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DECOMPOSITION FOR CARTAN'S SECOND CURVATURE TENSOR OF DIFFERENT ORDER IN FINSLER SPACES

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Abstract. The Cartan's second curvature tensor P^i_{jkh} is a positively homogeneous of degree-1 in y^i, where y^i represent a directional coordinate for the line element in Finsler space. In this paper, we discuss the decomposition of Cartan's second curvature tensor P^i_{jkh} in two spaces, a generalized $\mathfrak{B}P$-recurrent space and generalized $\mathfrak{B}P$-birecurrent space. We obtain different tensors which satisfy the recurrence and birecurrence property under the decomposition. Also, we prove the decomposition for different tensors are non-vanishing. As an illustration of the applicability of the obtained results, we finish this work with some illustrative examples.

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1. Introduction

Finsler geometry has many uses in relative physics and many of mathematicians contributed in this study and improved it. The decomposition of curvature tensor of recurrent manifold discussed initially by Takano [19], Sinha and Singh [17] and others. The decomposition of Berwald curvature tensor H^i_{jkh} and Cartan's fourth curvature tensor K^i_{jkh} for some spaces in sense of Berwald and Cartan discussed by Pandey [12]. The decomposition of Cartan's third curvature tensor R^i_{jkh} equipped with non-symmetric connection studied by Mishra et al. [9]. The decomposition of Riemannian curvature tensor field discussed by Gicheru and Ngari [6]. The decomposition of normal projective curvature tensor studied by Qasem [13]. Hit [7] introduced Berwald curvature tensor which be decomposable in the form $H^i_{jkh} = X^i Y_{jkh}$ and obtained several results, Pande and Khan [10] discussed Berwald curvature tensor which be decomposable in the form $H^i_{jkh} = X^i_j Y_{kh}$. Rawat and Chauhan [15] studied the decomposition of curvature tensor fields R^i_{jkh} in terms of two non-zero vectors and a tensor field in some spaces. Pande and Shukla [11] discussed the decomposition of curvature tensor field K^i_{jkh} and H^i_{jkh} which satisfy the recurrence property.

Assallal [4] studied the decomposition of Cartan's second curvature tensor P^i_{jkh} in generalized P^h-birecurrent Finsler space, Sinha and Tripathi [18] discussed the birecurrent Finsler space whose curvature tensor be decomposition. Recently, Bisht and Neg [5] studied decomposition of normal projective curvature tensor fields in Finsler manifolds.

The aim of this paper is to study some decomposition of Cartan's second curvature tensor P^i_{jkh} in various spaces. Additionally, several theorems have been established and proved. Finally, some examples have been discussed under the decomposition in $G(\mathfrak{B}P) - RF_n$ and $G(\mathfrak{B}P) - BRF_n$.

2. Preliminaries

In this section, some conditions and definitions will be provided for the purpose of this paper. The line element in Finsler geometry is (x, y), x and y are called positional and directional coordinate, respectively [12, 14].

An $n-$ dimensional space X_n equipped with a function $F(x, y)$ which denoted by $F_n = (X_n, F(x, y))$ called a Finsler space if the function $F(x, y)$ satisfies the following three conditions [9, 16]:

(i) The function $F(x, y)$ is positively homogeneous of degree one in y^i, that is,

$$F(x, ky) = kF(x, y),$$

where k is some positive scalar.

(ii) The function $F(x,y)$ is positive unless all y^i vanish simultaneously, that is, $F(x,y) > 0$, with $\sum_i \left(y^i\right)^2 \neq 0$.

(iii) The quadratic form

$$\left\{\dot{\partial}_i \dot{\partial}_j F^2\left(x,\ y\right)\right\} \xi^i\ \xi^j,\ \ \dot{\partial}_i := \frac{\partial}{\partial y^i}$$

is assumed to be positive definite for all variable ξ^i.

The vector y_i is defined by

$$y_i = g_{ij}(x,y) y^j, \tag{2.1}$$

where g_{ij} is a metric tensor of the space F_n. The vectors y^j, y_i are given by

$$\delta^i_j y^j = y^i \text{ and } \delta^i_j y_i = y_j. \tag{2.2}$$

Matsumoto [8] introduced a tensor C_{ijk} called it $(h)hv-$torsion tensor which is positively homogeneous of degree -1 in y^i and symmetric in all its indices and defined by

$$C_{ijk} = \frac{1}{2}\dot{\partial}_i g_{jk} = \frac{1}{4}\dot{\partial}_i \dot{\partial}_j \dot{\partial}_k F^2,$$

which satisfies the following

$$C_{ijk} y^i = C_{kij} y^i = C_{jki} y^i = 0. \tag{2.3}$$

Berwald covariant derivative $\mathfrak{B}_k T^i_j$ of an arbitrary tensor field T^i_j with respect to x^k is given by [15]

$$\mathfrak{B}_k T^i_j := \partial_k T^i_j - (\dot{\partial}_r T^i_j) G^r_k + T^r_j G^i_{rk} - T^i_r G^r_{jk}.$$

Berwald covariant derivative of the vector y^i vanish identically, that is,

$$\mathfrak{B}_k y^i = 0 \text{ and } \mathfrak{B}_k y_i = 0. \tag{2.4}$$

But, in general, Berwald covariant derivative of the metric tensor g_{ij} does not vanish and given by

$$\mathfrak{B}_k g_{ij} = -2C_{ijk|h} y^h = -2y^h \mathfrak{B}_h C_{ijk}. \tag{2.5}$$

Example 2.1. Let us consider the functions:

(1) $F(x,y) = \frac{|y|+|xy|}{1+|x|}$,

(2) $\vartheta(u,v) = \frac{\sqrt{|v|^2 + \left(|u|^2\,|v|^2 - |uv|^2\right)}}{1+|u|^2}$.

Then, it is obvious that the functions $F(x,y)$ and $\vartheta(u,v)$ satisfy the condition (i), (ii) and (iii). These functions are called the fundamental function or the metric function of the Finsler space F_n.

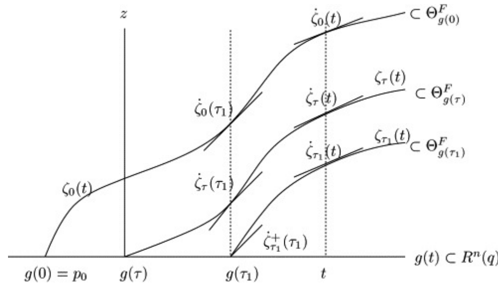

FIGURE 1. Relation Between Metric Spaces and Finsler Spaces

Definition 2.2. Let the current coordinates in the tangent space at the point x_0 be x^i. Then the indicatrix I_{n-1} is a hypersurface defined by [15] $F\left(x_0, x^i\right) = 1$ or by the parametric form defined by $x^i = x^i\left(u^a\right)$, $a = 1, 2, \ldots, n-1$.

Definition 2.3. The projection of any tensor T^i_j on indicatrix I_{n-1} given by [1, 15]

$$p.T^i_j = T^a_b h^i_a h^b_j, \tag{2.6}$$

where

$$h^i_c = \delta^i_c - l^i l_c. \tag{2.7}$$

The projection of the vector y^i, the unit vector l^i and the metric tensor g_{ij} on the indicatrix are given by $p.y^i = 0$, $p.l^i = 0$ and $p.g_{ij} = h_{ij}$, where $h_{ij} = g_{ij} - l_i l_j$.

Abdallah et al. [1, 2, 3] introduced the generalized $\mathfrak{B}P-$recurrent space and generalized $\mathfrak{B}P-$birecurrent space which are characterized by the conditions:

$$\mathfrak{B}_m P^i_{jkh} = \lambda_m P^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}) \tag{2.8}$$

and

$$\mathfrak{B}_l \mathfrak{B}_m P^i_{jkh} = a_{lm} P^i_{jkh} + b_{lm}(\delta^i_j g_{kh} - \delta^i_k g_{jh}) - 2y^t \mu_m \mathfrak{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}), \tag{2.9}$$

respectively. These spaces are denoted by $G(\mathfrak{B}P) - RF_n$ and $G(\mathfrak{B}P) - BRF_n$.

Let us consider a Finsler space which Cartan's second curvature tensor P^i_{jkh} is decomposition. Since the curvature tensor is a mixed tensor of the type $(1, 3)$, that is, rank 4, it may be written as product of contravariant (or covariant) vector and tensor of rank 3, that is, covariant tensor of the type (0,3) (or mixed tensor of the type (1,2)) as following [13, 14]:

$$P^i_{jkh} = X^i Y_{jkh}, \tag{2.10}$$

$$P^i_{jkh} = X_j Y^i_{kh}, \tag{2.11}$$

$$P^i_{jkh} = X_k Y^i_{jh} \tag{2.12}$$

and

$$P^i_{jkh} = X_h Y^i_{jk}, \tag{2.13}$$

as first case. Or in second case as product of two tensors each them of rank 2, that is, mixed tensors of the type $(1,1)$ and covariant tensor of the type $(0,2)$ as following [13, 14]

$$P^i_{jkh} = T^i_j \psi_{kh}, \tag{2.14}$$

$$P^i_{jkh} = T^i_k \psi_{jh} \tag{2.15}$$

and

$$P^i_{jkh} = T^i_h \psi_{jk}. \tag{2.16}$$

In next sections, we will discuss the possible forms in three decomposable of the tensor, two decompositions for the first case (the other are similar) and one decomposition for the second case (the other are similar). Obviously, from all several possibilities, we will study the possibilities which given by (2.10), (2.11) and (2.14).

3. Decomposition of Cartan's second curvature tensor in $G(\mathfrak{B}P) - RF_n$

In this section, we will discuss the decomposition of Cartan's second curvature tensor P^i_{jkh} in generalized $\mathfrak{B}P$-recurrent space. Let us consider Cartan's second curvature tensor P^i_{jkh} is decomposable as (2.10), where Y_{jkh} is non-zero covariant tensor field and homogeneous of degree-1 in its directional argument which called decomposition tensor field and X^i is independent of x^m.

In next theorem we will discuss the decomposition (2.10) for Cartan's second curvature tensor P^i_{jkh} which is generalized recurrent.

Theorem 3.1. *In $G(\mathfrak{B}P) - RF_n$, under the decomposition* (2.10) *and if X^i is covariant constant, then the decomposition tensor* $(X^i Y_{jkh})$ *satisfies the generalized recurrence property.*

Proof. Assume that X^i is covariant constant. Taking $\mathfrak{B}-$ covariant derivative for equation (2.10) with respect to x^m, we get

$$\mathfrak{B}_m P^i_{jkh} = (\mathfrak{B}_m X^i) Y_{jkh} + X^i \mathfrak{B}_m Y_{jkh}. \tag{3.1}$$

Since the decomposition vector field X^i is covariant constant, that is, $(\mathfrak{B}_m X^i = 0)$, therefore equation (3.1) can be written as

$$\mathfrak{B}_m P^i_{jkh} = X^i \mathfrak{B}_m Y_{jkh}.$$

By using the condition (2.8) in above equation and in view of (2.10), we get

$$X^i \mathfrak{B}_m Y_{jkh} = \lambda_m X^i Y_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}). \tag{3.2}$$

Since X^i is independent of x^m, equation (3.2) can be written as

$$\mathfrak{B}_m(X^i Y_{jkh}) = \lambda_m(X^i Y_{jkh}) + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}).$$

The last equation refers that the decomposition tensor $(X^i Y_{jkh})$ is generalized recurrent, that is, satisfies the condition (2.8). The proof for this theorem is completed. □

Now, from the Theorem 3.1, we can get the following corollary.

Corollary 3.2. *Under the decomposition* (2.10) *and if the tensor field* $\emptyset_{mjkh}$ *is skewsymmetric in second and third indicator, then the decomposition tensor* Y_{jkh} *is non-vanishing.*

Proof. In view of equation (3.2), we get

$$\mathfrak{B}_m Y_{jkh} = \lambda_m Y_{jkh} + \alpha_{mi}(\delta^i_j g_{kh} - \delta^i_k g_{jh}),$$

where $\alpha_{mi} = \mu_m / X^i$. Above equation can be written as

$$\mathfrak{B}_m Y_{jkh} = \lambda_m Y_{jkh} + (\emptyset_{mjkh} - \emptyset_{mkjh}),$$

where $\emptyset_{mjkh} = \alpha_{mj} g_{kh}$ and $\emptyset_{mkjh} = \alpha_{mk} g_{jk}$.

Now, if the tensor field $\emptyset_{mjkh}$ is skew-symmetric in second and third indicator, then above equation can be written as

$$\mathfrak{B}_m Y_{jkh} = \lambda_m Y_{jkh} + 2\emptyset_{mjkh}. \tag{3.3}$$

The equation (3.3) refers that the decomposition tensor Y_{jkh} is non-vanishing in $G(\mathfrak{B}P) - RF_n$. The proof for this corollary is completed. □

Let us consider a Finsler space which Cartan's second curvature tensor P^i_{jkh} is decomposition (2.11), where X_j is non-zero covariant vector field and Y^i_{kh} decomposition tensor field.

In next theorem we will discuss the decomposition (2.11) for Cartan's second curvature tensor P^i_{jkh} which be generalized recurrent.

Theorem 3.3. *In* $G(\mathfrak{B}P) - RF_n$*, under the decomposition* (2.11) *and if the covariant vector field* λ_m *is not equal the covariant vector field* v_m*, then the decomposition tensor* $(X_j Y^i_{kh})$ *satisfies the generalized recurrence property.*

Proof. Taking $\mathfrak{B}-$ covariant derivative for equation (2.11) with respect to x^m, we get

$$\mathfrak{B}_m P^i_{jkh} = (\mathfrak{B}_m X_j) Y^i_{kh} + X_j \mathfrak{B}_m Y^i_{kh} \tag{3.4}$$

or

$$\mathfrak{B}_m P^i_{jkh} = v_m X_j Y^i_{kh} + X_j \mathfrak{B}_m Y^i_{kh}, \tag{3.5}$$

where $\mathfrak{B}_m X_j = v_m X_j$. By using the condition (2.8) in equation (3.5), we obtain

$$v_m X_j Y^i_{kh} + X_j \mathfrak{B}_m Y^i_{kh} = \lambda_m P^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}). \tag{3.6}$$

In view of (2.11), then equation (3.6) can be written as

$$X_j \mathfrak{B}_m Y^i_{kh} = (\lambda_m - v_m) X_j Y^i_{kh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}). \tag{3.7}$$

Now, assume that the vector field λ_m is not equal to the vector field v_m, we get

$$X_j \mathfrak{B}_m Y^i_{kh} = \chi_m X_j Y^i_{kh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}), \tag{3.8}$$

where $\chi_m = \lambda_m - v_m$. Since X_j is independent of x^m, so equation (3.8) can be written as

$$\mathfrak{B}_m (X_j Y^i_{kh}) = \chi_m (X_j Y^i_{kh}) + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}).$$

The last equation refers that the decomposition tensor $(X_j Y^i_{kh})$ is generalized recurrent, that is, satisfies the condition (2.8). The proof for this theorem is completed. □

Now, from the Theorem 3.3, we can get the following corollary.

Corollary 3.4. *Under the decomposition* (2.11)*, if X_j is covariant constant and X is constant, then the behavior of decomposition tensors Y^i_{kh}, XY^i_h, Y^i_h and XY is recurrent.*

Proof. By using the condition (2.8) in equation (3.4) and in view of (2.11), we get

$$(\mathfrak{B}_m X_j) Y^i_{kh} + X_j \mathfrak{B}_m Y^i_{kh} = \lambda_m X_j Y^i_{kh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}). \tag{3.9}$$

Since X_j is covariant constant, equation (3.9) can be written as

$$\mathfrak{B}_m Y^i_{kh} = \lambda_m Y^i_{kh} + \omega^j_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}),$$

where $\omega^j_m = \mu_m X_j$. Above equation can be written as

$$\mathfrak{B}_m Y^i_{kh} = \lambda_m Y^i_{kh}, \tag{3.10}$$

where $\theta^i_{mkh} = \omega^j_m \delta^i_j g_{kh}$ and $\theta^i_{mkh} = \omega^j_m \delta^i_k g_{jh}$.

Transvecting equation (3.9) by y^j, using (2.1), (2.2) and (2.4), we get

$$(\mathfrak{B}_m X) Y^i_{kh} + X \mathfrak{B}_m Y^i_{kh} = \lambda_m X Y^i_{kh} + \mu_m(y^i g_{kh} - \delta^i_k y_h),$$

where $X = X_j y^j$. If X is constant, that is, $(\mathfrak{B}_m X = 0)$, then above equation can be written

$$\mathfrak{B}_m(XY^i_{kh}) = \lambda_m(XY^i_{kh}) + \mu_m(y^i g_{kh} - \delta^i_k y_h). \tag{3.11}$$

Transvecting equation (3.11) by y^k, using (2.1), (2.2) and (2.4), we get

$$\mathfrak{B}_m(XY^i_h) = \lambda_m(XY^i_h), \tag{3.12}$$

where $Y^i_h = Y^i_{kh} y^k$. Since X is constant, equation(3.12) can be written as

$$\mathfrak{B}_m Y^i_h = \lambda_m Y^i_h. \tag{3.13}$$

Contracting the indices i and h in eq. (3.12), we get

$$\mathfrak{B}_m(XY) = \lambda_m(XY), \tag{3.14}$$

where $Y = Y^i_i$. The equations (3.10), (3.12), (3.13) and (3.14) refer that the tensors Y^i_{kh}, XY^i_h, Y^i_h and XY satisfy the recurrence property in $G(\mathfrak{B}P) - RF_n$. The proof for this corollary is completed. □

Let us consider a Finsler space which Cartan's second curvature tensor P^i_{jkh} is decomposition (2.14), where T^i_j and ψ_{kh} are the decomposition tensors field.

In next theorem we will discuss the decomposition (2.14) for Cartan's second curvature tensor P^i_{jkh} which is generalized recurrent.

Theorem 3.5. *In $G(\mathfrak{B}P) - RF_n$, under the decomposition* (2.14) *and if the covariant vector field λ_m is not equal the covariant vector field v_m, then the decomposition tensor $(T^i_j \psi_{kh})$ satisfies the generalized recurrence property.*

Proof. Taking $\mathfrak{B}-$ covariant derivative for equation (2.14) with respect to x^m, we get

$$\mathfrak{B}_m P^i_{jkh} = (\mathfrak{B}_m T^i_j)\psi_{kh} + T^i_j \mathfrak{B}_m \psi_{kh} \tag{3.15}$$

or

$$\mathfrak{B}_m P^i_{jkh} = v_m T^i_j \psi_{kh} + T^i_j \mathfrak{B}_m \psi_{kh}. \tag{3.16}$$

where $\mathfrak{B}_m T^i_j = v_m T^i_j$.

Using the condition (2.8) in above equation, we obtain

$$v_m T^i_j \psi_{kh} + T^i_j \mathfrak{B}_m \psi_{kh} = \lambda_m P^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}). \tag{3.17}$$

In view of (2.14), equation (3.17) can be written as

$$T^i_j \mathfrak{B}_m \psi_{kh} = (\lambda_m - v_m) T^i_j \psi_{kh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}). \tag{3.18}$$

Now, assume that the vector field λ_m is not equal the vector field v_m, we get

$$T^i_j \mathfrak{B}_m \psi_{kh} = \chi_m T^i_j \psi_{kh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}), \tag{3.19}$$

where $\chi_m = \lambda_m - \upsilon_m$. Since T^i_j is independent of x^m, equation (3.19) can be written as

$$\mathfrak{B}_m(T^i_j\psi_{kh}) = \chi_m(T^i_j\psi_{kh}) + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}).$$

The last equation refers that the decomposition tensor $(T^i_j\psi_{kh})$ is generalized recurrent, that is, satisfies the condition (2.8). The proof for this theorem is completed. □

Now, from the Theorem 3.5, we can get the following corollary.

Corollary 3.6. *Under the decomposition* (2.14) *and if the tensor field* φ_{mkh} *is skew symmetric in first and second indices, then the decomposition tensor* ψ_{kh} *is non-vanishing.*

Proof. By using the condition (2.8) in equation (3.15), we get

$$(\mathfrak{B}_m T^i_j)\psi_{kh} + T^i_j\mathfrak{B}_m\psi_{kh} = \lambda_m P^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}).$$

In view of (2.14) and if the decomposition tensor field T^i_j is covariant constant, the above equation can be written as

$$\mathfrak{B}_m\psi_{kh} = \lambda_m\psi_{kh} + \chi^j_{im}(\delta^i_j g_{kh} - \delta^i_k g_{jh}),$$

where $\chi^j_{im} = \mu_m T^i_j$. Also above equation can be written as

$$\mathfrak{B}_m\psi_{kh} = \lambda_m\psi_{kh} + (\varphi_{mkh} - \varphi_{kmh}),$$

where $\varphi_{mkh} = \chi^j_{im}\delta^i_j g_{kh}$ and $\varphi_{kmh} = \chi^j_{im}\delta^i_k g_{jh}$.

Now, if the tensor field φ_{mkh} is skew symmetric in first and second indicator, then above equation can be written as

$$\mathfrak{B}_m\psi_{kh} = \lambda_m\psi_{kh} + 2\varphi_{mkh}. \tag{3.20}$$

The equation(3.20) refers that the decomposition tensor ψ_{kh} is non-vanishing in $G(\mathfrak{B}P) - RF_n$. The proof for this corollary is completed. □

In view the Theorems 3.1, 3.3 and 3.5, we can conclude that if $\delta^i_j g_{kh} = \delta^i_k g_{jh}$, then the decomposition tensors $(X^i Y_{jkh})$, $(X_j Y^i_{kh})$ and $(T^i_j\psi_{kh})$ behave as recurrent, clearly, satisfy the following conditions:

$$\mathfrak{B}_m(X^i Y_{jkh}) = \lambda_m(X^i Y_{jkh}), \tag{3.21}$$

$$\mathfrak{B}_m(X_j Y^i_{kh}) = \chi_m(X_j Y^i_{kh}) \tag{3.22}$$

and

$$\mathfrak{B}_m(T^i_j\psi_{kh}) = \chi_m(T^i_j\psi_{kh}), \tag{3.23}$$

respetively.

4. Decomposition of Cartan's second curvature tensor in $G(\mathfrak{B}P) - BRF_n$

In this section, we will discuss the decomposition of Cartan's second curvature tensor P^i_{jkh} in generalized $\mathfrak{B}P-$ birecurrent space.

In next theorem we will discuss the decomposition (2.10) for Cartan's second curvature tensor P^i_{jkh} which be generalized birecurrent.

Theorem 4.1. *In $G(\mathfrak{B}P) - BRF_n$, under the decomposition* (2.10) *and if X^i is covariant constant, then the decomposition tensor $(X^i Y_{jkh})$ satisfies the generalized birecurrence property.*

Proof. Assume that X^i is covariant constant. Taking $\mathfrak{B}-$ covariant derivative for equation (2.10) twice with respect to x^m and x^l, respectively, we get

$$\mathfrak{B}_l \mathfrak{B}_m P^i_{jkh} = X^i \mathfrak{B}_l \mathfrak{B}_m Y_{jkh}. \tag{4.1}$$

Using the condition (2.9) in equation (4.1) and in view of (2.10), we get

$$\begin{aligned} X^i \mathfrak{B}_l \mathfrak{B}_m Y_{jkh} \quad = \quad & a_{lm} X^i Y_{jkh} + b_{lm}(\delta^i_j g_{kh} - \delta^i_k g_{jh}) \\ & -2y^t \mu_m \mathfrak{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}). \end{aligned} \tag{4.2}$$

Since X^i is covariant constant, equation (4.2) can be written as

$$\mathfrak{B}_l \mathfrak{B}_m (X^i Y_{jkh}) = a_{lm}(X^i Y_{jkh}) + b_{lm}(\delta^i_j g_{kh} - \delta^i_k g_{jh}) - 2y^t \mu_m \mathfrak{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}).$$

The last equation refers that the decomposition tensor $(X^i Y_{jkh})$ is generalized birecurrent , that is, satisfies the condition (2.9). The proof for this theorem is completed. □

Now, from the Theorem 4.1, we can obtain the following corollary.

Corollary 4.2. *Under the decomposition* (2.10) *if the tensor field Φ_{lmjkh} is skew symmetric in third and fourth indicator, then the decomposition tensor field Y_{jkh} is non-vanishing.*

Proof. In view of equation (4.2), we get

$$\mathfrak{B}_l \mathfrak{B}_m Y_{jkh} = a_{lm} Y_{jkh} + \gamma_{lmi}(\delta^i_j g_{kh} - \delta^i_k g_{jh}) - 2y^t \alpha_{mi} \mathfrak{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}),$$

where $\gamma_{lmi} = b_{lm} X^i$ and $\alpha_{mi} = \mu_m X^i$. The above equation can be written as

$$\mathfrak{B}_l \mathfrak{B}_m Y_{jkh} = a_{lm} Y_{jkh} + (\Phi_{lmjkh} - \Phi_{lmkjh}) - 2y^t \alpha_{mi} \mathfrak{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}),$$

where $\Phi_{lmjkh} = \gamma_{lmi} \delta^i_j g_{kh}$ and $\Phi_{lmkjh} = \gamma_{lmi} \delta^i_k g_{jh}$.

Now, if the tensor field Φ_{lmjkh} is skew symmetric in third and fourth indicator, above equation can be written as

$$\mathfrak{B}_l \mathfrak{B}_m Y_{jkh} = a_{lm} Y_{jkh} + 2\Phi_{lmjkh} - 2y^t \alpha_{mi} \mathfrak{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}). \tag{4.3}$$

The equation (4.3) refers that the decomposition tensor Y_{jkh} is non-vanishing in $G(\mathfrak{B}P) - BRF_n$. The proof for this corollary is completed. □

In next theorem we will discuss the decomposition (2.11) for Cartan's second curvature tensor P^i_{jkh} which be generalized birecurrent.

Theorem 4.3. *In $G(\mathfrak{B}P) - BRF_n$, under the decomposition* (2.11) *and if X_j is covariant constant, then the decomposition tensor $(X_j Y^i_{kh})$ satisfies the generalized birecurrence property.*

Proof. Assume that X_j is covariant constant. Taking $\mathfrak{B}-$ covariant derivative for equation (2.11) twice with respect to x^m and x^l, respectively, we get

$$\mathfrak{B}_l \mathfrak{B}_m P^i_{jkh} = X_j \mathfrak{B}_l \mathfrak{B}_m Y^i_{kh}. \tag{4.4}$$

Using the condition (2.9) in equation (4.4) and in view of (2.11), we get

$$\begin{aligned} X_j \mathfrak{B}_l \mathfrak{B}_m Y^i_{kh} \quad = \quad & a_{lm} X_j Y^i_{kh} + b_{lm}(\delta^i_j g_{kh} - \delta^i_k g_{jh}) \\ & -2y^t \mu_m \mathfrak{B}_t (\delta^i_j C_{khl} - \delta^i_k C_{jhl}). \end{aligned} \tag{4.5}$$

Since X_j is covariant constant, equation (4.5) can be written as

$$\mathfrak{B}_l \mathfrak{B}_m (X_j Y^i_{kh}) = a_{lm}(X_j Y^i_{kh}) + b_{lm}(\delta^i_j g_{kh} - \delta^i_k g_{jh}) - 2y^t \mu_m \mathfrak{B}_t (\delta^i_j C_{khl} - \delta^i_k C_{jhl}).$$

The last equation refers that the decomposition tensor $(X_j Y^i_{kh})$ is generalized birecurrent, that is, satisfies the condition (2.9). The proof for this theorem is completed. □

Now, from the Theorem 4.3, we can obtain the following corollary.

Corollary 4.4. *Under the decomposition* (2.11) *and if X is constant, then the tensors XY^i_h, Y^i_h and XY behave as birecurrent.*

Proof. Transvecting equation (4.5) by y^j, using (2.1), (2.2), (2.3) and (2.4), we get

$$X \mathfrak{B}_l \mathfrak{B}_m Y^i_{kh} = a_{lm} X Y^i_{kh} + b_{lm}(y^i g_{kh} - \delta^i_k y_h) - 2y^t \mu_m \mathfrak{B}_t (y^i C_{khl}),$$

where $X = X_j y^j$. If X is constant, that is, $(\mathfrak{B}_m X = 0)$, then above equation reduces to

$$\mathfrak{B}_l \mathfrak{B}_m (X Y^i_{kh}) = a_{lm}(X Y^i_{kh}) + b_{lm}(y^i g_{kh} - \delta^i_k y_h) - 2y^t \mu_m \mathfrak{B}_t (y^i C_{khl}). \tag{4.6}$$

Transvecting equation (4.6) by y^k , using (2.1), (2.2), (2.3) and (2.4), we get

$$\mathfrak{B}_l \mathfrak{B}_m (X Y^i_h) = a_{lm}(X Y^i_h), \tag{4.7}$$

where $Y^i_h = Y^i_{kh} y^k$. Since X is constant, equation (4.7) can be written as

$$\mathfrak{B}_l \mathfrak{B}_m Y^i_h = a_{lm} Y^i_h. \tag{4.8}$$

Contracting the indices i and h in equation (4.7), we get

$$\mathfrak{B}_l\mathfrak{B}_m(XY) = a_{lm}(XY), \tag{4.9}$$

where $Y = Y^i_h$. The equations (4.7), (4.8) and (4.9) refer that the tensors XY^i_h, Y^i_h and XY satisfy the birecurrent property in $G(\mathfrak{B}P) - BRF_n$. The proof for this corollary is completed. □

In next theorem we will discuss the decomposition (2.14) for Cartan's second curvature tensor P^i_{jkh} which be generalized birecurrent.

Theorem 4.5. *In $G(\mathfrak{B}P) - BRF_n$, under the decomposition* (2.14) *and if T^i_j is covariant constant, then the decomposition tensor $(T^i_j\psi_{kh})$ satisfies the generalized birecurrence property.*

Proof. Assume that T^i_j is covariant constant. Taking $\mathfrak{B}-$ covariant derivative for equation (2.14) twice with respect to x^m and x^l, respectively, we get

$$\mathfrak{B}_l\mathfrak{B}_m P^i_{jkh} = T^i_j\mathfrak{B}_l\mathfrak{B}_m\psi_{kh}. \tag{4.10}$$

Using the condition (2.9) in equation (4.10) and in view of (2.14), we get

$$T^i_j\mathfrak{B}_l\mathfrak{B}_m\psi_{kh} = a_{lm}T^i_j\psi_{kh} + b_{lm}(\delta^i_j g_{kh} - \delta^i_k g_{jh}) - 2y^t\mu_m\mathfrak{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}). \tag{4.11}$$

Since T^i_j is covariant constant, therefore equation (4.11) can be written as

$$\mathfrak{B}_l\mathfrak{B}_m(T^i_j\psi_{kh}) = a_{lm}(T^i_j\psi_{kh}) + b_{lm}(\delta^i_j g_{kh} - \delta^i_k g_{jh}) - 2y^t\mu_m\mathfrak{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}).$$

The last equation refers that the decomposition tensor $(T^i_j\psi_{kh})$ is generalized birecurrent, that is, satisfies the condition (2.9). The proof for this theorem is completed. □

Now, from the Theorem 4.5, we can obtain the following corollary.

Corollary 4.6. *Under the decomposition* (2.14) *and if the decomposition tensor field T^i_j is covariant constant, then the decomposition ψ_{kh} behaves as birecurrent if and only if $2y^t\chi^j_{im}\mathfrak{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}) = 0$.*

Proof. In view of equation (4.11) and the decomposition tensor field T^i_j is covariant constant, we get

$$\mathfrak{B}_l\mathfrak{B}_m\psi_{kh} = a_{lm}\psi_{kh} + \alpha^j_{ilm}(\delta^i_j g_{kh} - \delta^i_k g_{jh}) - 2y^t\chi^j_{im}\mathfrak{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}),$$

where $\alpha^j_{ilm} = b_{lm}T^i_j$ and $\chi^j_{im} = \mu_m T^i_j$. Above equation can be written as

$$\mathfrak{B}_l\mathfrak{B}_m\psi_{kh} = a_{lm}\psi_{kh} - 2y^t\chi^j_{im}\mathfrak{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}),$$

where $\alpha^j_{ilm}\delta^i_j g_{kh} = \alpha^j_{ilm}\delta^i_k g_{jh}$. This shows that $\mathfrak{B}_l\mathfrak{B}_m\psi_{kh} = a_{lm}\psi_{kh}$ if and only if

$$2y^t\chi^j_{im}\mathfrak{B}_t(\delta^i_j C_{khl} - \delta^i_k C_{jhl}) = 0. \tag{4.12}$$

Then, the decomposition tensor ψ_{kh} is birecurrent in $G(\mathfrak{B}P) - BRF_n$ if and only if equation (4.12) hold. The proof for this corollary is completed. □

In view the theorems 4.1, 4.3 and 4.5, we can conclude that if $\delta^i_j g_{kh} = \delta^i_k g_{jh}$ and $\delta^i_j C_{khl} = \delta^i_k C_{jhl}$, then the decomposition tensors $(X^i Y_{jkh})$, $(X_j Y^i_{kh})$ and $(T^i_j \psi_{kh})$ behave as birecurrent, clearly, satisfy the following conditions

$$\mathfrak{B}_l \mathfrak{B}_m (X^i Y_{jkh}) = a_{lm}(X^i Y_{jkh}), \tag{4.13}$$

$$\mathfrak{B}_l \mathfrak{B}_m (X_j Y^i_{kh}) = a_{lm}(X_j Y^i_{kh}) \tag{4.14}$$

and

$$\mathfrak{B}_l \mathfrak{B}_m (T^i_j \psi_{kh}) = a_{lm}(T^i_j \psi_{kh}), \tag{4.15}$$

respetively.

5. Examples

In order to illustrate the effectiveness of the proposed findings, we consider some examples of the recurrence and birecurrence properties.

Example 5.1. The decomposition tensor $(X^i Y_{jkh})$ is recurrent if and only if it satisfies

$$\mathfrak{B}_m \left[p.(X^i Y_{jkh})\right] = \lambda_m \left[p.(X^i Y_{jkh})\right].$$

Firstly, since the decomposition tensor $(X^i Y_{jkh})$ is recurrent, the condition (3.21) is satisfied. In view of (2.6), the decomposition tensor $(X^i Y_{jkh})$ on indicatrix given by

$$p.(X^i Y_{jkh}) = X^a Y_{bcd} h^i_a h^b_j h^c_k h^d_h. \tag{5.1}$$

By using $\mathfrak{B}$-covariant derivative for eq. (5.1) with respect to x^m, using equation (3.21) and the fact that h^a_b is covariant constant in above equation, we get

$$\mathfrak{B}_m \left[p.(X^i Y_{jkh})\right] = \lambda_m X^a Y^a_{bcd} h^i_a h^b_j h^c_k h^d_h.$$

Using equation (5.1) in above equation, we get

$$\mathfrak{B}_m \left[p.(X^i Y_{jkh})\right] = \lambda_m \left[p.(X^i Y_{jkh})\right]. \tag{5.2}$$

Above equation means the projection on indicatrix for the decomposition tensor $(X^i Y_{jkh})$ behaves as recurrent.

Secondly, let the projection on indicatrix for the decomposition tensor $(X^i Y_{jkh})$ is recurrent, that is, it satisfies equation (5.2). By using (2.6) in equation (5.2), we get

$$\mathfrak{B}_m \left(X^a Y_{bcd}\, h^i_a h^b_j h^c_k h^d_h\right) = \lambda_m X^a Y_{bcd}\, h^i_a h^b_j h^c_k h^d_h.$$

Using (2.7) in above equation, we get

$$\begin{aligned}
&\mathfrak{B}_m\Big[(X^iY_{jkh}) - (X^iY_{jkd})l^dl_h - (X^iY_{jch})l^cl_k + (X^iY_{jcd})l^cl_kl^dl_h \\
&\quad - (X^iY_{bkh})l^bl_j + (X^iY_{bkd})l^bl_jl^dl_h + (X^iY_{bch})l^bl_jl^cl_k - (X^iY_{bcd})l^bl_jl^cl_kl^dl_h \\
&\quad - (X^aY_{jkh})l^il_a + (X^aY_{jkd})l^il_al^dl_h + (X^aY_{jch})l^il_al^cl_k - (X^aY_{jcd})l^il_al^cl_kl^dl_h \\
&\quad + (X^aY_{bkh})l^il_al^bl_j - (X^aY_{bkd})l^il_al^bl_jl^dl_h - (X^aY_{bch})l^il_al^bl_jl^cl_k \\
&\quad + (X^aY_{bcd})l^il_al^bl_jl^cl_kl^dl_h\Big] \\
&= \lambda_m\Big[(X^iY_{jkh}) - (X^iY_{jkd})l^dl_h - (X^iY_{jch})l^cl_k + (X^iY_{jcd})l^cl_kl^dl_h \\
&\quad - (X^iY_{bkh})l^bl_j + (X^iY_{bkd})l^bl_jl^dl_h + (X^iY_{bch})l^bl_jl^cl_k - (X^iY_{bcd})l^bl_jl^cl_kl^dl_h \\
&\quad - (X^aY_{jkh})l^il_a + (X^aY_{jkd})l^il_al^dl_h + (X^aY_{jch})l^il_al^cl_k - (X^aY_{jcd})l^il_al^cl_kl^dl_h \\
&\quad + (X^aY_{bkh})l^il_al^bl_j - (X^aY_{bkd})l^il_al^bl_jl^dl_h - (X^aY_{bch})l^il_al^bl_jl^cl_k \\
&\quad + (X^aY_{bcd})l^il_al^bl_jl^cl_kl^dl_h\Big].
\end{aligned}$$

From, $l^i = \frac{y^i}{F}$ and $l_i = \frac{y_i}{F}$, if $(X^aY_{bcd})\ y_a = (X^aY_{bcd})\ y^b = (X^aY_{bcd})\ y^c = (X^aY_{bcd})\ y^d = 0$, then above equation can be written as

$$\mathfrak{B}_m(X^iY_{jkh}) = \lambda_m(X^iY_{jkh}).$$

Above equation means the decomposition tensor (X^iY_{jkh}) behaves as recurrent.

Also, we can use same technique for showing the decomposition tensors $(X_jY^i_{kh})$ and $(T^i_j\psi_{kh})$ are recurrent if and only if the projection on indicatrix for them behave as recurrent.

Example 5.2. The decomposition tensor (X^iY_{jkh}) is birecurrent if and only if it satisfies

$$\mathfrak{B}_l\mathfrak{B}_m\left[p.(X^iY_{jkh})\right] = a_{lm}\left[p.(X^iY_{jkh})\right].$$

Firstly, since the decomposition tensor (X^iY_{jkh}) is birecurrent, that, the condition (4.13) is satisfied. By using $\mathfrak{B}-$ covariant derivative for equation (5.1) with respect to x^m and x^l, using equation (4.13) and the fact that h^a_b is covariant constant, we get

$$\mathfrak{B}_l\mathfrak{B}_m\left[p.(X^iY_{jkh})\right] = a_{lm}X^aY^a_{bcd}h^i_ah^b_jh^c_kh^d_h.$$

Using equation (5.1) in above equation, we get

$$\mathfrak{B}_l\mathfrak{B}_m\left[p.(X^iY_{jkh})\right] = a_{lm}\left[p.(X^iY_{jkh})\right]. \tag{5.3}$$

Equation (5.3) means the projection on indicatrix for the decomposition tensor (X^iY_{jkh}) behaves as birecurrent.

Secondly, let the projection on indicatrix for the decomposition tensor (X^iY_{jkh}) is birecurrent, that is, it satisfies equation (5.3). By using (2.6) in equation (5.3), we get

$$\mathfrak{B}_l\mathfrak{B}_m\left(X^aY_{bcd}\ h^i_ah^b_jh^c_kh^d_h\right) = a_{lm}X^aY_{bcd}\ h^i_ah^b_jh^c_kh^d_h.$$

Using (2.7) in above equation, we get

$$\begin{aligned}
&\mathfrak{B}_l\mathfrak{B}_m\Big[(X^iY_{jkh}) - (X^iY_{jkd})l^dl_h - (X^iY_{jch})l^cl_k + (X^iY_{jcd})l^cl_kl^dl_h \\
&\quad - (X^iY_{bkh})l^bl_j + (X^iY_{bkd})l^bl_jl^dl_h + (X^iY_{bch})l^bl_jl^cl_k - (X^iY_{bcd})l^bl_jl^cl_kl^dl_h \\
&\quad - (X^aY_{jkh})l^il_a + (X^aY_{jkd})l^il_al^dl_h + (X^aY_{jch})l^il_al^cl_k - (X^aY_{jcd})l^il_al^cl_kl^dl_h \\
&\quad + (X^aY_{bkh})l^il_al^bl_j - (X^aY_{bkd})l^il_al^bl_jl^dl_h - (X^aY_{bch})l^il_al^bl_jl^cl_k \\
&\quad + (X^aY_{bcd})l^il_al^bl_jl^cl_kl^dl_h\Big] \\
&= a_{lm}\Big[(X^iY_{jkh}) - (X^iY_{jkd})l^dl_h - (X^iY_{jch})l^cl_k + (X^iY_{jcd})l^cl_kl^dl_h \\
&\quad - (X^iY_{bkh})l^bl_j + (X^iY_{bkd})l^bl_jl^dl_h + (X^iY_{bch})l^bl_jl^cl_k - (X^iY_{bcd})l^bl_jl^cl_kl^dl_h \\
&\quad - (X^aY_{jkh})l^il_a + (X^aY_{jkd})l^il_al^dl_h + (X^aY_{jch})l^il_al^cl_k - (X^aY_{jcd})l^il_al^cl_kl^dl_h \\
&\quad + (X^aY_{bkh})l^il_al^bl_j - (X^aY_{bkd})l^il_al^bl_jl^dl_h - (X^aY_{bch})l^il_al^bl_jl^cl_k \\
&\quad + (X^aY_{bcd})l^il_al^bl_jl^cl_kl^dl_h\Big].
\end{aligned}$$

From $l^i = \frac{y^i}{F}$ and $l_i = \frac{y_i}{F}$, if $(X^aY_{bcd})\ y_a = (X^aY_{bcd})\ y^b = (X^aY_{bcd})\ y^c = (X^aY_{bcd})\ y^d = 0$, then above equation can be written as $\mathfrak{B}_l\mathfrak{B}_m(X^iY_{jkh}) = a_{lm}(X^iY_{jkh})$. Last equation means the decomposition tensor (X^iY_{jkh}) behaves as birecurrent.

Also, we can use same technique for showing the decomposition tensors $(X_jY^i_{kh})$ and $(T^i_j\psi_{kh})$ are birecurrent if and only if the projection on indicatrix for them behave as birecurrent.

6. Conclusion

This article contributed in particular to the growth of the decomposition of Cartan's second curvature tensor P^i_{jkh} in the generalized $\mathfrak{B}P-$recurrent space and generalized $\mathfrak{B}P-$birecurrent space. We obtained some tensors satisfy the recurrence and birecurrence property under the decomposition. Also, different identities and several theorems have been discussed under the decomposition in $G(\mathfrak{B}P)-RF_n$ and $G(\mathfrak{B}P)-BRF_n$. The topic examined in this manuscript can be expanded to a greater extent by the use of decomposition of Cartans second curvature tensor P^i_{jkh} in generalized $\mathfrak{B}P-$trirecurrent space and we find some theorems under the decomposition in $G(\mathfrak{B}P) - TRF_n$.

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