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ROBUST PORTFOLIO OPTIMIZATION UNDER HYBRID CEV AND STOCHASTIC VOLATILITY

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Abstract. In this paper, we investigate the portfolio optimization problem under the SVCEV model, which is a hybrid model of constant elasticity of variance (CEV) and stochastic volatility, by taking into account of minimum-entropy robustness. The Hamilton-Jacobi-Bellman (HJB) equation is derived and the first two orders of optimal strategies are obtained by utilizing an asymptotic approximation approach. We also derive the first two orders of practical optimal strategies by knowing that the underlying Ornstein-Uhlenbeck process is not observable. Finally, we conduct numerical experiments and sensitivity analysis on the leading optimal strategy and the first correction term with respect to various values of the model parameters.

1. Introduction

In financial economics, a very active line of research focuses on the question of how to allocate a portfolio between a risky and risk-free asset in a dynamical environment. The research of portfolio optimization selections in continuous-time for a market under the Black-Scholes frame was first investigated by Merton in [13] and [14], through the dynamic programming method. After nearly twenty years, Cox and Huang [5] conducted a parallel study of this topic when there exist non-negative constraints on consumption and final wealth by using a martingale method. As we know, the Black-Scholes market framework assumes the volatility of a risky asset is a constant. However, some empirical studies such as Rubinstein [16] and Jackwerth and Rubinstein [10] have indicated that the implied volatility of a risky asset is characterized by a smile or skew effect. In order to reflect this relationship, in 1975, Cox [4] developed a local stochastic volatility model, currently known as the constant elasticity of variance (CEV) model. Further empirical evidence tested by Macbeth and Merville [11] has shown that Cox's CEV model is superior to

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the Black-Scholes model in call option valuation. Along this direction, Gao [8] utilized the CEV model to study the portfolio optimization problem of pension plans. One thing that we should mention is that in the CEV model, volatility is perfectly correlated either positively or negatively with the underlying asset price. However, there is no clear evidence that there is a perfect definite correlation all the time, as shown in Harvey [9]. This motivated Choi et al. to incorporate stochastic volatility driven by a hidden process into the CEV model in [3]. In the sequel, we shall call this model the SVCEV model. In 2015, Yang et al. [17] specified the SVCEV model for an underlying process of volatility as a fast mean-reverting Ornstein-Uhlenbeck (OU) process and applied this model to optimal portfolio selection for pension plans under the CRRA utility. Furthermore, Yang et al. [18] in 2014 incorporated the stochastic volatility driven by a hidden OU process in a slightly different setting and the resultant model is sometime called the SEV model in the literature.

A fundamental assumption in aforementioned work on portfolio choice is the absence of any uncertainty about the return process. Typically, we obtain point estimates for the asset return parameters and subsequently assume these are known and fixed. However, as mentioned in [12], there is a lack of consensus concerning the expected risk premium or even the model generating excess return among financial economists. Some authors believe that the future expected returns should be lower, while some others are skeptical about the reliability of historical estimates of equity premium, based on the argument that historical studies suffer from severe ex post survival bias. Due to these and many reasons, it is desirable to take uncertainty about the return process into account when studying optimal dynamic portfolio decisions. The continuous-time methodology on robustness developed in Anderson et al. [1] provides a natural framework for portfolio choice problems involving uncertainty about the return process for equities. Using this methodology, Maenhout derived consumption and portfolio rules that are robust to a particular type of model specification, stemming from uncertainty about the return process. Historically, this type of robustness is based on the key assumption that the decision-maker worries about some worse-case scenario. Here, we shall adopt the approach employed by Maenhout in [12], in which the disparity between the reference model that the decision-maker is skeptical about and the worst-case alternative model that he or she considers is constrained by a preference parameter, quantifying the strength of the preference for robustness.

In a recent paper [15], Peng et al. studied the portfolio selection problem under the SVCEV model with the exponential utility function (CARA). In this paper, we extend this study by taking into account of minimum-entropy robustness, in order to allow decision makers to doubt the model for misspecification and consider alternative models. To the best of our knowledge, this has not been considered before under the SVCEV model framework. Thus, our contributions in this paper fill some gap in the literature. The rest of the paper is structured as follows. Section 2 briefly introduces the basic model setup for exponential utility without taking account of robustness. Section 3 describes the definition of robustness in terms of minimum-entropy and establishes the relevant Hamiton-Jacobi-Bellman (HJB) equation with robustness factor by dynamic programming. Furthermore, a nonlinear partial differential equation of the value function is derived. In Section 4, we use an asymptotic expansion approach to derive an approximation of the value function. In Sections 5 and 6, we derive asymptotic approximations of optional strategies, including the leading term of optimal strategy and the first correction term. In Section 6, we conduct numerical experiments and investigate the behaviour and sensitivity of the leading term of optimal strategy and the first correction term with respect to various model parameters. Finally, Section 7 concludes.

2. Basic portfolio selection under SVCEV

In this section, we present a simple financial market setup and formulate a basic portfolio selection problem. We consider a market structure that consists of a risk-free asset (treasury bond or bank account) whose price dynamics are driven by the following ordinary differential equation

$$dB_t = rB_t dt, \tag{2.1}$$

and a risky asset whose price dynamics satisfy the stochastic different equations

$$\frac{dS_t}{S_t} = \mu dt + \sigma(Y_t) S_t^{\beta/2} dW_t^s, \tag{2.2}$$

$$dY_t = \alpha(m - Y_t)dt + \delta dW_t^y, \tag{2.3}$$

where $r > 0$, $\alpha > 0$, $\delta > 0$, $\beta < 0$, μ and m are constants, $\sigma(\cdot)$ is assumed to be a smooth, bounded and positive function, and $\{W_t^s : t \geq 0\}$ and $\{W_t^y : t \geq 0\}$ are two correlated standard Brownian motions with correlation coefficient $-1 < \rho < 1$. Here, $r > 0$ is the risk-free rate. The Ornstein-Uhlenbeck (OU) process $\{Y_t : t \geq 0\}$ given in Eq. (2.3) is characterized by its typical time to obtain back to the mean level m of its long-run distribution. The parameter α determines the speed of mean-reversion and δ controls the volatility of Y_t. In [2] and [3], the model described in Eq. (2.2) and Eq. (2.3) is called the *SVCEV model*.

Let X_t be the wealth of an investor at time $t \geq 0$ and π_t be the proportion of the wealth invested in the risky asset. Then, the proportion of the wealth invested in the risk-free asset is $1 - \pi_t$. Under the SVCEV model, the wealth evolves according to the following stochastic different equation

$$dX_t = \pi_t X_t \frac{dS_t}{S_t} + (1 - \pi_t) X_t \frac{dB_t}{B_t}, \quad 0 \leq t \leq T.$$

Applying Eq. (2.1) and Eq. (2.2), we obtain

$$dX_t = [r + (\mu - r)\pi_t] X_t dt + \pi_t \sigma(Y_t) S_t^{\beta/2} X_t dW_t^s, \quad 0 \leq t \leq T. \tag{2.4}$$

As usual, the interest of the investor is to find a strategy π_t^* which maximizes the conditional expectation of the utility of the terminal wealth, given by

$$\mathbb{E}\left[U(X_T) : S_t = s, X_t = x, Y_t = y\right]. \tag{2.5}$$

To find the optimal strategy π_t^*, we introduce the value function V defined by

$$V(t, s, x, y) := \max_{\pi_t} \mathbb{E}\left[U(X_T) : S_t = s, X_t = x, Y_t = y\right]. \tag{2.6}$$

Then the *Hamilton-Jacob-Bellman* (HJB) equation associated with the optimization problem is given by

$$0 = \max_{\pi_t} \mathcal{D}^{\pi_t} V(t, s, x, y) \tag{2.7}$$

with the terminal condition $V(T, s, x, y) = U(x)$, where $\mathcal{D}^{\pi_t} V(t, s, x, y)$ is the infinitesimal generator given by

$$\begin{aligned}
\mathcal{D}^{\pi_t} V(t, s, x, y) = {} & V_t + \mu s V_s + r x V_x + \alpha(m - y) V_y \\
& + \frac{1}{2}\sigma(y)^2 s^{\beta+2} V_{ss} + \frac{1}{2}\delta^2 V_{yy} + \delta\rho\sigma(y) s^{\frac{\beta}{2}+1} V_{sy} \\
& + \pi_t \left((\mu - r) x V_x + \sigma(y)^2 s^{\beta+1} x V_{sx} + \delta\rho\sigma(y) s^{\frac{\beta}{2}} x V_{xy} \right) \\
& + \frac{1}{2}\pi_t^2 \sigma(y)^2 s^{\beta} x^2 V_{xx}.
\end{aligned}$$

In [17], Yang et al. considered this problem under the same model framework with the power utility function (CRRA) given by

$$U(x) = \frac{x^p}{p}, \quad p \neq 0, \quad p < 1.$$

Using an asymptotic analysis approach, they derived a correction to the optimal strategy and subsequently the fine structure of the corrected optimal strategy is revealed. Their result is a generalization of Merton's strategy in terms of the stochastic volatility and the elasticity of variance. In this paper, we consider this problem under a more general model framework than the one mentioned with the exponential utility function (CARA) given by

$$U(x) = -\frac{1}{\gamma} e^{-\gamma x}, \quad \gamma > 0.$$

Our model framework involves with minimum-entropy robustness.

3. Portfolio selection under SVCEV with minimum-entropy robustness

Following [1] and [12], we consider portfolio rules that are robust to a particular type of model misspecification, stemming from uncertainty about the return process. The main goal for the robustness of decision rules to model misspecification is to design decision rules that not only work well when the underlying model for the state variable holds exactly, but also perform reasonably well if there is some form of model misspecification. Here, we assume that the decision maker worries about some worst-case scenario. In particular, the disparity between the "referenced model" that the agent is skeptical and the worse-case alternative model that he considers is constrained by a (preference) parameter, quantifying the strength of the preference for robustness.

Similar to the discussions in [12, Section 1.1], the objective $\mathcal{D}^{\pi_t}V$ in Eq. (2.7) is essentially the mechanism through robustness is introduced. Heuristically, $\mathcal{D}^{\pi_t}V$ can be thought of as $\frac{1}{dt}\mathbb{E}[dV]$. A key insight in [1] is that this differential expectation operator, used to calculate the differential continuation payoff in the HJB equation, reflects a particular underlying model for the state variable X_t. The decision maker accepts this "reference model" as useful, but doubts it to be misspecified. So, he or she likes to consider alternative models when calculating his/her continuation payoff. A preference for robustness is achieved by having the decision maker guard against an adverse alternative model that is reasonably similar to the reference model. In a pure diffusion setting like Eq. (2.4), Anderson et al. showed in [1, Section 2] that this adverse alternative model simply adds an endogenous drift $u(X_t)$ to the dynamics of X_t. This approach was further employed by Maenhout in [12], where the diffusion term is adjusted. Here, we follow the method of Maenhout [12, Section 1.1] to introduce robustness of model misspecification by adjusting the diffusion part dW_t^s term in Eq. (2.4) to $\sigma(\pi_t, X_t, S_t)u(X_t)dt + dW_t^s$, where the term $\sigma(\pi_t, X_t, S_t)u(X_t)dt$ captures the uncertainty of return process. Thus, the adjusted wealth process dynamics evolve according to the following SDE:

$$dX_t = \mu(\pi_t, X_t)dt + \sigma(\pi_t, X_t, S_t)\left[\sigma(\pi_t, X_t, S_t)u(X_t)dt + dW_t^s\right], \tag{3.1}$$

where $\mu(\pi_t, X_t)$ and $\sigma(\pi_t, X_t, S_t)$ denote the drift and diffusion coefficients in Eq. (2.4), respectively.

The main problem that we tackle in this section is still to find a strategy π_t^* which maximizes the conditional expectation of the utility of the terminal wealth given by Eq. (2.5), via the value function defined in Eq. (2.6), but with respect to the adjusted wealth process dynamics in Eq. (3.1). The drift adjustment $u(X_t)$ will be chosen to minimize the sum of the expected continuous payoff of Eq. (3.1). In order to reflect the additional drift component in Eq. (3.1), we need to add an entropy penalty term. Based upon this analysis, the infinitesimal generator is adjusted to

$$\min_u \left(\mathcal{D}^{\pi_t}V(t,s,x,y) + u(X_t)\sigma(\pi_t, X_t, S_t)^2 V_x + \frac{1}{2\hat{\theta}}u(X_t)^2\sigma(\pi_t, X_t, S_t)^2\right),$$

where the entropy penalty is weighted by $\frac{1}{\hat{\theta}}$. The adjusted HJB equation becomes:

$$0 = \max_{\pi_t}\min_u \left(\mathcal{D}^{\pi_t}V(t,s,x,y) + \pi_t^2\sigma(y)^2u(x)s^\beta x^2 V_x + \frac{1}{2\hat{\theta}}\pi_t^2\sigma(y)^2u(x)^2 s^\beta x^2\right). \tag{3.2}$$

The minimization part yields $u^* = -\hat{\theta} V_x$. Then, we obtain the optimal choice π_t^* by calculating the maximization part in Eq. (3.2) with u^* as follows:

$$\pi_t^* = -\frac{(\mu - r)V_x + \sigma(y)^2 s^{\beta+1} V_{sx} + \delta\rho\sigma(y) s^{\frac{\beta}{2}} V_{xy}}{\sigma(y)^2 s^\beta x \left(V_{xx} - \hat{\theta} V_x^2\right)}. \tag{3.3}$$

The parameter $\hat{\theta} \geq 0$ measures the strength of the preference for robustness. In [1], $\hat{\theta}$ is fixed and state-independent. In [12], Maenhout replaced it with a state-dependent version of θ. In this paper, we follow [12] to replace $\hat{\theta}$ by $\Psi(t, s, x, y)$. Moreover, for the desired homotheticity property, we set

$$\Psi(t, s, x, y) = \frac{\theta}{(1-\gamma)V(t, s, x, y)}, \tag{3.4}$$

where θ is a constant.

Theorem 3.1. *Under the previous assumptions, the option strategy π_t^* under the SVCEV model with minimum-entropy robustness can take the following form*:

$$\pi_t^* = \frac{(1-\gamma)(\mu - r)}{\gamma(1-\gamma-\theta)a\sigma^2 s^\beta x} - \frac{\gamma(1-\gamma) s g_s}{(1-\gamma-\theta)ax} - \frac{(1-\gamma)\delta\rho g_y}{(1-\gamma-\theta)a\sigma s^{\beta/2} x}, \tag{3.5}$$

where $a = \exp(r(T-t))$ and g satisfies the following PDE:

(3.6)

$$\begin{aligned}
& g_t + \frac{r\gamma - r + \theta\mu}{\gamma - 1 + \theta} s g_s - \frac{\gamma\theta\sigma^2 s^{\beta+2}}{2(\gamma-1+\theta)} g_s^2 + \frac{\alpha((\gamma-1)(m-y) + \theta(m-y))}{\gamma - 1 + \theta} g_y \\
& + \frac{\delta\rho(\mu - r)(1-\gamma)}{(\gamma-1+\theta)\sigma s^{\beta/2}} g_y + \frac{\gamma\delta^2((1-\gamma)(1-\rho^2) - \theta)}{2(\gamma-1+\theta)} g_y^2 + \frac{1}{2}\sigma^2 s^{\beta+2} g_{ss} + \frac{1}{2}\delta^2 g_{yy} \\
& + \delta\rho\sigma s^{\beta/2+1} g_{sy} - \frac{\gamma\delta\theta\rho\sigma s^{\beta/2+1}}{\gamma-1+\theta} g_y g_s + \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)\sigma^2 s^\beta} = 0
\end{aligned}$$

with the terminal condition $g(T, s, y) = 0$.

Proof. We conjecture the value function with the following form:

$$V(t, s, x, y) = -\frac{1}{\gamma} \exp\left\{-\gamma\left(a(t)(x - b(t)) + g(t, s, y)\right)\right\}. \tag{3.7}$$

Substituting V, u^* and π_t^* into Eq. (3.2), we can obtain the following PDE:

(3.8)

$$\begin{aligned}
& a_t(x-b) - ab_t + arx + g_t + \frac{r\gamma - r + \theta\mu}{\gamma - 1 + \theta} s g_s - \frac{\gamma\theta\sigma^2 s^{\beta+2}}{2(\gamma-1+\theta)} g_s^2 \\
& + \frac{\alpha((\gamma-1)(m-y) + \theta(m-y))}{\gamma-1+\theta} g_y + \frac{\delta\rho(\mu-r)(1-\gamma)}{(\gamma-1+\theta)\delta\ s^{\beta/2}} g_y \\
& + \frac{\gamma\delta^2((1-\gamma)(1-\rho^2) - \theta)}{2(\gamma-1+\theta)} g_y^2 + + \frac{1}{2}\sigma^2 s^{\beta+2} g_{ss} + \frac{1}{2}\delta^2 g_{yy} + \delta\rho\sigma s^{\beta/2+1} g_{sy}
\end{aligned}$$

$$-\frac{\gamma\delta\theta\rho\sigma s^{\beta/2+1}}{\gamma-1+\theta}g_s g_y+\frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)\sigma^2 s^\beta}=0,$$

where a, b and σ are short notations for functions $a(t)$, $b(t)$ and $\sigma(y)$. Eq. (3.8) can be decomposed into three differential equations

$$a_t+ra=0,\qquad a(T)=1, \tag{3.9}$$

$$a_t b+ab_t=0,\qquad b(T)=0, \tag{3.10}$$

and Eq. (3.6). Note that Eq. (3.9) and Eq. (3.10) have closed-form solutions $a(t)=\exp(r(T-t))$ and $b(t)=0$, respectively, but Eq. (3.6) does not have a closed-form solution. Finally, substituting Ψ and V in Eq. (3.4) and Eq. (3.7) into Eq. (3.3) gives formula (3.5). $\square$

4. Asymptotic value function

Since it is impossible to obtain a closed-form solution to Eq. (3.6), in this section, we apply the approximation approach used in [6], [7], [17] and [18] to obtain an asymptotic approximation of the value function. Note that the process $\{Y_t : t\geq 0\}$ is characterized by an infinitesimal generator $\mathcal{A}=\alpha\mathcal{L}_Y$, where

$$\mathcal{L}_Y:=\nu^2\frac{\partial^2}{\partial y^2}+(m-y)\frac{\partial}{\partial y}.$$

Here, ν is given by $\nu:=\frac{\delta}{\sqrt{2\alpha}}$. The long-term distribution of $\{Y_t : t\geq 0\}$ is the normal distribution $\mathcal{N}(m,\nu^2)$ whose probability density function is denoted by $\Phi(y)$.

Now, we express α and δ in terms of a small and positive parameter ϵ such that $\alpha=\frac{1}{\epsilon}$ and $\delta=\frac{\sqrt{2}\nu}{\sqrt{\epsilon}}$. Then, Eq. (3.6) becomes

$$\begin{aligned}&\frac{1}{\epsilon}\left(\mathcal{L}_Y g+\frac{\nu^2\gamma((1-\gamma)(1-\rho^2)-\theta)}{\gamma-1+\theta}g_y^2\right)+\frac{1}{\sqrt{\epsilon}}\Bigg(\sqrt{2}\nu\rho\sigma s^{\beta/2+1}g_{sy}\\&-\frac{\sqrt{2}\nu\gamma\theta\rho\sigma s^{\beta/2+1}}{\gamma-1+\theta}g_y g_s+\frac{\sqrt{2}\nu\rho(\mu-r)(1-\gamma)}{\sigma(\gamma-1+\theta)s^{\beta/2}}g_y\Bigg)+g_t+\frac{r\gamma-r+\theta\mu}{\gamma-1+\theta}sg_s\\&-\frac{\gamma\theta\sigma^2 s^{\beta+2}}{2(\gamma-1+\theta)}g_s^2+\frac{1}{2}\sigma^2 s^{\beta+2}g_{ss}+\frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)\sigma^2 s^\beta}=0.\end{aligned} \tag{4.1}$$

Next, we consider the power series representation of g as increasing powers of ϵ as follows:

$$g=g_0+\sqrt{\epsilon}g_1+\epsilon g_2+\cdots. \tag{4.2}$$

Substituting the representation of g into Eq. (4.1), then the $\mathcal{O}(\frac{1}{\epsilon})$-term yields

$$\mathcal{L}_Y g_0+\frac{\nu^2\gamma((1-\gamma)(1-\rho^2)-\theta)}{\gamma-1+\theta}g_{0,y}^2=0. \tag{4.3}$$

The solution to Eq. (4.3) can be expressed in the following form:

$$g_0 = \frac{\gamma - 1 + \theta}{\gamma((1-\gamma)(1-\rho^2) - \theta)} \ln\left(c_1(s,t) \int_0^y e^{\frac{(u-m)^2}{2\nu^2}} du + c_2(s,t)\right),$$

where c_1 and c_2 are some functions of s and t. Hence, g_0 grows unreasonably fast with respect to $\ln y$ unless $c_1 = 0$. To avoid this, we assume that $c_1 = 0$ and consequently g_0 is independent of y. Substituting the representation of g into Eq. (4.1), the $\mathcal{O}\left(\frac{1}{\sqrt{\epsilon}}\right)$-term yields $\mathcal{L}_Y g_1 = 0$, whose solution can be expressed as

$$g_1 = c_3(s,t) \int_0^y e^{\frac{(u-m)^2}{2nu^2}} du + c_4(s,t).$$

Similar to the case of g_0, we assume that $c_3 = 0$ in order to avoid that g_1 grows unreasonably fast with respect to y. As a result, g_1 is also independent of y.

In our next result, we present an explicit closed-form formula for g_0.

Theorem 4.1. *For the power series representation of g as increasing powers of ϵ,*

$$g = g_0 + \sqrt{\epsilon} g_1 + \epsilon g_2 + \cdots,$$

we assume that g_0 and g_1 do not grow unreasonably fast. Then we can obtain the solution g_0 independent of y with the following form:

$$g_0(t,s) = C(t) + D(t) s^{-\beta},$$

where for $0 < t < T$, $C(t)$ and $D(t)$ are given by

$$C(t) = \frac{1}{2}\bar{\sigma}^2 \beta(\beta+1)\left\{m_2(T-t) - \frac{1}{\lambda}\ln\left(\frac{m_1 - m_2}{m_1 - m_2 e^{-a(m_2 - m_1)(T-t)}}\right)\right\},$$

$$D(t) = \frac{m_2 - m_2 e^{\lambda(m_1 - m_2)(T-t)}}{1 - \frac{m_2}{m_1} e^{\lambda(m_1 - m_2)(T-t)}},$$

with

$$\lambda = \frac{\beta^2 \gamma \theta \bar{\sigma}^2}{2(\gamma - 1 + \theta)},$$

$$m_1 = \frac{-(r\gamma - r + \theta\mu) + \sqrt{(r\gamma - r + \theta\mu)^2 + \theta\left(\frac{\bar{\sigma}}{\tilde{\sigma}}\right)^2 (\gamma - 1)(\mu - r)^2}}{\beta\gamma\theta\bar{\sigma}^2},$$

$$m_2 = \frac{-(r\gamma - r + \theta\mu) - \sqrt{(r\gamma - r + \theta\mu)^2 + \theta\left(\frac{\bar{\sigma}}{\tilde{\sigma}}\right)^2 (\gamma - 1)(\mu - r)^2}}{\beta\gamma\theta\bar{\sigma}^2}.$$

Proof. From the $\mathcal{O}(1)$-term in Eq. (4.1), the y-independence of g_0 and g_1 yields

$$\mathcal{L}_Y g_2 + g_{0,t} + \frac{r\gamma - r + \theta\mu}{\gamma - 1 + \theta} s g_{0,s} - \frac{\gamma\theta\sigma^2 s^{\beta+2}}{2(\gamma - 1 + \theta)} g_{0,s}^2 + \frac{1}{2}\sigma^2 s^{\beta+2} g_{0,ss} + \frac{(\gamma-1)(\mu - r)^2}{2\gamma(\gamma - 1 + \theta)\sigma^2 s^\beta} = 0. \tag{4.4}$$

Note that Eq. (4.4) is a Poisson equation of the following type:

$$\mathcal{L}_Y g_2 + f(t,s,y) = 0.$$

The necessary condition for this Poisson equation to have a solution is the centring condition $\langle f\rangle = 0$, which leads to

$$\begin{aligned} g_{0,t} &+ \frac{r\gamma - r + \theta\mu}{\gamma - 1 + \theta} s g_{0,s} - \frac{\gamma\theta\bar{\sigma}^2 s^{\beta+2}}{2(\gamma-1+\theta)} g_{0,s}^2 \\ &+ \frac{1}{2}\bar{\sigma}^2 s^{\beta+2} g_{0,ss} + \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)\breve{\sigma}^2 s^{\beta}} = 0, \end{aligned} \tag{4.5}$$

with the terminal condition $g_0(T) = 0$, where $\bar{\sigma} := \langle \sigma^2\rangle^{\frac{1}{2}}$ and $\breve{\sigma} := \left\langle \frac{1}{\sigma^2}\right\rangle^{-\frac{1}{2}}$. Here, $\langle\cdot\rangle$ denotes the expectation with respect to $\mathcal{N}(m,\nu^2)$, i.e.,

$$\langle f\rangle = \int_{-\infty}^{+\infty} f(t,s,y)\Phi(y)dy.$$

In order to solve Eq. (4.5), we conjecture that its solution has the following form:

$$g_0(t,s) = C(t) + D(t)s^{-\beta}.$$

Substituting g_0 into Eq. (4.5), we obtain two ordinary differential equations:

$$D_t - \frac{\beta^2\gamma\theta\bar{\sigma}^2}{2(\gamma-1+\theta)}D^2 - \frac{\beta(r\gamma - r+\theta\mu)}{\gamma-1+\theta}D + \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)\breve{\sigma}^2} = 0, \tag{4.6}$$

$$C_t + \frac{1}{2}\bar{\sigma}^2\beta(\beta+1)D = 0, \tag{4.7}$$

with terminal conditions $C(T) = 0$ and $D(T) = 0$, respectively. The solution to Eq. (4.6) is given by

$$D(t) = \frac{m_2 - m_2 e^{\lambda(m_1-m_2)(T-t)}}{1 - \frac{m_2}{m_1}e^{\lambda(m_1-m_2)(T-t)}},$$

where λ, m_1 and m_2 are defined by

$$\begin{aligned} \lambda &= \frac{\beta^2\gamma\theta\bar{\sigma}^2}{2(\gamma-1+\theta)}, \\ m_1 &= \frac{-(r\gamma - r+\theta\mu) + \sqrt{(r\gamma-r+\theta\mu)^2 + \theta\left(\frac{\bar{\sigma}}{\breve{\sigma}}\right)^2(\gamma-1)(\mu-r)^2}}{\beta\gamma\theta\bar{\sigma}^2}, \\ m_2 &= \frac{-(r\gamma - r+\theta\mu) - \sqrt{(r\gamma-r+\theta\mu)^2 + \theta\left(\frac{\bar{\sigma}}{\breve{\sigma}}\right)^2(\gamma-1)(\mu-r)^2}}{\beta\gamma\theta\bar{\sigma}^2}, \end{aligned}$$

respectively. The solution to Eq. (4.7) is given by

$$C(t) = \frac{1}{2}\bar{\sigma}^2\beta(\beta+1)\left\{m_2(T-t) - \frac{1}{\lambda}\ln\left(\frac{m_1-m_2}{m_1 - m_2 e^{-a(m_2-m_1)(T-t)}}\right)\right\}.$$

□

Unlike the case for g_0, we cannot obtain an explicit closed-form formula for g_1. Nevertheless, we provide a result related to g_1 below.

Theorem 4.2. *For the power series representation of g as increasing powers of ϵ,*

$$g = g_0 + \sqrt{\epsilon} g_1 + \epsilon g_2 + \cdots ,$$

we assume that g_0 and g_1 do not grow unreasonably fast. Then g_1 is the solution to the following PDE:

$$\begin{aligned}
& g_{1,t} + \frac{r\gamma - r + \theta\mu}{\gamma - 1 + \theta} s g_{1,s} + \frac{1}{2}\bar{\sigma}^2 s^{\beta+2} g_{1,ss} + \sqrt{2}\nu\rho s^{\beta/2+1} \times \left\{ \frac{1}{2}\bigg((\beta+2) s^{\beta+1}\mathcal{E}(g_0) \right. \\
& \left. + s^{\beta+2}\mathcal{F}(g_0) \bigg) \langle \sigma\varphi'(y)\rangle + \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)} \beta s^{-\beta-1} \langle \sigma\psi'(y)\rangle \right\} \\
& + \frac{\sqrt{2}\nu\beta\gamma\theta\rho B(t) s^{-\beta/2}}{\gamma-1+\theta} \left(\frac{1}{2} s^{\beta+2} \langle \sigma\varphi'(y)\rangle \mathcal{E}(g_0) - \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)s^{\beta}} \langle \sigma\psi'(y)\rangle \right) \\
& + \frac{\sqrt{2}\nu\rho(\mu-r)(1-\gamma)}{(\gamma-1+\theta)s^{\beta/2}} \left(\frac{1}{2} s^{\beta+2} \langle \frac{1}{\sigma}\varphi'(y)\rangle \mathcal{E}(g_0) - \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)s^{\beta}} \langle \frac{1}{\sigma}\psi'(y)\rangle \right) \\
& - \frac{\gamma\theta\bar{\sigma}^2 s^{\beta+2}}{\gamma-1+\theta} g_{0,s} g_{1,s} = 0,
\end{aligned}$$

where $\mathcal{E}(g_0)$ and $\mathcal{F}(g_0)$ are given by

$$\begin{aligned}
\mathcal{E}(g_0) &= \frac{\gamma\theta}{\gamma-1+\theta} g_{0,s}^2 - g_{0,ss}, \\
\mathcal{F}(g_0) &= \frac{2\gamma\theta g_{0,s} g_{0,ss}}{\gamma-1+\theta} - g_{0,sss},
\end{aligned}$$

respectively.

Proof. Considering the $\mathcal{O}(\sqrt{\epsilon})$-term in Eq. (4.1), we obtain the following PDE for g_1:

$$\begin{aligned}
\mathcal{L}_Y g_3 &+ g_{1,t} + \frac{r\gamma - r + \theta\mu}{\gamma-1+\theta} s g_{1,s} + \frac{1}{2}\sigma^2 s^{\beta+2} g_{1,ss} \\
&+ \sqrt{2}\nu\rho\sigma s^{\beta/2+1} g_{2,sy} - \frac{\sqrt{2}\nu\gamma\theta\rho\sigma s^{\beta/2+1}}{\gamma-1+\theta} g_{2,y} g_{0,s} \\
&+ \frac{\sqrt{2}\nu\rho(\mu-r)(1-\gamma)}{(\gamma-1+\theta)\sigma s^{\beta/2}} g_{2,y} - \frac{\gamma\theta\sigma^2 s^{\beta+2}}{\gamma-1+\theta} g_{0,s} g_{1,s} = 0.
\end{aligned} \tag{4.8}$$

Note that Eq. (4.8) is a Poisson equation, which can be used to determine g_1 if we apply the centring condition. However, we need to determine g_2 first. For this purpose, we subtract Eq. (4.3) from Eq. (4.4) to get a PDE of g_2 as

$$\mathcal{L}_Y g_2 = \frac{1}{2} s^{\beta+2}(\sigma^2 - \bar{\sigma}^2)\mathcal{E}(g_0) - \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)s^{\beta}} \left(\frac{1}{\sigma^2} - \frac{1}{\breve{\sigma}^2} \right),$$

where $\mathcal{E}(g_0)$ is given by

$$\mathcal{E}(g_0) = \frac{\gamma\theta}{\gamma - 1 + \theta} g_{0,s}^2 - g_{0,ss}.$$

Then, g_2 can be expressed as

$$g_2 = \frac{1}{2} s^{\beta+2} \varphi(y) \mathcal{E}(g_0) - \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)s^\beta} \psi(y) + k(s,t),$$

where $k(s,t)$ is some function independent of y, and φ and ψ are solutions to the following two differential equations:

$$\mathcal{L}_Y \varphi = \sigma(y)^2 - \bar{\sigma}^2,$$
$$\mathcal{L}_Y \psi = \frac{1}{\sigma(y)^2} - \frac{1}{\breve{\sigma}^2},$$

respectively. Consequently, the partial derivatives $g_{2,y}$ and $g_{2,sy}$ of g_2 can be expressed as

$$g_{2,y} = \frac{1}{2} s^{\beta+2} \varphi'(y) \mathcal{E}(g_0) - \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)s^\beta} \psi'(y),$$
$$g_{2,sy} = \frac{1}{2}\Big((\beta+2)s^{\beta+1}\mathcal{E}(g_0) + s^{\beta+2}\mathcal{F}(g_0)\Big)\varphi'(y) + \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)} \beta s^{-\beta-1} \psi'(y),$$

respectively, where $\mathcal{F}(g_{0,s}, g_{0,ss})$ is given by

$$\mathcal{F}(g_0) = \frac{2\gamma\theta g_{0,s} g_{0,ss}}{\gamma - 1 + \theta} - g_{0,sss}.$$

Finally, if we plug $g_{2,y}$ and $g_{2,sy}$ into Eq. (4.8) and then apply the centring condition as we did for Eq. (4.4), we can derive the following PDE for g_1:

(4.9)

$$\begin{aligned}
&g_{1,t} + \frac{r\gamma - r + \theta\mu}{\gamma - 1 + \theta} s g_{1,s} + \frac{1}{2}\bar{\sigma}^2 s^{\beta+2} g_{1,ss} + \sqrt{2}\nu\rho s^{\beta/2+1} \times \Big\{ \frac{1}{2}\Big((\beta+2)s^{\beta+1}\mathcal{E}(g_0) \\
&+ s^{\beta+2}\mathcal{F}(g_0)\Big)\langle\sigma\varphi'(y)\rangle + \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)}\beta s^{-\beta-1}\langle\sigma\psi'(y)\rangle\Big\} \\
&+ \frac{\sqrt{2}\nu\beta\gamma\theta\rho B(t) s^{-\beta/2}}{\gamma-1+\theta}\Big(\frac{1}{2}s^{\beta+2}\langle\sigma\varphi'(y)\rangle\mathcal{E}(g_0) - \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)s^\beta}\langle\sigma\psi'(y)\rangle\Big) \\
&+ \frac{\sqrt{2}\nu\rho(\mu-r)(1-\gamma)}{(\gamma-1+\theta)s^{\beta/2}}\Big(\frac{1}{2}s^{\beta+2}\langle\frac{1}{\sigma}\varphi'(y)\rangle\mathcal{E}(g_0) - \frac{(\gamma-1)(\mu-r)^2}{2\gamma(\gamma-1+\theta)s^\beta}\langle\frac{1}{\sigma}\psi'(y)\rangle\Big) \\
&- \frac{\gamma\theta\bar{\sigma}^2 s^{\beta+2}}{\gamma-1+\theta} g_{0,s} g_{1,s} = 0.
\end{aligned}$$

It is not possible to solve Eq. (4.9) analytically. So, we will solve it numerically. ∎

5. Optimal strategies

In this section, we discuss asymptotic approximation of the solution to the portfolio selection problem under the SVCEV model with minimum-entropy robustness.

First, we write π_t^* in an asymptotic expansion form as

$$\pi_t^* = \pi_0^* + \sqrt{\epsilon}\pi_1^* + \cdots .$$

Substituting the expansions of π_t^* and g in Eq. (4.2) to Eq. (3.5), we obtain

$$\pi_0^* = \frac{(1-\gamma)(\mu-r)}{\gamma(1-\gamma-\theta)a\sigma^2 s^\beta x} - \frac{(1-\gamma)sg_{0,s}}{(1-\gamma-\theta)ax}, \tag{5.1}$$

and

$$\pi_1^* = -\frac{(1-\gamma)sg_{1,s}}{(1-\gamma-\theta)ax} - \frac{\sqrt{2}(1-\gamma)\rho\nu g_{2,y}}{(1-\gamma-\theta)a\sigma s^{\beta/2}x}, \tag{5.2}$$

respectively. Here, we call π_0^* the *leading order optimal strategy* and π_1^* the *first correction term*.

If the agent desires no robustness (or has complete faith in validity of the model), then $\theta = 0$. In addition, if σ is also a constant, then π_0^* becomes

$$\pi_0^* = \frac{\mu-r}{\gamma a\sigma^2 s^\beta x} - \frac{sg_{0,s}}{ax}, \tag{5.3}$$

which is consistent with the result in Theorem 5.2 of [18]. Further, if $\beta = 0$, then π_0^* in Eq. (5.3) corresponds to the result for the case in which the risky asset follows geometric Brownian motion, that is, under the Black-Scholes framework.

Note that formulas of π_0^* in Eq. (5.1) and π_1^* in Eq. (5.2) contain the σ-terms. In practice, the stochastic volatility level given by the hidden process $\{Y_t : t \geq 0\}$ is not directly observable. So, following the work of Yang et al. in [18] and [17], we derive trading strategies which do not depend upon the unobserved variable.

Theorem 5.1. *Under the SVCEV model with minimum-entropy robustness, the leading order optimal strategy π_0^* can take the practical form of*

$$\pi_0^* = \frac{(1-\gamma)(\mu-r)}{\gamma(1-\gamma-\theta)a\bar{\sigma}^2 s^\beta x} - \frac{(1-\gamma)sg_{0,s}}{(1-\gamma-\theta)ax}, \tag{5.4}$$

and the first correction term π_1^* can take the practical form of

$$\pi_1^* = -\frac{(1-\gamma)sg_{1,s}}{(1-\gamma-\theta)ax} - \frac{\sqrt{2}(1-\gamma)\rho\nu\langle\sigma g_{2,y}\rangle}{(1-\gamma-\theta)a\bar{\sigma}^2 s^{\beta/2}x}. \tag{5.5}$$

Proof. We substitute V, Ψ and $b(t) = 0$ into Eq. (3.2), the HJB equation becomes

$$g_t - \frac{1}{2}\gamma\delta^2 g_y^2 - \delta\rho\sigma s^{\frac{\beta}{2}+1}(\gamma g_s g_y - g_{sy}) + \mu s g_s - \frac{1}{2}\sigma^2 s^{\beta+2}(\gamma g_s^2 - g_{ss}) \tag{5.6}$$

$$+ \alpha\mathcal{L}_Y g + \max_{\pi_t}\bigg(-\frac{\gamma(1-\gamma-\theta)}{2(1-\gamma)}a^2\sigma^2 s^\beta x^2\pi_t^2$$
$$+ \Big((\mu-r)ax - \gamma a\sigma^2 s^{\beta+1}xg_s - \delta\rho\gamma a\sigma s^{\frac{\beta}{2}}xg_y\Big)\pi_t\bigg) = 0.$$

Substituting the expansion of g and π_t into Eq. (5.6) and considering the $\mathcal{O}(1)$-term, we can derive the following equation:

$$\mathcal{L}_Y g_2 + g_{0,t} + \max_{\pi_0}\bigg(-\frac{\gamma(1-\gamma-\theta)}{2(1-\gamma)}a^2\sigma^2 s^\beta x^2\pi_0^2 + (\mu-r)ax\pi_0 + \mathcal{A}^{\sigma,\pi_0}g_0\bigg) = 0,$$

where the operator $\mathcal{A}^{\sigma,\pi_0}$ is defined by

$$\mathcal{A}^{\sigma,\pi_0} := \mu s\partial_s - \frac{1}{2}\sigma^2 s^{\beta+2}(\gamma\partial_s^2 - \partial_{ss}) - \pi_0\gamma a\sigma^2 s^{\beta+1}x\partial_s,$$

and the centering condition leads to

$$g_{0,t} + \max_{\pi_0}\bigg\langle -\frac{\gamma(1-\gamma-\theta)}{2(1-\gamma)}a^2\sigma^2 s^\beta x^2\pi_0^2 + (\mu-r)ax\pi_0 + \mathcal{A}^{\sigma,\pi_0}g_0\bigg\rangle = 0.$$

The maximization part yields the leading order optimal strategy given in Eq. (5.4).

The $\mathcal{O}(\sqrt{\epsilon})$-term in Eq. (5.6) leads to the following equation:

(5.7)
$$\mathcal{L}_Y g_3 + g_{1,t} - \sqrt{2}\rho\nu\sigma s^{\frac{\beta}{2}+1}(\gamma g_{0,s}g_{2,y} - g_{2,sy}) - \frac{1}{2}\sigma^2 s^{\beta+2}(2\gamma g_{0,s}g_{1,s} - g_{1,ss})$$
$$+ \mu s g_{1,s} + \max_{\pi_0,\pi_1}\bigg(-\gamma a^2\sigma^2 s^\beta x^2\pi_0\pi_1 + \frac{\gamma\theta}{1-\gamma}a^2\sigma^2 s^\beta x^2\pi_0\pi_1$$
$$+ ((\mu-r)ax - \gamma a\sigma^2 s^{\beta+1}xg_{0,s})\pi_1$$
$$- \gamma a\sigma^2 s^{\beta+1}xg_{1,s}\pi_0 - \sqrt{2}\gamma\rho\nu a\sigma s^{\frac{\beta}{2}}xg_{2,y}\pi_0\bigg) = 0.$$

Note that Eq. (5.7) can be re-written as

(5.8)
$$\mathcal{L}_Y g_3 + g_{1,t} + \max_{\pi_0,\pi_1}\bigg((\mu-r)ax\pi_1 + \mathcal{A}^{\sigma,\pi_0}(g_0,g_1) + \mathcal{A}^{\sigma,\pi_0}(g_0,g_2) + \mathcal{A}^{\sigma,\pi_1}(g_0)$$
$$- \gamma a^2\sigma^2 s^\beta x^2\pi_0\pi_1 + \frac{\gamma\theta}{1-\gamma}a^2\sigma^2 s^\beta x^2\pi_0\pi_1\bigg) = 0,$$

where $\mathcal{A}^{\sigma,\pi_0}(g_0,g_1)$, $\mathcal{A}^{\sigma,\pi_0}(g_0,g_2)$ and $\mathcal{A}^{\sigma,\pi_1}(g_0)$ are defined by

$$\mathcal{A}^{\sigma,\pi_0}(g_0,g_1) := \mu s g_{1,s} + \frac{1}{2}\sigma^2 s^{\beta+2}g_{1,ss} - \gamma a\sigma^2 s^{\beta+1}xg_{1,s}\pi_0 - \gamma\sigma^2 s^{\beta+2}g_{0,s}g_{1,s},$$
$$\mathcal{A}^{\sigma,\pi_0}(g_0,g_2) := \sqrt{2}\rho\nu\sigma s^{\frac{\beta}{2}+1}(g_{2,sy} - \gamma g_{2,y}g_{0,s}) - \sqrt{2}\gamma\rho\nu a\sigma s^{\frac{\beta}{2}}xg_{2,y}\pi_0,$$
$$\mathcal{A}^{\sigma,\pi_1}(g_0) := -\gamma a\sigma^2 s^{\beta+1}xg_{0,s}\pi_1,$$

respectively. Similarly, the centering condition of Eq. (5.8) is equivalent to

$$g_{1,t} + \mu s g_{1,s} - \sqrt{2}\rho\nu\tilde{\sigma}s^{\frac{\beta}{2}+1}(\gamma g_{0,s}g_{2,y} - g_{2,sy}) - \frac{1}{2}\bar{\sigma}^2 s^{\beta+2}(2\gamma g_{0,s}g_{1,s} - g_{1,ss})$$
$$+ \max_{\pi_0,\pi_1}\Bigg(-a^2\bar{\sigma}^2 s^{\beta}x^2\gamma\pi_0\pi_1 + ax((\mu - r) - \bar{\sigma}^2 s^{\beta+1}\gamma g_{0,s})\pi_1 - \gamma a\bar{\sigma}^2 s^{\beta+1}xg_{1,s}\pi_0$$
$$- \sqrt{2}\gamma\rho\nu axs^{\frac{\beta}{2}}\langle\sigma g_{2,y}\rangle\pi_0 + \frac{\gamma\theta}{1-\gamma}a^2\bar{\sigma}^2 s^{\beta}x^2\pi_0\pi_1\Bigg) = 0,$$

where $\tilde{\sigma} := \langle\sigma\rangle$. Again, by working on the maximization part, we can obtain the first correction term π_1^* given by (5.5). □

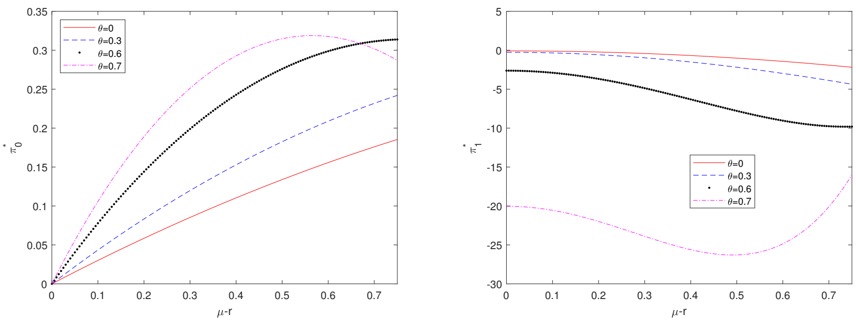
6. Numerical results

In this section, we conduct a numerical study to investigate the behaviour and sensitivity of the approximations π_0^* and π_1^* of the optimal strategy with respect to variations of parameter values. The values of parameters used in this section are given in Table 1, whenever they are required to be fixed.

TABLE 1. The role and numerical value of parameters.

Parameter	Role	Value
s	risky asset price	100
r	interest rate	0.05
μ	mean return of risky asset	0.12
α	mean reversion rate	10^4
t	initial time	0
T	terminal time	1.5
γ	utility coefficient	0.05
x	instaneous wealth	1000
ρ	correlation coefficient	0.5
δ	diffusion coefficient of OU process	0.03
θ	robust factor	0.1
β	elasticity of variance	-0.1

Note that in Eq. (5.5), the first correction term π_1^* requires the value of g_1, which is the solution to Eq. (4.9). Using the values of parameters involved in this PDE as given in Table 1, we use a finite difference method to solve this equation. After that, we are able to plot π_0^* and π_1^* against various parameters.



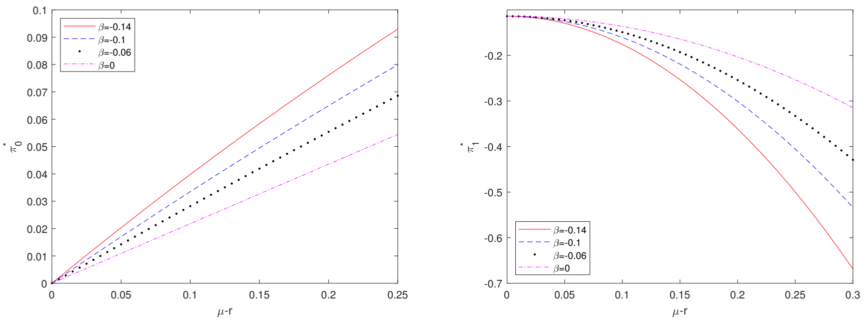
(A) (B)

FIGURE 1. Plot of π_0^* and π_1^* against $\mu - r$ for different values of θ

First, we consider how the leading order strategy and the first correction term, corresponding to different values of robustness parameter θ, vary with respect to the excess return $\mu - r$. As Figure 1 shows, for a fixed θ value, π_0^* increases monotonically as the excess return increases to some extent. After reaching its peak, π_0^* begins to descend, due to risk aversion effect. In contrast, for a fixed θ value, π_1^* decreases monotonically as the excess return increases to some extent. After reaching its trough, π_1^* begins to increase. We can image that π_1^* is the correction due to the OU process underneath, so that as excess return increases, the effect on optimal strategy from the underlying OU process $\{Y_t : t \geq 0\}$ also diminishes to some extent and then increases. Furthermore, when all other parameters are fixed, π_1^* decreases as θ increases. This indicates that more anxiety causes more negative proportion in the correction term.

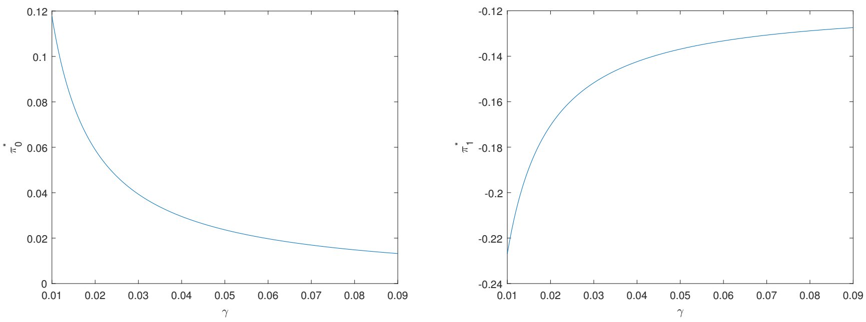
As mentioned previously, the case $\theta = 0$ corresponds to optimal portfolio strategies without considering robustness. As θ increases, there is more robustness from the model. When the excess return $\mu - r$ is less than 60% and all other parameters are fixed, π_0^* increases when θ increases. As π_0^* is the leading order strategy, this means that when $\mu - r$ is less than 60%, the portfolio weight allocated to risky asset increases. This finding is opposite to that result in [12] for the case of power utility, which says that the portfolio weight allocated to risky asset decreases as θ increase when $\mu - r$ is 6%, refer to page 967 of [12].

In Figures 2, 3 and 4, we set $\theta = 14$ as it is done in Maenhout [12]. In Figure 2, β has three selected negative values as well as 0. When $\beta = 0$ and α is large enough (e.g., $\alpha = 10^4$), our model in Eq. (2.2)-(2.3) can be regarded as an approximation of geometric Brownian motion, that is, the case under the Black-Scholes framework. In particular, π_0^* corresponds to Merton's optimal strategy in the Black-Scholes case and π_1^* is the first correction term perturbed by the OU process $\{Y_t : t \geq 0\}$. The choice of negative values of β is to reflect the leverage effect, where as the price of risky asset (e.g., a stock) increases the volatility decreases. For the chosen fixed value of risky asset price $s = 100$, a bigger value of β represents a bigger volatility, and thus less proportion in the leading term π_0^*. The correction term π_1^* is less negative as the volatility increases, because σ with the underlying stochastic process $\{Y_t : t \geq 0\}$ is relative less influential as $s^{\beta/2}$ increases. Figure 3 displays that the leading optimal strategy π_0^* decreases as γ increases in a way similar to that of an exponential decay graph; whereas π_1^* has a positive correlation with γ. Figure 4 shows a negative correlation of π_0^* to β, because π_0^* decreases as volatility increases. The reason for positive correlation between π_1^* and β is because that π_1^* becomes less significant as volatility increases.



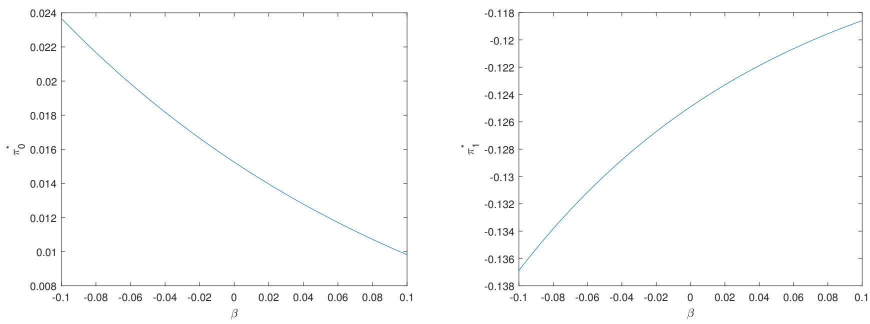
(A) (B)

FIGURE 2. Plot of π_0^* and π_1^* against $\mu - r$



(A) (B)

FIGURE 3. Plot of the leading-term optimal strategy and the first correction term against γ



(A) (B)

FIGURE 4. Plot of the leading-term optimal strategy and the first correction term against β

7. Conclusion

In this paper, we have investigated the optimal portfolio allocation problem under the SVCEV model with taking account of minimum-entropy robustness. The CARA utility function is implemented in particular. Applying an asymptotic approximation approach, we have derived the leading term optimal strategy and the first correction term for the problem. We have also undertook numerical experiments, and investigated how the leading optimal strategy and the first correction term vary with the excess return in terms of different values of the robustness parameter θ. After fixing the best calibrated value of θ, we have conducted sensitivity analysis on the leading term optimal strategy and the first correction term with respect to the elasticity parameter β and the CARA coefficient γ.

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