices $u_1, u_5, u_9, \cdots, u_{n-1}$ and assign the labels $\frac{n+6}{4}, \frac{n+10}{4}, \frac{n+14}{4}, \cdots, \frac{n}{2}$ respectively to the vertices $u_3, u_7, u_{11}, \cdots, u_{n-3}$. Next assign the labels $\frac{n+2}{2}, \frac{n+4}{2}, \frac{n+6}{2}, \cdots, n-3$ respectively to the vertices $u_2, u_6, u_{10}, \cdots, u_n$ and assign the labels $\frac{3n+6}{4}, \frac{3n+10}{4}, \frac{3n+14}{4}, \cdots, n$ respectively to the vertices $u_4, u_8, u_{12}, \cdots, u_{n-2}$.

Assign the labels $-1,-2,-3,\cdots,-(\frac{n+6}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n+6}{2}}$ and assign the labels $-(\frac{n+10}{2})$,$-(\frac{n+14}{2})$, $-(\frac{n+18}{2}),\cdots,-(n)$ respectively to the vertices $v_{\frac{n+8}{2}}, v_{\frac{n+10}{2}}, v_{\frac{n+12}{2}}, \cdots, v_{\frac{3n+6}{4}}$. Next assign the labels $-(\frac{n+8}{2})$,$-(\frac{n+12}{2})$, $-(\frac{n+16}{2}),\cdots,-(n-1)$ respectively to the vertices $v_{\frac{3n+10}{4}}, v_{\frac{3n+14}{4}}, v_{\frac{3n+18}{4}}, \cdots, v_n$.

Case 4. $n \equiv 3(mod4)$.

Subcase 1. $n = 11$.
Assign the labels $1,2,3,4,5,7,6,8,9,11,10$ respectively to the vertices $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}$ and assign the labels $-1,-4,-7,-10,-2,-5,-8,-11,-3, -6,-9$ to the vertices $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}$ respectively.

Subcase 2. $n > 11$.

Assign the labels $1,2,3,\cdots,\frac{n+1}{4}$ respectively to the vertices $u_1, u_5, u_9, \cdots, u_{n-2}$ and assign the labels $\frac{n+5}{4}$,$\frac{n+9}{4}$, $\frac{n+11}{4},\cdots,\frac{n+1}{2}$ respectively to the vertices $u_2, u_6, u_{10}, \cdots, u_{n-1}$. Next assign the labels $\frac{n+3}{2}$,$\frac{n+5}{2}$, $\frac{n+7}{2},\cdots,\frac{3n-5}{4}$ respectively to the vertices $u_3, u_7, u_{11}, \cdots, u_n$ and assign the labels $\frac{3n-1}{4}$,$\frac{3n+3}{4}$, $\frac{3n+7}{4}$, $\cdots, n$ respectively to the vertices $u_4, u_8, u_{12}, \cdots, u_{n-3}$.

Assign the labels $-1,-2,-3,\cdots,-(\frac{n-1}{2})$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-1}{2}}$ and assign the labels $-(\frac{n+3}{2})$,$-(\frac{n+7}{2})$, $-(\frac{n+11}{2}),\cdots,-(n)$ respectively to the vertices $v_{\frac{n+1}{2}}, v_{\frac{n+3}{2}}, v_{\frac{n+5}{2}}, \cdots, v_{\frac{3n+7}{4}}$. Next assign the labels $-(\frac{n+1}{2})$,$-(\frac{n+5}{2})$, $-(\frac{n+9}{2}),\cdots,-(n-1)$ respectively to the vertices $v_{\frac{3n+11}{4}}, v_{\frac{3n+15}{4}}, v_{\frac{3n+19}{4}}, \cdots, v_n$.

Table 5 shows that $P(n,4)$ is pair difference cordial for all values of $n \geq 8$.

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
$n \equiv 0 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1 \pmod 4$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$
$n \equiv 2 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 3 \pmod 4$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$

TABLE 5

□

6. Discussion

The pair sum labeling was introduced by Ponraj and Parthipan in [20]. The concept of difference cordial labeling of graphs was introduced in [21]. Motivated by these two concepts, we have defined a new graph labeling called pair difference cordial labeling of graphs [12]. Accordingly,the pair difference cordial labeling behaviour of some Petersen graphs $P(n,2), P(n,3), P(n,4)$ have investigated in this paper.

7. Limitation of Research

Presently, it is difficult to investigate the pair difference cordial labeling behaviour of Swastik graph on large number of vertices.

8. Conclusion

In this paper we have investigated pair difference cordial labeling behaviour of some Petersen graphs $P(n,2), P(n,3), P(n,4)$. The pair difference cordial labeling behaviour of Petersen graphs $P(n,k), n \geq 5$ are the open problems.

Conflicts of interest : The authors declare no conflict of interest.

Data availability : Not applicable

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