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# NUMERICAL MODELING OF NON-CAPACITY MODEL FOR SEDIMENT TRANSPORT BY CENTRAL UPWIND SCHEME

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Abstract. This work deals with the numerical modeling of dam-break flow over erodible bed. The mathematical model consists of the shallow water equations, the transport diffusion and the bed morphology change equations. The system is solved by central upwind scheme. The obtained results of the resolution of dam-beak problem is presented in order to show the performance of the numerical scheme. Also a comparison of central upwind and Roe schemes is presented.

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## 1. Introduction

Numerical modeling of alluvial rivers is getting attention these last years, specially dam-break flows and problems of sediment transports. Mainly, for their impact on land use planning and other economic and security reasons.

The majority of the first mathematical models modelizing dam-break problem considered a fixed bottom see [6] [8] and [26] among others. A few years later and based on many researchers, the coupled model saw birth in [5]. The mathematical model consists of four equations; the mass and the momentum conservation equation for the water-sediment mixture, the transport diffusion equation for sediment particles and bed morphology change equation, complemented by the empirical formulations for bed friction and sediment exchange between the water column and the bed [24]. The coupled model interlink between flow, sediment transport and morphological evolution, in which entrainment and deposition of sediments are treated as independent processes this property is called non-capacity model [2] and [24]. Since there, this mathematical model is used by many researchers see among others [2], [10], [12], [27], [25], [22] and [11].

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Central-upwind schemes were introduced at first in [13, 14, 15] for one dimensional hyperbolic systems of conservation laws and its multidimensional extensions. The central-upwind schemes belong Godunov central schemes family, therefore they enjoy the main advantages of central schemes for solving time-dependent differential equations in different fields like robustness, simplicity and high-resolution. At the same time, in central upwind a more careful estimate of the one-sided local speeds of propagation and integration over Riemann fans with variable sizes is used (see [17], for instance). This decreases the numerical dissipation and results in increased resolution of the computed solution. Central-upwind schemes have been proposed for general hyperbolic system of conversation law in [17, 18] and extended to the shallow water equations and related models in [15], [16] and [19].

The governing equations are solved numerically using central upwind scheme. The main contribution is to propose a discretisization of the source term which agree with the central upwind scheme and compare it with Roe scheme presented in [10]. The MUSCL method with generalized minmod limiter and the Runge-Kutta are used to achieve a second order accuracy. We focus in this work on the evolution of dam-break flow, sediment transport and bed morphological development. Many comparisons are reported in order to enhance the performance of central upwind scheme: the numerical scheme in use is compared vis-a-vis previous evolutionary behaviors of dam-break over erodible bed addressed in [3], [7] and [2], and vis-a-vis Roe scheme with a new discretization of the source term developed in [10].

This work is organized as follows. Section 2 presents the governing equations for a dam-break over erodible sediment bed. In Section 3 the central upwind scheme is formulated and discretized. Section 4 covers the resolution of dam-break problem. Finally concluding remarks are given in Section 5.

## 2. The mathematical model

This work consists of on one-dimensional flow in a channel with rectangular cross section of constant width, over an erodible bed composed of uniform and noncohesive sediment particles. This mathematical model can be extendable to natural rivers with complex geometries, nonuniform sediments and multidimensional problems. The governing equations are subject of four equations: the mass and the momentum conservation equations for the water-sediment mixture, the mass conservation equation for the sediment and the mass conservation equation for the bed material are written as [1], [2], [10], [23] and [24]:

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = \frac{E - D}{1 - p} \tag{1}$$

$$\frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2 + \frac{1}{2}gh^2)}{\partial x} = B \tag{2}$$

$$\frac{\partial (hc)}{\partial t} + \frac{\partial (huc)}{\partial x} = E - D \tag{3}$$

$$\frac{\partial z}{\partial t} = -\frac{E-D}{1-p} \tag{4}$$

where $B$ is the source term defined by :

$$B = -gh\frac{\partial z}{\partial x} - \frac{\rho_s-\rho_w}{2\rho}gh^2\frac{\partial c}{\partial x} - ghS_f - \frac{\rho_0-\rho}{\rho}\,\frac{E-D}{1-p}\,u \tag{5}$$

$t$ is the time, $x$ the streamwise coordinate, $h$ the flow depth, $u$ the depth-averaged streamwise velocity, $z$ the bed elevation, $c$ the flux-averaged volumetric sediment concentration, $g$ the gravitational acceleration, $p$ the bed sediment porosity. $D$ and $E$ are the sediment deposition and entrainment fluxes across the bottom boundary of flow, they represent the exchange between water column and bed. $S_f$ is the friction slope, $\rho = \rho_w(1-c)+\rho_s c$ is the density of water-sediment mixture,
$\rho_0 = \rho_w p + \rho_s(1-p)$ is the density of the saturated bed, $\rho_w$ and $\rho_s$ are the densities of water and sediment, respectively.

To avoid repetition, we refer the reader to [10] for extra detail concerning empirical functions.

## 3. Central-upwind scheme

It is well known that Godunov-type central schemes are Riemann-problem-solver-free and are robust, simple and high-resolution methods for solving time-dependent differential equations in different fields. The central-upwind schemes belong Godunov central schemes family, where a more careful estimate of the one-sided local speeds of propagation and integration over Riemann fans with variable sizes is used. This decreases the numerical dissipation and results in increased resolution of the computed solution. Another advantage of these schemes, as opposed to the earlier developed staggered central schemes, is that they can be used for steady state computations. Central-upwind schemes have been proposed for general hyperbolic system of conversation law in [17, 18] and extended to the shallow water equations and related models in [15], [16] and [19].

Equations (1-4) can be arranged in the conservative form:

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = S + Q \tag{6}$$

or non-conservative form:

$$\frac{\partial U}{\partial t} + \mathcal{A}(U)\frac{\partial U}{\partial x} = Q \tag{7}$$

where

$$U = \begin{pmatrix} h \\ hu \\ hc \\ z \end{pmatrix},\; F = \begin{pmatrix} hu \\ hu^2+\frac{1}{2}gh^2 \\ huc \\ 0 \end{pmatrix},\; S = \begin{pmatrix} 0 \\ -gh\frac{\partial z}{\partial x} - \frac{(\rho_s-\rho_w)}{2\rho}gh^2\frac{\partial c}{\partial x} \\ 0 \\ 0 \end{pmatrix}, \tag{8}$$

$$Q = \begin{pmatrix} \frac{E-D}{1-p} \\ -ghS_f - \frac{\rho_0-\rho}{\rho}\frac{E-D}{1-p}u \\ E-D \\ -\frac{E-D}{1-p} \end{pmatrix} \tag{9}$$

the matrix $\mathcal{A}(U)$ is given by

$$\mathcal{A}(U) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ gh - u^2 - \frac{\rho_s-\rho_w}{2\rho}ghc & 2u & \frac{\rho_s-\rho_w}{2\rho}gh & gh \\ -uc & c & u & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

$\mathcal{A}(U)$ has the four following distinct real eigenvalues

$$\lambda_1 = 0, \quad \lambda_2 = u, \quad \lambda_3 = u - \sqrt{gh}, \quad and\ \lambda_4 = u + \sqrt{gh} \tag{11}$$

The spatial domain is discretisized into finite volume cells $\mathcal{C}_i = [x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}]$ with the same length $\Delta x$. The time interval is divided into subintervals $[t_n, t_{n+1}]$ with uniform size $\Delta t$. We suppose that at certain time $t$, the solution is given in terms of its cell averages $U_i = \frac{1}{\Delta x}\int_{\mathcal{C}_i} U(x,t)dx$, which are given in time according to the semi-discrete central-upwind scheme, see for instance [14, 15] as follows

$$\frac{\partial U_i}{\partial t} = -\frac{H_{i+\frac{1}{2}}(t) - H_{i-\frac{1}{2}}(t)}{\Delta x} + S_i(t) + Q_i(t), \tag{12}$$

where $S_i(t)$ and $Q_i(t)$ are respectively the cell average of $S(t)$ and $Q(t)$ on $\mathcal{C}_i$ at the time $t$. The central-upwind numerical flux $H_{i+\frac{1}{2}}(t)$ are given by

$$H_{i+\frac{1}{2}}(t) = \frac{a^+_{i+\frac{1}{2}}F^-_{i+\frac{1}{2}}(t) - a^-_{i+\frac{1}{2}}F^+_{i+\frac{1}{2}}(t)}{a^+_{i+\frac{1}{2}} - a^-_{i+\frac{1}{2}}} + \frac{a^+_{i+\frac{1}{2}}a^-_{i+\frac{1}{2}}}{a^+_{i+\frac{1}{2}} - a^-_{i+\frac{1}{2}}}(U^+_{i+\frac{1}{2}}(t) - U^-_{i+\frac{1}{2}}(t)). \tag{13}$$

due to the hyperbolicity of the system of differential equations 7, the discontinuities appearing at the reconstruction step at the interface points $x_{i+1/2}$ propagate at finite speeds estimated by

$$a^+_{i+\frac{1}{2}} = \max\left(0, u^+_{i+\frac{1}{2}} + \sqrt{gh^+_{i+\frac{1}{2}}}, u^-_{i+\frac{1}{2}} + \sqrt{gh^-_{i+\frac{1}{2}}}\right) \tag{14}$$

$$a^-_{i+\frac{1}{2}} = \min\left(0, u^+_{i+\frac{1}{2}} - \sqrt{gh^+_{i+\frac{1}{2}}}, u^-_{i+\frac{1}{2}} - \sqrt{gh^-_{i+\frac{1}{2}}}\right) \tag{15}$$

**3.1. Second order approximation in space.** When $U^-_{i+\frac{1}{2}}$ and $U^+_{i+\frac{1}{2}}$ are approximated by $U_i$ and $U_{i+1}$ respectively, the semi-discrete central-upwind scheme is only first-order accurate in space. However, if we take them as the right and the left point values of the piecewise linear reconstruction on the cell $\mathcal{C}_i$, the scheme is second order in space. In our study, we adopt the linear reconstruction given by [4] and then for each $i$ we put

$$\bar{U}_i(x) = U_i + \left(\frac{\partial U}{\partial x}\right)_i (x - x_i), \quad \forall x \in [x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}] \tag{16}$$

$U^+_{i+\frac{1}{2}}$ and $U^-_{i+\frac{1}{2}}$ are the right and left point values of the piecewise linear reconstruction at $x = x_{i+\frac{1}{2}}$, then

$$U_{i+\frac{1}{2},L} = U_i + \frac{\Delta x}{2}\left(\frac{\partial U}{\partial x}\right)_i \tag{17}$$

$$U_{i+\frac{1}{2},R} = U_{i+1} - \frac{\Delta x}{2}\left(\frac{\partial U}{\partial x}\right)_{i+1} \tag{18}$$

The numerical derivatives $(U_x)_i$ are to be computed using a nonlinear limiter. In this paper the generalized minmod limiter in order to warrant the second order accuracy and a non-oscillatory nature of the reconstruction is used :

$$\left(\frac{\partial U}{\partial x}\right)_i = minmod\left(\theta\frac{U_{i+1} - U_i}{\Delta x}; \theta\frac{U_i - U_{i-1}}{\Delta x}; \frac{U_{i+1} - U_{i-1}}{2\Delta x}\right) \tag{19}$$

where the minmod function is defined by:

$$minmod(\alpha_1, \alpha_2, \alpha_3) = \begin{cases} \min(\alpha_1, \alpha_2, \alpha_3) & if \quad \alpha_i > 0, \forall i \\ \max(\alpha_1, \alpha_2, \alpha_3) & if \quad \alpha_i < 0, \forall i \\ 0 & otherwise \end{cases} \tag{20}$$

The central-upwind framework allows one to decrease a relatively large amount of numerical dissipation present at the staggered central schemes. In [16], the authors present a modification of the one-dimensional semi-discrete central-upwind scheme, in which the numerical dissipation is more reduced. In this case the central-upwind numerical flux $H_{i+\frac{1}{2}}(t)$ are given by

$$H_{i+\frac{1}{2}}(t) = \frac{a^+_{i+\frac{1}{2}} F^-_{i+\frac{1}{2}}(t) - a^-_{i+\frac{1}{2}} F^+_{i+\frac{1}{2}}(t)}{a^+_{i+\frac{1}{2}} - a^-_{i+\frac{1}{2}}} + \frac{a^+_{i+\frac{1}{2}} a^-_{i+\frac{1}{2}}}{a^+_{i+\frac{1}{2}} - a^-_{i+\frac{1}{2}}} (U^+_{i+\frac{1}{2}}(t) - U^-_{i+\frac{1}{2}}(t)) - \mathbf{d}_{i+\frac{1}{2}}, \tag{21}$$

where $\mathbf{d}_{i+\frac{1}{2}}$ is called the correction term or built-in anti-diffusion term and is defined by

$$\mathbf{d}_{i+\frac{1}{2}} = \frac{a^+_{i+\frac{1}{2}} a^-_{i+\frac{1}{2}}}{a^+_{i+\frac{1}{2}} - a^-_{i+\frac{1}{2}}} minmod\left(U^+_{i+\frac{1}{2}}(t) - U^*_{i+\frac{1}{2}}(t), U^*_{i+\frac{1}{2}}(t) - U^-_{i+\frac{1}{2}}(t)\right). \tag{22}$$

The intermediate value $U^*_{i+\frac{1}{2}}(t)$ is given by

$$U^*_{i+\frac{1}{2}}(t) = \frac{a^+_{i+\frac{1}{2}} U^+_{i+\frac{1}{2}}(t) - a^-_{i+\frac{1}{2}} U^-_{i+\frac{1}{2}}(t) - (F^+_{i+\frac{1}{2}}(t) - F^-_{i+\frac{1}{2}}(t))}{a^+_{i+\frac{1}{2}} - a^-_{i+\frac{1}{2}}}. \tag{23}$$

**3.2. Second order approximation in time.** To reach second order approximation in time, we rewrite relation (12) as:

$$\frac{\partial U_i}{\partial t} = \mathcal{L}(U_i) \tag{24}$$

then we use the Runge-Kutta second order scheme [9] and [20]:

$$\begin{cases} U^* = U^n + \Delta t \ \mathcal{L}(U^n) \\ U^{**} = U^* + \Delta t \ \mathcal{L}(U^*) \\ U^{n+1} = \frac{1}{2}\left(U^n + U^{**}\right) \end{cases} \tag{25}$$

**3.3. Discretization of the source term.** We propose the following decomposition and discretization of the source terms given in Equation (6) by

$$S_i^n = \begin{pmatrix} 0 \\ -g\frac{z_{i+\frac{1}{2},R} - z_{i-\frac{1}{2},R}}{\Delta x} h_i - \frac{(\rho_s - \rho_w)}{2\rho} g h_i^2 \frac{c_{i+\frac{1}{2},R} - c_{i-\frac{1}{2},R}}{\Delta x} \\ 0 \\ 0 \end{pmatrix} \tag{26}$$

$$Q_i^n = \begin{pmatrix} \frac{E-D}{1-p} \\ -g h_i S_f - \frac{(\rho_0 - \rho)}{\rho} \frac{(E-D)}{(1-p)} u_i \\ E - D \\ -\frac{E-D}{1-p} \end{pmatrix} \tag{27}$$

## 4. Numerical results

In this section, we solve numerically the coupled model (Equations 1-2-3-4) applying on the dam-break over mobile bed problem by the second order central-upwind-Runge-Kutta scheme (see Section 3) with the diffusion correction term. Attention is given to the behavior of the dam-break flow over a mobile bed. The channel length is $50,000$m, the dam is initially located at the middle of the channel $x = 25,000$m, the test is largely used in the literature [1], [23], [2] and [10]. The initial conditions are:

$$h(x) = \begin{cases} 40m, & x \leq 25,000m \\ 2m, & x > 25,000m \end{cases}, \ u(0,x) = 0m/s, \ c(0,x) = 0.001 \tag{28}$$

Initially, the channel bed is considered horizontal and composed of noncohesive uniform sediments. Step size space is $\Delta x = 10$m and $\Delta t$ is computed according to a specified value of $C_{FL}$ number equal to 0.85.

**4.1. Dam-break problem by central upwind scheme.** In order to show the effect of sediment size on the behavior of the flow, we resolve the dam-break problem described earlier using different diameters by central upwind scheme. Figures 3 and 4 present the evolution of water free surface, bed, concentration and velocity profiles. Several observations can be made:

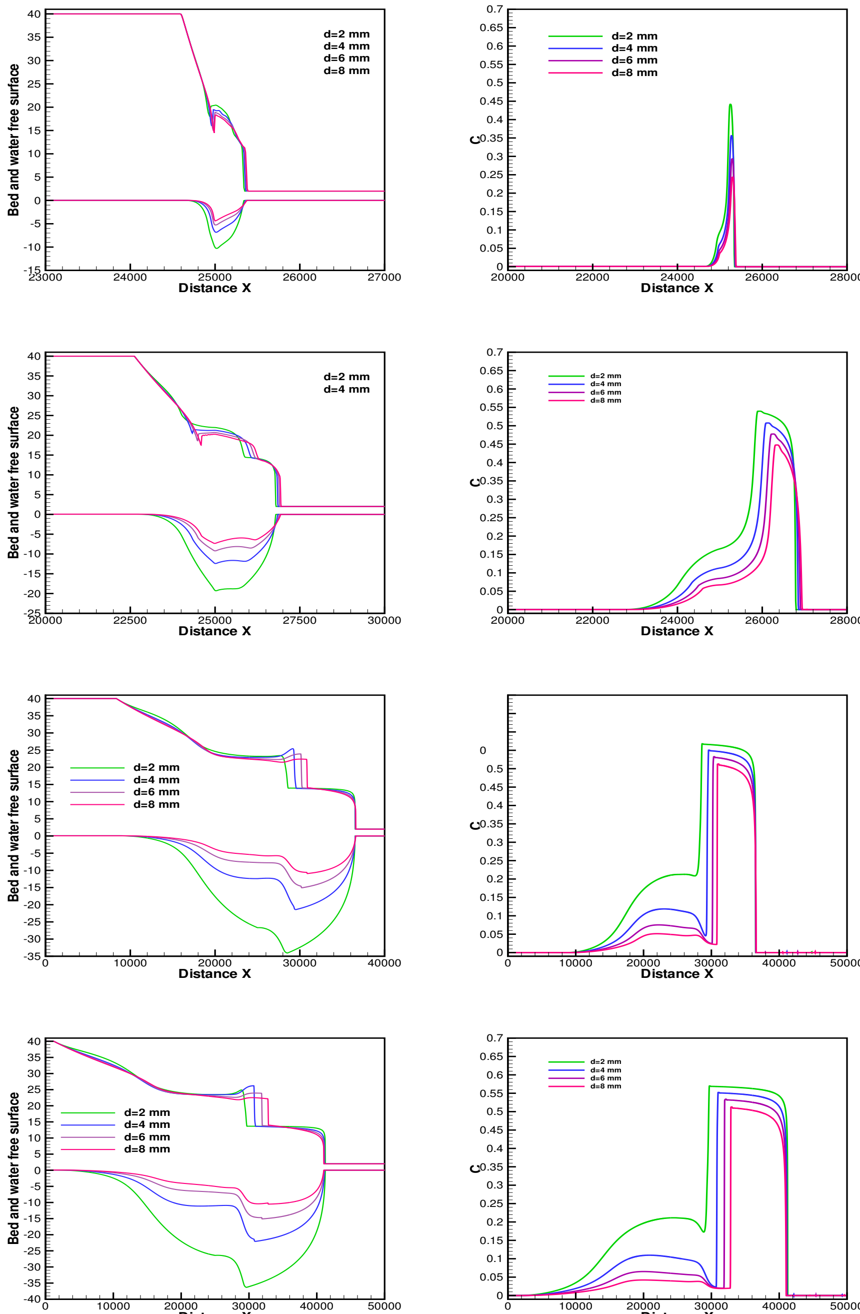


FIGURE 1. Water free surface, bed, and concentration profiles at several times (from top to bottom $t = 1$min, $t = 2$min, 14min and 20min) using $d = 2$mm.

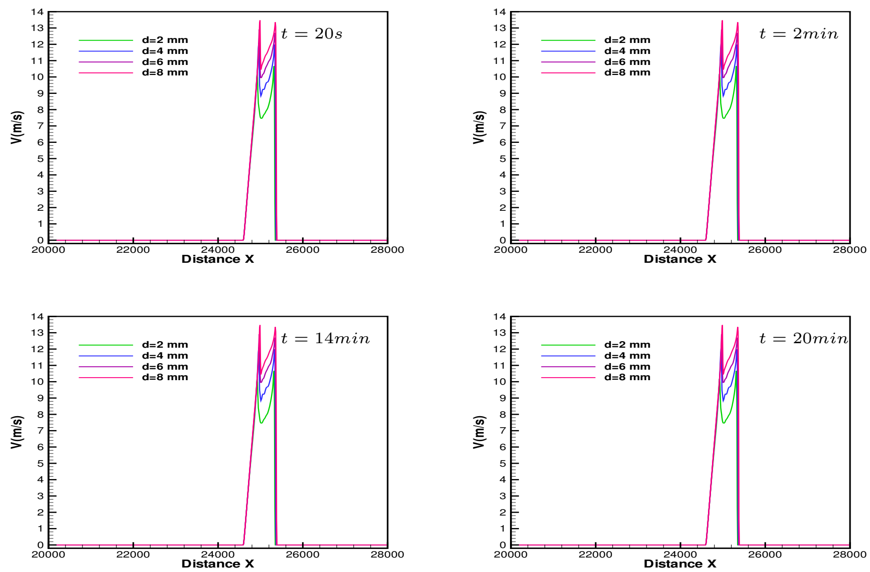


Figure 2. Velocity profiles at several times using $d = 2$mm.

- We can observe a hydraulic jump located in the initial position of the dam, then the jump gradually decreases and disappears as it propagates upstream. This is in agreement with observations reported in [3] and [21].
- The bed deformation is significant comparing of that of water free surface.
- The concentration profiles increase progressively in parallel with the changes brought on the erodible bottom.
- Sediment size has a great effect on dam-break flow:
  - The hydraulic jump is more pronounced when the sediment size is greater;
  - Bed rate change is more significant when sediment size is smaller;
  - Concentration profiles reach higher level when also sediment size is smaller;
  - Higher velocities back to greater sediments.

Theses remarks are in agreement with those reported in [1], [23], [2] and [10]

**4.2. Central upwind versus Roe scheme.** In order to enhance the performance of central upwind, we compare it with Roe scheme with the new discretization satisfing the C-property presented in [10]. Figures 3 and 4 present the evolution of water free surface, bed, concentration and velocity profiles.

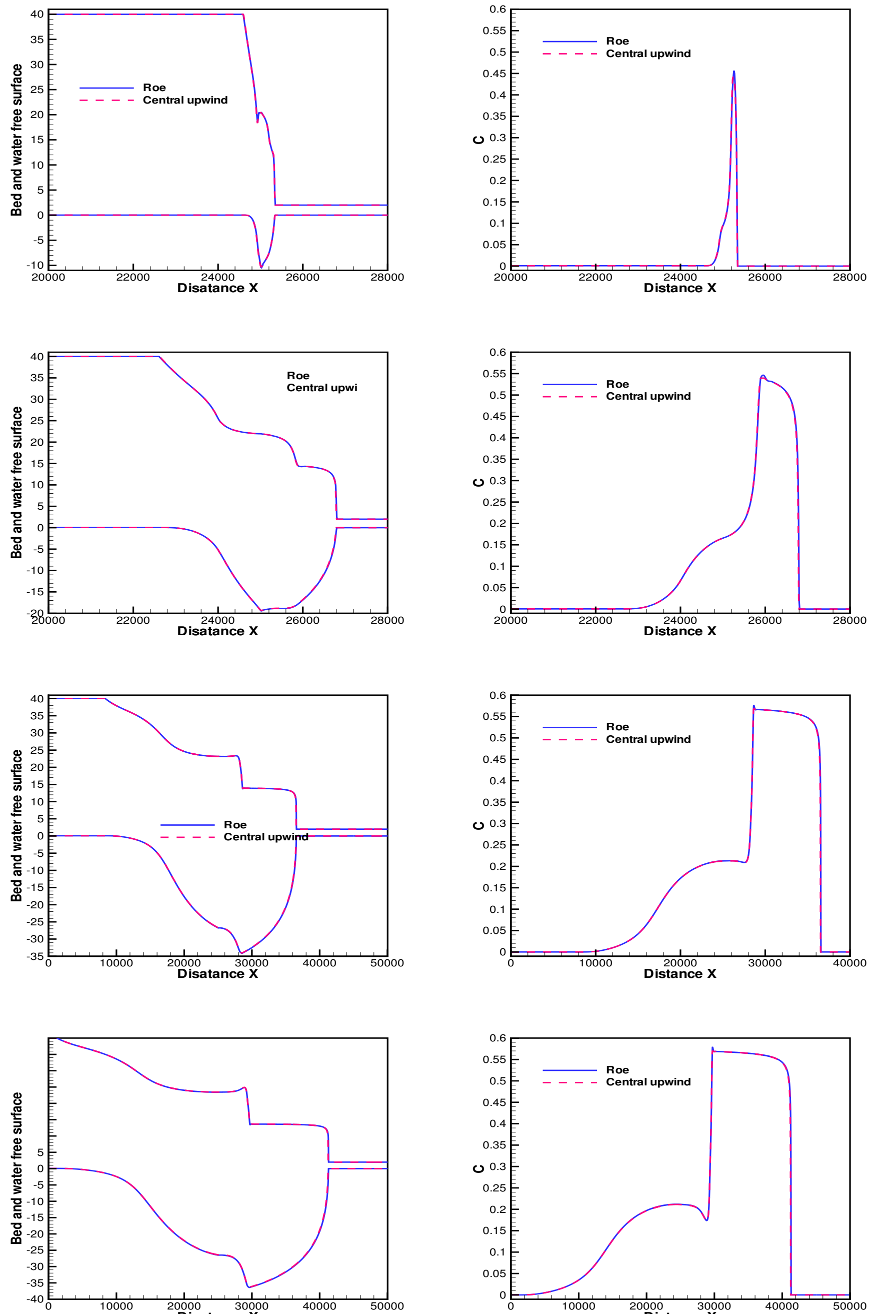


Figure 3. Water free surface, bed, and concentration profiles of central upwind versus Roe scheme at several times (from top to bottom $t$ = 1min, $t$ = 2min, 14min and 20min) using $d = 2$mm.

Roe scheme with the new discretiztion proposed by [10] has demonstrated a great performance and robustness based on performance and conservation verification tests. We remark Figures 3 and 4 that the profiles provided by central upwind coincide with those provided by Roe scheme. Therefore, central upwind has shown a great level of performance and robustness.

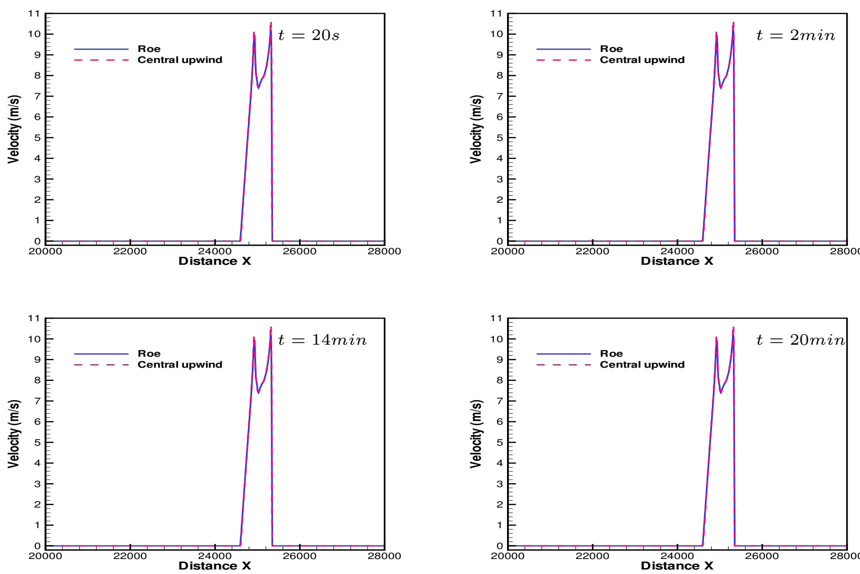


FIGURE 4. Velocity profiles at several times using $d = 8$mm.

## 5. Conclusion

In this study, dam-break problem with sediment transport is resolved using central upwind scheme. Through the obtained results, the numerical scheme detected the bed rate change and changes on velocity, concentration and free water-surface profiles. Also, central upwind scheme is compared to Roe scheme with the new discretization of the source term which satisfies the C-property introduced by [10], and the results were very satisfying. Therefore, central upwind scheme has shown a great level of performance and robustness. Subsequently, central upwind scheme can be a very good approximation tool to solve the shallow water equations coupled with the sediment transport equations.

**Conflicts of interest** : The authors declare no conflict of interest.

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