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Analytical model for the formation of electric fields in parallel-plate capacitors

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Abstract: In this study, we propose an analytical model to elucidate the formation of electric fields between two parallel conducting plates. Using nine Gaussian surfaces, we investigated the charge redistributions and electric fields formed by parallel conducting plates when two charged plates get close together. The electric charges are redistributed via a new electrostatic equilibrium to create the electric field of each plates. As a result, the electric field start from + electrode plate to - electrode plate via inducing a new electrostatic equilibrium, implying that the application of Gaussian surfaces to only one of the electrodes of parallel-plate capacitors is deserved. The results will help undergraduate students understand the charge redistribution and the electric field formation in parallel-plate capacitors in a reasonable manner.

keywords: Charge, Electric field, Parallel conducting plate, Parallel-plate capacitor, Gaussian surface

Ⅰ. Introduction

A parallel-plate capacitor is a device which stores electrical charge, widely applied in various electronic devices. Besides being applied in electronics such as computer keyboards or electrostatic microphones, parallel-plate capacitors are also used to measure the relative permittivity (dielectric constant) of materials (Grove, Masters, & Miers 2005; Radivojević, Rupčić, Srnović, & Benšić, 2018). Therefore, it is important that students understand how charges are stored and how electric fields form in these capacitors. When two parallel conducting plates of equal area S and separated by a distance d carry equal charges Q of opposite sign, an electric field (pointing toward the negative plate) is created and the plates experience a potential difference V. Assuming that the conductor plates are infinitely wide,(Halliday, Resnick, & Walker, 2014; Knight, 2017) the electric field strength E is expressed as:

$$E = \frac{\sigma}{\epsilon_0} \tag{1}$$

where $\sigma = Q/S$ is the charge density per unit area and ϵ_0 is the vacuum permittivity. Since E is constant between two infinite, parallel conducting plates, the potential difference V between the plates is given by:

$$V = Ed. \tag{2}$$

Therefore, the capacitance C of the parallel-plate capacitor can be expressed as:

$$C = \frac{Q}{V} = \frac{\epsilon_0 S}{d} \tag{3}$$

These are well-known formulas describing an ideal parallel-plate capacitor.

In this paper, we evaluate the methods used to obtain Eq. (1) in general physics textbooks used in the college and propose a more appropriate derivation process. Based on six

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general physics textbooks (Halliday, Resnick, & Walker, 2014; Knight, 2017; Serway & Vuille, 2017; Tipler & Moska, 2004, 2008; Wolfson & Pasachoff, 2017) and questions argued in physics forums, (Gerard, 2014; Pricklebush, 2013) we analyzed the issues related to the electric field between the electrode plates of parallel-plate capacitors found in each textbook, and developed a correct theoretical approach to solve these by applying the Gauss's law to calculate these electric fields.

As covered in most college textbooks, Gauss's law can be used to derivate the formulas for the electric field near a thin infinite charged plate and near the surface of a conductor as:

$$E = \frac{\sigma}{2\epsilon_0} \tag{4}$$

and

$$E = \frac{\sigma}{\epsilon_0} \tag{5}$$

respectively, where σ denotes a uniform positive surface charge density. Some general physics textbooks (Knight, 2017; Serway & Vuille, 2017) explain the electric field inside a parallel-plate capacitor as the superposition of the electric fields generated by two thin charged plates - one positively and the other negatively charged - which are infinitely large and of negligible thickness. In other words, since the opposite charges on the plates are attracted to each other, the charges are distributed only on the inner surfaces, resulting in two infinite plane charges with opposite sign. Hence, the superposition of these electric fields results in twice the electric field strength of Eq. (4) inside the capacitor, and no electric field outside the capacitor. Similar to the textbooks, (Knight, 2017; Serway & Vuille, 2017) in the 5th edition of the textbook (Tipler & Moska, 2004), Eq. (5) was described as the superposition of the electric fields generated by each plate. However, in the 6th edition (Tipler & Moska, 2008), it was explained by discontinuities in the electric field. That is, because the opposite charges on the two adjacent conducting plates are attracted to each other, therefore, the electric charges distribute uniformly on the inner surface of the plates. Because the electric field inside the conductor becomes zero at electrostatic equilibrium and; the electric field at the boundary of the surface charge is discontinuous by σ/ϵ_0, the electric field between capacitors is described by Eq. (5).

On the other hand, in one textbook (Wolfson & Pasachoff, 2017) the electric field expressed by Eq. (5) is obtained using Gauss's law. But a parallel-plate capacitor is composed of a pair of conducting plates which carry opposite charge and, consequently, the author wonders that the electric field between the capacitors will be twice that of Eq. (5) if Gauss's law is applied for both conductor plates. Furthermore, the parallel-plate capacitor is not isolated and, therefore, the symmetry of the charge distribution is broken and the charge accumulates only on the inner surfaces of the two plates. Hence, one can consider the two conducting plates as two plane charge distributions, and the electric field inside the capacitor becomes twice as large as that of Eq. (4), while the electric field outside the capacitor is zero, due to the superposition of the electric field. The electric field inside the parallel-plate capacitor was also calculated by applying the Gauss's law to only one conducting plate, and not as a superposition of electric fields generated by the two thin charge plates (Halliday, Resnick, & Walker, 2014). Although these methods lead to the same result, this brings on the possibility that the electric field inside the capacitor may be twice that of Eq. (5) when superposing the electric field generated by each plate, if we apply Gauss's law to both plates. In this textbook (Halliday, Resnick, & Walker, 2014), the electric field between two equally but opposite charged-conducting plates was also discussed, based on the attraction of excess charges σ_1 and a chage redistribution σ. But, in this textbook, we never find out the mathematical proof for $\sigma = 2\sigma_1$ in a new electrostatic equilibrium. Most of these textbooks refer to a fringing electric field that occurs due to edge effects near the edges of the plates, and a fringing electric field that is due to external charges on the electrode plate (Invchenko, 2021). However, for parallel-plate capacitors with finite size plates and a small gap between the plates, they consider that the charge and electric field except in the vicinity of edges can be approximated by Eqs. (3) and (5). Analyzing the electric field calculations in the six textbooks considered, we found that the overall issue stems from the fact that the electric field inside the parallel-plate capacitor is derived using different charge distributions on the plates, making it difficult to find consistency. Students refer to several textbooks while studying and can get confused by different ways of explaining parallel-plate capacitors. To begin with, it is confusing that the electric field inside a parallel-plate capacitor (composed of two conductor plates) is calculated considering either the superposition of electric fields $\sigma/(2\epsilon_0)$ near two thin infinite plates, or only the electric field σ/ϵ_0 on the surface of one conductor. Moreover, there is no clear explanation of whether or not the Gauss's law should be applied to both the positive and negative electrode plates of the capacitor. If it is, we should obtain the electric field by superposition after setting two Gaussian surfaces, and the electric field is $2\sigma/\epsilon_0$, that is what makes one question these approaches. To avoid doubting, most textbooks derive the electric field of the parallel-plate capacitor by the superposition of the electric field around the infinite plate charge expressed by Eq. (4).

The above question is also heavily discussed in the physics forum site (Gerard, 2014; Pricklebush, 2013) “Stack Exchange.” There, a search for “What is the electric field in a parallel capacitor?” and “Field between the plates of a parallel-plate capacitor using Gauss's Law” revealed that learners have a keen interest and many questions about how to use the electric field of a thin infinite plate and the electric field of a conductor surface to calculate the electric field inside the parallel plates (187,000 and 89,000 views, respectively).

A comprehensive analysis of the physics textbooks (Grove, Masters, & Miers 2005; Radivojević, Rupčić, Srnović, & Benšić, 2018; Serway & Vuille, 2017; Tipler & Moska, 2004, 2008; Wolfson & Pasachoff, 2017) and the Stack Exchange forum (Gerard, 2014; Pricklebush, 2013) indicated that the general problem of the parallel-plate capacitor is the avoidance of the following concepts. If two isolated conductors are brought close to each other, a new electrostatic equilibrium is achieved by the redistribution of the charges on each plate which produces electric fields. The initially asymmetrical charge distribution in an electrostatic equilibrium is rearranged on each electrode plate due to the proximity of the charges of the other plate, and electric field lines form between the two electrode plates (pointing from the + plate to the − plate) in a new electrostatic equilibrium. Therefore, Gauss’s law should only be applied to either of the two electrode plates and the superposition of fields is not needed in this case.

The purpose of this study is to elucidate the formation mechanism of the electric field in the parellel plate capacior covered in college textbooks and propose an anaytical model to explain it in reasonable manner.

Ⅱ. Theoretical Calculations: A new analysis of two parallel conducting plates

To summarize, we can get some questions: is it reasonable to calculate the electric field inside a parallel-plate capacitor by the superposition the electric fields generated around two infinite charge plates? In other words, is it preferable to obtain the electric field of the parallel-plate capacitor by applying the Gauss's law to only one electrode plate, based on the concept of a newly formed electrostatic equilibrium of the two facing conductors? Aren't there a more general mathematical approach to settle the problems for the charge redistributions and electric fields formed by the proximity of two charged parallel conducting plates.

These questions are the motivation for this research. Considering these aspects, in this study, we set around the two charged conducting plates nine Gaussian surfaces(Fig. 1) to calculate the electric field and the charge distribution inside and outside the plates. And we show that when two charged conductor plates with charge densities σ_1 and σ_2 are brought close to each other, the charge distribution rearranges. Concomitantly, we attempt to show that the electric field is determined by the rearranged electric charges. Further, we intend to answer the questions related to the electric field of the parallel-plate capacitor which have been controversial so far by calculating the electric field and the charge distribution for a parallel-plate capacitor in a special case, such as $\sigma_1 = -\sigma_2$.

As shown in Fig. 1, the conducting plates are assumed ideal, very large, and located close to each other so that fringe effects can be neglected and electric field lines are perpendicular to the plane of the plates. When these plates, charged with $+Q_1$ and $+Q_2$ (surface area $2S$), are brought close, assuming that $\sigma_1(=Q_1/2S)$ and $\sigma_2(=Q_2/2S)$ on the inner and outer surfaces of both conductor plates are distributed by $\sigma_1'(=Q_1'/S)$ and $\sigma_1''(=Q_1''/S)$, and $\sigma_2'(=Q_2'/S)$ and $\sigma_2''(=Q_2''/S)$, respectively. they can be expressed as:

$$Q_1 = Q_1' + Q_1'', \quad Q_2 = Q_2' + Q_2'', \tag{6}$$

$$\sigma_1' + \sigma_1'' = 2\sigma_1, \quad \sigma_2' + \sigma_2'' = 2\sigma_2. \tag{7}$$

We apply the Gauss's law by defining nine Gaussian surfaces as seen in Fig. 1, setting the normal to the plane (A) to point toward the outside of the capacitor. At the electrostatic equilibrium, the electric fields E_1 and E_1' inside the conductor vanish with respect to the Gaussian surface (1), and the electric flux ϕ_1 can be written as:

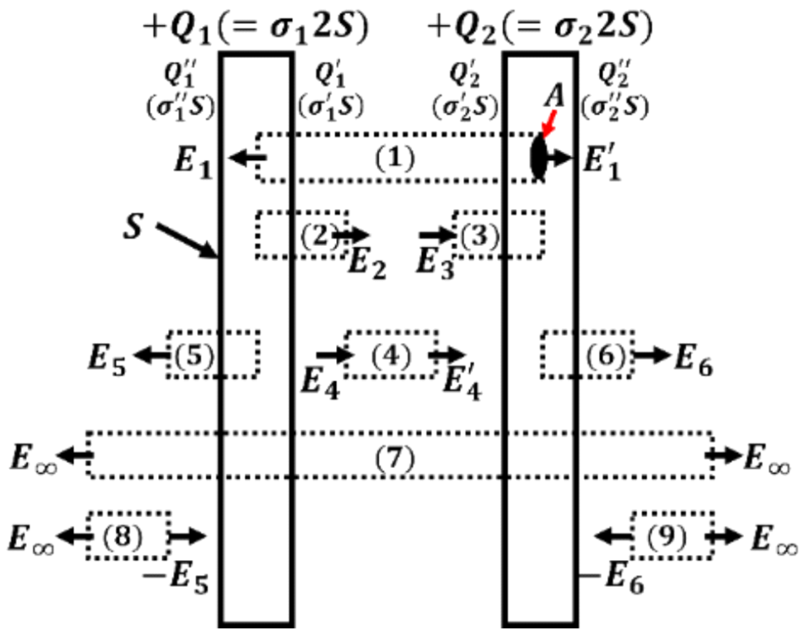

Figure 1. Gaussian surfaces used to find the electric field and charge distribution of two facing conductor plates with area S.

$$\phi_1 = \int EdA = E_1A + E_1'A = 0$$
$$= \frac{Q_{in}}{\epsilon_0} = \frac{\sigma_1'A + \sigma_2'A}{\epsilon_0} \quad (8)$$

from where:

$$\sigma_1' + \sigma_2' = 0. \quad (9)$$

Therefore, the charge sign is opposite on the inner side of the two conducting plates, which are charged with the same amount of charge.

Using the Gaussian surface (2), ϕ_2 becomes:

$$\phi_2 = \int EdA = E_2A = \frac{\sigma_1'A}{\epsilon_0} \quad (10)$$

from where we deduce that:

$$E_2 = \frac{\sigma_1'}{\epsilon_0}. \quad (11)$$

The Gaussian surface (3) yields a ϕ_3 described by:

$$\phi_3 = \int EdA = -E_3A = \frac{\sigma_2'A}{\epsilon_0} \quad (12)$$

from where:

$$E_3 = \frac{-\sigma_2'}{\epsilon_0}. \quad (13)$$

In Eq. (9), σ_1' and σ_2' are equal and of opposite sign, so E_2 of Eq. (11) and E_3 of Eq. (13) are also equal and have the same direction (as indicated in Fig. 1).

Since the Gaussian surface (4) lies between the Gaussian surfaces (2) and (3), ϕ_4 is given by:

from where:

$$\phi_4 = \int EdA = -E_4A + E_4'A = \frac{Q_{in}}{\epsilon_0} = 0 \quad (14)$$

$$E_4 = E_4' \quad (15)$$

in accordance to the requirement of electric field continuity in the same medium. That is, E_4 and E_4' are equal and point in the same direction, and are described by:

$$E_4 = E_2 = \frac{\sigma_1'}{\epsilon_0}, \qquad E_4' = E_3 = \frac{-\sigma_2'}{\epsilon_0} \quad (16)$$

due to the continuity of the electric field in the same medium with no free local charges. Combining the Eqs. (15) and (16), we obtain $\sigma_1' = -\sigma_2'$, same as Eq. (9).

Using the Gaussian surface (5), ϕ_5 is derived as follows:

$$\phi_5 = \int EdA = E_5A = \frac{\sigma_1''A}{\epsilon_0} \quad (17)$$

and the resultant electric field is:

$$E_5 = \frac{\sigma_1''}{\epsilon_0} \quad (18)$$

whose direction is left if $\sigma_1 > 0$ and $\sigma_1'' > 0$.

Using the Gaussian surface (6) and Eqs. (9) and (16), if $\sigma_2 > 0$, than $\sigma_2' < 0$ and $\sigma_2'' > 0$, and the flux is derived as:

$$\phi_6 = \int EdA = E_6A = \frac{\sigma_2''A}{\epsilon_0}. \quad (19)$$

Thus, we obtain:

$$E_6 = \frac{\sigma_2''}{\epsilon_0}. \quad (20)$$

In this case, the direction of E_6 becomes the right direction.

The Gaussian surface (7) shown in Fig. 1 can

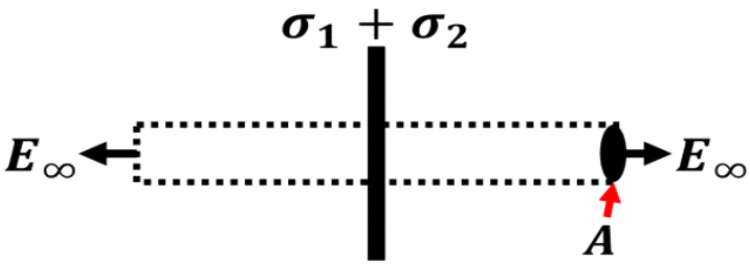

Figure 2. Electric charge distribution of two conducting plates seen from an infinite distance.

be regarded at infinity as a planar charge distribution as seen in Fig. 2. Therefore, ϕ_7 for the Gaussian surface (7) is given by:

$$\phi_7 = \int EdA = E_\infty 2A = \frac{Q_{in}}{\epsilon_0} = \frac{(\sigma_1+\sigma_2)2A}{\epsilon_0} \quad (21)$$

and, thus, the field at infinity becomes:

$$E_\infty = \frac{(\sigma_1+\sigma_2)}{\epsilon_0}. \quad (22)$$

For the Gaussian surface (8), ϕ_8 is expressed as:

$$\phi_8 = \int EdA = E_\infty A - E_5 A = \frac{Q_{in}}{\epsilon_0} = 0 \quad (23)$$

and therefore

$$E_\infty = E_5. \quad (24)$$

Similarly, for the Gaussian surface (9), ϕ_9 is expressed as:

$$\phi_9 = \int EdA = E_\infty A - E_6 A = 0 \quad (25)$$

and, therefore,

$$E_\infty = E_6. \quad (26)$$

Using all Eqs. (7)~(26), we derive the relationships between charge densities as follows:

$$\sigma_1'' = \sigma_2'' = \sigma_1 + \sigma_2, \quad \sigma_1' = -\sigma_2' = \sigma_1 - \sigma_2 \quad (27)$$

and the relationship between electric fields as:

$$E_2 = E_3 = \frac{\sigma_1 - \sigma_2}{\epsilon_0}, \quad E_5 = E_6 = \frac{\sigma_1 + \sigma_2}{\epsilon_0} \quad (28)$$

Eqs. (27) and (28) have a rather lengthy derivation procedure using nine Gaussian surfaces, but they are very important relations in elucidating quantitatively the charge distributions and the electric field formation in two conducting parallel-plate system. As can be seen in Eqs. (27) and (28), the amount of charge and the electric field strength inside and outside two conducting plates facing each other at close distance depend on the charge density on each plate when they are far apart. That is, even the electric field formed in the vicinity of each plate is determined by the charge density distributed on both plates. Thus, the two plates influence each other resulting in a new electrostatic equilibrium state, electric fields form inside and outside the plates due to the newly formed electric charge density in a new electrostatic equilibrium, and E_2 and E_3 become equal like Eq. (28). Even if Eq. (28) shows that the field can be expressed by the superposition of electric fields from Eq. (4), since the charge distribution of the two plates arises from the concept of plates at electrostatic equilibrium affecting each other, a good analysis method should not introduce the concept of superposition. It is noted that Eqs. (27) and (28), that is, the charge redistributions and electric fields formed by the proximity of two charged parallel conducting plates can be derived using nine Gaussian surfaces.

Thus, when two isolated charged conducting plates are brought close together, the electric fields generated by each electric charge redistribute the electric charge density achieved at electrostatic equilibrium by the isolated conductor plate, forming a new charge density with a corresponding electric field. Therefore, the charge redistribution and the electric field inside and outside two charged conducting plates having an arbitrary charge distribution may appear different, depending on the amount and polarity of the charge density of the two isolated conducting plates at electrostatic equilibrium.

On the other hand, since in a parallel-plate capacitor $\sigma_1 = -\sigma_2$, using Eq. (27) we obtain:

$$\sigma_1'' = \sigma_2'' = 0, \sigma_1' = 2\sigma_1, \sigma_2' = 2\sigma_2 \tag{29}$$

Therefore, Eq. (28) becomes:

$$E_2 = E_3 = \frac{2\sigma_1}{\epsilon_0}, \quad E_5 = E_6 = 0 \tag{30}$$

Eqs. (29) and (30) are well-known formulas describing the ideal parallel-plate capacitor. In Eqs. (29) and (30), factor 2 originates from the definition of $\sigma_1(= Q_1/2S)$ and $\sigma_2(= Q_2/2S)$, and $\sigma_1'(= Q_1'/S)$ and $\sigma_2'(= Q_2'/S)$. From Eq. (29), one can find easily, $\sigma_1'(= Q_1'/S) = 2\sigma_1(= 2Q_1/2S)$, i.e. $Q_1 = Q_1'$, and $\sigma_2'(= Q_2'/S) = 2\sigma_2(= 2Q_2/2S)$, i.e. $Q_2 = Q_2'$. This means that the charges accumulate only on inner surfaces of plates in parallel-plate capacitors. Similarly, from Eq. (30), it becomes that $E_2 = E_3 = 2\sigma_1/\epsilon_0 = 2Q_1/(2S\epsilon_0) = Q_1/(S\epsilon_0)$. Therefore, we can obtain

$$E_2 = E_3 = \frac{Q_1}{\epsilon_0 S} = \frac{\sigma}{\epsilon_0} \tag{31}$$

where $\sigma = Q_1/S$. Eq. (31) are equivalent to Eq. (1).

Thus, we found that the charge of the parallel-plate capacitor exists only on the inner surfaces of the conducting plates, and that the electric field occurs only between the plates and E_2 and E_3 due to $\sigma_1(= \sigma_1')$ are equal in both strength and direction. Further, a new electrostatic equilibrium between the two conducting plates of the parallel-plate capacitor is achieved due to redistribution of charges between the plates, and the electric field lines start from $\sigma_1(= \sigma_1')$ and end on $\sigma_2(= \sigma_2' = -\sigma_1' = -\sigma_1)$. Therefore, when employing Gauss's law to find the electric field inside the parallel-plate capacitor, the Gaussian surface must be applied to only one of the conducting plates. For the same reason, in a coaxial cylindrical capacitor, the electric field is obtained by setting the Gaussian surface only around the inner conducting cylindrical plate. These are why Eqs. (27) and (28) are crucial in elucidating the electric fields in the parallel-plate capacitors. They are key equations to explain the charge distribution, a new electrostatic equilibrium, the application of Gauss' law in the parallel- plate capacitors. In addition, Eqs. (27) and (28) are more generalized fomulas for the charge distributions and electric fields formed by two charged parallel conducting plates getting close together. It should be kept in mind that Eqs. (27)~ (30) could be obtained in a bit long mathematical procedure using nine Gaussian surfaces. Nevertheless, they give more realistic and quantitative intuitions on the the charge distributions and electric fields in two conducting plate system without relying on the superposition of electric fields.

Ⅲ. Application of Theoretical Calculations and Discussions

To apply the theoretical calculations of sec.2 to realistic systems, let's examine the charge redistribution of two distant, charged conducting plates which were in an initially electrostatic equilibrium. They are brought

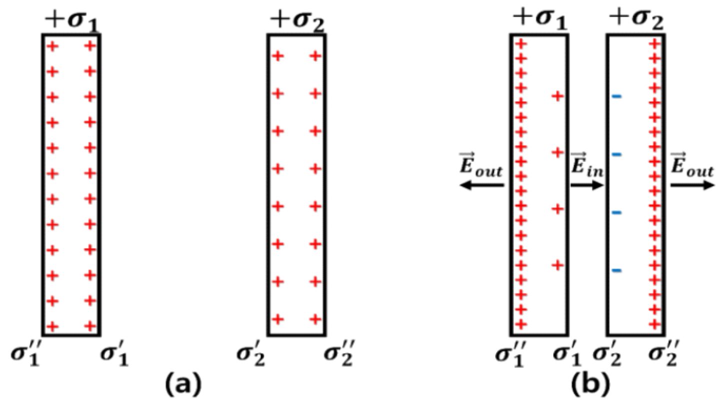

Figure 3. Electric charge redistribution of two conducting plates for $|+\sigma_1| > |+\sigma_2|$: before (a) and after (b) a new electrostatic equilibrium.

closer together and achieve a new electrostatic equilibrium due to the electric fields formed by the charge redistribution. Further, let's show the charge redistributions and the electric fields of the plates given by Eqs. (27) and (28) before and after reaching a new electrostatic equilibrium schematically, with the positive and negative charges denoted by + and -, and the charge density amount represented by the number of + and - symbols illustrated. The positive(+) charges are due to the transfer of negative(-) charges.

Fig. 3 shows the charge redistribution of the two conducting plates before (a) and after (b) reaching a new electrostatic equilibrium, for $|+\sigma_1| > |+\sigma_2|$; in case (a), the + charges are distributed with the same density on the outer surfaces of the two conducting plates, and in case (b), the + and - charges are distributed with the same density on the inner surfaces of the conductor plates. In the latter case, the electric field is strong outside the capacitor ($\vec{E}_{out}$) and weak inside ($\vec{E}_{in}$), pointing out to the change from a conducting plate with high initial charge density to one with small initial charge density.

Fig. 4 shows the charge redistribution of two conducting plates before (a) and after (b) reaching a new electrostatic equilibrium, for $|+\sigma_1| > |-\sigma_2|$; the inner electric field is large, in this case, because plenty of the positive and negative charges are arranged with the same density on the inner surfaces ($\vec{E}_{in}$) of the plates. The outer electric field ($\vec{E}_{out}$) is weak, because few positive charges are distributed on

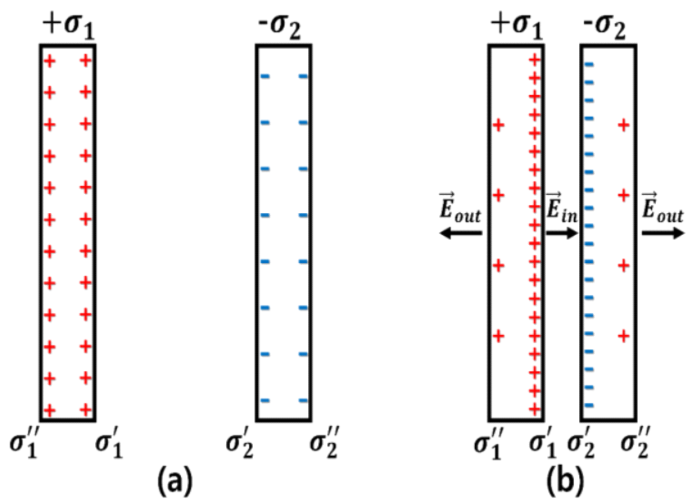

Figure 4. Electric charge redistribution of two conducting plates for $|+\sigma_1| > |-\sigma_2|$: before (a) and after (b) a new electrostatic equilibrium.

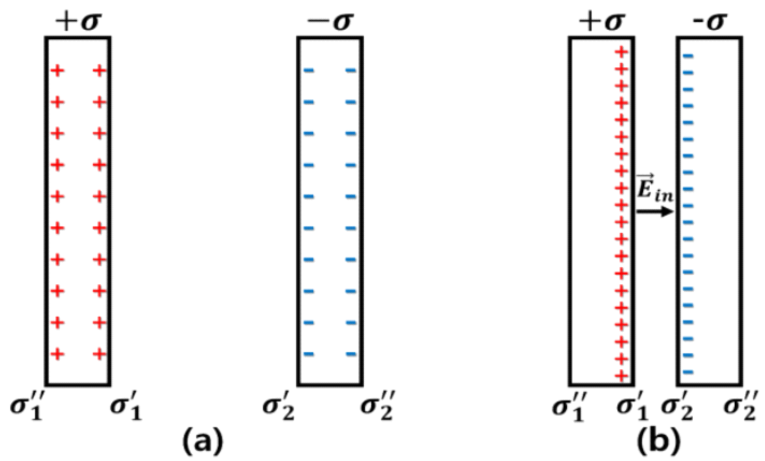

Figure 5. Electric charge redistribution of two conducting plates for $|+\sigma_1| = |-\sigma_2|$: before (a) and after (b) a new electrostatic equilibrium.

the outside surfaces of the two conducting plates. (Although not shown, in the case of $|+\sigma_1| < |-\sigma_2|$ the sign of the charge density of the outer surfaces of the two plates is reversed, but the sign of the inner surface charges does not change.)

Fig. 5 shows the charge redistribution of two conducting plates before (a) and after (b) reaching a new electrostatic equilibrium for $|+\sigma_1| = |-\sigma_2|$; the inner electric field ($\vec{E}_{in}$) starts from the + charges and ends at the negative charges, and the outer electric field vanishes because all the charges, either + or -, are distributed only inside the two conducting plates. These facts are well-known for ideal parallel-plate capacitors.

There are no ways to confirm the results, Eqs. (27)~(30) experimentally because the charges and electric fields between two conducting plates or in a parallel-plate capacitor with finite sizes cannot be measured without the fringing effects. Most of all, we have no tools to measure electric fields directly. Also, to measure the charges on the plates without the fringing effect, we need a parallel-plate capacitor with finite size plates and a small gap between the plates. But, in such case, we cannot insert a charge sensor into the between of conducting plate electrodes. Nevertheless, Eqs. (27)~(30) will be useful formulas to settle the currently issued problems concerning the parallel-plate capacitor in the college textbooks and the physics forums.

Ⅳ. Conclusion

In conclusion, after analyzing the methods by which the electric field inside and outside parallel-plate capacitors are described using Gaussian law in college-level physics textbooks, we propose a different but more reasonable theoretical approach. Our approach consists of a method to calculate the electric fields between two charged conducting parallel plates, as well as evaluate their electric charge density redistributions.

We achieve this goal by setting nine Gaussian surfaces for two closely facing conducting plates and getting Eqs. (27) and (28). When two isolated charged conducting plates are brought close together, the electric fields formed by each of the charge densities redistribute the electric charges, leading to a new electrostatic equilibrium state, resulting in new charge distributions and electric fields. Also, from Eqs. (29) and (30), in the parallel-plate capacitor, the charge of both conducting plates are rearranged only on the inner surfaces, via inducing a new electrostatic equilibrium by the charges on each conductor plate and the electric field inside is formed by the rearranged charge density. As a result, the electric field lines start at the + electrode plate and end at the - electrode plate. Therefore, when calculating the electric field between the conducting plates via inducing a new electrostatic equilibrium, Gaussian surface should be applied to only one of the electrodes of parallel-plate capacitors. Thus, the derived Eqs. (27)~(30) will be expected to lead the students to not only elucidating the charge distribution on inner surfaces of plates but explaing ' why we apply Gauss' law to only one of the plates'. Counting on the derived Eqs. (27)~(30), the undergradute students could understand easily the charge distributions and electric fields in two charged conducting parallel plate system such as parallel-plate capacitors without relying on the superposition of electric fields.

The results of this study can provide the undergradute students with an analytical explanation for the formation mechanism of electric fields in parallel-plate capacitors, but it is necessary to follow-up studies to utilize them in the field of the physics education. We hope that, in the near future, a study on the application of our results to the field of physics education will be conducted.

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