Design of a Novel Polishing Tool Mechanism with 3-axis Compliance 993

# Design of a Novel Polishing Tool Mechanism with 3-axis Compliance

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In this paper, a novel polishing tool mechanism with 3-axis compliance is presented, which consists of 2-axis rotational and 1-axis linear compliances in series. The 2-axis rotational compliance mechanism is made up of four cantilever beams for adjusting rotational stiffness and one flexure universal joint at the center for constraining the z-axis deflection. The 2-axis rotational compliance can mechanically adjust the polishing tool to machined surfaces. The polishing press force can be simply controlled by using a linear spring along the z-axis. The 2-axis rotational and 1-axis linear compliance design is decoupled. The stiffness analysis of the 2-axis compliance mechanism was performed based on link compliance matrix and rigid body transformation. A 3-axis polishing tool was designed by configuring the 2-axis compliance mechanism and one linear spring.

### *Keywords : Polishing Tool, Compliance, Stiffness, Decoupled, Flexure Universal Joint*

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### 1. Introduction

Polishing refers to as the process of smoothing or giving gloss to the surface of a processed product. It is mainly used for trimming and finishing various products such as metal, plastic, and glass. To ensure accurate and precise work, polishing tasks require skilled workers [1]. Polishing tasks are repetitive and require only a small number of skilled workers due to the high intensity of the work. Additionally, it is difficult for even skilled workers to work for long periods of time [2]. Recently, polishing tasks using robots become important, and research has been conducted to apply a constant force by providing polishing tools with compliance similar to that of workers [3]. In order to increase the precision of polishing, research has been conducted on position/force control of robots based on force sensors [4] and force control considering the rigidity of the robot [5]. In addition, research was also conducted on simultaneous position/force control using AI to create machining paths [6]. In previous studies, the position/force control was performed by using a 6-axis compliance device [7-10].

In this paper, a polishing tool mechanism with 3-axis compliance is presented, which consists of 2-axis rotational and 1-axis linear compliances in series. First, the stiffness of the 2-axis rotational compliance mechanism is derived in an analytical manner. Second, the stiffness matrix becomes diagonal by changing design parameters. Finally, the prototype of the polishing tool with 3-axis compliance is designed by simply adding linear springs to the 2-axis rotational compliance mechanism in series.

## 2. Stiffness Analysis of a 2-axis Rotational Compliance Mechanism

This chapter presents the stiffness analysis of a 2-axis rotational compliance mechanism. First, the working direction of the polishing tool is defined as shown in Fig. 1. The







Fig. 2 Conceptual design of a 2-axis rotational compliance mechanism with four cantilever beams and one flexure universal joint

direction along the z-axis is the direction in which the polishing tool applies force to the workpiece, and the directions about the xand y-axes are the directions of rotational compliance for the polishing tool to be in close contact with the workpiece. The polishing press force is defined as  $f_z$ , and the reaction moments are defined as  $n_x$ ,  $n_y$ . The 2-axis rotational compliance mechanism is shown in Fig. 2. Additionally, the frame  $\{B_i\}$ is  $B_i - x_i y_i z_i$  for i = 1,2,3,4. Fig. 3 [11] is modeled as Euler-Bernoulli beams with the frame ki as the origin. The compliance matrix of a cantilever beam can be obtained by Eq. (1).

$$F_{ki} = \begin{bmatrix} \frac{L^3}{3EI_y} & 0 & 0 & 0 & \frac{L^2}{2EI_y} & 0 \\ 0 & \frac{L^3}{3EI_x} & 0 & -\frac{L^2}{2EI_x} & 0 & 0 \\ 0 & 0 & \frac{L}{AE} & 0 & 0 & 0 \\ 0 & -\frac{L^2}{2EI_x} & 0 & \frac{L}{EI_x} & 0 & 0 \\ \frac{L^2}{2EI_y} & 0 & 0 & 0 & \frac{L}{EI_y} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{L}{GI_p} \end{bmatrix}$$
(1)

where A and L denote link area and length; E and G are the modulus of elasticity and



Fig. 3 Elastic model of link ki

shear modulus; and  $I_x$ ,  $I_y$  and  $I_p$  are the area moments of inertia about the x- and y-axes, and the polar area moment of inertia, respectively.

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The structure of four cantilever beams in Fig. 2 has linear compliance along the z-axis as well as rotational compliance, according to Eq. (1). By employing a flexure universal joint at the center, the linear compliance along the z-axis can be almost eliminated. The resulting structure has only 2-axis rotational compliance about the x- and y-axes.

Quantities specified in a local frame L are combined with others in another reference frame G by introducing  $6 \times 6$  rigid body transformations.

$${}^{G}E_{L} = \begin{bmatrix} {}^{G}R_{L} {}^{G}\hat{\mathbf{p}}_{L} {}^{G}R_{L} \\ 0 {}^{G}R_{L} \end{bmatrix}$$
(2)

where  ${}^{G}R_{L}$  is the rotation matrix in frame *G* of frame *L*,  ${}^{G}\hat{p}_{L}$  is the vector in frame *G* from origin *G* to origin *L* expressed as a  $3 \times 3$  skew-symmetric matrix.

Referring to Fig. 4(a), the compliance matrix  ${}^{B_i}F_{1i}$  for the  $i^{th}$  cantilever beam in frame  $\{B_i\}$  can be expressed in frame  $\{B\}$  by the following transformation.

$${}^{B}F_{1i} = {}^{B}E_{Bi} {}^{Bi}F_{1i} {}^{B}E_{Bi}^{T} \text{ for } i = 1, 2, 3, 4$$
(3)

Since the four cantilever beams are connected in parallel to the moving platform, the stiffness matrix by four cantilever beams,  ${}^{B}K_{B} = {}^{B}F_{B}^{-1}$  is obtained by

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$${}^{B}F_{B}^{-1} = \sum_{i=1}^{4} {}^{B}F_{1i}^{-1} \tag{4}$$

The one rotational axis of a flexure universal joint can be modeled as two small cantilever beams as shown in Fig. 4(b). The compliance matrix  ${}^{B_{5j}}F_{2j}$  for the  $j^{th}$  small beam in frame  $\{B_{5j}\}$  can be expressed in frame  $\{B\}$  by the following transformation.

$${}^{B}F_{2j} = {}^{B}E_{B5j} {}^{B5j}F_{2j} {}^{B}E_{B5j}^{T} \text{ for } j = 1,2$$
(5)

Since two small beams are connected in parallel, the compliance matrix of one rotational axis of a flexure universal joint is obtained by



Fig. 4 Frame definitions of a 2-axis rotational compliance mechanism

$${}^{B}F_{2}^{-1} = \sum_{j=1}^{2} {}^{B}F_{2j}^{-1} \tag{6}$$

Since the two perpendicular rotational axes of a universal joint are connected in series, the resulting compliance matrix of a flexure universal joint,  ${}^{B}F_{U}$  can be obtained by the sum of two compliances.

$${}^{B}F_{U} = {}^{B}F_{2} + {}^{B}R_{z} {}^{B}F_{2} {}^{B}R_{z} {}^{T}$$

$$\tag{7}$$

where  ${}^{B}R_{z}$  is the rotational matrix about the z-axis by 90°.

Therefore, the stiffness matrix of the 2-axis rotational compliance mechanism can be obtained as follows.

$${}^{B}K = {}^{B}F_{B}^{-1} + {}^{B}F_{U}^{-1} \tag{8}$$

## 3. Design of a 3-axis Polishing Tool

In this chapter, a polishing tool with 3-axis compliance was designed based on the stiffness analysis in the previous chapter. The 3-axis compliance can be designed by the 2-axis rotational compliance mechanism and the 1-axis linear compliance with compression springs.

The stiffness matrix  ${}^{B}K$  for a 2-axis rotational compliance mechanism is expressed by

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$${}^{B}K = \begin{bmatrix} k_{11} & 0 & 0 & 0 & k_{15} & 0 \\ 0 & k_{22} & 0 & k_{24} & 0 & 0 \\ 0 & 0 & k_{33} & 0 & 0 & 0 \\ 0 & k_{42} & 0 & k_{44} & k_{45} & 0 \\ k_{51} & 0 & 0 & k_{54} & k_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{66} \end{bmatrix}$$
(9)

In the case of  $p_{ix} = p_{iy}$  of the position vector,  ${}^{B}\boldsymbol{p}_{i} = [p_{ix}, p_{iy}, p_{iz}]^{T}$  in Fig. 4(a), the stiffness matrix becomes  $k_{45} = k_{54} = 0$ . For numerical calculation, the four cantilever beams (material: SUS304) are selected with 52 mm length, 15 mm width, and 1.5 mm thickness. The overall cantilever beam dimensions are designed considering 100×100mm polishing tool size. Depending on the z coordinate,  $p_{5jz}$ of  ${}^{B}\boldsymbol{p}_{5j}$  in Fig. 4(b), the off-diagonal elements  $k_{15} = k_{51}$  and  $k_{24} = k_{42}$  can converge to 0. The z coordinate indicates the center of the universal joint. Therefore, the stiffness matrix become a diagonal matrix when the frame  $\{B\}$  is at the center of the flexure universal joint and  $p_{ix} = p_{iy}$ . The design variables for a







Fig. 6 3D modeling of the 3-axis polishing tool

diagonal stiffness matrix are  $p_{ix} = p_{iy} = 50.5$ mm and  $p_{5jz} = 7.5$ mm.

In Fig. 6, 3D modeling for the 3-axis polishing tool is presented. In addition, it is possible to attach a displacement sensor along the compression springs and to calculate polishing press force by spring constant  $(k_z)$ . Since the 2-axis rotational compliance mechanism is connected in series with the linear spring, the total stiffness can be obtained by

$${}^{B}K_{T} = diag[k_{11}, k_{22}, k_{33}, k_{44}, k_{55}, k_{66}]$$
(10)

where  $k_{33}^{'-1} = k_{33}^{-1} + k_z^{-1}$ . The total compliance matrix  $({}^BF_T = {}^BK_T^{-1})$  in frame  $\{B\}$  can be obtained by

$${}^{B}F_{T} = diag[f_{11}, f_{22}, f_{33}, f_{44}, f_{55}, f_{66}]$$
(11)

Table 1. Diagonal elements of  ${}^{B}F_{T}$ 

Total compliance	Unit	Value
$f_{11}$		$5.61 \times 10^{-6}$
$f_{22}$	mm/N	$5.61  imes 10^{-6}$
$f_{33}$		$3.89  imes 10^{-2}$
$f_{44}$		$1.96  imes 10^{-3}$
$f_{55}$	rad/Nm	$1.96 \times 10^{-3}$
$f_{66}$		$1.16  imes 10^{-6}$

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The total compliance expressed in frame  $\{B\}$  is shown in Table 1. The total compliance matrix of the 3-axis compliance mechanism in frame  $\{C\}$  at the polishing tool tip can be expressed by

$${}^{C}F_{T} = {}^{C}E_{B} {}^{B}F_{T} {}^{C}E_{B}^{T}$$
(12)

The total compliance matrix in frame  $\{C\}$  has off-diagonal elements. Off-diagonal elements are much smaller than diagonal ones. In Table 2, only diagonal elements of the total compliance matrix at the tool tip are presented. Comparing Table 2 with Table 1,  $f_{11}$  and  $f_{22}$  are increased due to the distance from  $\{B\}$  to  $\{C\}$  and the rotational compliances of  $f_{44}$  and  $f_{55}$ . The proposed polishing tool has finite 2-axis rotational

Table	2.	Diagonal	elements	of	$^{C}F_{T}$
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Total compliance	Unit	Value
$f_{11}$		$7.07  imes 10^{-3}$
$f_{22}$	mm/N	$7.07  imes 10^{-3}$
$f_{33}$		$3.89 \times 10^{-2}$
$f_{44}$		$1.96  imes 10^{-3}$
$f_{55}$	rad/Nm	$1.96  imes 10^{-3}$
$\overline{f_{66}}$		$1.16 \times 10^{-7}$

Table	3.	Compliance	analysis	accuracy
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Rotational compliance	$f_{44}$ [rad/Nm]	$f_{55}$ [rad/Nm]
Theory	$1.96 \times 10^{-4}$	$1.96  imes 10^{-4}$
FEM	$2.06  imes 10^{-4}$	$2.06  imes 10^{-4}$
Error	4.85%	4.85%



Fig. 7. Prototype of the 3-axis polishing tool

compliance values of  $f_{44}$  and  $f_{55}$  and 1-axis linear compliance value of  $f_{33}$ . The other diagonal compliance elements have smaller values as expected.

Table 3 shows the error between the analytical calculation and finite element method (FEM) analysis for  $f_{44}$  and  $f_{55}$ , and the error is less than 5%.

Based on the design method, a prototype of the 3-axis polishing tool is developed and attached at the UR10e robot as shown in Fig. 7.

### 4. Conclusions

In this paper, a novel 3-axis compliance mechanism is proposed for a robotic polishing tool. It is noted that the 2-axis rotational and 1-axis linear compliance design is decoupled. The analytical stiffness analysis for the 2-axis rotational compliance mechanism is performed and verified through FEM analysis. The prototype of the polishing tool with 3-axis compliance is developed. In future research, precision polishing control based on the force/moment measurements will be conducted.

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