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NOTES ON $(LCS)_n$-MANIFOLDS SATISFYING CERTAIN CONDITIONS

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Abstract. The object of the present paper is to study the properties of conharmonically flat $(LCS)_n$-manifold, special weakly Ricci symmetric and generalized Ricci recurrent $(LCS)_n$-manifold. The existence of such a manifold is ensured by non-trivial example.

1. Introduction

As a generalization of LP-Sasakian manifold ([17, 18]), Shaikh ([25, 26]) introduced the notion of $(LCS)_n$-manifold along with their existence and applications to the general theory of relativity and cosmology. Moreover, Shaikh and his coauthors ([25–27]) studied $(LCS)_n$-manifolds by imposing various curvature restrictions. The $(LCS)_n$-manifolds have also been studied by Atceken [2], Hui et al. ([3, 5, 11–13]), Narain and Yadav [20], Prakasha [22], Sreenivasa et al. [30], Venkatesha and Kumar [31], Yadav et al. [32]. Certain conditions on trans-Sasakian manifolds were studied by S. K. Chaubey [6].

Locally symmetric manifolds were weakened by many geometers in different extents. In those, the idea of recurrent manifolds was introduced by Walker in 1950. On the other hand, De and Guha [7] introduced generalized recurrent manifold (GK_n) with the non-zero 1-form A and another non-zero associated 1-form B. If the associated 1-form B becomes zero, then the manifold GK_n is reduced to a recurrent manifold (K_n) introduced by Ruse [23].

The notion of recurrent manifolds has been generalized by various authors as Ricci recurrent manifolds (R_n) by Patterson [21], 2-recurrent manifolds by Lichnerowicz [16], projective 2-recurrent manifolds by Ghosh [10] and generalized Ricci recurrent manifold (GR_n) by De et al. [8], and Kim et al. [15].

Recently, semi generalized recurrent condition was introduced and studied on Lorentzian α-Sasakian manifolds and P-Sasakian manifolds by Dey and Bhattacharyya [9] and Singh et al. [29], respectively.

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Definition. A Riemannian manifold (M, g) is said to be a semi-generalized Ricci recurrent manifold if

$$(\nabla_X S)(Y, Z) = A(X)S(Y, Z) + B(X)g(Y, Z)$$

holds, where A and B are 1-forms associated with the vector fields P_1, P_2, respectively, on M, i.e.,

$$A(X) = g(X, P_1);\ B(X) = g(X, P_2).$$

Our work is structured as follows: In Section 2, we give a brief information about $(LCS)_n$-manifold. In Section 3, we study conharmonically flat $(LCS)_n$-manifold. Section 4 deals with special weakly Ricci-symmetric $(LCS)_n$-manifold. In Section 5, we study generalized Ricci-recurrent $(LCS)_n$-manifold.

2. Preliminaries

A $(2n+1)$-dimensional Lorentzian manifold M is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric g, that is, M admits a smooth symmetric tensor field g of type $(0, 2)$ such that for each point $p \in M$, the tensor $g_p : T_pM \times T_pM \to \mathbb{R}$ is a non-degenerate inner product of signature $(-, +, \ldots, +)$, where T_pM denotes the tangent vector space of M at p and $\mathbb{R}$ is the real number space. A non-zero vector $v \in T_pM$ is said to be time like (resp., non-spacelike, null, spacelike) if it satisfies $g_p(v, v) < 0$ (resp, $\leq 0, = 0, > 0$).

Definition. In a Lorentzian manifold (M, g) a vector field P defined by

$$g(X, P) = A(X)$$

for any X on M, is said to be a concircular vector field if

$$(D_X A)(Y) = \alpha\{g(X, Y) + \omega(X)A(Y)\},$$

where α is a non-zero scalar and ω is a closed 1-form and D denotes the operator of covariant differentiation with respect to the Lorentzian metric g.

Let M be a $(2n+1)$-dimensional Lorentzian manifold admitting a unit time like concircular vector field ξ called the characteristic vector field of the manifold. Then we have

$$g(\xi, \xi) = -1.$$

Since ξ is a unit concircular vector field, it follows that there exists a non-zero 1-form η such that for

$$g(X, \xi) = \eta(X),$$

the equation of the following form holds:

$$(D_X \eta)(Y) = \alpha\{g(X, Y) + \eta(X)\eta(Y)\},\ \alpha \neq 0 \tag{2.1}$$

for any vector fields X, Y on M, where D denotes the operator of covariant differentiation with respect to the Lorentzian metric g and α is a non-zero scalar function satisfying

$$D_X \alpha = (X\alpha) = d\alpha(X) = \rho\eta(X),$$

ρ is a certain scalar function given by $\rho = -(\xi\alpha)$. Let us take

$$\phi X = X(\alpha) = \frac{1}{\alpha} D_X \xi. \tag{2.2}$$

Then by virtue of (2.1) and (2.2), we have

$$\phi X = X + \eta(X)\xi,$$

from which it follows that ϕ is a symmetric $(1,1)$ tensor field called the structure tensor of the manifold. Thus the Lorentzian manifold M together with the unit time like concircular vector field ξ, its associated 1-form η and an $(1,1)$ tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly, $(LCS)_n$-manifold) [24]. Especially, if we take $\alpha = 1$, then we obtain the LP-Sasakian structure of Matsumoto [17]. This leads to the following expression:

$$(D_X\phi)Y = \alpha\{g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\} \tag{2.3}$$

for smooth functions α of M. If $\alpha = 0$, then (2.3) gives

$$(D_X\phi)(Y) = 0.$$

From (2.3), we conclude that

$$D_X\xi = \alpha[X + \eta(X)\xi]. \tag{2.4}$$

The following relations hold in an $(LCS)_n$-manifold $(n > 2)$ ([24]):

$$\phi^2 = I + \eta \circ \xi, \tag{2.5}$$

$$\eta(\xi) = -1, \phi\xi = 0, \eta \circ \phi = 0, g(X,\xi) = \eta(X), \tag{2.6}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.7}$$

$$R(X,Y)Z = \phi R(X,Y)Z + (\alpha^2 - \rho)\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}\xi, \tag{2.8}$$

$$\eta(R(X,Y)Z) = (\alpha^2 - \rho)\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}, \tag{2.9}$$

$$R(X,Y)\xi = (\alpha^2 - \rho)\{\eta(Y)X - \eta(X)Y\}, \tag{2.10}$$

$$R(\xi,X)Y = (\alpha^2 - \rho)\{g(X,Y)\xi - \eta(Y)X\}, \tag{2.11}$$

$$R(\xi,X)\xi = (\alpha^2 - \rho)[\eta(X)\xi + X], \tag{2.12}$$

$$S(\phi X, \phi Y) = S(X,Y) + 2n(\alpha^2 - \rho)\eta(X)\eta(Y), \tag{2.13}$$

$$S(X,\xi) = 2n(\alpha^2 - \rho)\eta(X), \tag{2.14}$$

$$QX = 2n(\alpha^2 - \rho)X, \tag{2.15}$$

$$Q\xi = 2n(\alpha^2 - \rho)\xi. \tag{2.16}$$

A Riemannian manifold M is said to be η-Einstein if its Ricci tensor S is of the form

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y)$$

for arbitrary vector fields X and Y, where a and b are smooth functions on (M,g) [4].

In 1976, Mishra and Pandey [19] defined a tensor of type $(1,3)$ on a Riemannian manifold as

$$(X,Y)Z = R(X,Y)Z - \frac{1}{2n-1}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY], \tag{2.17}$$

so that $(X,Y,Z,U) = g((X,Y)Z,U) = (Z,U,X,Y)$, where Q is the Ricci operator defined by $S(X,Y) = g(QX,Y)$ and S is the Ricci tensor for arbitrary vector fields X,Y and Z. Such a tensor field is known as conharmonic curvature tensor. Asghari and Taleshian [1], Khan [14] and other geometers studied the properties of conharmonic curvature tensor.

3. Conharmonically flat $(LCS)_n$-manifolds

Theorem 3.1. *A conharmonically flat $(LCS)_n$-manifold of dimension $(2n+1)$ is an η-Einstein manifold.*

Proof. In view of $= 0$, (2.17) becomes

$$R(X,Y)Z = \frac{1}{2n-1}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY], \tag{3.1}$$

replacing Z by ξ in (3.1) and then using (2.6), (2.10) and (2.14), we obtain

$$(2n-1)(\alpha^2-\rho)\eta(Y)\xi + (2n-1)(\alpha^2-\rho)Y = 4n(\alpha^2-\rho)\eta(Y)\xi + 2n(\alpha^2-\rho)Y + QY.$$

Again substituiting $X = \xi$ in the above equation and using (2.5), (2.6) and (2.16), we have

$$QY = (-1-2n)(\alpha^2-\rho)\eta(Y)\xi - (\alpha^2-\rho)Y,$$

which gives

$$S(Y,Z) = J_1 g(Y,Z) + J_2\eta(Y)\eta(Z),$$

where

$$J_1 = -(\alpha^2-\rho), \qquad J_2 = (-1-2n)(\alpha^2-\rho).$$ □

4. On special weakly Ricci-symmetric $(LCS)_n$-manifolds

Theorem 4.1. *An $(LCS)_n$-manifold (M,g) of dimension $(2n+1)$ can not be a special weakly Ricci-symmetric manifold $(SWRS)_{2n+1}$.*

Proof. A $(2n+1)$-dimensional $(LCS)_n$-manifold (M,g) is called a special weakly Ricci-symmetric manifold $(SWRS)_{2n+1}$ if

$$(D_X S)(Y,Z) = 2\pi(X)S(Y,Z) + \pi(Y)S(X,Z) + \pi(Z)S(X,Y), \tag{4.1}$$

where π is a 1-form and is defined by $\pi(X) = g(X, \rho)$ for associated vector field ρ ([14, 28]). Taking $Z = \xi$ in (4.1) and using (2.6) and (2.14), we get

$$(D_X S)(Y, \xi) = 4n(\alpha^2 - \rho)\pi(X)\eta(Y) + 2n(\alpha^2 - \rho)\pi(Y)\eta(X) + \pi(\xi)S(X, Y). \tag{4.2}$$

We also know that

$$(D_X S)(Y, \xi) = D_X S(Y, \xi) - S(D_X Y, \xi) - S(Y, D_X \xi). \tag{4.3}$$

In consequence of (2.4) and (2.14), (4.3) becomes

$$(D_X S)(Y, \xi) = D_X\{2n(\alpha^2 - \rho)\eta(Y)\} - 2n(\alpha^2 - \rho)\eta(D_X Y) - S(Y, \alpha(X + \eta(X)\xi)). \tag{4.4}$$

Equation (4.4) with equations (2.6), (2.1), (2.14), (4.2) and $X = \xi$ becomes

$$6n(\alpha^2 - \rho)\pi(\xi)\eta(Y) - 2n(\alpha^2 - \rho)\pi(Y) = 0, \tag{4.5}$$

putting $Y = \xi$ in (4.5) and using (2.6), we obtain

$$-8n(\alpha^2 - \rho)\pi(\xi) = 0,$$

which implies

$$\pi(\xi) = 0. \tag{4.6}$$

In view of (4.6), (4.5) gives

$$\pi(Y) = 0,$$

which is inadmissible. □

5. Generalized Ricci-Recurrent $(LCS)_n$-manifold

Theorem 5.1. *In a generalized Ricci-recurrent $(LCS)_n$-manifold of dimension $(2n+1)$, the associated vector fields of the 1-forms A and B are in the opposite or in same direction, according as $(\alpha^2 - \rho)$ is positive or negative, respectively.*

Proof. A non-flat Riemannian manifold M of dimension greater than two is called a generalized Ricci-recurrent manifold [8] if its Ricci tensor S satisfies the condition

$$(D_X S)(Y, Z) = A(X)S(Y, Z) + B(X)g(Y, Z), \tag{5.1}$$

where D is the Riemannian connection of the Riemannian metric g and A, B are 1-forms associated with the vector fields P_1, P_2, respectively, on M, i.e.,

$$A(X) = g(X, P_1);\ B(X) = g(X, P_2)$$

for arbitrary vector fields X, Y and Z. If the 1-form B vanishes identically, the manifold M is reduced to the well known Ricci-recurrent manifold [21].

Let M be a generalized Ricci-recurrent $(LCS)_n$-manifold. It is known that

$$(D_X S)(Y, Z) = XS(Y, Z) - S(D_X Y Z) - S(Y, D_X Z) \tag{5.2}$$

for arbitrary vector fields X, Y and Z. From equations (5.1) and (5.2), we get

$$A(X)S(Y,Z)+B(X)S(Y,Z)=XS(Y,Z)-S(D_XY,Z)-S(Y,D_XZ),$$

replacing Z by ξ in above equation and using (2.5), (2.6), (2.4) and (2.14), we get

$$[2n(\alpha^2-\rho)A(X)+B(X)]\eta(Y)+\alpha S(Y,\phi^2X)=2n(\alpha^2-\rho)(D_X\eta)(Y). \tag{5.3}$$

In consequence of (2.1), (5.3) becomes

$$\begin{aligned}&[2n(\alpha^2-\rho)A(X)+B(X)]\eta(Y)+\alpha S(Y,\phi^2X)\\&=2n(\alpha^2-\rho)\{\alpha[g(X,Y)+\eta(X)\eta(Y)]\},\end{aligned}$$

putting $Y=\xi$ in above equation and using (2.6), we obtain

$$2n(\alpha^2-\rho)A(X)+B(X)=0. \tag{5.4}$$

□

Theorem 5.2. *If a generalized Ricci-recurrent $(LCS)_n$-manifold of dimension $(2n+1)$ admits a cyclic Ricci tensor, then the manifold is an Einstein manifold, provided $A(\xi)\neq 0$.*

Proof. Let us consider that a generalized Ricci-recurrent $(LCS)_n$-manifold M admits a cyclic Ricci tensor S, i.e.,

$$(D_XS)(Y,Z)+(D_YS)(Z,X)+(D_ZS)(X,Y)=0 \tag{5.5}$$

for arbitrary vector fields X, Y and Z. In view of (5.1), (5.5) follows that

$$\begin{aligned}&A(X)S(Y,Z)+B(X)g(Y,Z)+A(Y)S(Z,X)\\&+B(Y)g(Z,X)+A(Z)S(X,Y)+B(Z)g(X,Y)=0,\end{aligned} \tag{5.6}$$

replacing Z by ξ in (5.6) and using (2.7) and (2.16), we get

$$\begin{aligned}&[2n(\alpha^2-\rho)A(X)+B(X)]\eta(Y)+[2n(\alpha^2-\rho)A(Y)\\&+B(Y)]\eta(X)+A(\xi)S(X,Y)+B(\xi)g(X,Y)=0.\end{aligned} \tag{5.7}$$

In view of (5.4), (5.7) gives

$$A(\xi)S(X,Y)=-B(\xi)g(X,Y),$$

which is an Einstein manifold, provided $A(\xi)\neq 0$. □

6. Example of $(LCS)_n$-manifolds

Example 6.1. We consider the 3-dimensional manifold $M=\{(X,Y,Z)\in\mathbb{R}^3\}$, where (X,Y,Z) are the standard coordinates in $\mathbb{R}^3$. Let $\{\varepsilon_1,\varepsilon_2,\varepsilon_3\}$ be a linearly independent global frame on M given by

$$\varepsilon_1=\varepsilon^{-Z}(\frac{\partial}{\partial X}+Y\frac{\partial}{\partial Y}),\ \varepsilon_2=\varepsilon^{-Z}\frac{\partial}{\partial Y},\ \varepsilon_3=\varepsilon^{-2Z}\frac{\partial}{\partial Z}.$$

Let g be the Lorentzian metric defined by $g(\varepsilon_1,\varepsilon_3)=g(\varepsilon_2,\varepsilon_3)=g(\varepsilon_1,\varepsilon_2)=0$, $g(\varepsilon_1,\varepsilon_1)=g(\varepsilon_2,\varepsilon_2)=1$, $g(\varepsilon_3,\varepsilon_3)=-1$. Let η be the 1-form defined by $\eta(U)=g(U,\varepsilon_3)$ for any $U\in\chi(M)$. Let ϕ be the $(1,1)$ tensor field defined by

$\phi\varepsilon_1 = \varepsilon_1$, $\phi\varepsilon_2 = \varepsilon_2$, $\phi\varepsilon_3 = 0$. Then using the linearity of ϕ and g we have $\eta(\varepsilon_3) = -1$, $\phi^2 U = U + \eta(U)\varepsilon_3$ and $g(\phi U, \phi W) = g(U, W) + \eta(U)\eta(W)$ for any $U, W \in \chi(M)$. Thus for $\varepsilon_3 = \xi$, (ϕ, ξ, η, g) defines a Lorentzian paracontact structure on M.

Let D be the Levi-Civita connection with respect to the Lorentzian metric g and R be the curvature tensor of g. Then we have

$$[\varepsilon_1, \varepsilon_1] = -\varepsilon^{-Z}\varepsilon_2, \quad [\varepsilon_1, \varepsilon_3] = \varepsilon^{-2Z}\varepsilon_1, \quad [\varepsilon_2, \varepsilon_3] = \varepsilon^{-2Z}\varepsilon_2.$$

Taking $\varepsilon_3 = \xi$ and using Koszul formula for the Lorentzian metric g, we can easily calculate

$$D_{\varepsilon_1}\varepsilon_3 = \varepsilon^{-2Z}\varepsilon_1, \quad D_{\varepsilon_1}\varepsilon_2 = 0, \quad D_{\varepsilon_1}\varepsilon_1 = \varepsilon^{-2Z}\varepsilon_3,$$
$$D_{\varepsilon_2}\varepsilon_3 = \varepsilon^{-2Z}\varepsilon_2, \quad D_{\varepsilon_2}\varepsilon_2 = \varepsilon^{-2Z}\varepsilon_3 - \varepsilon^{-Z}\varepsilon_1, \quad D_{\varepsilon_2}\varepsilon_1 = \varepsilon^{-2Z}\varepsilon_2,$$
$$D_{\varepsilon_3}\varepsilon_3 = 0, \quad D_{\varepsilon_3}\varepsilon_2 = 0, \quad D_{\varepsilon_3}\varepsilon_1 = 0.$$

From the above it can be easily seen that (ϕ, ξ, η, g) is an $(LCS)_3$ structure on M. Consequently $M^3(\phi, \xi, \eta, g)$ is an $(LCS)_3$-manifold with $\alpha = \varepsilon^{-2Z} \neq 0$ such that $(X\alpha) = \rho\eta(X)$, where $\rho = 2\varepsilon^{-4Z}$. Using the above relations, we can easily calculate the non-vanishing components of the curvature tensor as follows:

$$R(\varepsilon_2, \varepsilon_3)\varepsilon_3 = \varepsilon^{-4Z}\varepsilon_2, \; R(\varepsilon_1, \varepsilon_3)\varepsilon_3 = \varepsilon^{-4Z}\varepsilon_1, \; R(\varepsilon_1, \varepsilon_2)\varepsilon_2 = \varepsilon^{-4Z}\varepsilon_1 - \varepsilon^{-2Z}\varepsilon_1,$$
$$R(\varepsilon_2, \varepsilon_3)\varepsilon_2 = \varepsilon^{-4Z}\varepsilon_3, \; R(\varepsilon_1, \varepsilon_3)\varepsilon_1 = \varepsilon^{-4Z}\varepsilon_3, \; R(\varepsilon_1, \varepsilon_2)\varepsilon_1 = -\varepsilon^{-4Z}\varepsilon_2 + \varepsilon^{-2Z}\varepsilon_2$$

and the components which can be obtained from these by the symmetry properties from which, we can easily calculate the non-vanishing components of the Ricci tensor S as follows:

$$S(\varepsilon_1, \varepsilon_1) = 2\varepsilon^{-4Z} - \varepsilon^{-2Z}, \quad S(\varepsilon_2, \varepsilon_2) = 2\varepsilon^{-4Z} - \varepsilon^{-2Z}, \quad S(\varepsilon_3, \varepsilon_3) = 2\varepsilon^{-4Z}.$$

Since $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ is a frame field for $(LCS)_3$-manifold, any vector fields $X, Y \in \chi(M)$ can be written as:

$$X = a_1\varepsilon_1 + b_1\varepsilon_2 + c_1\varepsilon_3$$

and

$$Y = a_2\varepsilon_1 + b_2\varepsilon_2 + c_2\varepsilon_3,$$

where $a_i, b_i, c_i \in \mathbb{R}^+$ (the set of positive real numbers), $i = 1, 2, 3$, such that $c_1c_2 \neq a_1a_2 + b_1b_2$. Hence

$$S(X, Y) = 2(a_1a_2 + b_1b_2 + c_1c_2)\varepsilon^{-4Z} - (a_1a_2 + b_1b_2)\varepsilon^{-2Z}$$

and

$$g(X, Y) = a_1a_2 + b_1b_2 - c_1c_2.$$

By virtue of the above we have the following:

$$(D_{\varepsilon_1}S)(X, Y) = (a_1c_2 + a_2c_1)(\varepsilon^{-4Z} - 4\varepsilon^{-6Z}),$$
$$(D_{\varepsilon_2}S)(X, Y) = (b_1c_2 + b_2c_1)(\varepsilon^{-4Z} - 4\varepsilon^{-6Z})$$

and

$$(D_{\varepsilon_3}S)(X,Y)=0.$$

We shall show that this $(LCS)_3$-manifold is a generalized Ricci recurrent, i.e., it satisfies the relation (5.1). Let us now consider the 1-forms:

$$A(\varepsilon_1)=\frac{(a_1c_2+a_2c_1)}{2(a_1a_2+b_1b_2+c_1c_2)},$$
$$A(\varepsilon_2)=\frac{(b_1c_2+b_2c_1)}{2(a_1a_2+b_1b_2+c_1c_2)},$$
$$A(\varepsilon_3)=0,$$
$$B(\varepsilon_1)=\frac{\varepsilon^{-2Z}(a_1c_2+a_2c_1)[(a_1a_2+b_1b_2)\left(1-8\varepsilon^{-4Z}\right)-8c_1c_2\varepsilon^{-4Z}]}{2(a_1a_2+b_1b_2+c_1c_2)(a_1a_2+b_1b_2-c_1c_2)},$$
$$B(\varepsilon_2)=\frac{\varepsilon^{-2Z}(b_1c_2+b_2c_1)[(a_1a_2+b_1b_2)\left(1-8\varepsilon^{-4Z}\right)-8c_1c_2\varepsilon^{-4Z}]}{2(a_1a_2+b_1b_2+c_1c_2)(a_1a_2+b_1b_2-c_1c_2)},$$
$$B(\varepsilon_3)=0$$

at any point $X\in M$. In our M^3, (5.1) is reduced with these 1-forms to the following equations:

(1) $(D_{\varepsilon_1}S)(X,Y)=A(\varepsilon_1)S(X,Y)+B(\varepsilon_1)g(X,Y)$,
(2) $(D_{\varepsilon_2}S)(X,Y)=A(\varepsilon_2)S(X,Y)+B(\varepsilon_2)g(X,Y)$,
(3) $(D_{\varepsilon_3}S)(X,Y)=A(\varepsilon_3)S(X,Y)+B(\varepsilon_3)g(X,Y)$.

This shows that the manifold under consideration is a generalized Ricci recurrent $(LCS)_3$-manifold in which the associated vector fields of the 1-forms A and B are in same direction.

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