

## SOME RESULTS ON CENTRALIZERS OF SEMIPRIME RINGS

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ABSTRACT. The objective of this research paper is to prove that an additive mapping  $T$  from a semiprime ring  $R$  to itself will be centralizer having a suitable torsion restriction on  $R$  if it satisfy any one of the following algebraic equations

- (a)  $2T(x^n y^n x^n) = T(x^n) y^n x^n + x^n y^n T(x^n)$
- (b)  $3T(x^n y^n x^n) = T(x^n) y^n x^n + x^n T(y^n) x^n + x^n y^n T(x^n)$  for every  $x, y \in R$ .

Further, few extensions of these results are also presented in the framework of  $*$ -ring.

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### 1. Introduction

Throughout the present paper  $R$  will denote an associative ring with identity element. A ring  $R$  is said to be  $t$ -torsion free, if  $tx = 0$  implies  $x = 0$  for all  $x$  in  $R$ ,  $n$  is strictly greater than 1. A ring  $R$  is prime if  $xRy = \{0\}$  implies  $x = 0$  or  $y = 0$ , and particularly a ring  $R$  is said to be semiprime if it satisfies the condition that  $xRx = \{0\}$  implies  $x = 0$ . Helgosen [3] initiated to work on centralizers in Banach algebras. The concept of centralizers is also known multipliers (see [11]). Further, Wang [10] studied the concept of centralizers on commutative Banach algebras. Johnson also studied the centralizers on topological algebras and he has presented continuity of centralizers on Banach algebras (for the reference, see [6, 7]). Then, Johnson introduced the concept of centralizers in certain classes of rings. Following [5], any mapping  $T : R \rightarrow R$  is known as left centralizer if it satisfies two conditions: (i)  $T(x + y) = T(x) + T(y)$  and (ii)  $T(xy) = T(x)y$  and  $T$  is said to be right centralizer if  $T$  satisfies (i) and  $T(xy) = xT(y)$  for every pairs  $x, y \in R$ . If  $T$  is left as well as right centralizer, then  $T$  is known as centralizer. Particularly, additive mapping  $T$  is a Jordan left centralizer

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and Jordan right centralizer if  $T$  satisfies  $T(x^2) = T(x)x$  and  $T(x^2) = xT(x)$  respectively for each  $x \in R$ , and if  $T$  is both, then it is Jordan centralizer. Following the Theorem 2.3.2 in [2], if  $T$  from a semiprime ring  $R$  to itself is a centralizer, then there exists an element  $\lambda \in \mathcal{C}$  such that  $T(x) = \lambda x$  for all  $x \in R$ , where  $\mathcal{C}$  is extended centroid on  $R$ . Zalar [12] has given a remarkable result which says that any Jordan centralizer on a 2-torsion free semiprime ring is a centralizer. Later, Vukman [8] has presented a striking result which declares that any additive mapping  $T$  from  $R$  to itself is a centralizer if  $T$  satisfies the algebraic equation  $2T(x^2) = xT(x) + T(x)x$  on  $R$ , where  $R$  is 2-torsion free semiprime ring. Further, Vukman and Kosi Ulbl [9] established a result by taking algebraic equation  $3T(xy x) = T(x)yx + xT(y)x + xyT(x)$ . In fact they proved that any additive mapping  $T$  from 2-torsion free semiprime  $R$  to itself will be of the type  $T(x) = \lambda x$ , where  $\lambda \in \mathcal{C}$  if it satisfies the algebraic equation  $3T(xy x) = T(x)yx + xT(y)x + xyT(x)$  for each pair  $x, y \in R$ . Inspired by these results, we have extended the above cited results. More precisely, it is proved that: if  $R$  is a semiprime ring with  $n!$ -torsion restriction and  $T$  from  $R$  to itself is an additive mapping which satisfies  $2T(x^n y^n x^n) = T(x^n)y^n x^n + x^n y^n T(x^n)$  or  $3T(x^n y^n x^n) = T(x^n)y^n x^n + x^n T(y^n)x^n + x^n y^n T(x^n)$  for each  $x, y \in R$ , then  $T$  will be centralizer on  $R$ , where  $n \geq 1$  be any fixed integer. To achieve the final conclusion of the main theorems, we need the following result due to Vukman:

**Lemma 1.1** ([8, Theorem 1]). *Let  $R$  be a 2 torsion free semiprime ring and  $T : R \rightarrow R$  be an additive mapping satisfying the condition  $2T(x^2) = T(x)x + xT(x)$  for all  $x \in R$ , then  $T$  is a centralizer on  $R$ .*

## 2. Main results

**Theorem 2.1.** *Suppose that  $R$  is a  $n!$ -torsion free semiprime ring and  $n \geq 1$  be a fixed integer. If  $T : R \rightarrow R$  is an additive mapping which satisfies  $2T(x^n y^n x^n) = T(x^n)y^n x^n + x^n y^n T(x^n)$  for all  $x, y \in R$ , then  $T$  will be a centralizer on  $R$ .*

*Proof.* For all  $x, y \in R$ , we have given that

$$2T(x^n y^n x^n) = T(x^n)y^n x^n + x^n y^n T(x^n). \quad (1)$$

Replace  $x$  by  $e$  in (1) to obtain the following

$$2T(y^n) = T(e)y^n + y^n T(e) \quad \forall y \in R. \quad (2)$$

Replacing  $y$  by  $e$  in (1), to find

$$2T(x^{2n}) = T(x^n)x^n + x^n T(x^n) \quad \forall x \in R. \quad (3)$$

Next, put  $y + e$  in place of  $y$  in (2), we find

$$\sum_{i=0}^n {}^n C_i [2T(y^{n-i}) - T(e)y^{n-i} - y^{n-i}T(e)] = 0 \quad \forall y \in R. \quad (4)$$

Substituting  $ky$  for  $y$ , we get

$$\sum_{i=0}^n {}^n C_i k^{n-i} [2T(y^{n-i}) - y^{n-i}T(e) - T(e)y^{n-i}] = 0 \quad \forall y \in R. \quad (5)$$

Putting  $k = 1, 2, 3, \dots, n-1$  one by one and recognize the subsequent homogeneous system of  $n-1$  linear equations to obtain a Vander Monde matrix

$$\mathcal{V} = \begin{pmatrix} 1^1 & 1^2 & \dots & 1^{n-1} \\ 2^1 & 2^2 & \dots & 2^{n-1} \\ \vdots & \vdots & \dots & \vdots \\ (n-1)^1 & (n-1)^2 & \dots & (n-1)^{n-1} \end{pmatrix}.$$

Since  $|\mathcal{V}|$  is equal to the product of positive integers and every element of that is smaller than  $n-1$ , which yields that  ${}^n C_i [2T(y^{n-i}) - T(e)y^{n-i} - y^{n-i}T(e)] = 0$  for every  $y \in R$ . Particularly, putting  $i = n-1$  and use torsion restriction on  $R$  to find

$$2T(y) = yT(e) + T(e)y \quad \forall y \in R. \quad (6)$$

Now, replacing  $x$  by  $x + ke$  in equation (3), we get

$$\begin{aligned} & 2 \sum_{i=0}^{2n} {}^{2n} C_i T(x^{2n-i}(ke)^i) \\ &= \left[ \sum_{i=0}^n {}^n C_i T(x^{n-i}(ke)^i) \right] \left[ \sum_{i=0}^n {}^n C_i (x^{n-i}(ke)^i) \right] \\ &+ \left[ \sum_{i=0}^n {}^n C_i (x^{n-i}(ke)^i) \right] \left[ \sum_{i=0}^n {}^n C_i T(x^{n-i}(ke)^i) \right] \quad \forall x \in R. \end{aligned} \quad (7)$$

Rearranging the terms of  $k^i$  for all  $i = 1, 2, 3, \dots, 2n-1$ , we obtain

$$\begin{aligned} & k[2^{2n} C_1 T(x^{2n-1}) - {}^n C_1 T(x^n)x^{n-1} - {}^n C_1 T(x^{n-1})x^n - {}^n C_1 x^{n-1}T(x^n) \\ & - {}^n C_1 x^n T(x^{n-1})] + k^2[2^{2n} C_2 T(x^{2n-2}) - {}^n C_2 T(x^n)x^{n-2} - {}^n C_2 T(x^{n-1})x^{n-1} \\ & - {}^n C_2 x^{n-2}T(x^n) - {}^n C_2 x^{n-1}T(x^{n-1})] + \dots + k^{2n-2}[2^{2n} C_{2n-2} T(x^2) - {}^n C_{n-2} T(x)x \\ & - {}^n C_{n-2} T(e)x^2 - {}^n C_{n-2} xT(x) - {}^n C_{n-2} x^2 T(e)] + k^{2n-1}[2^{2n} C_{2n-1} T(x) \\ & - {}^n C_{n-1} T(x) - {}^n C_{n-1} T(e)x - {}^n C_{n-1} T(x) - {}^n C_{n-1} xT(e)] = 0 \end{aligned} \quad (8)$$

Again, replacing  $k = 1, 2, 3, \dots, 2n-2$  one by one and using the same interpretation as done earlier, we get a homogeneous system of  $2n-2$  linear equations with trivial solution. Hence all co-efficients of  $k^i$  are equal to zero. In particular,

$$\begin{aligned} 2^{2n} C_{2n-2} T(x^2) &= {}^n C_{n-2} T(x)x + {}^n C_{n-2} T(e)x^2 \\ &+ {}^n C_{n-2} xT(x) + {}^n C_{n-2} x^2 T(e) \quad \forall x \in R. \end{aligned} \quad (9)$$

Replace  $y$  by  $x^2$  to get

$$2T(x^2) = T(e)x^2 + x^2 T(e) \quad \forall x \in R. \quad (10)$$

Using (10) together with torsion restriction on  $R$  in (9), we obtain  $2T(x^2) = T(x)x + xT(x) \quad \forall x \in R$ . Hence, from Lemma 1.1, we obtain the required result.

**Theorem 2.2.** *Let  $R$  be a  $n!$ -torsion free semiprime ring and  $n \geq 1$  be any fixed integer. If  $T : R \rightarrow R$  is an additive mapping which satisfies  $3T(x^n y^n x^n) =$*

$T(x^n)y^n x^n + x^n T(y^n)x^n + x^n y^n T(x^n)$  for every  $x, y \in R$ , then  $T$  will be a centralizer on  $R$ .

*Proof.* Since

$$3T(x^n y^n x^n) = T(x^n)y^n x^n + x^n T(y^n)x^n + x^n y^n T(x^n) \quad \forall x, y \in R, \quad (11)$$

then, putting  $e$  for  $x$  in (11), we find

$$3T(y^n) = T(e)y^n + T(y^n) + y^n T(e) \quad \forall y \in R. \quad (12)$$

Substitute  $e$  by  $y$  in (11) to obtain

$$3T(x^{2n}) = T(x^n)x^n + x^n T(e)x^n + x^n T(x^n) \quad \forall x \in R. \quad (13)$$

Next, replacing  $y$  by  $y + e$  in equation (12), we have

$$\sum_{i=0}^n {}^n C_i [3T(y^{n-i}) - y^{n-i}T(e) - T(e)y^{n-i} - T(y^{n-i})] = 0 \quad \forall y \in R. \quad (14)$$

Replacing  $y$  by  $ky$ , we obtain

$$\sum_{i=0}^n {}^n C_i k^{n-i} [3T(y^{n-i}) - y^{n-i}T(e) - T(e)y^{n-i} - T(y^{n-i})] = 0 \quad \forall y \in R. \quad (15)$$

Putting  $k = 1, 2, 3, \dots, n-1$  one by one and using the same arguments as did earlier, we have a homogeneous system of  $n-1$  linear equations with trivial solution. Hence all co-efficients of  $k^i$  are equal to zero. Which yields that  ${}^n C_i k^{n-i} [3T(y^{n-i}) - y^{n-i}T(e) - T(e)y^{n-i} - T(y^{n-i})] = 0$  for every  $y \in R$ . Particularly, replace  $i = n-1$ , we obtain  $n[3T(y) - T(e)y - T(y) - yT(e)] = 0$  for all  $y \in R$ . Using torsion restriction on  $R$ , we obtain

$$2T(y) = yT(e) + T(e)y, \quad \forall y \in R. \quad (16)$$

Now, replacing  $x$  by  $x + ke$  in equation (13), we get

$$\begin{aligned} & 3 \sum_{i=0}^{2n} {}^{2n} C_i T(x^{2n-i}(ke)^i) \\ &= \left[ \sum_{i=0}^n {}^n C_i T(x^{n-i}(ke)^i) \right] \left[ \sum_{i=0}^n {}^n C_i (x^{n-i}(ke)^i) \right] \\ &+ \left[ \sum_{i=0}^n {}^n C_i (x^{n-i}(ke)^i) \right] T(e) \left[ \sum_{i=0}^n {}^n C_i (x^{n-i}(ke)^i) \right] \\ &+ \left[ \sum_{i=0}^n {}^n C_i (x^{n-i}(ke)^i) \right] \left[ \sum_{i=0}^n {}^n C_i T(x^{n-i}(ke)^i) \right], \quad \forall x \in R \end{aligned} \quad (17)$$

Reshuffling the terms of  $k^i$  for all  $i = 1, 2, 3, \dots, 2n-1$ , we obtain

$$\begin{aligned} & k[3^{2n} C_1 T(x^{2n-1}) - {}^n C_1 T(x^n)x^{n-1} - {}^n C_1 T(x^{n-1})x^n - {}^n C_1 x^n T(e)x^{n-1} \\ & - {}^n C_1 x T(e)x^n - {}^n C_1 x^{n-1} T(x^n) - {}^n C_1 x^n T(x^{n-1})] + \dots + k^{2n-2} [3^{2n} C_{2n-2} T(x^2) \\ & - {}^n C_{n-2} T(x)x - {}^n C_{n-2} T(e)x^2 - {}^n C_{n-1} {}^n C_{n-1} x T(e)x - {}^n C_{n-2} x T(x) \\ & - {}^n C_{n-2} x^2 T(e)] + k^{2n-1} [3^{2n} C_{2n-1} T(x) - {}^n C_{n-1} T(x) - {}^n C_{n-1} T(e)x \\ & - {}^n C_{n-1} x T(e) - {}^n C_{n-1} T(x) - {}^n C_{n-1} T(e)x - {}^n C_{n-1} x T(e)] = 0 \end{aligned} \quad (18)$$

Replacing  $k = 1, 2, 3, \dots, 2n - 2$  in tern and using the same algebraic arguments as done earlier we get that all co-efficient of  $k^i$  is equal to zero. Particularly,

$$3^{2n}C_{2n-2}T(x^2) - {}^nC_{n-2}T(x)x - {}^nC_{n-2}T(e)x^2 - {}^nC_{n-1}{}^nC_{n-1}xT(e)x - {}^nC_{n-2}xT(x) - {}^nC_{n-2}x^2T(e) = 0, \forall x \in R \quad (19)$$

Replacing  $y$  by  $x$  and  $y$  by  $x^2$  in (16), we find the following two equations

$$2T(x^2) = T(e)x^2 + x^2T(e), \forall x \in R. \quad (20)$$

$$2T(x) = T(e)x + xT(e), \forall x \in R. \quad (21)$$

Multiplying from left side by  $x$  and from right side by  $x$  one by one to (21), we have

$$2xT(x) = xT(e)x + x^2T(e), \forall x \in R. \quad (22)$$

$$2T(x)x = T(e)x^x + xT(e)x, \forall x \in R. \quad (23)$$

Adding the above two equations and using (20), we find

$$xT(e)x = xT(x) + T(x)x - T(x^2), \forall x \in R. \quad (24)$$

Using (20) and (24) in (19) and torsion restriction on  $R$ , we get  $2T(x^2) = T(x)x + xT(x)$  for all  $x \in R$ . Therefore by Lemma 1.1, we reached the desired conclusion.

Example 2.1 shows that the above theorems are not insignificant.

**Example 2.1.** Let  $R = \left\{ \begin{pmatrix} r & s \\ 0 & t \end{pmatrix} \mid r, s, t \in 2\mathbb{Z}_8 \right\}$  is a ring under matrix addition and matrix multiplication, where  $\mathbb{Z}_8$  denotes the ring of integers addition and multiplication modulo 8. Define mapping  $T : R \rightarrow R$  by  $T \left[ \begin{pmatrix} r & s \\ 0 & t \end{pmatrix} \right] = \begin{pmatrix} 0 & s \\ 0 & 0 \end{pmatrix}$ . One can easily see that  $R$  is not a 2-torsion free semiprime ring and  $T$  satisfy the algebraic identities (1) and (11) but  $T$  is not a centralizer on  $R$ , hence semiprimeness hypothesis is crucial for Theorem 2.1 and Theorem 2.2.

### 3. Results on involution

Before going the main results of this section, we fix some basic definitions and notions. An additive mapping  $*$  from  $R$  to itself is known as involution if it fascinate 2 conditions:  $(ab)^* = b^*a^*$  and  $(a^*)^* = a$  for every  $a, b \in R$ . A ring together with an involution  $*$  is said to be a  $*$ -ring (also known as ring with involution). An additive mapping  $T$  from  $R$  to itself is said to be a left  $*$ -centralizer if for all  $x, y \in R$ ,  $T$  satisfies  $T(xy) = T(x)y^*$  and  $T$  is said to be right  $*$ -centralizer if  $T(xy) = x^*T(y)$ . An additive mapping  $T$  from  $R$  to itself is known as  $*$ -centralizer if it is left  $*$ -centralizer along with right  $*$ -centralizer on  $R$ . Particularly,  $T$  is left Jordan  $*$ -centralizer and right Jordan  $*$ -centralizer if  $T$  satisfies  $T(x^2) = T(x)x^*$  and  $T(x^2) = x^*T(x)$  respectively. If  $T$  is both, then

it is known as Jordan  $*$ -centralizer. If  $T$  is  $*$ -centralizers on  $R$ , then obviously for all  $x, y \in R$ ,  $T$  satisfies  $2T(x^n y^n x^n) = T(x^n)(y^*)^n(x^*)^n + (x^*)^n(y^*)^n T(x^n)$  and  $3T(x^n y^n x^n) = T(x^n)(y^*)^n(x^*)^n + (x^*)^n T(y^n)(x^*)^n + (x^*)^n(y^*)^n T(x^n)$  but the converse of this statement does not hold generally. So, it legitimate to ask whether an additive mapping  $T$  from  $R$  to itself satisfying the above two algebraic conditions, will be a  $*$ -centralizer on  $R$ . The response of this mathematical sentence is in positive sense. So, this paper accord with the investigation of this result. Indeed, it is proved that an additive mapping  $T$  from a semiprime  $*$ -ring  $R$  to itself satisfying  $2T(x^n y^n x^n) = T(x^n)(y^*)^n(x^*)^n + (x^*)^n(y^*)^n T(x^n)$  or  $3T(x^n y^n x^n) = T(x^n)(y^*)^n(x^*)^n + (x^*)^n T(y^n)(x^*)^n + (x^*)^n(y^*)^n T(x^n)$  with suitable torsion restriction, will be a  $*$ -centralizer of  $R$ . To complete the proof of this result, we use the following result due to Ashraf and Mozumder.

**Lemma 3.1** ([1, Corollary 2.1]). *Any additive mapping  $T$  from a 2-torsion free semiprime  $*$ -ring  $R$  to itself is a  $*$ -centralizer, if it satisfies the condition  $2T(x^2) = T(x)x^* + x^*T(x)$  for each  $x \in R$ .*

Let us start main result of this part from the following theorem:

**Theorem 3.2.** *Any additive mapping  $T$  from a  $n!$ -torsion free semiprime  $*$ -ring  $R$  to itself is a  $*$ -centralizer, if it satisfies the algebraic condition  $2T(x^n y^n x^n) = T(x^n)(y^*)^n(x^*)^n + (x^*)^n(y^*)^n T(x^n)$ ,  $\forall x, y \in R$ , where  $n \geq 1$  be a fixed integer.*

*Proof.* Suppose that  $S : R \rightarrow R$  is a mapping which is defined as  $S(x) = T(x^*)$ ,  $\forall x \in R$ . It is clear that  $S$  is an additive mapping on  $R$ . Now, consider

$$\begin{aligned} 2S(x^n y^n x^n) &= 2T((x^n y^n x^n)^*) \\ &= 2T[(x^*)^n (y^*)^n (x^*)^n] \\ &= T(x^*)^n y^n x^n + x^n y^n T(x^*)^n \\ &= S(x^n) y^n x^n + x^n y^n S(x)^n \text{ for all } x, y \in R. \end{aligned}$$

Using Theorem 2.1, we obtain that  $S$  will be a centralizer on  $R$ . Hence,  $S(xy) = xS(y) = S(x)y$ ,  $\forall x, y \in R$ . Which yields that  $T(x^*)^2 = xT(x^*) = T(x^*)x$ ,  $\forall x \in R$ . Now, replacing  $x$  by  $x^*$  and using Lemma 3.1, we get required result.

**Theorem 3.3.** *Any additive mapping  $T$  from a  $n!$ -torsion free semiprime  $*$ -ring  $R$  to itself is a  $*$ -centralizer, if it satisfies the algebraic condition  $3T(x^n y^n x^n) = T(x^n)(y^*)^n(x^*)^n + (x^*)^n T(y^n)(x^*)^n + (x^*)^n(y^*)^n T(x^n)$ ,  $\forall x, y \in R$ , where  $n \geq 1$  be a fixed integer.*

*Proof.* Consider a mapping  $S$  from  $R$  to itself defined as  $S(x) = T(x^*)$ ,  $\forall x \in R$ . One can easily see that  $S$  is an additive mapping. Now, consider

$$\begin{aligned} 3S(x^n y^n x^n) &= 3T((x^n y^n x^n)^*) \\ &= 3T[(x^*)^n (y^*)^n (x^*)^n] \\ &= T(x^*)^n y^n x^n + x^n T(y^*)^n x^n + x^n y^n T(x^*)^n \\ &= S(x^n) y^n x^n + x^n T(y^n) x^n + x^n y^n S(x)^n, \forall x, y \in R. \end{aligned}$$

Using Theorem 2.2, we find that  $S$  will be a centralizer. Hence,  $S(xy) = xS(y) = S(x)y$ ,  $\forall x, y \in R$ , which gives that  $T(x^*)^2 = xT(x^*) = T(x^*)x$ ,  $\forall x \in R$ . Substituting  $x^*$  for  $x$  and applying Lemma 3.1, we arrive at our conclusion.

**Example 3.1.** Let  $R = \left\{ \begin{pmatrix} r & s \\ 0 & t \end{pmatrix} \mid r, s, t \in 2\mathbb{Z}_8 \right\}$  is a ring with involution from  $R$  to itself by  $\begin{pmatrix} r & s \\ 0 & t \end{pmatrix}^* = \begin{pmatrix} t & -s \\ 0 & r \end{pmatrix}$  for all  $r, s, t \in 2\mathbb{Z}_8$  under matrix addition and matrix multiplication, where  $\mathbb{Z}_8$  has its usual notation. Consider a mapping  $T : R \rightarrow R$  defined by  $T \left[ \begin{pmatrix} r & s \\ 0 & t \end{pmatrix} \right] = \begin{pmatrix} 0 & s \\ 0 & 0 \end{pmatrix}$  for all  $r, s, t \in 2\mathbb{Z}_8$ . It is clear that  $T$  satisfy the identities of Theorem 3.2 and Theorem 3.3 and  $R$  is neither a 2-torsion free semiprime ring nor  $T$  is a centralizer on  $R$ , hence semiprimeness hypothesis is crucial for the above theorems.

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