Print ISSN: 2288-4637 / Online ISSN 2288-4645
doi:10.13106/jafeb.2022.vol9.no10.0001

Forecasting Volatility of Stocks Return: A Smooth Transition Combining Forecasts

Jen Sim HO[1], Wei Chong CHOO[2], Wei Theng LAU[3], Choy Leng YEE[4], Yuruixian ZHANG[5], Cheong Kin WAN[6]

Received: September 15, 2022 Revised: November 26, 2022 Accepted: December 05, 2022

Abstract

This paper empirically explores the predicting ability of the newly proposed smooth transition (ST) time-varying combining forecast methods. The proposed method allows the "weight" of combining forecasts to change gradually over time through its unique feature of transition variables. Stock market returns from 7 countries were applied to Ad Hoc models, the well-known Generalized Autoregressive Conditional Heteroskedasticity (GARCH) family models, and the Smooth Transition Exponential Smoothing (STES) models. Of the individual models, GJRGARCH and STES-E&AE emerged as the best models and thereby were chosen for constructing the combined forecast models where a total of nine ST combining methods were developed. The robustness of the ST combining forecasts is also validated by the Diebold-Mariano (DM) test. The post-sample forecasting performance shows that ST combining forecast methods outperformed all the individual models and fixed weight combining models. This study contributes in two ways: 1) the ST combining methods statistically outperformed all the individual forecast methods and the existing traditional combining methods using simple averaging and Bates & Granger method. 2) trading volume as a transition variable in ST methods was superior to other individual models as well as the ST models with single sign or size of past shocks as transition variables.

Keywords: Stocks Volatility Forecasts, Combining Forecasts, Smooth Transition

JEL Classification Code: C22, C53, G10

1. Introduction

Over the years, the accuracy of volatility forecast has become the center of study for both practitioners and academicians as volatility forecast is essential in risk management, portfolio analysis, derivative pricing, hedging, and another financial decision-making process. Various rigorous models such as GARCH family models, Stochastic models, and ad hoc time series models with the application of realized volatility and implied volatility in the forecasting models have been developed, aimed at producing better predictive models. Nonetheless, there is no agreement on which method is the best approach for forecasting (Poon & Granger, 2003; Taylor, 2005; Andersen et al., 2006; Benavides & Capistran, 2012). This has led to the emergence of another strand of study, the combining forecasts method which combines useful information inherent in each method.

Motivated by the promising results from combining forecast methods, various combining forecasts methods have been derived in the past decades. However, not all kinds of

[1]First Author and Corresponding Author. Ph.D. Candidate, School of Business and Economics, Universiti Putra Malaysia, Malaysia. ORCID: https://orcid.org/0000-0001-6651-4686. [Postal Address: Jalan Universiti 1, 43400 Serdang, Selangor, Malaysia]
Email: jensimho@yahoo.com.my
[2][1] Associate Professor, School of Business and Economics, Universiti Putra Malaysia, Malaysia. ORCID: https://orcid.org/0000-0002-5797-7707. [2] Head of Laboratory of Computational Statistics and Operations Research, Institute for Mathematical Research, Universiti Putra Malaysia, Malaysia. Email: wcchoo@upm.edu.my
[3]Senior Lecturer, School of Business and Economics, Universiti Putra Malaysia, Malaysia. Email: lwtheng@upm.edu.my
[4]Senior Lecturer, School of Business and Economics, Universiti Putra Malaysia, Malaysia. Email: yee@upm.edu.my
[5]Ph.D. Candidate, School of Business and Economics, Universiti Putra Malaysia, Malaysia. ORCID: https://orcid.org/0000-0002-5413-6929. Email: yuruixianzhang@gmail.com
[6]Assistant Professor, Faculty of Business and Management, UCSI University, Malaysia. ORCID: https://orcid.org/0000-0003-4271-9884. Email: wancheongkin68@gmail.com

forecasts can be combined. There are two guiding criteria for the construction of combining forecasts: (i) the different forecasting models extract different predictive factors from essentially the same data, and (ii) the different models offer different predictions due to the usage of different data. Thereby, the overall aim of combining forecasts is to minimize post-sample forecast errors from different methods.

In combining forecasts, the "weight" employed in combining various individual forecasts is the main issue of concern in the literature. The key discussion topics include how the "weight" should be determined optimally for combining forecasts (Palm & Zellner, 1992), which combining methods are better (Makridakis, 1989), and how forecast errors are minimized (de Menezes et al., 2000). While many studies have proven that simple combining weight methods work well, some researchers argued that the combining weight should be allowed to change over time to adapt to the changing relative superiority of the forecasts (Diebold & Pauly, 1987; Sessions & Chatterjee, 1989).

The remainder of the paper is organized as follows. Section 2 presents a review of the literature on combining forecasts and methodology such as the volatility forecasting models are described in Section 3. Then, the results are illustrated in Section 4. The last section is the concluding remarks.

2. Literature Review

The concept of combining forecasts through regression was first introduced by Crane and Crotty (1967). In the same year, Zarnowitz (1967) proposed a simple approach to combining two forecasts which is known as the simple average (SA) combining forecasts method. Subsequently, Bates and Granger (1969) introduced a combining method which has since become the seminal work that serves as a foundation for the later development of combining methods. The combining method by Bates and Granger (B&G hereafter) uses an optimal method of combining where the linear weights are calculated to minimize the error variance of the combination (with the assumption that both the forecasts are unbiased). The B&G applies the weight with a value between 0–1 where the weight is obtained via minimizing the sum of squared forecast errors of the out–of the–sample. Gunter (1992) suggested that least square regression is one of the best procedures in this aspect.

Clement (1989) conducted an extensive review on combining forecasts works in the disciplines of forecasting, statistics, psychology, and management science. He concluded that combining forecasts improves predictive accuracy. Of these, the simple combining method was seen to outperform other complex combinations. In later development on combining forecasts, many researchers have subsequently provided evidence that simple average combining methods are robust in forecasting. A study by Aksu and Gunter (1992) using macroeconomic variables and firm-specific series revealed that the performance of the simple average method and Nonnegative Restricted Least Square (NRLS) are superior to the Ordinary Least Square method (OLS) and Equality Restricted Least Squares (ERLS) methods. Swanson and Zeng (2001) proposed selection criteria for combining forecasts based on Schwarz (SIC) and Akaike (AIC) Information Criteria. By applying nine US macroeconomic variables, the simple averaging method emerged as the best among the artificial neural network (ANN), linear models, and professional forecast models. Becker and Clements (2008) in another study of volatility forecasting of the S&P 500 series had successfully proven that combining forecast methods outperformed the individual forecast methods in terms of forecasting accuracy. Asadullah et al. (2021) further provided evidence of the robustness of combining techniques in exchange rate forecasting.

While the use of fixed weight combining methods has produced many encouraging results, it does have its weakness, thus subjected to criticism. Many researchers claimed that the combined weight should be allowed to change over time to adapt to the changing relative superiority of the forecasts. Diebold and Pauly (1987) applied this time-varying weight method in their study to capture the structural change in the combination forecasts. Furthermore, Sessions and Chatterjee (1989) propose recursive techniques with non-stationary weights to combine the forecast. LeSage and Magura (1992) extended the work by the Granger and Ramanathan (1984) which allows the weight in combining forecasts to vary according to time. By proposing a new method known as a multi-process mixture-model approach, they have concluded that the results using the new method were very promising.

In another study by Deutsch et al. (1994), the regime-switching weights and smooth transition weight were used in developing the time-varying weight combining forecasts in studying the US and UK inflation rates. It was found that the smooth transition method performed better than the regime-switching method. Nonetheless, the deficiency of this study lies in the difficulty of obtaining the optimal parameter estimation for the transition coefficient by minimizing the in-sample sum of squared errors.

Across other continents in Europe, Swanson and Zeng (2001) carried out a comprehensive study from linear, time-varying, and non-linear models by using 500 macroeconomic variables for the countries in the European Monetary Union. The results revealed that linear combination with equal or non-equal weight dominated all other combining methods.

Fameliti and Skintzi (2019) have performed various combinations on S&P 500 index. The methods included simple techniques to time-varying techniques. The empirical results concluded that combining methods outperform the single method but there was no specific technique producing the best results from statistical and economic perspectives. By and large, the review of the literature has shown that there are very limited studies in time-varying combining volatility forecasts. The performance of the available methods was also limited to certain stock markets. Therefore, this study extends the literature by proposing a new time-varying combining method in this study.

Meanwhile, trading volume has long been regarded as useful information in financial studies. There are two main theories developed to investigate the relationship between volatility and trading volume, the Mixture of Distributions (MDH) and the Sequential Information Hypothesis (SIH). MDH was proposed by Clark (1973) which suggested that volume and volatility are strongly positively correlated proposed. Since then, many subsequent works have validated the MDH theory (Karpoff, 1987; Chen et al., 2014; Slim & Dahmene, 2016; Zheng et al., 2019). On the other hand, Copeland (1976), Jennings et al. (1981) and Smirlock and Starks (1985) presented the Sequential Information Hypothesis (SIH) where the theory indicated that traders obtained the new information sequentially and randomly. Numerous results have concluded a positive lead-lag relationship exists between volume and volatility.

Based on the review, two hypotheses are formulated:

H1: *The combined forecasts method outperformed the individual forecast models or methods.*

H2: *There is a positive relationship between volume and volatility of stocks return*

In this paper, the objectives are two-fold. First, the study aims to investigate the performance of combining methods as compared to individual methods. Second, to explore the role of trading volume in combining methods. A details description of the methods and findings are presented in the following sections.

3. Volatility Forecasting Methods

Various volatility forecast approaches used in this study are described in detail in the following section. The models were divided into two sections: individual volatility forecasting methods and combined volatility forecasting methods.

3.1. Individual Volatility Forecasting Methods

The review of literature has indicated the superiority of individual volatility forecast models in some studies which are influenced by many factors such as time horizons, stylized facts in volatility behavior, news shocks, and others. Therefore, ad hoc methods, GARCH family models, and STES methods are used as benchmark models for the combining methods in this study.

3.1.1. *Ad Hoc* Methods

Two ad hoc methods are used in this study: The moving Average method (MA30) and the Exponential Weighted Moving Average method (EWMA). The Moving Average Method (MA30) is the moving window of the last 30 daily observations while the Exponential Weighted Moving Average (EWMA) is the moving average method with exponential weight. The equation of the EWMA is expressed below:

$$\sigma_t^2 = \beta\sigma_{t-1}^2 + (1-\beta)\frac{1}{L}\sum_{j}^{L}\sigma_{t-j}^2 \quad (1)$$

Where L is the length of the moving average and β is the decaying factor. The JP Morgan RiskMetrics model suggested β values of 0.94 and 0.97 for daily and weekly, respectively (Chuang et al., 2007).

3.1.2. GARCH Model

The GARCH model was introduced by Bollerslev (1986) and has become the center of volatility forecast research. Many studies found that GARCH (1,1) is sufficient to model the variance changing over long sample periods (see Franses & Van Dijk, 1996; Choo et al., 1999; Sahadudheen, 2015; Rahmi et al., 2016; Nguyen & Nguyen, 2019; Golder et al., 2022). In view of this, and for consistency, the GARCH (1,1) specification is chosen for all the GARCH models used in this study. The parameters in all GARCH models were estimated using the common procedure of maximum likelihood based on a Gaussian density function.

The GARCH (1,1) is expressed as:

$$r_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_t^2) \quad (2)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 \quad (3)$$

where r_t is the expected return of a financial asset, μ_t is the conditional mean, ε_t is the residual series, σ_t^2 is the conditional variance of the residual, ω, α and β are parameters to be estimated using the maximum likelihood estimation.

3.1.3. I-GARCH Model

Noting that the impact of past shocks decays in the short run at an exponential rate in a standard GARCH model. On contrary, the shock decays at a slow pace but persists infinitely in the long run. Engle and Bollerslev (1986) extended the standard GARCH to an Integrated GARCH (I-GARCH) to deal with this slow decay situation. The variance equation of IGARCH is expressed as follows:

$$r_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim N\ (0, \sigma^2) \tag{4}$$

$$\sigma_t^2 = \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 . \tag{5}$$

With the constant term ω omitted and restriction of $\alpha + \beta = 1$, the IGARCH model is capable of capturing the long memory of financial time series.

3.1.4. GJR- GARCH Model

Glosten et al. (1993) further extended the standard GARCH by including a dummy variable to discriminate the positive and negative past shock to capture the leverage effects. The model is known as the GJR-GARCH model as specified as follows:

$$r_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim N\ (0, \sigma^2) \tag{6}$$

$$\sigma_{t,n}^2 = \omega + \alpha \varepsilon_{t-1,n}^2 + \beta \sigma_{t-1,n}^2 + \gamma D_{t-1,n} \varepsilon_{t-1,n}^2 \tag{7}$$

where D is the dummy variable
The dummy variable is defined as:

a) If $\varepsilon_{t-i} < 0,\ D_{t-i} = 1$ (8)

b) If $\varepsilon_{t-i} > 0,\ D_{t-i} = 0$ (9)

It should be noted that the leverage effects only exist when $\gamma > 0$. Furthermore, the restriction on parameters is given *as* $\alpha + \beta + \frac{\gamma}{2} \leq 1$.

3.1.5. E-GARCH Model

Nelson (1991) extends the traditional symmetric linear GARCH models to the asymmetric non-linear Exponential GARCH (E-GARCH) model which is expressed as follows:

$$r_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim N\ (0, \sigma^2) \tag{10}$$

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{q} \left(\alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) + \sum_{j=1}^{p} \beta_j (\ln(\sigma_{t-j}^2) \tag{11}$$

where γ is the parameter of the asymmetric effect
The existence of an asymmetric effect is indicated by γ:

a) If $\gamma = 0$, there is no asymmetric effect.
b) If $\gamma > 0$, the volatility increases along with a positive past shock which implies that volatility is more sensitive to good news than bad news.
c) If $\gamma < 0$, the volatility increases along with a negative past shock or bad news, implying the existence of leverage effects.

The logarithmic form of conditional variance removes most of the restrictions given to parameters except the requirement of $|\beta| < 1$.

3.1.6. Smooth Transition Exponential Smoothing (STES) Method

STES method was proposed by Taylor (2004) where a logistic function of a user-specified transition variable is used as an adaptive smoothing parameter α_{t-1}. The formula is written as

$$\sigma_t^2 = \alpha_{t-1} \varepsilon_{t-1}^2 + \left(1 - \alpha_{t-1}\right) \sigma_{t-1}^2 \tag{12}$$

where

$$\alpha_{t-1} = \frac{1}{1 + \exp\left(\beta + \gamma V_{t-1}\right)} \tag{13}$$

The smoothing parameter varies between 0 and 1, and accommodates the changes in the transition variable, V_{t-1}. As the STES method is not guided by any statistical theory in the choice of parameter optimization, it is then recommended to minimize the sum of in-sample 1-step-ahead prediction errors as in the formula below.

$$\min \sum_{t=1}^{T} (\varepsilon_t^2 - \hat{\sigma}_t^2)^2 \tag{14}$$

Taylor (2004) proposed the use of ε_{t-1} and ε_{t-1}^2 as transition variables in the STES method. The ε_t or the sign of past shocks is used to model the asymmetry in stock return volatility while ε_t^2 is the size of the past shocks that permits more flexibility in modeling the dynamics of the conditional variance. Table 1 lists the five STES methods with the respective transition variables.

3.2. Combining Volatility Forecasting Methods

In this study, the combining methods are implemented as a second stage in the forecasting process after two or more individual conditional variance forecasts have been produced. Based on the review of the literature, the combining methods applied in this study can be concluded

Table 1: STES Methods and Their Choices of Transition Variables

STES Method	Transition Variables
STES-E	ε_{t-1}
STES-SE	ε^2_{t-1}
STES-AE	$\|\varepsilon_{t-1}\|$
STES-E&AE	ε_{t-1} and $\|\varepsilon_{t-1}\|$

into 2 categories, fixed weights combining methods and time-varying weight combining methods. Time-varying combining methods emerged following the argument that the combining weight should change according to time to adapt to the changing relative superiority of the forecasts. Several combining methods with varying weights were then proposed. For instance, Discounted Squared Forecasting Error (DSFE) was proposed by Diebold and Pauly (1987) and Stock and Watson (2004). LeSage and Magura (1992) suggested a mixture-model approach to combine forecasts. Deutsch et al. (1994) proposed a combining method with the weight derived from regime-switching weights or smooth transition weights for time-varying combining forecasts. The changing weight was varying in accordance with the fluctuation in a regime and the weight can be estimated from two approaches: first using the lagged forecast errors from the fundamental forecasts whilst the second is based on the regime on a related economic variable. Hardle (1990) introduced a non-parametric kernel regression approach with time-varying parameters. The weight as proposed by Yang (2004) was calculated using mean squared error (MSE).

3.2.1. Fixed Weights Combining Method

The simplest approach to combine the forecast methods is the Simple Average (SA) method proposed by Zarnowitz (1967). While this method is simple, various studies have documented that its performance is better than most complicated forecasting approaches (Hendry & Clements, 2004; Timmermann, 2006). The equation of SA is:

$$\hat{\sigma}^2_{t,c} = \frac{(\hat{\sigma}^2_{t,1} + \hat{\sigma}^2_{t,2})}{2} \tag{15}$$

where $\hat{\sigma}^2_{t,c}$ = combined forecast for period t,

$\hat{\sigma}^2_{t,1}$ = first individual forecast for period t,

$\hat{\sigma}^2_{t,2}$ = second individual forecast for period t.

In their seminal work, Bates and Granger (1969) proposed an optimal method of combining or also known as the minimum variance method. The linear weight of the combining is calculated by minimizing the error variance of the combination (by assuming unbiasedness for each forecast), specified as follows:

$$\hat{\sigma}^2_{t,c} = w\hat{\sigma}^2_{t,1} + (1-w)\,\hat{\sigma}^2_{t,2} \tag{16}$$

where the weights, *w,* take the value from 0 to 1.

3.2.2. Smooth Transition (ST) Combining Method

The smooth transition weights method is one of the time-varying combining forecasts using the regression model. The combining weight is allowed to change gradually as the system passes from one regime to another by using a continuous function for the values of the coefficient. The smooth transition combining model is in the form:

$$\hat{\sigma}^2_{t,c} = w\hat{\sigma}^2_{t,1} + (1-w)\,\hat{\sigma}^2_{t,2} \tag{17}$$

where $w_t = \dfrac{1}{1+\exp(b_o + b_1 V_{t-1})}$ and V_{t-1} is a transition variable.

The transition variables used in this study are listed in Table 2. The logistic function enables the value of the time-varying weight to evolve and adapt in relation to the changes in the characteristics of the transition variable, V_{t-1}. If the coefficient of V_{t-1} in the exponential expression is greater than 0 and b_0 is a positive value, the logistic function is a monotonically decreasing function of V_{t-1}. Thus, when V_{t-1} increases from a large negative value to a large positive value, the impact on combining forecast, $\hat{\sigma}^2_{t,c}$, will gradually shift from the first individual forecast, $\hat{\sigma}^2_{t,1}$ to the second individual variance forecast, $\hat{\sigma}^2_{t,2}$. In other words, more weight is put on the $\hat{\sigma}^2_{t,1}$ rather than $\hat{\sigma}^2_{t,2}$.

In the same manner, as the transition variable applied in STES methods explained above, trading volume is also considered a transition variable in this study. The effects of the trading volume on forecast performance are in the forms

Table 2: Transition Variable in Smooth Transition Combining Weight Method

Combining Method	Transition Variable
ST1	ε_{t-1}
ST2	ε^2_{t-1}
ST3	$\lvert\varepsilon_{t-1}\rvert$
ST4	ε_{t-1}, $\lvert\varepsilon_{t-1}\rvert$
ST5	ε_{t-1}, vol
ST6	ε_{t-1}, ε^2_{t-1}, indvol
ST7	ε_{t-1}, ε^2_{t-1}, vol
ST8	ε_{t-1}, $\lvert\varepsilon_{t-1}\rvert$, indvol
ST9	ε_{t-1}, $\lvert\varepsilon_{t-1}\rvert$, vol

of lagged trading volume, vol_{t-1} and, indicator volume. The trading volume has been scaled by dividing by 10^{10}. On the other hand, indicator volume, as suggested by Donaldson and Kamstra (2005) is a dummy variable to indicate if the high or low trading volume influences the volatility forecast. The description of indicator volume, indvol is shown in the equation below:

$$\text{ind vol}_{t-1} = \begin{cases} 1 & \text{if Volume}_{t-1} > \frac{1}{4}\sum_{i=2}^{5}\text{Volume}_{t-i} \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

Table 2 lists a total of nine (9) combination methods evaluated in this study. The ST1 and ST2 methods use the sign of past shock ε_{t-1} and the size of the past shock ε_{t-1}^2 as transition variables, respectively. ST3 adopts the past shock and the absolute past shock $|\varepsilon_{t-1}|$ |as the transition variables. The role of lagged volume as a transition variable was tested in ST5, ST7, and ST9 while indicator volume as a transition variable was assessed in ST6 and ST8. Parameters were estimated using the same minimization described for the exponential smoothing and minimum variance combining methods.

3.3. Post-Sample Forecasting Evaluation Criteria

Two criteria namely RMSE and MAE are used to evaluate post-sample forecast accuracy. Brooks (1998) mentioned that Root mean squared error (RMSE) is the most preferred accuracy criterion for evaluating the performance of volatility forecasting models. However, Franses and Ghijsels (1999) claimed that this principle gives a quadratic loss function that is brought to spurious estimation when outliers exist. Other than RMSE, mean absolute error (MAE) was used to measure the post-sample forecast accuracy. The MAE is said to be more robust to outliers. The root formula mean squared error (RMSE) and mean absolute error (MAE) are presented as:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}\left(\varepsilon_t^2 - \hat{\sigma}_t^2\right)^2} \quad (19)$$

$$\text{MAE} = \frac{1}{N}\sum_{t=1}^{N}\left|\varepsilon_t^2 - \hat{\sigma}_t^2\right| \quad (20)$$

3.4. Robustness Check

In addition to RMSE and MAE, the Diebold and Mariano (1995) test is employed to evaluate equal predictive ability among the ST methods. The DM statistic is defined as:

$$DM = \frac{\bar{d}}{\sqrt{\widehat{lrv}\left(\bar{d}\right)}} \quad (21)$$

Where $\bar{d} = \frac{1}{N}\sum_{t=1}^{N}\left(l\left(e_{jt}\right) - l\left(e_{it}\right)\right)$ is the average loss differential, and e is the error term. $\widehat{lrv}\left(\bar{d}\right)$ is a consistent estimate of the long-run asymptotic variance of $\bar{d}$ which is expressed in the equation:

$$\widehat{lrv}\left(\bar{d}\right) = \frac{1}{T}(\hat{\gamma}_0 + 2\sum_{k=1}^{h-1}\hat{\gamma}_k) \quad (22)$$

where $\hat{\gamma}_k$ is an estimator of the kth autocovariance of the d_t.

3.5. Data

This study used the daily stock indices and their respective trading volume for 7 major markets comprising Amsterdam (AEX), Frankfurt (DAX), Hong Kong (Hang Seng), New York (S&P 500), Paris (CAC 40), Tokyo Nikkei225 (Nik) and Shanghai (SSE). Each series contains of a total 2000 observations, covering about nine (9) years from November 2011 with the ending period on 31 December 2019 (for all the seven series). The first 1500 observations were used for model estimation and the remaining 500 observations were used for post-sample evaluation. The forecasts in this study were based on one-day-ahead volatility forecasting.

4. Results and Discussion

The in-sample and post-sample volatility performance of the respective model were evaluated under MAE and RMSE criteria and is summarized by the Mean Theil-U ranking. A lower value Mean Theil-U implies a better forecasting performance.

4.1. Performance of Individual Forecasting Methods

Tables 3 and 4 present the RMSE and MAE results calculated for 500 post-sample forecasts produced by the individual models: standard methods (MA30, EWMA, GARCH, GJR, IGARCH, EGARCH) and the STES methods. To ease the comparison, the mean value of a Theil-U measure was calculated for all seven series with reference to the GJR model, and the rank score is calculated based on the mean values where a smaller mean value denotes a better model, indicated in bold.

Table 3: Summary of RMSE for 500 Post-Sample Volatility Forecast Methods (x 10^6)

	NIK	SSEI	AEX	HSII	S&P	DAX	CAC	Mean Theil U	Rank
Standard Methods									
MA30	261	334	129	234	212	155	142	1.025	10
EWMA	256	330	126	233	207	153	139	1.008	8
GARCH	255	330	125	232	203	153	139	1.003	6
GJR	253	330	124	233	203	153	138	1.000	**3**
IGARCH	256	330	126	232	209	154	139	1.010	9
EGARCH	254	330	126	233	203	153	138	1.003	5
STES Methods									
STES-E	256	330	126	232	205	153	139	1.006	7
STES-SE	254	329	125	232	202	153	138	1.000	**4**
STES-AE	254	329	125	232	203	153	138	1.002	**2**
STES-E&AE	253	329	124	232	202	152	137	0.997	**1**

Table 4: Summary of MAE for 500 Post-Sample Volatility Forecast Methods (x 10^6)

	NIK	SSE	AEX	HSII	S&P	DAX	CAC	Mean Theil U	Rank
Standard Methods									
MA30	130	160	72	134	102	96	82	1.052	10
EWMA	127	159	70	133	99	94	80	1.030	6
GARCH	124	160	72	140	93	97	85	1.046	9
GJR	123	168	67	123	106	100	66	1.000	**4**
IGARCH	127	159	70	133	101	99	80	1.039	8
EGARCH	140	160	73	123	101	103	65	1.024	5
STES Methods									
STES-E	127	157	70	134	99	96	81	1.032	7
STES-SE	254	329	125	232	202	153	138	0.960	**3**
STES-AE	114	150	64	131	89	90	73	0.955	**2**
STES-E&AE	109	151	62	130	89	90	71	0.942	**1**

Overall, in terms of RMSE and MAE for individual models, the GJR model emerged as the best model in the standard method category while in the STES method category, the STES-E&AE model is the best performer. This illustrates that the combination of ε_{t-1} and $|\varepsilon_{t-1}|$ as transition variables produced a better forecasting method compared to using a single transition variable. Both the results were consistent with previous studies (Choo et al., 1999; Taylor, 2004; Liu et al., 2020; Wan et al., 2021). Based on this result, the best performing model for the standard method and STES method, GJRGARCH and STES-E&AE respectively, are thus selected for the construction of the combining forecasts models.

4.2. Performance of Combining Forecasting Methods

The results of the post-sample forecast for smooth transition combining methods as well as the individual methods and conventional combining methods are summarized in Tables 5 and 6. Mean Theil-U values were calculated for the total seven stock indices series with the GJR model as a reference. The values in bold in the rank column exhibit the best five performing models, of which generally the ST combining methods have better predictive power as compared to individual models and fixed weight combining methods.

In addition, as shown in Table 5 with RMSE as evaluation criteria, the best model is ST8 with ε_{t-1}, $|\varepsilon_{t-1}|$, indvol as transition variables and followed by ST9 where ε_{t-1}, $|\varepsilon_{t-1}|$, lagged volume *vol* as transition variables. The third best model is ST1 with ε_{t-1} as a transition variable.

Table 5: Summary of RMSE for 500 Post-Sample Volatility Forecast Methods (x 10^6)

RMSE	NIK	SSE	AEX	HSII	S&P	DAX	CAC	Mean Theil U	Rank
Individual Methods									
MA30	261	334	129	234	212	155	142	1.025	25
EWMA	256	330	126	233	207	153	139	1.008	22
GARCH	255	330	125	232	203	153	139	1.003	16
GJR	253	330	124	233	203	153	138	1.005	13
GJR-indvol	256	330	125	241	203	153	138	1.008	21
GJR-vol	257	330	125	236	203	153	139	1.005	19
IGARCH	256	330	126	232	209	154	139	1.010	23
EGARCH	254	330	126	233	203	153	138	1.003	15
STES-E	256	330	126	232	205	153	139	1.006	20
STES-SE	254	329	125	232	202	153	138	1.000	12
STES-AE	254	329	125	232	203	153	138	1.002	14
STES-E&AE	253	329	124	232	202	152	137	0.997	11
STES-indvol	256	330	125	232	205	153	138	1.004	17
STES-vol	256	330	126	232	204	153	138	1.004	18
Fixed Weight Combining Forecast Methods									
SA _GJR + STES -EAE	252	329	124	232	199	151	137	0.993	9
B&G _GJR + STES EAE	253	329	125	240	203	160	139	1.013	24
Smooth Transition Weight Combining Forecast Methods									
ST1 ε_{t-1}	253	329	123	233	192	150	138	0.988	**3**
ST2 ε^2_{t-1}	252	329	123	233	197	150	139	0.992	8
ST3 $\|\varepsilon_{t-1}\|$	251	329	123	233	199	150	140	0.995	10
ST4 ε_{t-1}, $\|\varepsilon_{t-1}\|$	252	329	123	232	192	150	140	0.989	**5**
ST5 ε_{t-1}, vol	252	329	123	232	192	150	139	0.989	7
ST6 ε_{t-1}, ε^2_{t-1}, indvol	252	329	125	232	191	149	140	0.989	6
ST7 ε_{t-1}, ε^2_{t-1}, vol	252	329	123	232	192	150	139	0.989	**4**
ST8 ε_{t-1}, $\|\varepsilon_{t-1}\|$, indvol	251	329	123	232	192	149	140	0.988	**1**
ST9 ε_{t-1}, $\|\varepsilon_{t-1}\|$, vol	251	329	123	232	192	149	140	0.988	**2**

The 4th and 5th best models were ST7 and ST4. It is also interesting to note that Bates and Granger (1969) combining method did not perform well as compared to other models except MA30. On the other hand, it seems that the inclusion of trading volume in the form of *indvol* and *vol* as transition variables in ST methods gave the best results. It is interesting to note that the simple average combining method (SA) performed better than Bates & Granger (B&G) combining method. The performance of SA in this study as compared to other individual methods is consistent with the findings of Timmermann (2006).

Similarly, ST combining methods stand out among all the individual methods, simple combining method as well as Bates & Granger method in terms of MAE. The best model is ST7 with ε_{t-1}, ε_{t-1}^2, *vol* as transition variables. ST5 emerged as the second-best performing model where the transition variables consist of ε_{t-1} and lagged volume, *vol* while the third is the ST6 which formed by the ε_{t-1}, ε_{t-1}^2 and *indvol* transition variables. The results demonstrate the role of trading volume in improving forecast accuracy which is in line with the studies by Brooks (1998), Donaldson and Kamstra (2005), and Liu et al. (2020). The performance of fixed weight combining methods based on the MAE criterion indicates opposing results compared to the RMSE criterion. Bates and Granger's (1969) combined method outperformed all the individual methods except STES-E&AE.

4.3. DM Test

The Diebold-Mariano (DM) test was further employed to validate the equal predictive power of ST methods as compared to other forecast methods. The results in terms of MAE and RMSE were summarized in Tables 7 and 8. The numbers in the tables were the total number of stock indices series that perform better as compared to the tested reference model.

In terms of MAE, the results generally demonstrate that the combining methods were significantly better than the majority of the individual methods, particularly the MA30 methods and GJR-*indvol*. For STES models, STES-E&AE and STES-AE were found as good as the combining methods either the fixed weight combining methods or ST combining methods.

When using the RMSE as an evaluation criterion, it was found that Bates and Granger (1969) combining method was generally not significantly better than most of the individual methods across all the seven series except MA30 and IGARCH models. It is interesting to note that while the forecast errors of the GJR, EGARCH, and GJR-*vol* models are bigger than other ST combining methods, the DM tests indicated the opposite results.

5. Conclusion

Forecasting volatility of return of financial assets has attracted extensive attention from both researchers and practitioners globally. The work in the past few decades had laid the foundation for the development of volatility forecast models of high accuracy. Combining forecasts method is one of the methodologies proposed to improve forecasting performance. Combining forecast methods can be classified into two categories: fixed weight combining methods and time-varying weight combining methods. The former uses the fixed weight in combining while the latter applies the weight that changes over time to cater to the changing relative superiority of the forecasts.

This study aims to examine the performance of the time-varying weight-combining method in volatility forecasts. Smooth transition (ST) combining methods with ε_{t-1}, $|\varepsilon_{t-1}|, \varepsilon_{t-1}^2$, *indvol* and *vol* as transition variables were developed and compared with individual methods (MA30, EWMA, GARCH family models, and STES Models) as well as fixed weight combining forecasts methods (Simple Average and Bates & Granger methods) in volatility forecast.

Using seven major stock indices, a total of nine ST combining methods were developed. The performance of post-sample forecasting was evaluated using RMSE and MAE. In general, the post-sample forecasting performance shows that ST combining forecast methods outperformed all the individual models and fixed weight combining models. The inclusion of the lagged trading volume as a transition variable in the form of high/low volume indicators and volume has contributed substantially towards the forecasting performance. It is also surprising to note that the Bates & Granger combining method was the second lowest ranked based on the RMSE evaluation criterion, implying that ST combining forecasts methods, Simple Average combining forecasts method, individual GARCH family models, and individual STES methods are better forecasting models.

In terms of MAE evaluation criterion, the inclusion of trading volume as a transition variable in the model has improved the predictive power in the ST combining forecasts methods as well as the individual models such as GJR-*vol*, STES-*involve,* and STES-*vol* as documented in past studies (Brooks, 1998; Donaldson & Kamstra, 2005; Liu et al., 2020).

The results of the DM Test concluded that the predictive power of all the ST combining methods is significantly better than all the individual methods particularly the MA30 and fixed weight combining method based on both RMSE and MAE.

The results concluded that the time-varying combining weight approach to combining forecasts is superior to all

Table 6: Summary of MAE for 500 Post-Sample Volatility Forecast Methods (x 10^6)

MAE	NIK	SSE	AEX	HSII	S&P	DAX	CAC	Mean Theil U	Rank
Individual Methods									
MA30	130	160	72	134	102	96	82	1.052	24
EWMA	127	159	70	133	99	94	80	1.030	20
GARCH	124	160	72	140	93	97	85	1.046	23
GJR	123	168	67	123	106	100	66	1.000	17
GJR-indvol	136	166	68	141	118	96	87	1.094	25
GJR-vol	109	162	60	134	120	89	79	1.010	18
IGARCH	127	159	70	133	101	99	80	1.039	22
EGARCH	140	160	73	123	101	103	65	1.024	19
STES-E	127	157	70	134	99	96	81	1.032	21
STES-SE	254	329	125	232	202	153	138	0.960	23
STES-AE	114	150	64	131	89	90	73	0.955	12
STES-E&AE	109	151	62	130	89	90	71	0.942	9
STES-indvol	120	152	64	133	95	90	74	0.979	15
STES-vol	127	158	67	132	91	93	73	0.994	16
Fixed Weight Combining Forecast Methods									
SA _GJR + STES EAE	115	159	64	126	96	94	68	0.962	14
B&G _GJR + STES EAE	113	153	67	140	91	78	65	0.942	10
Smooth Transition Weight Combining Forecast Methods									
ST1 ε_{t-1}	112	153	62	118	90	89	65	0.920	6
ST2 ε^2_{t-1}	112	153	62	122	94	89	65	0.930	8
ST3 $\|\varepsilon_{t-1}\|$	116	154	61	123	102	87	65	0.944	11
ST4 ε_{t-1}, $\|\varepsilon_{t-1}\|$	111	153	61	119	91	86	65	0.915	**4**
ST5 ε_{t-1}, vol	112	153	61	118	90	85	65	0.913	**2**
ST6 ε_{t-1}, ε^2_{t-1}, indvol	112	153	59	118	89	90	65	0.913	**3**
ST7 ε_{t-1}, ε^2_{t-1}, vol	112	153	62	119	89	85	65	0.911	**1**
ST8 ε_{t-1}, $\|\varepsilon_{t-1}\|$, indvol	113	153	61	118	92	88	65	0.921	7
ST9 ε_{t-1}, $\|\varepsilon_{t-1}\|$, vol	115	153	61	118	91	85	65	0.918	**5**

Table 7: The Results of the DM Test in Terms of MAE

Model		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13	M14
		MA30	EWMA	GARCH	GJR	IGARCH	EGARCH	GJR-indvol	GJR-vol	STES-E	STES-AE	STES-SE	STES-E&AE	STES-indvol	STES-vol
M15	SA _GJR + STES -EAE	**6**	5	**6**	5	5	4	**7**	4	**6**	2	2	2	3	4
M16	B&G _STES EAE - GJR	**6**	**6**	**6**	4	5	4	**6**	4	**6**	3	3	2	4	4
M17	ST1 ε_{t-1}	**7**	**7**	**6**	**7**	**6**	5	**7**	4	**7**	4	4	2	5	5
M18	ST2 ε^2_{t-1}	**7**	**7**	**6**	**6**	**6**	5	**7**	4	**7**	4	4	2	4	5
M19	ST3 $\lvert\varepsilon_{t-1}\rvert$	**6**	**6**	**6**	**5**	**5**	**3**	**7**	4	**6**	3	3	3	4	5
M20	ST4 ε_{t-1}, $\lvert\varepsilon_{t-1}\rvert$	**7**	**7**	**6**	**6**	**6**	5	**7**	4	**7**	4	4	3	4	5
M21	ST5 ε_{t-1}, vol	**7**	**6**	**6**	**7**	**6**	5	**7**	4	**6**	4	4	2	5	5
M22	ST6 ε_{t-1}, ε^2_{t-1}, indvol	**6**	**6**	**6**	**6**	**6**	5	**6**	4	**6**	4	4	3	5	5
M23	ST7 ε_{t-1}, ε^2_{t-1}, vol	**7**	**6**	**7**	**6**	**6**	5	**7**	4	**6**	4	4	2	5	5
M24	ST8 ε_{t-1}, $\lvert\varepsilon_{t-1}\rvert$, indvol	**7**	**7**	**6**	**6**	**6**	5	**7**	4	**7**	3	4	2	4	**6**
M25	ST9 ε_{t-1}, $\lvert\varepsilon_{t-1}\rvert$, vol	**6**	**6**	**6**	**6**	**6**	5	**6**	4	**6**	3	3	3	4	5

Note: The highest number is 7 as the total series used in this study are seven series. The model M1 to M14 is the individual model; M15 & M16 are the fixed weight models; M17 – M25 are the ST combining models. The DM test is a pair test for the individual method with the combining method.

Table 8: The Results of the DM Test in Terms of MAE

Model		M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13	M14
		MA30	EWMA	GARCH	GJR	IGARCH	EGARCH	GJR-indvol	GJR-vol	STES-E	STES-AE	STES-SE	STES-E&AE	STES-indvol	STES-vol
M15	SA _GJR + STES -EAE	**6**	5	**6**	5	5	4	**7**	4	**6**	2	2	2	3	4
M16	B&G _STES EAE - GJR	**6**	**6**	**6**	4	5	4	**6**	4	**6**	3	3	2	4	4
M17	ST1 ε_{t-1}	**7**	**7**	**6**	**7**	**6**	5	**7**	4	**7**	4	4	2	5	5
M18	ST2 ε^2_{t-1}	**7**	**7**	**6**	**6**	**6**	5	**7**	4	**7**	4	4	2	4	5
M19	ST3 $\lvert\varepsilon_{t-1}\rvert$	**6**	**6**	**6**	**5**	**5**	**3**	**7**	4	**6**	3	3	3	4	5
M20	ST4 ε_{t-1}, $\lvert\varepsilon_{t-1}\rvert$	**7**	**7**	**6**	**6**	**6**	5	**7**	4	**7**	4	4	3	4	5
M21	ST5 ε_{t-1}, vol	**7**	**6**	**6**	**7**	**6**	5	**7**	4	**6**	4	4	2	5	5
M22	ST6 ε_{t-1}, ε^2_{t-1}, indvol	**6**	**6**	**6**	**6**	**6**	5	**6**	4	**6**	4	4	3	5	5
M23	ST7 ε_{t-1}, ε^2_{t-1}, vol	**7**	**6**	**7**	**6**	**6**	5	**7**	4	**6**	4	4	2	5	5
M24	ST8 ε_{t-1}, $\lvert\varepsilon_{t-1}\rvert$, indvol	**7**	**7**	**6**	**6**	**6**	5	**7**	4	**7**	3	4	2	4	**6**
M25	ST9 ε_{t-1}, $\lvert\varepsilon_{t-1}\rvert$, vol	**6**	**6**	**6**	**6**	**6**	5	**6**	4	**6**	3	3	3	4	5

Note: The highest number is 7 as the total series used in this study are seven series. The model M1 to M14 is the individual model; M15 & M16 are the fixed weight models; M17 – M25 are the ST combining models. The DM test is a pair test for the individual method with the combining method.

other individual methods as well as fixed weight combining forecasts methods. Based on the RMSE as the evaluation criterion, the DM test result shows that Bates & Granger combining method performed poorly across the seven stock indices series where GJR, EGARCH, STES-AE, and STES-E&AE are significantly better than the Bates & Granger combining method.

In future research, it would be interesting to study other transition variables such as implied volatility and realized volatility. As only a one-step-ahead forecast was considered in this study, it is also recommended to investigate multi-step-ahead prediction in ST combining methods. Another potential research is to extend the ST combining methods from two models to three models.

References

Aksu, C., & Gunter, S. I. (1992). An empirical analysis of the accuracy of SA, OLS, ERLS, and NRLS combination forecasts. *International Journal of Forecasting*, *8*(1), 27–43. https://doi.org/10.1016/0169-2070(92)90005-T

Andersen, T. G., Bollerslev, T., Christoffersen, P. F., & Diebold, F.X. (2006). Volatility and correlation forecasting. In G. Elliott, C. W. J. Granger & A. Timmermann (Eds.), *Handbook of economic forecasting* (pp. 777–878). Netherlands: North Holland.

Asadullah, M., Uddin, I., Qayyam, A., Ayubi, S., & Sabri, R. (2021). Forecasting Chinese yuan/USD, via combination techniques during COVID-19. *Journal of Asian Finance, Economics, and Business*, *8*(5), 221–229. https://doi.org/10.13106/jafeb.2021.vol8.no5.0221

Bates, J. M., & Granger, C. W. J. (1969). The combination of forecasts. *Journal of the Operational Research Society*, *20*(4), 451–468. https://doi.org/10.1057/jors.1969.103

Becker, R., & Clements, A. E. (2008). Are combination forecasts of S&P 500 volatility statistically superior? *International Journal of Forecasting*, *24*(1), 122–133. https://doi.org/10.1016/j.ijforecast.2007.09.001

Benavides, G., & Capistrán, C. (2012). Forecasting exchange rate volatility: The superior performance of conditional combinations of time series and option implied forecasts. *Journal of Empirical Finance*, *19*(5), 627–639. https://doi.org/10.1016/j.jempfin.2012.07.001

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, *31*(3), 307–327. https://doi.org/10.1016/0304-4076(86)90063-1

Brooks, C. (1998). Predicting stock market volatility: Can market volume help? *Journal of Forecasting*, *17*(1), 59–80. https://doi.org/10.1002/(SICI)1099-131X(199801)17:1<59::AID-FOR676>3.0.CO;2-H

Chen, M., Chen, P., & Lee, C. (2014). Frontier stock market integration and the global financial crisis. *North American Journal of Economics and Finance*, *29*, 84–103. https://doi.org/10.1016/j.najef.2014.05.004

Choo, W. C., Ahmad, M. I., & Abdullah, M. Y. (1999). Performance of GARCH models in forecasting Stock Market Volatility. *Journal of Forecasting*, *18*(5), 333–343. https://doi.org/10.1002/(SICI)1099-131X(199909)18:5<333::AID-FOR742>3.0.CO;2-K

Chuang, I. Y., Lu, J., & Lee, P. (2007). Forecasting volatility in the financial markets: A comparison of alternative distributional assumptions. *Applied Financial Economics*, *17*(13), 1051–1060. https://doi.org/10.1080/09603100600771000

Clark, P. K. (1973). A subordinate stochastic process model with finite variance for speculative prices. *Econometrica*, *41*(1), 135–155. https://doi.org/10.2307/1913889

Clement, R. T. (1989). Combining forecasts: A review and annotated bibliography. *International Journal of Forecasting*, *5*(4), 559–583. https://doi.org/10.1016/0169-2070(89)90012-5

Copeland, T. E. (1976). A model of asset trading under the assumption of sequential information arrival. *Journal of Finance*, *31*(4), 1149–1168. https://doi.org/10.2307/2326280

Crane, D. B., & Crotty, J. R. (1967). A two-stage forecasting model: Exponential smoothing and multiple regression. *Management Science*, *13*(8), 501. https://doi.org/10.1287/mnsc.13.8.B501

de Menezes, L.M., Bunn, D.W., & Taylor, J. W. (2000). Review of guidelines for the use of combined forecasts. *European Journal of Operational Research, 120*(1), 190–204. https://doi.org/10.1016/S0377-2217(98)00380-4.

Deutsch, M., Granger, C. W. J., & Teräsvirta, T. (1994). The combination of forecasts using changing weights. *International Journal of Forecasting*, *10*(1), 47–57. https://doi.org/10.1016/0169-2070(94)90049-3

Diebold, F. X., & Pauly, P. (1987). Structural change and the combination of forecasts. *Journal of Forecasting*, *6*(1), 21–40. https://doi.org/10.1002/for.3980060103

Diebold, F. X., & Mariano, R. S. (1995). Comparing predictive accuracy. *Journal of Business and Economic Statistics*, *13*(3), 253–263. https://doi.org/10.1080/07350015.1995.10524599

Donaldson, R. G., & Kamstra, M. J. (2005). Volatility forecasts, trading volume, and the arch versus option-implied volatility trade-off. *Journal of Financial Research*, *28*(4), 519–538. https://doi.org/10.1111/j.1475-6803.2005.00137.x

Engle, R. F., & Bollerslev, T. (1986). Modeling the persistence of conditional variances. *Econometric Reviews*, *5*(1), 1–50. http://doi.org/10.1080/07474938608800095

Fameliti, S. P., & Skintzi, V. D. (2019). Predictive ability and economic gains from volatility forecast combinations. *Journal of Forecasting*, *5,* 1–20. https://doi.org/10.1002/for.2622

Franses, P. H., & Van Dijk, D. (1996). Forecasting stock market volatility using (non-linear) GARCH models. *Journal of Forecasting*, *15*(3), 229–235. https://doi.org/10.1002/(SICI)1099-131X(199604)15:3<229::AID-FOR620>3.0.CO;2-3

Franses, P. H., & Ghijsels, H. (1999). Additive outliers, GARCH, and forecasting volatility. *International Journal of Forecasting*, *15*(1), 1–9. https://doi.org/10.1016/S0169-2070(98)00053-3

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance*, *48*(5), 1779–1801. https://doi.org/10.1111/j.1540-6261.1993.tb05128.x

Golder, U., Rumaly, N., Shahriar, A. H. M., & Alam, M. J. (2022). The impact of COVID-19 on the volatility of Bangladeshi stock market: Evidence from GJR-GARCH model. *Journal of Asian Finance, Economics, and Business*, *9*(4), 29–38. https://doi.org/10.13106/jafeb.2022.vol9.no4.0029

Granger, C. W. J., & Ramanathan, R. (1984). Improved Methods of Combining Forecasts. *Journal of Forecasting, 3*: 197–204. https://doi.org/10.1002/for.3980030207

Gunter, S. I. (1992). Nonnegativity restricted least squares combinations. *International Journal of Forecasting*, *8*(1), 45–59. https://doi.org/10.1016/0169-2070(92)90006-U

Hardle, W. (1990) Applied Nonpararnetric Regression. Cambridge: Cambridge University Press.

Hendry, D. F., & Clements, M. P. (2004). Pooling of forecasts. *Econometric Reviews*, *122*, 47–79. https://doi.org/10.1111/j.1368-423X.2004.00119.x

Jennings, R. H., Starks, L. T., & Fellingham, J. C. (1981). An equilibrium model of asset trading with sequential information arrival. *Journal of Finance*, *36*(1), 143–161. https://doi.org/10.1111/j.1540-6261.1981.tb03540.x

Karpoff, J. M. (1987). The relation between price changes and trading volume: A survey. *Journal of Financial and Quantitative Analysis*, *22*(1), 109–126. https://doi.org/10.2307/2330874

LeSage, J. P., & Magura, M. (1992). A mixture model approach to combining forecasts. *Journal of Business and Economic Statistics*, *10*(4), 445–452. https://doi.org/10.1080/07350015.1992.10509920

Liu, M., Taylor, J. W., & Choo, W. C. (2020). Further empirical evidence on the forecasting of volatility with smooth transition exponential smoothing. *Economic Modelling*, *93*, 651–659. https://doi.org/10.1016/j.econmod.2020.02.021

Makridakis, S. (1989). Why combining works?. *International Journal of Forecasting, 5*(4), 601-603. https://doi.org/10.1016/0169-2070(89)90017-4

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, *59*(2), 347–370. https://doi.org/10.2307/2938260

Nguyen, C. T., & Nguyen, M. H. (2019). Modeling stock price volatility: Empirical evidence from the Ho Chi Minh City stock exchange in Vietnam. *Journal of Asian Finance, Economics, and Business*, *6*(3), 19–26. https://doi.org/10.13106/jafeb.2019.vol6.no3.19. vol6.no

Palm, F., & Zellner, A. (1992). To combine or not to combine? issues of combining forecasts. *Journal of Forecasting, 11*(8), 687–701. https://doi.org/10.1002/for.3980110806

Poon, S., & Granger, C. W. J. (2003). Forecasting volatility in financial markets: A review. *Journal of Economic Literature*, *41*(2), 478–539. https://doi.org/10.1257/jel.41.2.478

Rahmi, M., Azma, N., Muttaqin, A. A., Jazil, T., & Rahman, M. (2016). Risk volatility measurement: Evidence from Indonesian stock market. *Journal of Asian Finance, Economics, and Business*, *3*(3), 57–65. https://doi.org/10.13106/jafeb.2016.vol3.no3.57.

Sahadudheen, I. (2015). An exponential GARCH approach to the effect of the impulsiveness of the euro on the Indian stock market. *Journal of Asian Finance, Economics, and Business*, *2*(3), 17–22. https://doi.org/10.13106/jafeb.2015.vol2.no3.17

Sessions, D. N., & Chatterjee, S. (1989). The combining of forecasts using recursive techniques with non-stationary weights. *Journal of Forecasting*, *8*(3), 239–251. https://doi.org/10.1002/for.3980080309

Slim, S., & Dahmene, M. (2016). Asymmetric information, volatility components, and the volume-volatility relationship for the CAC40 stocks. *Global Finance Journal*, *29*, 70–84. https://doi.org/10.1016/j.gfj.2015.04.001

Smirlock, M., & Starks, L. (1985). A further examination of stock price changes and transaction volume. *Journal of Financial Research*, *8*(3), 217–226. https://doi.org/10.1111/j.1475-6803.1985.tb00404.x

Stock, J. H., & Watson, M. W. (2004). Combination forecasts of output growth in a seven-country data set. *Journal of Forecasting*, *23*(6), 405–430. https://doi.org/10.1002/for.928

Swanson, N. R., & Zeng, T. (2001). Choosing among competing econometric forecasts: Regression-based forecast combination using model selection. *Journal of Forecasting*, *20*(6), 425–440. https://doi.org/10.1002/for.784

Taylor, J. W. (2004). Volatility forecasting with smooth transition exponential smoothing. *International Journal of Forecasting*, *20*(2), 273–286. https://doi.org/10.1016/j.ijforecast.2003.09.010

Taylor, J. W. (2005). Generating volatility forecasts from the value at risk estimates. *Management Science*, *51*(5), 712–725. https://doi.org/10.1287/mnsc.1040.0355

Timmermann, M. (2006). Forecast combinations. In G. Elliot, C. W. J. Granger & A. Timmermann (Eds.), *Handbook of economic forecasting* (pp. 135–196). Netherlands: North-Holland Publishers.

Wan. C. K., Choo, W. C., Nassir, A. M., Habibullah, M. S., & Yusop, Z. (2021). Volatility forecasting performance of smooth transition exponential smoothing method: Evidence from mutual fund indices in Malaysia. *Asian Economic and Financial Review*, *11*(10), 829–859. https://doi.org/10.18488/journal.aefr.2021.1110.829.859

Yang, Y. (2004). Combining forecasting procedures: Some theoretical results. *Econometric Theory*, *20*(1), 176–222. https://doi.org/10.1017/S0266466604201086

Zarnowitz, R. (1967). *An appraisal of short-term economic forecasts*. MA: National Bureau of Economic Research.

Zheng, Z., Gui, J., Qiao, Z., Fu, Y., Stanley, H. E., & Li, B. (2019). New dynamics between volume and volatility. *Physica*, *525*(114), 1343–1350. https://doi.org/10.1016/j.physa.2019.03.100