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POSITIVE IMPLICATIVE MBJ-NEUTROSOPHIC IDEALS IN BCK-ALGEBRAS

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Abstract. The notion of positive implicative MBJ-neutosophic ideal of BCK-algebras is defined and some properties of it are investigated. Relations between positive implicative MBJ-neutrosophic ideal and positive implicative ideal are discussed. In a BCK-algebra, the extension property for positive implicative MBJ-neutrosophic ideal is established.

1. Introduction

In order to handle uncertainties in many real applications, the fuzzy set was introduced by L.A. Zadeh [22] in 1965. The intuitionistic fuzzy set on a universe X was introduced by K. Atanassov in 1983 as a generalization of fuzzy set. As a more general platform that extends the notions of classic set, (intuitionistic) fuzzy set and interval valued (intuitionistic) fuzzy set, the notion of neutrosophic set is developed by Smarandache ([15]–[17]). Neutrosophic algebraic structures in BCK/BCI-algebras are discussed in the papers [1]–[18]. In [20], the notion of MBJ-neutrosophic sets is introduced as another generalization of neutrosophic set, it is applied to BCK/BCI-algebras. Mohseni et al. [20] introduced the concept of MBJ-neutrosophic subalgebras in BCK/BCI-algebras, and investigated related properties. They gave a characterization of MBJ-neutrosophic subalgebra, and established a new MBJ-neutrosophic subalgebra by using an MBJ-neutrosophic subalgebra of a BCI-algebra. They considered the homomorphic inverse image of MBJ-neutrosophic subalgebra, and discussed translation of MBJ-neutrosophic subalgebra. Jun and Roh [9] applied the notion of MBJ-neutrosophic sets to ideals of BCK/BI-algebras. They introduced the concept of MBJ-neutrosophic ideals in BCK/BCI-algebras, and investigated several properties. They provided a condition for an MBJ-neutrosophic subalgebra to be an MBJ-neutrosophic ideal in a BCK-algebra. They provided conditions for an MBJ-neutrosophic set to be an MBJ-neutrosophic ideal in a

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BCK/BCI-algebra. They discussed relations between MBJ-neutrosophic subalgebras, MBJ-neutrosophic $\circ$-subalgebras and MBJ-neutrosophic ideals. In a BCI-algebra, they provided conditions for an MBJ-neutrosophic ideal to be an MBJ-neutrosophic subalgebra. In an (S)-BCK-algebra, they considered a characterization of an MBJ-neutrosophic ideal. Hur et al. [4] introduced the notion of positive implicative MBJ-neutrosophic ideal, and investigated several properties. They discussed relations between MBJ-neutrosophic ideal and positive implicative MBJ-neutrosophic ideal. They provided characterizations of positive implicative MBJ-neutrosophic ideal.

In this paper, we apply the notion of MBJ-neutrosophic sets to positive implicative MBJ-neutrosophic ideals of BCK-algebras, and investigate some properties. We discuss relations between positive implicative MBJ-neutrosophic ideals and positive implicative ideals in BCK-algebra. In a BCK-algebra, the extension property for positive implicative MBJ-neutrosophic ideal is established.

2. Preliminaries

By a *BCK-algebra*, we mean a set X with a binary operation $*$ and a special element 0 that satisfies the following conditions:

(I) $((x*y)*(x*z))*(z*y)=0$,
(II) $(x*(x*y))*y=0$,
(III) $x*x=0$,
(IV) $x*y=0,\ y*x=0\ \Rightarrow\ x=y$,
(V) $(\forall x\in X)\ (0*x=0)$

for all $x,y,z\in X$.

Every BCK-algebra X satisfies the following conditions:

$$(\forall x\in X)\,(x*0=x)\,, \tag{1}$$

$$(\forall x,y,z\in X)\,(x\le y\ \Rightarrow\ x*z\le y*z,\ z*y\le z*x)\,, \tag{2}$$

$$(\forall x,y,z\in X)\,((x*y)*z=(x*z)*y)\,, \tag{3}$$

$$(\forall x,y,z\in X)\,((x*z)*(y*z)\le x*y) \tag{4}$$

where $x\le y$ if and only if $x*y=0$.

A subset I of a BCK-algebra X is called an *ideal* of X if it satisfies:

$$0\in I, \tag{5}$$

$$(\forall x\in X)\,(\forall y\in I)\,(x*y\in I\ \Rightarrow\ x\in I)\,. \tag{6}$$

A subset I of a BCK-algebra X is called a *positive implicative ideal* of X if it satisfies (5) and

$$(\forall x,y,z\in X)\,((x*y)*z\in I, y*z\in I\ \Rightarrow\ x*z\in I)\,. \tag{7}$$

Any positive implicative ideal must be an ideal, but the inverse is not true ([11]).

By an *interval number* we mean a closed subinterval $\tilde{a} = [a^-, a^+]$ of I, where $0 \leq a^- \leq a^+ \leq 1$. Denote by $[I]$ the set of all interval numbers. Let us define what is known as *refined minimum* (briefly, rmin) and *refined maximum* (briefly, rmax) of two elements in $[I]$. We also define the symbols "$\succeq$", "$\preceq$", "$=$" in case of two elements in $[I]$. Consider two interval numbers $\tilde{a}_1 := \left[a_1^-, a_1^+\right]$ and $\tilde{a}_2 := \left[a_2^-, a_2^+\right]$. Then

$$\begin{aligned}
&\operatorname{rmin}\{\tilde{a}_1, \tilde{a}_2\} = \left[\min\left\{a_1^-, a_2^-\right\}, \min\left\{a_1^+, a_2^+\right\}\right], \\
&\operatorname{rmax}\{\tilde{a}_1, \tilde{a}_2\} = \left[\max\left\{a_1^-, a_2^-\right\}, \max\left\{a_1^+, a_2^+\right\}\right], \\
&\tilde{a}_1 \succeq \tilde{a}_2 \;\Leftrightarrow\; a_1^- \geq a_2^-,\; a_1^+ \geq a_2^+,
\end{aligned}$$

and similarly we may have $\tilde{a}_1 \preceq \tilde{a}_2$ and $\tilde{a}_1 = \tilde{a}_2$. To say $\tilde{a}_1 \succ \tilde{a}_2$ (resp. $\tilde{a}_1 \prec \tilde{a}_2$) we mean $\tilde{a}_1 \succeq \tilde{a}_2$ and $\tilde{a}_1 \neq \tilde{a}_2$ (resp. $\tilde{a}_1 \preceq \tilde{a}_2$ and $\tilde{a}_1 \neq \tilde{a}_2$). Let $\tilde{a}_i \in [I]$ where $i \in \Lambda$. We define

$$\operatorname*{rinf}_{i\in\Lambda} \tilde{a}_i = \left[\inf_{i\in\Lambda} a_i^-, \inf_{i\in\Lambda} a_i^+\right] \quad \text{and} \quad \operatorname*{rsup}_{i\in\Lambda} \tilde{a}_i = \left[\sup_{i\in\Lambda} a_i^-, \sup_{i\in\Lambda} a_i^+\right].$$

Let X be a nonempty set. A function $A : X \to [I]$ is called an *interval-valued fuzzy set* (briefly, an *IVF set*) in X. Let $[I]^X$ stand for the set of all IVF sets in X. For every $A \in [I]^X$ and $x \in X$, $A(x) = [A^-(x), A^+(x)]$ is called the *degree* of membership of an element x to A, where $A^- : X \to I$ and $A^+ : X \to I$ are fuzzy sets in X which are called a *lower fuzzy set* and an *upper fuzzy set* in X, respectively. For simplicity, we denote $A = [A^-, A^+]$.

Let X be a non-empty set. A *neutrosophic set* in X (see [16], [14], [21]) is a structure of the form:

$$A := \{\langle x; A_T(x), A_I(x), A_F(x)\rangle \mid x \in X\}$$

where $A_T : X \to [0, 1]$ is a truth membership function, $A_I : X \to [0, 1]$ is an indeterminate membership function, and $A_F : X \to [0, 1]$ is a false membership function. For the sake of simplicity, we shall use the symbol $A = (A_T, A_I, A_F)$ for the neutrosophic set

$$A := \{\langle x; A_T(x), A_I(x), A_F(x)\rangle \mid x \in X\}.$$

We refer the reader to the books [3, 11] for further information regarding BCK/BCI-algebras, and to the site "http://fs.gallup.unm.edu/neutrosophy.htm" for further information regarding neutrosophic set theory.

Let X be a non-empty set. By an *MBJ-neutrosophic set* in X (see [20]), we mean a structure of the form:

$$\mathcal{A} := \{\langle x; M_A(x), \tilde{B}_A(x), J_A(x)\rangle \mid x \in X\}$$

where M_A and J_A are fuzzy sets in X, which are called a truth membership function and a false membership function, respectively, and $\tilde{B}_A$ is an IVF set in X which is called an indeterminate interval-valued membership function.

For the sake of simplicity, we shall use the symbol $\mathcal{A} = (M_A,\ \tilde{B}_A,\ J_A)$ for the MBJ-neutrosophic set

$$\mathcal{A} := \{\langle x; M_A(x), \tilde{B}_A(x), J_A(x)\rangle \mid x \in X\}.$$

Let X be a BCK-algebra. An MBJ-neutrosophic set $\mathcal{A} = (M_A,\ \tilde{B}_A,\ J_A)$ in X is called an *MBJ-neutrosophic ideal* of X ([9]) if it satisfies:

$$(\forall x \in X) \begin{pmatrix} M_A(0) \geq M_A(x) \\ \tilde{B}_A(0) \succeq \tilde{B}_A(x) \\ J_A(0) \leq J_A(x) \end{pmatrix} \tag{8}$$

and

$$(\forall x, y \in X) \begin{pmatrix} M_A(x) \geq \min\{M_A(x * y), M_A(y)\} \\ \tilde{B}_A(x) \succeq \mathrm{rmin}\{\tilde{B}_A(x * y), \tilde{B}_A(y)\} \\ J_A(x) \leq \max\{J_A(x * y), J_A(y)\} \end{pmatrix}. \tag{9}$$

3. Positive implicative MBJ-neutrosophic ideals

In what follows, let X be a BCK-algebra unless otherwise specified.

Definition 3.1. ([4]) *An MBJ-neutrosophic set $\mathcal{A} = (M_A,\ \tilde{B}_A,\ J_A)$ in X is called a positive implicative MBJ-neutrosophic ideal of X if it satisfies* (8) *and*

$$(\forall x, y, z \in X) \begin{pmatrix} M_A(x * z) \geq \min\{M_A((x * y) * z), M_A(y * z)\} \\ \tilde{B}_A(x * z) \succeq \mathrm{rmin}\{\tilde{B}_A((x * y) * z), \tilde{B}_A(y * z)\} \\ J_A(x * z) \leq \max\{J_A((x * y) * z), J_A(y * z)\} \end{pmatrix}. \tag{10}$$

Lemma 3.2. ([4]) *Every positive implicative MBJ-neutrosophic ideal is an MBJ-neutrosophic ideal, but the converse is not true.*

Definition 3.3. *Let $\mathcal{A} = (M_A,\ \tilde{B}_A,\ J_A)$ and $\mathcal{B} = (M_B,\ \tilde{B}_B,\ J_B)$ be MBJ-neutrosophic sets on X. Then the intersection of $\mathcal{A}$ and $\mathcal{B}$ is denoted by $\mathcal{A} \cap \mathcal{B}$ and defined as*

$$\mathcal{A} \cap \mathcal{B} = (M_{A\cap B}, \tilde{B}_{A\cap B}, J_{A\cap B}), \tag{11}$$

where $M_{A\cap B}(x) = \min\{M_A(x), M_B(x)\}$, $\tilde{B}_{A\cap B}(x) = \mathrm{rmin}\{\tilde{B}_A(x), \tilde{B}_B(x)\}$, and $J_{A\cap B}(x) = \max\{J_A(x),\ J_B(x)\}$ for any $x \in X$.

Theorem 3.4. *Let $\mathcal{A} = (M_A,\ \tilde{B}_A,\ J_A)$ and $\mathcal{B} = (M_B,\ \tilde{B}_B,\ J_B)$ be positive implicative MBJ-neutrosophic ideals of X. Then $\mathcal{A} \cap \mathcal{B}$ is a positive implicative MBJ-neutrosophic ideal of X*

Proof. By Lemma 3.2 and (11), the proof is straightforward. □

Definition 3.5. *Let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ and $\mathcal{B} = (M_B, \tilde{B}_B, J_B)$ be MBJ-neutrosophic sets on X. Then $\mathcal{A}$ is contained in $\mathcal{B}$, denoted as $\mathcal{A} \subseteq \mathcal{B}$ and defined by $\mathcal{A}(x) \leq \mathcal{B}(x)$. This means that*

$$M_A(x) \leq M_B(x), \tilde{B}_A(x) \preceq \tilde{B}_B, J_A(x) \geq J_B(x). \tag{12}$$

Two MBJ-neutrosophic sets $\mathcal{A}(x)$ and $\mathcal{B}(x)$ are called equal, i.e., $\mathcal{A}(x) = \mathcal{B}(x)$ if and only if $\mathcal{A} \subseteq \mathcal{B}$ and $\mathcal{A} \supseteq \mathcal{B}$.

Theorem 3.6. *Given a positive implicative ideal I of X, let $\mathcal{A} = (M_A, \tilde{B}_A, J_A)$ be an MBJ-neutrosophic set in X defined by*

$$\begin{aligned} M_A(x) &= \begin{cases} t & \text{if } x \in I, \\ 0 & \text{otherwise,} \end{cases} \\ \tilde{B}_A(x) &= \begin{cases} [\gamma_1, \gamma_2] & \text{if } x \in I, \\ [0,0] & \text{otherwise,} \end{cases} \\ J_A(x) &= \begin{cases} s & \text{if } x \in I, \\ 1 & \text{otherwise,} \end{cases} \end{aligned} \tag{13}$$

where $t \in (0,1]$, $s \in [0,1)$ and $\gamma_1, \gamma_2 \in (0,1]$ with $\gamma_1 < \gamma_2$. Then $\mathcal{A}$ is a positive implicative MBJ-neutrosophic ideal of X.

Proof. Let $x, y, z \in X$. If $(x * y) * z \in I$ and $y * z \in I$, then $x * z \in I$ and so

$$\begin{aligned} &M_A(x * z) = t = \min\{M_A((x * y) * z), M_A(y * z)\} \\ &\tilde{B}_A(x * z) = [\gamma_1, \gamma_2] = \text{rmin}\{[\gamma_1, \gamma_2], [\gamma_1, \gamma_2]\} = \text{rmin}\{\tilde{B}_A((x * y) * z), \tilde{B}_A(y * z)\}, \\ &J_A(x * z) = s = \max\{J_A((x * y) * z), J_A(y * z)\}. \end{aligned}$$

If any one of $(x * y) * z$ and $y * z$ is contained in I, say $(x * y) * z \in I$, then $M_A((x*y)*z) = t$, $\tilde{B}_A((x*y)*z) = [\gamma_1, \gamma_2]$, $J_A((x*y)*z) = s$, $M_A(y*z) = 0$, $\tilde{B}_A(y * z) = [0,0]$ and $J_A(y * z) = 1$. Hence

$$\begin{aligned} &M_A(x * z) \geq 0 = \min\{t, 0\} = \min\{M_A((x * y) * z), M_A(y * z)\} \\ &\tilde{B}_A(x * z) \succeq [0,0] = \text{rmin}\{[\gamma_1, \gamma_2], [0,0]\} = \text{rmin}\{\tilde{B}_A((x * y) * z), \tilde{B}_A(y * z)\}, \\ &J_A(x * z) \leq 1 = \max\{s, 1\} = \max\{J_A((x * y) * z), J_A(y * z)\}. \end{aligned}$$

If $(x * y) * z, y * z \notin I$, then $M_A((x * y) * z) = 0 = M_A(y * z)$, $\tilde{B}_A((x * y) * z) = [0,0] = \tilde{B}_A(y * z)$ and $J_A((x * y) * z) = 1 = J_A(y * z)$. It follows that

$$\begin{aligned} &M_A(x * z) \geq 0 = \min\{0, 0\} = \min\{M_A((x * y) * z), M_A(y * z)\} \\ &\tilde{B}_A(x * z) \succeq [0,0] = \text{rmin}\{[0,0], [0,0]\} = \text{rmin}\{\tilde{B}_A((x * y) * z), \tilde{B}_A(y * z)\}, \\ &J_A(x * z) \leq 1 = \max\{1, 1\} = \max\{J_A((x * y) * z), J_A(y * z)\}. \end{aligned}$$

It is obvious that $\mathcal{A}(0) \geq \mathcal{A}(x)$ for all $x \in X$. Therefore $\mathcal{A}$ is a positive implicative MBJ-neutrosophic ideal of X. □

Theorem 3.7. *For any non-empty subset I of X, let $\mathcal{A} = (M_A,\ \tilde{B}_A,\ J_A)$ be an MBJ-neutrosophic set in X which is given in* (13). *If $\mathcal{A}$ is a positive implicative MBJ-neutrosophic ideal of X, then I is a positive implicative ideal of X.*

Proof. Obviously, $0 \in I$. Let $x, y, z \in X$ be such that $(x * y) * z \in I$ and $y * z \in I$. Then $M_A((x * y) * z) = t = M_A(y * z)$, $\tilde{B}_A((x * y) * z) = [\gamma_1, \gamma_2] = \tilde{B}_A(y * z)$ and $J_A((x * y) * z) = s = J_A(y * z)$. Thus

$$M_A(x * z) \geq \min\{M_A((x * y) * z), M_A(y * z)\} = t,$$
$$\tilde{B}_A(x * z) \succeq \text{rmin}\{\tilde{B}_A((x * y) * z), \tilde{B}_A(y * z)\} = [\gamma_1, \gamma_2],$$
$$J_A(x * z) \leq \max\{J_A((x * y) * z), J_A(y * z)\} = s,$$

and hence $x * z \in I$. Therefore I is a positive implicative ideal of X. □

Theorem 3.8. *Let $\mathcal{A} = (M_A,\ \tilde{B}_A,\ J_A)$ be a positive implicative MBJ-neutrosophic ideal of X. Then the set*

$$K := \{x \in X | \mathcal{A}(x) = \mathcal{A}(0)\}$$

is a positive implicative ideal of X.

Proof. Clearly $0 \in K$. Let $x, y, z \in X$ be such that $(x * y) * z \in K$ and $y * z \in K$. Then

$$M_A(x * z) \geq \min\{M_A((x * y) * z), M_A(y * z)\} = M_A(0),$$
$$\tilde{B}_A(x * z) \succeq \text{rmin}\{\tilde{B}_A((x * y) * z), \tilde{B}_A(y * z)\} = \tilde{B}_A(0),$$
$$J_A(x * z) \leq \max\{J_A((x * y) * z), J_A(y * z)\} = J_A(0),$$

and so $\mathcal{A}(x * z) \geq \mathcal{A}(0)$. Using (8) and Definition 3.5, we have $x * z \in K$. Therefore K is a positive implicative ideal of X. □

Note that an ideal might not be a positive implicative ideal ([11]), but we know that the extension property holds for positive implicative ideal in BCK-algebra. The next lemma decribe a distributive case of all positive implicative ideals in a BCK-algebra, by means of which we will obtain a nice characterization for positive implicative BCK-algebras by ideals.

Lemma 3.9. ([11]) *Let I and A be ideals of X, and $I \subseteq A$. If I is positive implicative, then so is A.*

Does Lemma 3.9 hold true for the relationship between MBJ-neutrosophic ideal and MBJ-neutrosophic positive implicative ideal? But it does not hold. Let's look at the following example.

Example 3.10. *Consider a set $X = \{0, 1, 2, a\}$ with the binary operation $*$ which is given in Table 1. Then $(X; *, 0)$ is a BCK-algebra (see* [11]*). Let $\mathcal{A} = (M_A,\ \tilde{B}_A,\ J_A)$ and $\mathcal{B} = (M_B,\ \tilde{B}_B,\ J_B)$ be MBJ-neutrosophic sets in X defined by Table 2.*

Table 1. Cayley table for the binary operation "$*$"

$*$	0	1	2	a
0	0	0	0	0
1	1	0	0	1
2	2	1	0	2
a	a	a	a	0

Table 2. MBJ-neutrosophic sets $\mathcal{A}$ and $\mathcal{B}$

X	$M_A(x)$	$\tilde{B}_A(x)$	$J_A(x)$	$M_B(x)$	$\tilde{B}_B(x)$	$J_B(x)$
0	0.6	$[0.4, 0.9]$	0.7	0.7	$[0.4, 0.9]$	0.2
1	0.6	$[0.3, 0.8]$	0.7	0.6	$[0.3, 0.8]$	0.6
2	0.6	$[0.3, 0.8]$	0.7	0.6	$[0.3, 0.8]$	0.6
a	0.4	$[0.1, 0.3]$	0.4	0.4	$[0.1, 0.3]$	0.4

It is routine to verify that $\mathcal{A}$ is a positive implicative MBJ-neutrosophic ideal of X, $\mathcal{B}$ is an MBJ-neutrosophic ideal of X and $\mathcal{A} \subseteq \mathcal{B}$. But $\mathcal{B}$ is not a positive implicative MBJ-neutrosophic ideal of X because

$$M_B(2 * 1) = 0.6 < 0.7 = \min\{M_B((2 * 1) * 1), M_B(1 * 1)\}.$$

Lemma 3.11. ([4]) *An MBJ-neutrosophic set $\mathcal{A} = (M_A,\ \tilde{B}_A,\ J_A)$ in X is a positive implicative MBJ-neutrosophic ideal of X if and only if it is an MBJ-neutrosophic ideal of X satisfying the following condition:*

$$(\forall x, y, z \in X)(\mathcal{A}((x * z) * (y * z)) \geq \mathcal{A}((x * y) * z)) \tag{14}$$

We have the following extension property for positive implicative MBJ-neutrosophic ideal.

Theorem 3.12. *Let $\mathcal{A} = (M_A,\ \tilde{B}_A,\ J_A)$ and $\mathcal{B} = (M_B,\ \tilde{B}_B,\ J_B)$ be MBJ-neutrosophic ideals on X such that $\mathcal{A} \subseteq \mathcal{B}$ and $M_A(0) = M_B(0), \tilde{B}_A(0) = \tilde{B}_B(0), J_A(0) = J_B(0)$. If $\mathcal{A}$ is a positive implicative MBJ-neutrosophic ideal of X, then so is $\mathcal{B}$.*

Proof. Suppose that $\mathcal{A}$ is a positive implicative MBJ-neutrosophic ideal of X. Let $x, y, z \in X$. Using (3), (14) and (III), we have

$$\begin{aligned}
&M_B(((x*z)*(y*z))*((x*y)*z)) \\
&= M_B(((x*z)*((x*y)*z))*z)*(y*z)) \\
&= M_B(((x*((x*y)*z))*z)*(y*z)) \\
&\geq M_A(((x*((x*y)*z))*z)*(y*z)) \\
&\geq M_A(((x*((x*y)*z))*y)*z) \\
&= M_A(((x*y)*((x*y)*z))*z) \\
&= M_A(((x*y)*z)*((x*y)*z)) \\
&= M_A(0) = M_B(0),
\end{aligned}$$

$$\begin{aligned}
&\tilde{B}_B(((x*z)*(y*z))*((x*y)*z)) \\
&= \tilde{B}_B(((x*z)*((x*y)*z))*z)*(y*z)) \\
&= \tilde{B}_B(((x*((x*y)*z))*z)*(y*z)) \\
&\succeq \tilde{B}_A(((x*((x*y)*z))*z)*(y*z)) \\
&\succeq \tilde{B}_A(((x*((x*y)*z))*y)*z) \\
&= \tilde{B}_A(((x*y)*((x*y)*z))*z) \\
&= \tilde{B}_A(((x*y)*z)*((x*y)*z)) \\
&= \tilde{B}_A(0) = \tilde{B}_B(0),
\end{aligned}$$

and

$$\begin{aligned}
&J_B(((x*z)*(y*z))*((x*y)*z)) \\
&= J_B(((x*z)*((x*y)*z))*z)*(y*z)) \\
&= J_B(((x*((x*y)*z))*z)*(y*z)) \\
&\leq J_A(((x*((x*y)*z))*z)*(y*z)) \\
&\leq J_A(((x*((x*y)*z))*y)*z) \\
&= J_A(((x*y)*((x*y)*z))*z) \\
&= J_A(((x*y)*z)*((x*y)*z)) \\
&= J_A(0) = J_B(0).
\end{aligned}$$

It follows from (8) and (9) that

$$\begin{aligned}
M_B(((x*z)*(y*z)) &\geq \min\{M_B(((x*z)*(y*z))*((x*y)*z)), M_B((x*y)*z))\} \\
&\geq \min\{M_B(0), M_B((x*y)*z))\} = M_B((x*y)*z)),
\end{aligned}$$

$$\begin{aligned}
\tilde{B}_B(((x*z)*(y*z)) &\succeq \min\{\tilde{B}_B(((x*z)*(y*z))*((x*y)*z)), \tilde{B}_B((x*y)*z))\} \\
&\succeq \min\{\tilde{B}_B(0), \tilde{B}_B((x*y)*z))\} = \tilde{B}_B((x*y)*z)),
\end{aligned}$$

and

$$J_B(((x*z)*(y*z)) \leq \max\{J_B(((x*z)*(y*z))*((x*y)*z)), J_B((x*y)*z))\}$$
$$\leq \max\{J_B(0), J_B((x*y)*z))\} = J_B((x*y)*z))$$

for all $x, y, z \in X$. It follows from (14) that $\mathcal{B}$ is a positive implicative MBJ-neutrosophic ideal of X. □

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