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A NOTE ON THE UNIQUENESS OF MEROMORPHIC FUNCTIONS SHARING A UNIQUE RANGE SETS IM

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Abstract. In this paper, we study the uniqueness of meromorphic functions sharing a unique range sets Ignoring multiplicities. This paper improves the result of Pulak sahoo and Anjan Sarkar [15].

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1. Introduction

Let f and g be two non-constant meromorphic functions defined in the complex plane $\mathbb{C}$. We adopt the standard notations of Nevanlinna value distribution theory as explained in [6] and [14]. For any non-constant meromorphic function f, the symbol $S(r,f)$ stands for any quantity satisfying $S(r,f) = o(T(r,f))$ as $r \to \infty$, possibly outside a set of finite linear measure. We denote by $\mathbb{M}(C)$ the class of meromorphic functions defined in $\mathbb{C}$ and by $\mathbb{M}_1(C)$ the class of meromorphic functions which have finitely many poles in $\mathbb{C}$.

For $a \in S(f) \cap S(g)$, we say that f and g share the function $a = a(z)$ CM (counting multiplicity) or IM (ignoring multiplicity) if $f - a$ and $g - a$ have the same set of zeros counting multiplicities or ignoring multiplicities respectively. We define,

$$\rho(f) = \limsup_{r\to\infty} \frac{log\ T(r,f)}{logr} \quad \text{and} \quad \Gamma(f) = \limsup_{r\to\infty} \frac{T(r,f)}{r^{\rho(f)}}$$

as *order* and *type* of f respectively.
Before presenting the outcome of our study we need the following definition.

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Definition 1.1. For a family of functions $\mathbb{G}$, the subsets $S_1, S_2, \cdots, S_q$ of $\mathbb{C} \cup \{\infty\}$ such that for any $f, g \in \mathbb{G}$, f and g share S_j CM for $j = 1, 2, \cdots, q$ imply $f \equiv g$, are called unique range sets (URS, in brief) for the functions in $\mathbb{G}$.

Definition 1.2. Let k be a positive integer and $S_1 = \{\alpha_1, \alpha_2, \cdots, \alpha_k\} \subset \mathbb{C}\backslash\{0\}$. Suppose that

$$P(z) = \frac{z^k - (\sum \alpha_i)z^{k-1} + \cdots + (-1)^{k-1}((\sum \alpha_{i_1}\alpha_{i_2} \cdots \alpha_{i_{k-1}})z}{(-1)^{k+1}\alpha_1\alpha_2 \cdots \alpha_k}. \tag{1}$$

where $\alpha_i \in S_1$ for $i = 1, 2, \cdots, k$. Let m_1 be the number of simple zeros and m_2 be the number of multiple zeros of $P(z)$. Then we define $\Gamma_1 := m_1 + m_2$ and $\Gamma_2 := m_1 + 2m_2$.

Definition 1.3. For $a \in \mathbb{C} \cup \{\infty\}$, we denote by $\overline{N}(r, a; f| = k)$ the reduced counting function of those $a-$points of f whose multiplicities are exactly k. In particular, $\overline{N}(r, a; f| = 1)$ or $N(r, a; f| = 1)$ is the counting function of the simple $a-$points of f.

Definition 1.4. For a positive integer k, we denote by $N(r, a; f| \leq k)(N(r, a; f| \geq k))$ the counting function of those $a-$point of f whose multiplicities are not greater(less) than k, where each $a-$point is counted according to its multiplicity. $\overline{N}(r, a; f| \leq k)$ and $\overline{N}(r, a; f| \geq k)$ are the corresponding reduced counting functions.

Definition 1.5. We denote by $N_2(r, a; f)$ the sum $\overline{N}(r, a; f) + \overline{N}(r, a; f| \geq 2)$.

Most of the research works related to set sharing problems was broadly initiated due to the following question raised by Gross [5].

Question 1.1. Can one find two finite sets $S_i (i = 1, 2)$ of $\mathbb{C} \cup \{\infty\}$ such that any two nonconstant entire functions f and g satisfying $E_f(S_i) = E_g(S_i)$ for $i = 1, 2$ must be identical ?

In 1994, Yi [17] gave an affirmative answer to the above question by proving the following theorem.

Theorem 1.1. Let $S_1 = \{\omega | \omega^n - 1 = 0\}$ and $S_2 = \{a\}$, where $n \geq 5$ is an integer, $a \neq 0$ and $a^{2n} \neq 1$. If f and g are entire functions such that $E_f(S_j) = E_g(S_j)$ for $j = 1, 2$ then $f \equiv g$.

In this direction, Yi and Yang [21] proved the following two theorems.

Theorem 1.2. Let $S_1 = \{\omega \mid \omega^n - 1 = 0\}$ and $S_2 = \{\infty\}$. Also, let f and g be two nonconstant meromorphic functions such that $E_f(S_1) = E_g(S_1)$ and $\overline{E}_f(S_2) = \overline{E}_g(S_2)$. If $n \geq 6$, then either $f = tg$ or $fg = s$, where $t^n = 1$, $s^n = 1$ and $0, \infty$ are lacunary values of f and g.

Theorem 1.3. Let S_1 and S_2 be defined as in Theorem 1.2. Also, let f, g be two non constant meromorphic functions such that $\overline{E}_f(S_1) = \overline{E}_g(S_1)$ and $E_f(S_2) = E_g(S_2)$. If $n \geq 10$, then the conclusion of theorem 1.2 hold.

In [5], Gross also expressed his quest about how large the sets can be if the answer of Question 1.1 is affirmative.
In 1998, Yi [19] proved the following theorem regarding to the above comment.

Theorem 1.4. Let $S_1 = \{0\}$ and $S_2 = \{\omega | w^2(\omega + a) - b = 0\}$, where a and b are two nonzero constants such that the algebraic equation $\omega^2(\omega + a) - b = 0$ has no multiple roots. If f and g are two entire functions satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2$, then $f \equiv g$.

In the last two decades, a lot of research works have been done in this direction(see [4, 9, 10, 16, 20]). We recall the following recent result due to Chen [2].

Theorem 1.5. Let k be a positive integer and let $S_1 = \{\alpha_1, \alpha_2, \cdots, \alpha_k\},\ S_2 = \{\beta_1, \beta_2\}$ where $\alpha_1, \alpha_2, \cdots, \alpha_k, \beta_1, \beta_2$ are $k + 2$ distinct finite complex numbers satisfying

$$(\beta_1 - \alpha_1)^2(\beta_1 - \alpha_2)^2 \cdots (\beta_1 - \alpha_k)^2 \neq (\beta_2 - \alpha_1)^2(\beta_2 - \alpha_2)^2 \cdots (\beta_2 - \alpha_k)^2.$$

If two nonconstant meromorphic functions f and g in $\mathbb{M}_1(\mathbb{C})$ share S_1 CM, S_2 IM and if the order of f is neither an integer nor infinite, then $f \equiv g$.

In [2], the author proved result concerning unique range sets which is defined as follows.

Theorem 1.6. Let k be a positive integer and let $S_1 = \{\alpha_1, \alpha_2, \cdots, \alpha_k\},\ S_2 = \{\beta_1, \beta_2\}$, where $\alpha_1, \alpha_2, \cdots, \alpha_k, \beta_1, \beta_2$ are $k + 2$ distinct finite complex numbers satisfying

$$(\beta_1 - \alpha_1)^2(\beta_1 - \alpha_2)^2 \cdots (\beta_1 - \alpha_k)^2 \neq (\beta_2 - \alpha_1)^2(\beta_2 - \alpha_2)^2 \cdots (\beta_2 - \alpha_k)^2.$$

If the order of f is neither an integer nor infinite, then the sets S_1 and S_2 are the URS of meromorphic functions in $\mathbb{M}_1(\mathbb{C})$.

The necessity of the condition $(\beta_1 - \alpha_1)^2(\beta_1 - \alpha_2)^2 \cdots (\beta_1 - \alpha_k)^2 \neq (\beta_2 - \alpha_1)^2(\beta_2 - \alpha_2)^2 \cdots (\beta_2 - \alpha_k)^2$ in Theorems 1.5 and 1.6 can be shown by the following example.

Example 1.1. [2] For a positive integer k, let $f(z) = \Sigma_{n=1}^{\infty} \frac{z^n}{n^{3n}}, g(z) = -f(z), S_1 = \{-1, 1, -2, 2, \cdots, -k, k\}$ and $S_2 = \{-(k + 1), k + 1\}$. Then using the result of [3](see p.288) we deduce

$$\lambda(f) = \frac{1}{lim\ inf_{n\to\infty} \frac{logn^{3n}}{nlogn}} = lim\ sup_{n\to\infty} \frac{nlogn}{logn^{3n}} = \frac{1}{3}.$$

Clearly $f(z), g(z) \in \mathbb{M}_1(\mathbb{C})$ and $f(z), g(z)$ share S_1, S_2 CM. But $f(z) \not\equiv g(z)$.
The next example shows that the assumption " non constant meromorphic functions f and g in $\mathbb{M}_1(\mathbb{C})$ " in Theorems 1.5 and 1.6 cannot be relaxed to " nonconstant meromorphic functions f and g in $\mathbb{M}(\mathbb{C})$".

Example 1.2. [2] For a positive integer k, let $f(z) = \Sigma_{n=1}^{\infty}\frac{z^n}{n^{3n}}, g(z) = \frac{1}{f(z)}, S_1 = \{2, \frac{1}{2}, 3, \frac{1}{3}, \cdots, k, \frac{1}{k}\}$ and $S_2 = \{k+1, \frac{1}{k+1}\}$. From Example 1.1 we note that $\lambda(f) = \frac{1}{3}$ and, therefore, using the result of [3](see p.293) we see that $g(z)$ has infinitely many poles in $\mathbb{C}$. Moreover, $f(z)$ and $g(z)$ share the sets S_1, S_2 CM. But $f(z) \not\equiv g(z)$.

The necessity of the assumption in Theorem 1.5 and 1.6 that the order of f is neither an integer nor infinite can be easily verified by the following example given in [2].

Example 1.3. For a positive integer k, let $f(z) = e^z$ (resp. $f(z) = e^{e^z}$), $g(z) = \frac{1}{f(z)}, S_1 = \{2, \frac{1}{2}, 3, \frac{1}{3}, \cdots, k, \frac{1}{k}\}$ and $S_2 = \{k+1, \frac{1}{k+1}\}$. Then using Lemma 2.6 in Section 2 we see that $\lambda(f) = 1$(resp. $\lambda(f) = \infty$). Though all other conditions of Theorems 1.5 and 1.6 are satisfied, $f(z) \not\equiv g(z)$.

Regarding Theorem 1.5, it is natural to ask the following question:

Question 1.2. Does the conclusion of Theorem 1.5 hold if f and g share both S_1 and S_2 IM instead of sharing S_1 CM and S_2 IM ?

In 2019, P. Sahoo and A. Sarkar [15] try to find out possible answers to the above question and prove the following theorems.

Theorem 1.7. Let $f, g \in \mathbb{M}_1(\mathbb{C})$ and $S_1 = \{\alpha_1, \alpha_2, \cdots, \alpha_k\}$, $S_2 = \{\beta_1, \beta_2\}$, where $\alpha_1, \alpha_2, \cdots, \alpha_k, \beta_1, \beta_2$ are $k+2$ distinct non zero complex constants satisfying $k > 2\Gamma_2 + 3\Gamma_1$. If f, g share S_1 and S_2 IM, then $f \equiv g$, provided

$$(\beta_1 - \alpha_1)^2(\beta_1 - \alpha_2)^2 \cdots (\beta_1 - \alpha_k)^2 \neq (\beta_2 - \alpha_1)^2(\beta_2 - \alpha_2)^2 \cdots (\beta_2 - \alpha_k)^2$$

and f is non-integer finite order.

Theorem 1.8. Let S_1 and S_2 be stated as in theorem 1.7 with $k > 2\Gamma_2 + 3\Gamma_1$. If $\mathbb{M}_2(\mathbb{C})$ denote the subclass of meromorphic functions of non-integer finite order in $\mathbb{M}_1(\mathbb{C})$ then the sets S_1 and S_2 are the URS of meromorphic functions in $\mathbb{M}_2(\mathbb{C})$, provided

$$(\beta_1 - \alpha_1)^2(\beta_1 - \alpha_2)^2 \cdots (\beta_1 - \alpha_k)^2 \neq (\beta_2 - \alpha_1)^2(\beta_2 - \alpha_2)^2 \cdots (\beta_2 - \alpha_k)^2.$$

2. Fundamental Lemmas

In this section, we present some lemmas which will be needed in the sequel. Let us define H as follows :

$$H = \left(\frac{F''}{F'} - \frac{2F'}{F-1}\right) - \left(\frac{G''}{G'} - \frac{2G''}{G-1}\right).$$

where F and G are two meromorphic functions in $\mathbb{M}(\mathbb{C})$.

Lemma 2.1. [13] Let f be a nonconstant meromorphic functions and $P(f) = a_0 + a_1 f + a_2 f^2 + \cdots + a_n f^n$, where $a_0, a_1, \cdots, a_n$ are constants and $a_n \neq 0$. Then $T(r, P(f)) = nT(r, f) + O(1)$.

Lemma 2.2. [18] If $H \equiv 0$, then $T(r,G) = T(r,F) + O(1)$. If, in addition,

$$\lim_{r\to\infty, r\notin E} \frac{\overline{N}(r,0;F) + \overline{N}(r,\infty;F) + \overline{N}(r,0;G) + \overline{N}(r,\infty;G)}{T(r)} < 1.$$

where $T(r) = max\{T(r,F), T(r,G)\}$, then either $F \equiv G$ or $F.G \equiv 1$.

Remark 2.1. We observe that the above lemma holds for $F, G \in \mathbb{M}(\mathbb{C})$. As our discussion is restricted in $\mathbb{M}_1(\mathbb{C})$, we may drop the terms $\overline{N}(r,\infty;F)$ and $\overline{N}(r,\infty;G)$ while using this result.

Lemma 2.3. Let $F, G \in \mathbb{M}_1(\mathbb{C})$. If F and G share 1 IM and $H \not\equiv 0$, then

i $T(r,F) \leq N_2(r,0;F) + N_2(r,0;G) + 2\overline{N}(r,0;F) + \overline{N}(r,0;G) + S(r,F) + S(r,G)$;

ii $T(r,G) \leq N_2(r,0;G) + N_2(r,0;F) + 2\overline{N}(r,0;G) + \overline{N}(r,0;F) + S(r,F) + S(r,G)$;

Proof. This lemma can easily be obtained from Lemma 2.14 of [1] by considering the terms $N_2(r,\infty;F), \overline{N}(r,\infty;F)$ as $S(r,F)$ and the terms $N_2(r,\infty;G), \overline{N}(r,\infty;G)$ as $S(r,G)$ since we are dealing with functions of class $\mathbb{M}_1(\mathbb{C})$ here. ☐

Lemma 2.4. Let $f, g \in \mathbb{M}_1(\mathbb{C})$. If f, g share the set $\{\beta_1, \beta_2\}$ IM, then $\lambda(f) = \lambda(g)$.

Proof. Proof of this lemma is very similar to the first part of the proof of Theorem 1.3 in [2] (see p. 1247). Hence we omit the details here. ☐

Lemma 2.5. (see [14], p.65) Let h be an entire function and $f(z) = e^{h(z)}$. Then

i if $h(z)$ is a polynomial of $deg\ h$, then $\lambda(f) = deg\ h$;

ii if $h(z)$ is a transcendental entire function, then $\lambda(f) = \infty$.

Lemma 2.6. (see [14], p. 115) Let a_1, a_2 and a_3 be three distinct complex numbers in $\mathbb{C} \cup \{\infty\}$. If two nonconstant meromorphic functions f and g share a_1, a_2 and a_3 CM, and if the order of f and g is neither an integer nor infinity, then $f \equiv g$.

3. Main results

Now we prove our Main results

Theorem 3.1. Let $f, g \in \mathbb{M}_1(\mathbb{C})$ and $S_1 = \{\alpha_1, \alpha_2, \cdots, \alpha_k\}$, $S_2 = \{\beta_1, \beta_2\}$, where $\alpha_1, \alpha_2, \cdots, \alpha_k, \beta_1, \beta_2$ are $k+2$ distinct non zero complex constants satisfying $k > 3l + 2\Gamma_2 + 3\Gamma_1 + 4 > (2\Gamma_1 + l)$. If f, g share S_1 and S_2 IM, then $f \equiv g$, provided

$$(\beta_1 - \alpha_1)^2(\beta_1 - \alpha_2)^2 \cdots (\beta_1 - \alpha_k)^2 \neq (\beta_2 - \alpha_1)^2(\beta_2 - \alpha_2)^2 \cdots (\beta_2 - \alpha_k)^2$$

and f is non-integer finite order.

Proof of Theorem 3.1. Let $F = f^l P(f)$ and $G = g^l P(g)$, where $P(z)$ is defined as in eqn (1). Clearly F, G share 1 IM as f, g share S_1 IM. From Lemma 2.1, we obtain

$$T(r,F) = (l+k)T(r,f) + S(r,f); \tag{2}$$

$$T(r,G) = (l+k)T(r,g) + S(r,g) \tag{3}$$

Let $H \not\equiv 0$. Then by Lemma 2.3, we have

$$\begin{aligned} T(r,F) \le\ & N_2(r,0;f^l) + N_2(r,0;P(f)) + 2\overline{N}(r,0;g^l) + N_2(r,0;P(g)) + \\ & 2\overline{N}(r,0;f^l) + 2\overline{N}(r,0;P(f)) + \overline{N}(r,0;g^l) + \overline{N}(r,0;P(g)) \\ & + S(r,f) + S(r,g). \end{aligned} \tag{4}$$

Now

$$N_2(r,0;P(f)) \le \Gamma_2 \overline{N}(r,0,f),$$
$$\overline{N}(r,0;P(f)) \le \Gamma_1 \overline{N}(r,0,f),$$
$$\begin{aligned} T(r,F) \le\ & (2l+2) + (\Gamma_2 + 2\Gamma_1)\overline{N}(r,0;f) + 2l\overline{N}(r,0;g) + (\Gamma_2 + \Gamma_1)\overline{N}(r,0;g) + \\ & 2\overline{N}(r,0;f) + 2\overline{N}(r,0;g) + S(r,f) + S(r,g). \end{aligned}$$

Substituting these values in 4, we get

$$\begin{aligned} T(r,F) \le\ & \{(2l+2) + (\Gamma_2 + 2\Gamma_1)\}T(r,f) + \{(2l+1) + (\Gamma_2 + \Gamma_1)\}T(r,g) \\ & + S(r,f) + S(r,g). \end{aligned} \tag{5}$$

Similarly,

$$\begin{aligned} T(r,G) \le\ & \{(2l+2) + (\Gamma_2 + 2\Gamma_1)\}T(r,g) + \{(2l+1) + (\Gamma_2 + \Gamma_1)\}T(r,f) \\ & + S(r,f) + S(r,g). \end{aligned} \tag{6}$$

From $(2\breve{\ })$,(3),(5) and (6), we obtain

$$(l+k)\{T(r,f)+T(r,g)\} \le (4l+3+2\Gamma_2+3\Gamma_1)\{T(r,f)+T(r,g)\}+S(r,f)+S(r,g),$$

which is a contradiction as $k > 3l + 2\Gamma_2 + 3\Gamma_1 + 3$. Hence $H \equiv 0$.
Let $T(r) = max\{T(r,F), T(r,G)\}$. Now

$$\begin{aligned} \overline{N}(r,0;F) + \overline{N}(r,0;G) &\le (\Gamma_1+1)\overline{N}(r,0;f) + (\Gamma_1+1)\overline{N}(r,0;g) \\ &\le (\Gamma_1+1)\{T(r,f) + T(r,g)\} + S(r,f) + S(r,g) \\ &= \frac{(\Gamma_1+1)}{(l+k)}\{T(r,F) + T(r,G)\} + S(r,F) + S(r,G) \\ &\le \frac{2\Gamma_1+1}{l+k}T(r) + o\{T(r)\}. \end{aligned}$$

As $k > 3l + 2\Gamma_2 + 3\Gamma_1 + 4 > (2\Gamma_1 + 1)$, using Lemma 2.2 we obtain either $F \equiv G$ or $F.G \equiv 1$. Let $F.G \equiv 1$. Then $P(f).P(g) \equiv 1$. As $g \in \mathbb{M}_1(\mathbb{C})$, we have $P(g) \in \mathbb{M}_1(\mathbb{C})$. Hence $P(f)$ has at most finitely many zeros. Therefore $P(f) = \eta_1(z)e^{\varsigma_1(z)}$, where $\eta_1(z)$ is a rational function and $\varsigma_1(z)$ is an entire function, which is a contradiction by Lemma 2.5 as the order of f is neither an integer not infinity. Similarly, if we consider the case when $P(g)$ has at most finitely many zeros, we arrive at a contradiction as $\lambda(g) = \lambda(f)$, by Lemma 2.4.

Hence the case $F.G \equiv 1$ can not occur.
If $F \equiv G$, then we have $P(f) \equiv P(g)$, which gives $f^l P(f) \equiv g^l P(g)$ for $l = 0$, we have

$$\frac{(f(z)-\alpha_1)(f(z)-\alpha_2)\cdots(f(z)-\alpha_k)}{(g(z)-\alpha_1)(g(z)-\alpha_2)\cdots(g(z)-\alpha_k)} \equiv 1. \tag{7}$$

From (7) and the assumption

$$(\beta_1-\alpha_1)^2(\beta_1-\alpha_2)^2\cdots(\beta_1-\alpha_k)^2 \neq (\beta_2-\alpha_1)^2(\beta_2-\alpha_2)^2\cdots(\beta_2-\alpha_k)^2,$$

we obtain that $f(z) = \beta_1$ if and only if $g(z) = \beta_1$ since f and g share S_2 IM. Similarly, we see that $f(z) = \beta_2$ if and only if $g(z) = \beta_2$. Consequently, we have f and g share β_1 and β_2 IM. Again, from 7 we see that f and g share β_1, β_2 and ∞ CM. Noting that the order of f is neither an integer nor infinity, the conclusion of the theorem follows from Lemma 2.4 and Lemma 2.6.

Theorem 3.2. Let S_1 and S_2 be stated as in theorem 3.1 with $k > 3l + 2\Gamma_2 + 3\Gamma_1 + 4 > (2\Gamma_1 + l)$. If $\mathbb{M}_2(\mathbb{C})$ denote the subclass of meromorphic functions of non-integer finite order in $\mathbb{M}_1(\mathbb{C})$ then the sets S_1 and S_2 are the URS of meromorphic functions in $\mathbb{M}_2(\mathbb{C})$, provided

$$(\beta_1-\alpha_1)^2(\beta_1-\alpha_2)^2\cdots(\beta_1-\alpha_k)^2 \neq (\beta_2-\alpha_1)^2(\beta_2-\alpha_2)^2\cdots(\beta_2-\alpha_k)^2.$$

Proof of Theorem 3.2 If f, g share S_1 and S_2 CM, then f, g certainly share S_1 and S_2 IM, which satisfies the conditions of Theorem 3.1 and hence the conclusion follows. Here we omit the details.

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