# Joint Antenna Selection and Multicast Precoding in Spatial Modulation Systems

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## Abstract

In this paper, the downlink of the multicast based spatial modulation systems is investigated. Specifically, physical layer multicasting is introduced to increase the number of access users and to improve the communication rate of the spatial modulation system in which only single radio frequency chain is activated in each transmission. To minimize the bit error rate (BER) of the multicast based spatial modulation system, a joint optimizing algorithm of antenna selection and multicast precoding is proposed. Firstly, the joint optimization is transformed into a mixed-integer non-linear program based on single-stage reformulation. Then, a novel iterative algorithm based on the idea of branch and bound is proposed to obtain the quasi-optimal solution. Furthermore, in order to balance the performance and time complexity, a low-complexity deflation algorithm based on the successive convex approximation is proposed which can obtain a sub-optimal solution. Finally, numerical results are showed that the convergence of our proposed iterative algorithm is between 10 and 15 iterations and the signal-to-noise-ratio (SNR) of the iterative algorithm is 1-2dB lower than the exhaustive search based algorithm under the same BER accuracy conditions.

***Keywords:*** Spatial modulation, antenna selection, multicast, successive convex approximation.

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## 1. Introduction

Spatial modulation is an energy efficient transmission scheme, in which only one antenna is activated during each transmission. Hence only one radio frequency (RF) chain is needed in the spatial modulation systems (SMSs). In SMSs, the input signals are transmitted at both the transmit antenna (TA) domain and amplitude-phase modulation (APM) domain [1]. Although SMSs utilize the spatial dimension to transmit information bits, they lack transmit diversity due to limited number of active transmit antennas. Therefore, the achievable transmission rate is imposed a few limitations at the APM domain. On the other hand, physical layer multicasting is a spectral efficient transmission scheme, in which a common message is multicasted to a couple of users [2]. The integration of multicast and spatial modulation is highly desired. The multicast based SMSs can achieve good balance between the energy efficiency and spectral efficiency.

When channel state information is available at the base station, transmit antenna selection and transmit precoding can be employed to enhance the performance of SMSs. Specifically, the diversity order can be obtained by employing transmit antenna selection [3], [4]. Moreover, for SMSs, when the total number of transmit antennas is not the power of 2 in practice, the antenna selection process is also needed [4]. On the other hand, the precoding can be optimized to shape the received APM signal constellation and boost the bit error rate (BER) performance [5]–[7]. Note that in SMSs, only one antenna is activated in each transmission period and the precoding matrix should be diagonal [7].

In the references of multicast spatial modulation, reference [9] firstly analyzes the BER performance of the multicast SMSs and provide a closed-form asymptotic BER upper by exploiting the system statistics. Finally, reference [9] studies the impact of the receiver number on BER in the multicast spatial modulation. Furthermore, reference [10] proposes two heuristic transmit mode selection algorithms, including tree pruning technique and approximating the signal constellation. Then reference [10] also design a precoder approach to improve the system. Similarly, reference [8] considers both antenna selection and the precoding coefficients, and gives the optimization problem. However, the optimization problem was not solved directly and reference [8] uses exhaustive search algorithm and convex optimization tools in turn to solve antenna selection and precoding design problems.

In this paper, antenna selection and multicast precoding in multicast based SMSs are design jointly. To facilitate the joint optimization of antenna selection and precoding, a single-stage reformulation is proposed to reconstruct the problem of joint optimization into a mix-integer non-linear program. Secondly, a branch and bound based algorithm is designed to obtain the quasi-optimal solution, whose convergence behavior and performance superiority are verified by the numerical results. Specifically, when the number of iterations reaches 10 to 15, the proposed branch and bound based algorithm converges with the accuracy of 0.0001, which is defined as the gap between the obtained upper bounds and lower bounds. This shows that the proposed algorithm converges fast, thus proving the effectiveness of joint antenna selection and multicast precoding in SMS. Moreover, a low-complexity deflation algorithm based on successive convex approximation (SCA) is proposed to obtain a suboptimal solution, which can achieve BER performance close to the exhaustive search based algorithm. To the best of our knowledge, this is the first work studying the single-stage reformulation for jointly solving the antenna selection and precoding problems. Meanwhile, under the same BER condition, the SNR corresponding to the deflation algorithm achieve 1dB lower than the result of the exhaustive search based algorithm.

Notations: We employ uppercase boldface letters for matrices and lowercase boldface for vectors. $\mathbb{E}(\bullet)$, $Tr(\bullet)$, $\|\bullet\|_p$, $(\cdot)^T$, $(\cdot)^H$ and $\mathrm{Re}(\bullet)$ return the expectation, trace, $l_p$-norm, transpose, conjugate transpose and the real part of the input respectively.

## 2. System Description

As shown in **Fig. 1**, we consider the downlink of a multicasting SMS where a base station (BS) multicasts a common message to K users simultaneously using both the TA domain and APM domain signal transmission [1]. The BS is equipped with $N_{\text{tot}}$ antennas whereas each user equipment (UE) has $N_R$ antennas. For brevity, we denote $\mathcal{K}=\{1,2,...,K\}$ and $\mathcal{N}_{\text{tot}}=\{1,2,...,N_{\text{tot}}\}$ as the user set and transmit antenna set, respectively.

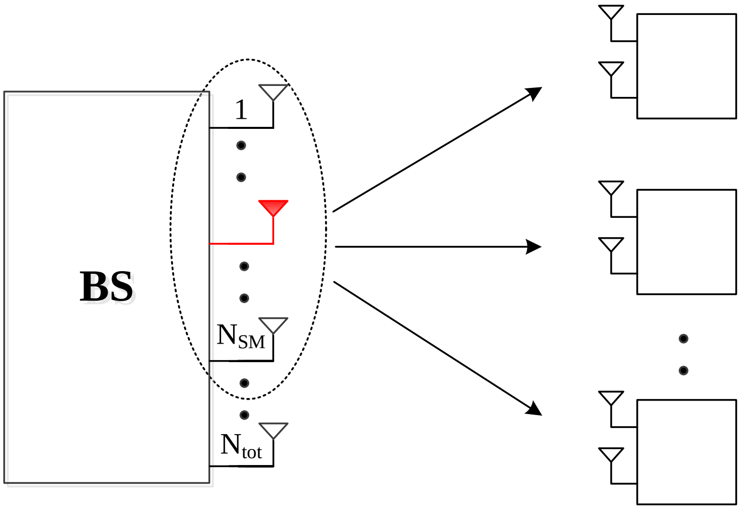


**Fig. 1.** Downlink of a multicasting SMS.

### 2.1 Signal Model

In each transmission, the BS selects $N_{\text{SM}}$ antennas from $N_{\text{tot}}$ to perform spatial modulation, where $N_{\text{SM}}$ is power of 2 [3]. There are total $C_{N_{\text{tot}}}^{N_{\text{SM}}}$ possible combinations of antenna selection. $I$ is denoted as the set of selected $N_{\text{SM}}$ antennas with $|I|=N_{\text{SM}}$, and $\mathcal{I}$ as the set of enumerations of all possible antenna selections with $|\mathbb{I}|=C_{N_{\text{tot}}}^{N_{\text{SM}}}$. Let $\mathbf{H}_k\in\mathbb{C}^{N_R\times N_{\text{tot}}}$ be the channel matrix from the BS to the $k$-th user. Then after antenna selection, the channel of user $k$ used for spatial modulation becomes $\mathbf{H}_{k,I}\in\mathbb{C}^{N_{\text{SM}}\times N_{\text{tot}}}$ where $I\in\mathcal{I}$. In each transmission, the first $\log_2 N_{\text{SM}}$ bits of the transmitted signal are mapped to one point in the spatial constellation set defined as

$$\mathcal{S}=\{e_1,...,e_i,...,e_{N_{SM}}\} \tag{1}$$

where $e_i\in\mathbb{C}^{N_{\text{SM}}\times 1}$ is an all-zero vector except for the $i$-th entry, which is 1. The selection of $e_i$ indicates that the $i$-th antenna among all the selected $N_{\text{SM}}$ antennas is activated. Next, the last $\log_2 M$ bits are transmitted from the activated antenna by M-QAM or M-PSK signal constellation defined by

$$\mathcal{M}=\{s_1,...,s_m,...,s_M\} \tag{2}$$

where $s_m\in\mathbb{C}$ with $\mathbb{E}\left\{|s_m|^2\right\}=1$

The spectral efficiency in the described system can be computed as $\log_2 N_{\mathrm{SM}} M$ bits per channel use. The transmitted spatial modulation constellation $\mathcal{SM}$ with cardinality $N_{\mathrm{SM}}M$ is the Cartesian product of $\mathcal{S}$ and $\mathcal{M}$ which is given by

$$\mathcal{SM} = \{e_1 s_1, \ldots, e_i s_m, \ldots, e_{N_{SM}} s_M\} \tag{3}$$

Next, denote the transmitted spatial modulation symbol as $x_l = e_i s_m \in \mathbb{C}^{N_{\mathrm{SM}} \times 1}$ with $x_l \in \mathcal{SM}$. Then $x_l$ is precoded by a diagonal matrix $\mathbf{W} = \mathrm{diag}(\mathbf{w}) \in \mathbb{C}^{N_{\mathrm{SM}} \times N_{\mathrm{SM}}}$ with $\mathbf{w} = \left[\mathrm{w}_{I_1}, \ldots, \mathrm{w}_{I_{\mathrm{SM}}}\right]^T \in \mathbb{C}^{N_{\mathrm{SM}} \times 1}$ given $I$. Then the received signal at user $k$ given antenna selection $I \in \mathcal{I}$ can be written as

$$\mathbf{y}_k = \mathbf{H}_{k,I} \mathbf{W} x_l + \mathbf{n}_k = \mathbf{h}_{k,I_i} \mathrm{w}_{I_i} s_m + \mathbf{n}_k \tag{4}$$

Where, $I_i$ is the $i$-th element in $I$ and $\mathbf{h}_{k,I_i}$ is the $i$-th column in $\mathbf{H}_{k,I}$ which also is the $I_i$-th column in the total channel matrix $\mathbf{H}_k$. And $\mathrm{w}_{I_i}$ is the precoding coefficient for antenna $I_i$ in $\mathcal{N}_{\mathrm{tot}}$. Moreover, $\mathbf{n}_k \sim \mathcal{CN}\left(0, \sigma^2 \mathbf{I}_{N_{\mathrm{R}}}\right)$ denotes the additive noise at user $k$.

At the receiver side, user $k$ decodes the TA domain signal (i.e., index of the activated antenna) and APM domain signal jointly with maximum likelihood (ML) detector as follows

$$\hat{x}_l^{\mathrm{ML}} = \arg \min_{\hat{x}_l \in \mathcal{SM}} \| \mathbf{y}_k - \mathbf{H}_{k,I} \mathbf{W} \hat{x}_l \|_2^2, \forall k \in \mathcal{K} \tag{5}$$

The error performance of the ML detector at user $k$ can be approximated by the sum of the pairwise error probability (PEP), which is given by [5]

$$P_k^{\mathrm{Error}}(I, \mathbf{w}) \leq \sum_{l=1}^{N_{\mathrm{SM}}M} \sum_{t=1, t \neq l}^{N_{\mathrm{SM}}M} Q\left(\sqrt{\frac{1}{2\sigma^2} d_{k,lt}(I, \mathbf{w})}\right) \tag{6}$$

where, $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{y^2}{2}} dy$ denotes the Gaussian tail probability [1], and $d_{k,lt}(I, \mathbf{w}) = \left\| \mathbf{H}_{k,I} \mathbf{W}(x_l - x_t) \right\|_2^2$ is the squared Euclidian distance between two different spatial modulation symbols $x_l \in \mathcal{SM}$ and $x_t \in \mathcal{SM}$ observed at receiver $k$. Similar to [1] and [5], we employ the nearest neighbor approximation for PEP in (6) and we have

$$\tilde{p}_k^{\mathrm{e}}(I, \mathbf{w}) = \lambda Q\left(\sqrt{\frac{1}{2\sigma^2} d_{\min}^k(I, \mathbf{w})}\right) \tag{7}$$

where, $\lambda$ is the number of neighboring constellation points [1] and

$$\begin{aligned} d_{\min}^k(I, \mathbf{w}) &= \min_{\substack{x_l, x_t \in \mathcal{SM} \\ x_l \neq x_t}} \left\| \mathbf{H}_{k,I} \mathbf{W}(x_l - x_t) \right\|_2^2 \\ &= \min_{\substack{i, j \in \mathcal{N}_{\mathrm{SM}}, i \leq j \\ s_m, s_n \in \mathcal{M} \\ (i,m) \neq (j,n)}} \left\| \mathbf{h}_{k,I_i} \mathrm{w}_{I_i} s_m - \mathbf{h}_{k,I_j} \mathrm{w}_{I_j} s_n \right\|_2^2 \end{aligned} \tag{8}$$

In multicast system, the system performance is dominated by the worst user. Similar to (9), the BER performance of the considered multicasting SMS is defined by the worst BER among all the users, and can be written by

$$\tilde{P}^{\mathrm{e}}(I, \mathbf{w}) = \max_{k \in \mathcal{K}} \tilde{P}_k^{\mathrm{e}}(I, \mathbf{w}) \tag{9}$$

## 2.2 Problem Formulation

In this paper, the BER of the multicasting SMS by jointly optimizing the transmit antenna selection and multicast precoding subject is minimized to the power constraint at the BS, which can be formulated as

$$\min_{I,\mathbf{w}} \max_{k\in\mathcal{K}} \tilde{P}_k^{\mathrm{e}}\left(I,\mathbf{w}\right) \tag{10a}$$

$$s.t.\quad \left\|\mathbf{w}\right\|_2^2 \le P_T \tag{10b}$$

where, $P_T$ is the total power constraint at the BS.

Since $\tilde{P}_k^{\mathrm{e}}\left(I,\mathbf{w}\right)$ in (7) is a monotonically decreasing function of $d_{\min}^k(I,\mathbf{w})$, the objective (10a) is equivalent to $\max_{I,\mathbf{w}} \min_{k\in\mathcal{K}} d_{\min}^k(I,\mathbf{w})$. Hence, problem (10) can be equivalently written as

$$\max_{I,\mathbf{W}} \min_{k\in\mathcal{K}} \min_{\substack{i,j\in\mathcal{N}_{\mathrm{SM}},i\le j\\ s_m,s_n\in\mathcal{M}\\ (i,m)\ne(j,n)}} \left\|\mathbf{h}_{k,I_i} w_{I_i} s_m - \mathbf{h}_{k,I_j} w_{I_j} s_n\right\|_2^2 \tag{11}$$

$$s.t.\quad \left\|\mathbf{w}\right\|_2^2 \le P_T \tag{11b}$$

## 3. Single-stage Reformulation

In the formulation (11), the antenna selection variable $I$ is a set of antenna indices, which cannot be designed jointly with precoding coefficients $w$ unless enumerating all possible antenna combinations as in formulation (8). In what follows, we integrate the antenna selection and multicast precoding into an equivalent single-stage reformulation of (11), by which we can leverage the tools of optimization to design them jointly. To this end, we introduce a binary vector $\mathbf{b}=\left[b_1,\ldots,b_{N_{\mathrm{tot}}}\right]$ to denote the antenna selection, i.e., $b_i=1$ if antenna $i$ is selected; otherwise, $b_i=0$. Since there are $N_{\mathrm{SM}}$ antennas selected, we have $\sum_{i=1}^{N_{\mathrm{tot}}} b_i = N_{\mathrm{SM}}$. Note that there is an one-to-one mapping between $\mathbf{b}$ and $I$ .Next, using the idea of Big-M formulation (11), formulation (11) is reformulated as follows

$$\max_{I,\mathbf{W}} \min_{k\in\mathcal{K}} \min_{\substack{i,j\in\mathcal{N}_{\mathrm{SM}},i\le j\\ s_m,s_n\in\mathcal{M}\\ (i,m)\ne(j,n)}} \left\|\mathbf{h}_{k,i} v_i s_m - \mathbf{h}_{k,j} v_j s_n\right\|_2^2 + \Phi_{ij}^k \tag{12a}$$

$$s.t.\quad b_i \in \left\{0,1\right\}, \forall i \in \mathcal{N}_{\mathrm{tot}} \tag{12b}$$

$$\sum_{i=1}^{N_{\mathrm{tot}}} b_i = N_{SM} \tag{12c}$$

$$\left|v_i\right|^2 \le b_i P_T, \forall i \in \mathcal{N}_{\mathrm{tot}} \tag{12d}$$

$$\left\|\mathbf{v}\right\|_2^2 \le P_T \tag{12e}$$

where $\Phi_{ij}^k = \left(1-b_i\right)\Omega_k + \left(1-b_j\right)\Omega_k$. Constant $\Omega_k$ has a sufficiently large value which should be chosen such that when $b_i=0$ or $b_j=0$ the term $\left\|\mathbf{h}_{k,i} v_i s_m - \mathbf{h}_{k,j} v_j s_m\right\|_2^2 + \Phi_{ij}^k$ is inactive in the second min function of (12a) for every feasible channel realization, precoding coefficients, and APM symbols. A simple choice could be $\Omega_k = \max_{i\in\mathcal{N}_{\mathrm{tot}}} 2P_T \left\|\mathbf{h}_{k,i}\right\|_2^2$. The variable $v_i$ is the precoding coefficient. Besides, $\mathbf{v}=\left[v_1,\ldots,v_{N_{\mathrm{tot}}}\right]^T \in \mathbb{C}^{N_{\mathrm{tot}}\times 1}$. Since there are $N_{\mathrm{SM}}$ antennas selected from $N_{\mathrm{tot}}$, we have $\left\|\mathbf{v}\right\|_0 = N_{\mathrm{SM}}$ which is guaranteed by constraints (12c) and (12d). We note that the indices $i$, $j$ in (12) take values from 1 to $N_{\mathrm{tot}}$, which is different from $i$, $j$ in (11) taking values from 1 to $N_{\mathrm{SM}}$.

Proposition 1: Problems (11) and (12) are equivalent.

Proof: First, based on the definition of binary variable $b_i = 1$ and constraint (12c), we can see that there is an one-to-one mapping between and $I \in \mathcal{I}$ in (11) and b in (12). Also, constraints (12c) and (12d) ensure $\|\mathbf{v}\|_0 = N_{\text{SM}}$ and only the selected antennas have non-zero precoding coefficients. Hence, there is an one-to-one mapping between $\mathbf{w}$ in (11) and $\mathbf{v}$ in (12).

On the other hand, for the objective in (12a), we denote

$$\tilde{d}_{\min}^{k}(\mathbf{b},\mathbf{v}) = \min_{\substack{i,j\in\mathcal{N}_{\text{SM}}, i\le j\\ s_m,s_n\in\mathcal{M}\\ (i,m)\neq(j,n)}} \left\|\mathbf{h}_{k,i}v_i s_m - \mathbf{h}_{k,j}v_j s_n\right\|_2^2 + \Phi_{ij}^{k} \tag{13}$$

Then to prove that (11) and (12) are equivalent, we only need to prove $\tilde{d}_{\min}^{k}(\mathbf{b},\mathbf{v}) = \tilde{d}_{\min}^{k}(I,\mathbf{w})$ for any given antenna selection, precoding coefficients and user index. Suppose $I = \{I_1,\dots,I_{N_{\text{SM}}}\}$ are the selected $N_{\text{SM}}$ antennas and the corresponding precoding coefficients are $\mathbf{w} = \{\mathrm{w}_{I_1},\dots,\mathrm{w}_{I_{N_{\text{SM}}}}\}$. Let $I^c = \mathcal{N}_{\text{tot}} \setminus I$, i.e., the relative complement of set $I$ with respect to set $N_{\text{tot}}$, denote the set of unused antennas. Hence we have $b_i = 1$ and $v_i = \mathrm{w}_i$ if $i \in I$; otherwise, if $i \in I^c$, we have $b_i = 0$ and $v_i = 0$. Then it follows that $\tilde{d}_{\min}^{k}(\mathbf{b},\mathbf{v}) = \min\left\{\tilde{d}_{\min}^{k,I}(\mathbf{b},\mathbf{v}), \tilde{d}_{\min}^{k,I^c}(\mathbf{b},\mathbf{v})\right\}$, where

$$\tilde{d}_{\min}^{k,I}(\mathbf{b},\mathbf{v}) = \min_{\substack{i,j\in I, i\le j\\ s_m,s_n\in\mathcal{M}\\ (i,m)\neq(j,n)}} \left\|\mathbf{h}_{k,i}v_i s_m - \mathbf{h}_{k,j}v_j s_n\right\|_2^2 \tag{14}$$

$$\tilde{d}_{\min}^{k,I^c}(\mathbf{b},\mathbf{v}) = \min_{\substack{i\in I^c \| j\in I^c,\\ i\le j,\\ s_m,s_n\in\mathcal{M}\\ (i,m)\neq(j,n)}} \left\|\mathbf{h}_{k,i}v_i s_m - \mathbf{h}_{k,j}v_j s_n\right\|_2^2 + \Phi_{ij}^{k} \tag{15}$$

where $i \in I^C \,\|\, j \in I^C$ means that at least one element in $\{i, j\}$ belongs to $I^c$, i.e., at least one element in $\{b_i, b_j\}$ is zero.

Note that in (14), $i$ and $j$ are the indices of selected transmit antennas and $b_i = b_j = 1$. Hence, $(1-b_i)\Omega_k + (1-b_j)\Omega_k = 0$.

Recalling (8), we can obtain that $\tilde{d}_{\min}^{k,I}(\mathbf{b},\mathbf{v}) = \tilde{d}_{\min}^{k}(\mathbf{b},\mathbf{v})$. Since $\Omega_k = \max_{i\in\mathcal{N}_{\text{tot}}} 2P_T\left\|\mathbf{h}_{k,i}\right\|_2^2$, we have $\tilde{d}_{\min}^{k,I}(\mathbf{b},\mathbf{v}) < \tilde{d}_{\min}^{k,I^c}(\mathbf{b},\mathbf{v})$ which results in $\tilde{d}_{\min}^{k}(\mathbf{b},\mathbf{v}) = \tilde{d}_{\min}^{k,I}(\mathbf{b},\mathbf{v}) = \tilde{d}_{\min}^{k}(I,\mathbf{w})$. Hence, (11) and (12) are equivalent.

## 4. Joint Antenna Selection and Multicast Precoding Design

The single-stage reformulation in problem (12) is a mixed-integer non-linear program (MINLP), which is NP-hard to solve. Besides, the objective in (12a) is non-smooth and non-convex. To proceed, we first show that problem (12) can be further reformulated into a mixed-integer quadratically constrained quadratic programming (MI-QCQP) problem.

### 4.1 Equivalent formulation

Proposition 2: The MINLP problem in (12) is equivalent to the following MI-QCQP problem

$$\max_{\mathbf{b},\mathbf{v}} \min_{k\in\mathcal{K}} \min_{\substack{i,j\in\mathcal{N}_{\text{tot}} \\ i\le j \\ s_m,s_n\in\mathcal{M} \\ (i,m)\neq(j,n)}} \mathbf{v}^H \mathbf{R}^k_{im,jn}\mathbf{v} + \Phi^k_{ij} \tag{16a}$$

$$s.t.\quad b_i \in \{0,1\}, \forall i \in \mathcal{N}_{\text{tot}} \tag{16b}$$

$$\sum_{i=1}^{N} b_i = N_{SM} \tag{16c}$$

$$|v_i|^2 \le b_i P_T, \forall i \in \mathcal{N}_{\text{tot}} \tag{16d}$$

$$\|\mathbf{v}\|_2^2 \le P_T \tag{16e}$$

where, $\mathbf{R}^k_{im,jn} = \mathbf{H}^H_k \mathbf{H}_k \circ \mathbf{\Theta}^T_{im,jn}$ is positive semidefinite matrix in which the matrix $\mathbf{\Theta}_{im,jn} = (\mathbf{e}_i s_m - \mathbf{e}_j s_n)(\mathbf{e}_i s_m - \mathbf{e}_j s_n)^H$. The operator $\circ$ denotes the Hadamard product. Moreover, $\mathbf{e}_i \in \mathbb{C}^{N_{\text{tot}}\times 1}$ is an all-zero vector except for the $i$ -th entry, which is 1.

Proof: In order to see the equivalence between the objective functions in (16a) and (12a), we denote the distance as $d^k_{im,jn} = \|\mathbf{h}_{k,i} v_i s_m - \mathbf{h}_{k,j} v_j s_n\|^2$. Next, introduce a diagonal matrix $\mathbf{V} = \text{diag}(\mathbf{v}) \in \mathbb{C}^{N_{\text{tot}}\times N_{\text{tot}}}$ where $\text{diag}(\mathbf{v})$ represents the diagonal matrix which its main diagonal elements are the components of the vector $\mathbf{v}$. Then we have

$$\begin{aligned} d^k_{im,jn} &= \left\|\mathbf{H}_k \mathbf{V}(\mathbf{e}_i s_m - \mathbf{e}_j s_n)\right\|^2 \\ &= \text{Tr}\left\{\mathbf{V}^H \mathbf{H}^H_k \mathbf{H}_k \mathbf{V}(\mathbf{e}_i s_m - \mathbf{e}_j s_n)(\mathbf{e}_i s_m - \mathbf{e}_j s_n)^H\right\} \\ &= \text{Tr}\left\{\mathbf{V}^H \mathbf{H}^H_k \mathbf{H}_k \mathbf{V}\mathbf{\Theta}_{im,jn}\right\} \\ &\overset{(a)}{=} \mathbf{v}^H\left(\mathbf{H}^H_k \mathbf{H}_k \circ \mathbf{\Theta}^T_{im,jn}\right)\mathbf{v} \\ &= \mathbf{v}^H \mathbf{R}^k_{im,jn}\mathbf{v} \end{aligned} \tag{17}$$

where, (a) follows from Eq. (1.10.6) in (12). Hence, the equivalence of problems (16) and (12) has been proved.

Next, the MI-QCQP problem (16) can be further converted (16) to the following equivalent problem

$$\max_{\mathbf{b},\mathbf{v},t} \quad t \tag{18a}$$

$$\begin{aligned} s.t.\quad & \mathbf{v}^H \mathbf{R}^k_{im,jn}\mathbf{v} + \Phi^k_{ij} \ge t, \forall k \in \mathcal{K}, i,j \in \mathcal{N}, i \le j \\ & s_m, s_n \in \mathcal{M}, (i,m) \neq (j,n) \end{aligned} \tag{18b}$$

$$(16\text{b}) - (16\text{e}) \tag{18c}$$

### 4.2 Quasi-Optimal Solution Based on Branch and Bound

In this subsection, we design a quasi-optimal algorithm for solving (18) by employing the branch and bound framework, in which branching and bounding are main tasks. In general, the branching step divides the feasible region into subsets and constructs sub-problems with those subsets. The bounding step finds the upper and lower bounds for those sub-problems within the corresponding subset. Specifically, we denote $Q=\{\mathbf{b},\mathbf{v}|(16\text{b})-(16\text{e})\}$ as the feasible region. Then we branch the feasible region $Q$ according to integer variables $\mathbf{b}$, and divide the original problem into sub-problems accordingly.

***Branching Step***: Pick any index $i$, and we can divide the origin set $Q$ into two subsets $Q_1=\{\mathbf{b},\mathbf{v}|b_i=0,(16\text{b})-(16\text{e})\}$ and $Q_2=\{\mathbf{b},\mathbf{v}|b_i=1,(16\text{b})-(16\text{e})\}$. Then the bounding steps described below are performed to obtain the upper and lower bounds of the original problem in these subsets respectively. Next, the subset with respect to the current overall upper bound is chosen to be further branched.

***Bounding Step***: In this step, we calculate the upper and lower bounds for the objective of (18) in different subsets. For convenience, we introduce $\boldsymbol{\xi}=[\xi_1,\ldots,\xi_{N_{\text{tot}}}]\in\mathbb{R}^{N_{\text{tot}}\times 1}$ as the state vector for the integer variables b and the corresponding subset $Q_\xi$ in the branching step, the elements of which take values from $\{0,1,-1\}$. Specifically, $\xi_i=1$ or 0 indicates that $b_i=1$ or 0. If $\xi_i=-1$, $b_i$ has not been branched yet. Denote $\mathcal{A}_\xi=\{i\in\mathcal{N}_{\text{tot}}|\xi_i=0\text{or}1\}$. Then subset $Q_\xi$ which is divided by $\mathcal{A}_\xi$ can be written as $Q_\xi=\{\mathbf{b},\mathbf{v}\,|\,b_i=\xi_i(i\in A_\xi),(16\text{b})-(16\text{e})\}$.

Next, to obtain the upper bound in subset $Q_\xi$, we relax the undetermined binary variables $b_i(i\in\mathcal{N}_{\text{tot}}\setminus A_\xi)$ into $0\le b_i\le 1$, and then solve problem (18) using the SCA framework [13], which convexity the nonconvex constraint (18b) and iteratively approximate the nonconvex problem to convex ones. Specifically, at SCA iteration $l$, the left-hand-side of (18b) can be lower bounded by its first-order Taylor series expansion around $\mathbf{v}^l$, and (18b) can approximated by

$$2\,\text{Re}\left(\mathbf{v}^{l,H}R_{im,jn}^{k}\mathbf{v}\right)-\mathbf{v}^{l,H}R_{im,jn}^{k}\mathbf{v}^{l}+\mathbf{\Phi}_{i,j}^{k}\ge t \tag{19}$$

Then the approximated convex problem at SCA iteration $l$ can be written as

$$\max_{\mathbf{b},\mathbf{v},t}\quad t \tag{20a}$$

$$s.t.\quad b_i=\xi_i, i\in\mathcal{A}_\xi \tag{20b}$$

$$0\le b_i\le 1, i\in\mathcal{N}_{\text{tot}}\setminus\mathcal{A}_\xi \tag{20c}$$

$$(16\text{c})-(16\text{e}),(19) \tag{20d}$$

which is convex and can be solved by modern solvers. The SCA algorithm is summarized for obtaining $\mathbf{v}$ giving $\boldsymbol{\xi}$ in **Algorithm 1**. The value of t at convergence is the upper bound.

**Algorithm 1** SCA Algorithm for Obtaining $(\mathbf{v},t)$ Given $\boldsymbol{\xi}$

1: Set $l:=0$. Initialize starting points $\mathbf{v}^0$.
2: **Repeat**
3: Solve problem (20) at $\mathbf{v}^l$ to get the optimal solution $(\mathbf{v}^*,t^*)$.
4: Update $\mathbf{v}^{l+1}=\mathbf{v}^*$.
5: Set $l:=l+1$.
6: **End**

Function $[U^*, \mathbf{b}^*] = \mathrm{UB}(\xi)$ is defined to represent the processing for computing the upper bound, in which $U^*$ is the obtained upper bound and $\mathbf{b}^* = [b_1^*, \ldots, b_{N_{\mathrm{tot}}}^*]$ is not integral. In order to get the lower bound for the objective of (18) in $Q_\xi$, round $b_i^* (i \in A_\xi)$ to 0 or 1, and denote the obtained integer variables as be $\tilde{\mathbf{b}}$. Then similar to **Algorithm 1**, we apply SCA algorithm to solve (18) given be $\tilde{\mathbf{b}}$, and the obtained lower bound is denoted as $L^* = \mathrm{LB}(\xi, \tilde{\mathbf{b}})$. If it is infeasible, the lower bound in subset $Q_\xi$ is $L^* = \infty$.

The branching and bounding steps are performed iteratively until the overall upper bound and overall lower bound converge. The details of the proposed branch and bound algorithm for solving problem (18) is summarized in **Algorithm 2**.

---

**Algorithm 2** Algorithm Based on Branch and Bound

---

1: Set the index of the Branch and Bound process $r := 0$.

2: Initialize state vector $\xi_{\mathrm{init}} = [-1, \ldots, -1] \in \mathbb{R}^{N_{\mathrm{tot}} \times 1}$.

3: Initialize search space $\mathcal{F} = \{\xi_{\mathrm{init}}\}$.

4: Get upper bound $U_0$ using $(U_0, \mathbf{b}_0) = \mathrm{UB}(\xi_{\mathrm{init}})$.

5: Round $\mathbf{b}_0$ to get integer vector $\tilde{\mathbf{b}}_0$, Get lower bound using $L_0 = \mathrm{UB}(\xi_{\mathrm{init}}, \tilde{\mathbf{b}}_0)$.

6: **While** $U_r - L_r > \varepsilon$

7: Pick $\xi \in \mathcal{F}$ for which $U_r = \mathrm{UB}(\xi)$.

8: Pick any index $i$ with $\xi_i = -1$.

9: Initialize state vector $\xi^{\mathrm{I}} = \xi^{\Pi} = \xi$.

10: Branch the selected subset by setting $\xi_i^{\mathrm{I}} = 0$, $\xi_i^{\Pi} = 1$.

11: Add $\xi^{\mathrm{I}}$ and $\xi^{\Pi}$ into $\mathcal{F}$, remove $\xi$ from $\mathcal{F}$.

12: Get upper bounds $(U^{\mathrm{I}}, \mathbf{b}^{\mathrm{I}}) = \mathrm{UB}(\xi^{\mathrm{I}})$, $(U^{\Pi}, \mathbf{b}^{\Pi}) = \mathrm{UB}(\xi^{\Pi})$.

13: Round $\mathbf{b}^{\mathrm{I}}$ and $\mathbf{b}^{\Pi}$ to get $\tilde{\mathbf{b}}^{\mathrm{I}}$ and $\tilde{\mathbf{b}}^{\Pi}$ respectively.

14: Get lower bounds: $L^{\mathrm{I}} = \mathrm{UB}(\xi^{\mathrm{I}}, \tilde{\mathbf{b}}^{\mathrm{I}})$, $L^{\Pi} = \mathrm{UB}(\xi^{\Pi}, \tilde{\mathbf{b}}^{\Pi})$.

15: Update upper bound and lower bound $U_{r+1} = \min\{U^{\mathrm{I}}, U^{\Pi}, U_r\}$,

$L_{r+1} = \min\{L^{\mathrm{I}}, L^{\Pi}, L_r\}$.

16: Set $r := r + 1$.

17: **End**

---

### 4.3 Low-complexity Suboptimal Deflation Algorithm Based on SCA

In this subsection, we propose a low-complexity algorithm for solving (18) in which the integer variables $\mathbf{b}$ are treated by the deflation process [14]. At first, all $N_{\mathrm{tot}}$ antennas are assumed to be selected. Next, we drop some poor communication link one by one until there remains $N_{\text{SM}}$ antennas, i.e., $\|\mathbf{b}\|_0 = N_{\text{SM}}$ . In each step of the deflation process, after the remaining set of antennas is determined, i.e., giving $\mathbf{b}$, the precoding vector $\mathbf{v}$ is obtained by solving (18) using the SCA framework similar to **Algorithm 1**.

Then the approximated convex problem at SCA iteration $l$ can be formulated as

$$\max_{\mathbf{v},t} \quad t \tag{21a}$$

$$s.t. \quad (16\text{c})-(16\text{e}),(19) \tag{21b}$$

The overall procedure of deflation SCA algorithm for solving problem (18) is summarized in **Algorithm 3.**

**Algorithm 3** Deflation Algorithm Based on SCA

1: Initialize: $b_i = 1, \forall i \in \mathcal{N}_{\text{tot}}$ , i.e., $\|\mathbf{b}\|_0 = N_{\text{tot}}$ .

2: **While** $\|\mathbf{b}\|_0 > N_{\text{SM}}$

3: Obtain $\mathbf{v}$ using **Algorithm 1**.

4: Get the index $\rho = \arg\max_{i \in \mathcal{N}_{\text{tot}}} \|v_i\|_2^2$.

5: Set $b_\rho := 0$.

6: **End**

7: Obtain $\mathbf{v}$ using **Algorithm 1.**

### 4.4 Complexity Analysis

In this section, the complexity of the proposed algorithms is discussed. Firstly, in Algorithm 3, the number of the subsets are needed to be considered increases exponentially with the problem dimension. Problem (20) approximately consist of $2N_{\text{tot}}+1$ variables and 1 second-order cone constraint of dimension $N_{\text{tot}}$. Hence, In the worst case, we need to traverse all $2N_{\text{tot}}-2$ subsets and the worst-case complexity is given by $O\left(2^{N_{\text{tot}}+1}N_{\text{tot}}^2\right)$. Secondly, the iteration times in Algorithm 3 is $N_{\text{tot}} - N_{\text{SM}}$, and the constraints in problem (21) consist of $N_{\text{tot}}+1$ variables and 1 second-order cone constraint of dimension $N_{\text{tot}}$ approximately. Thus, the complexity of Algorithm 3 can be given by $O\left(N_{\text{tot}}^3\right)$.

## 5. Numerical results

In this section, we present numerical results to evaluate the performance of the proposed algorithms, where $K=2$, $N_R=2$, $N_{SM}=2$, $\varepsilon=10^{-4}$ and $P_T = N_{SM}\,\text{Watt}$ . Moreover, QPSK constellations are employed to modulate the source information bits.

Firstly, we investigate the convergence behaviors of the branch and bound based algorithm in **Algorithm 2** and the SCA algorithm in **Algorithm 3** as depicted in **Fig. 2** and **Fig. 3**, respectively. The parameters are set as $N_{\text{tot}}=6$ and SNR = 3dB. The results in **Fig. 2** corroborate the fact that the proposed branch and bound algorithm converges to its optimal value exactly. Note that in the first three iterations, the value of the lower bound is ∞, so it is not drawn in **Fig. 2**. Then, as the number of iterations increases, the upper bound decreases and the lower bound increases until convergence. Moreover, as shown in **Fig. 3**, we can see that the SCA algorithm converges fast toward the optimal value illustrated by the dotted line therein.

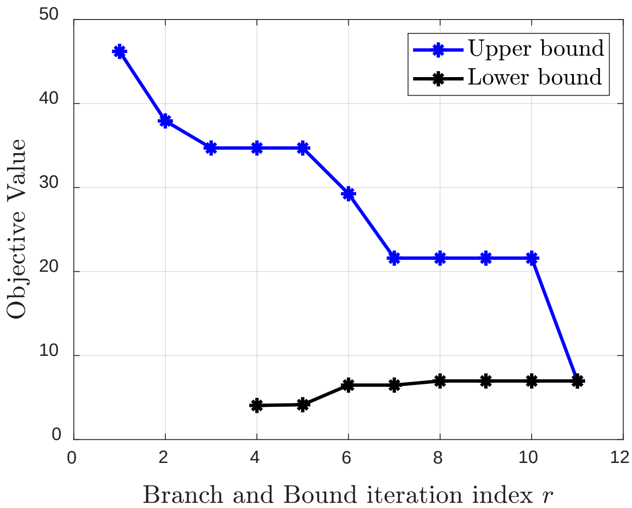


**Fig. 2.** Convergence behavior of **Algorithm 2** giving $N_{\text{tot}} = 6$, SNR = 3dB with QPSK.

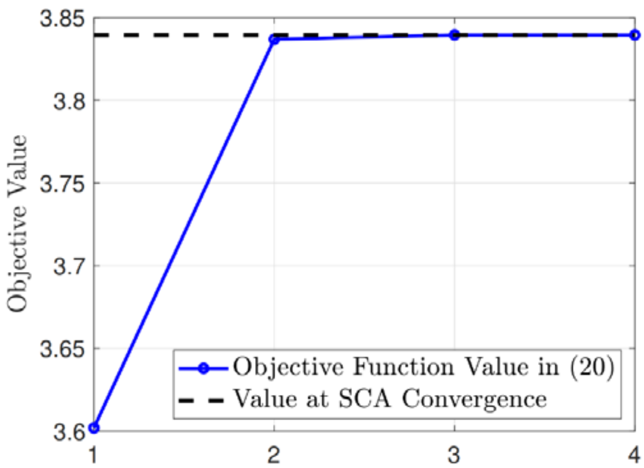


**Fig. 3.** Convergence behavior of SCA in **Algorithm 3** giving $N_{\text{tot}} = 6$, SNR = 3dB with QPSK.

Next, we investigate the BER comparison versus the signal-to-noise-ratio (SNR) in **Fig. 4** and **Fig. 5** with $N_{\text{tot}} = 4$ or 6. As shown in **Fig. 4** and **Fig. 5**, the branch and bound algorithm can achieve the best BER performance among all the algorithms since it can get the optimal solution, and the superiority becomes more pronounced as $N_{\text{tot}}$ increases. Moreover, the performance of the deflation SCA algorithm is close to the exhaustive search based algorithm while avoiding enumerating all the antenna combinations. Furthermore, the BER performance of these algorithms with $N_{\text{tot}} = 6$ in **Fig. 4** is better than that with $N_{\text{tot}} = 4$ in **Fig. 5**.

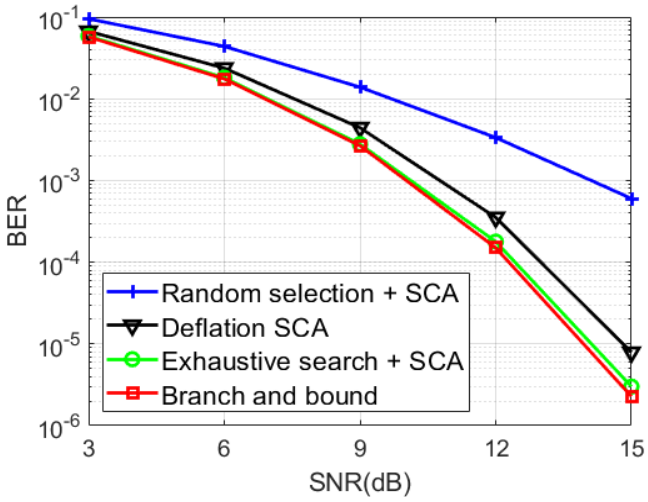


**Fig. 4.** BER for SMS giving $N_{\text{SM}} = 2$ with QPSK, $N_{\text{tot}} = 4$.

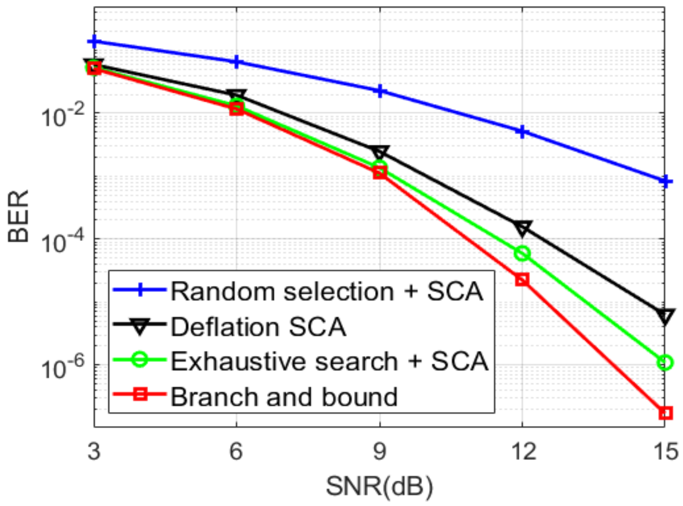


**Fig. 5.** BER for SMS giving $N_{\text{SM}} = 2$ with QPSK, $N_{\text{tot}} = 4$.

## 6. Conclusion

In this paper, the transmit antenna selection and multicast precoding problem has been treated in multicast based SMSs. To this end, an equivalent single-stage reformulation of the original problem has been proposed to leverage the tools of optimization to design antenna selection and precoding coefficients jointly and simultaneously. Then, two algorithms to solve the reformulated mixed-integer non-linear problem have been proposed, where a branch and bound based iterative algorithm that can achieve quasi-optimal solution, and a deflation SCA algorithm that achieves elegant performance with low-complexity. The outcomes of this study highlight the significance of our approaches in improving the performance of transmit antenna selection and multicast precoding in SMSs. Numerical results demonstrate the convergence and efficiency of the proposed schemes, the convergence of our proposed iterative algorithm is between 10 and 15 iterations. Moreover, when the iterative algorithm and the exhaustive search based algorithm take the same BER accuracy conditions, the signal-to-noise-ratio (SNR) of the iterative algorithm is 1-2dB lower than the exhaustive search based algorithm.

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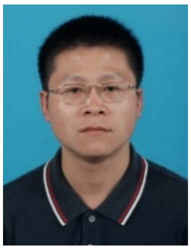

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