J. Appl. Math. & Informatics Vol. **40***(2022), No. 3 - 4, pp. 709 - 717*
https://doi.org/10.14317/jami.2022.709

SEMIPRIME RINGS WITH INVOLUTION AND CENTRALIZERS

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ABSTRACT. The objective of this research is to prove that an additive mapping $T : R \to R$ is a left as well as right centralizer on R if it satisfies any one of the following identities:
(i) $T(x^n y^n + y^n x^n) = T(x^n)y^n + y^n T(x^n)$
(ii) $2T(x^n y^n) = T(x^n)y^n + y^n T(x^n)$
for each $x, y \in R$, where $n \geq 1$ is a fixed integer and R is any $n!$-torsion free semiprime ring. In addition, we talk over above identities in the setting of $*$-ring(ring with involution).

AMS Mathematics Subject Classification : 16N60, 16B99, 16W25.
Key words and phrases : Semiprime ring, left(right) centralizer, involution.

1. Introduction

Our present interpretation is encouraged by the research work of Helgosen [4], who intimated the concept of centralizers (multipliers) on Banach algebras. A potential conception on centralizers of commutative Banach algebra posed by J.K. Wang [12]. Further B.E. Johnson [5] explore previous such ideas on centralizers for topological algebras and continuity of centralizers on Banach algebra. For exhaustive knowledge of related matter, one refer to [6, 7] and references therein.

First of all we need to recollect some basic notions that is useful for our concept. Throughout R will represent a ring possessing associativity and identity. A ring R is said to be n-torsion free for an integer $n > 1$, if $nw = 0$ implies $w = 0$ for all w in R. A ring R is called prime if $cRd = \{0\}$ gives that either $c = 0$ or $d = 0$, and is known as semiprime if $cRc = \{0\}$ gives that $c = 0$. A mapping $d : R \to R$ is called a derivation if it is additive and the condition $d(xw) = d(x)w + xd(w)$ holds for every $w, x \in R$ and we say d is a Jordan derivation if $d(y^2) = d(y)y + yd(y)$ is holds good for every y in R. Johnson [5]

Received August 11, 2021. Revised February 5, 2022. Accepted February 8, 2022.
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inaugrated the concept of centralizers in rings as follows: an additive mapping $T : R \to R$ is called a left (right) centralizer if $T(wu) = T(w)u$ $\big(T(wu) = wT(u)\big)$ holds for every w, u in R and is called a Jordan left (Jordan right) centralizer if $T(w^2) = T(w)w$ $\big(T(w^2) = wT(w)\big)$ holds for each $w \in R$. The concept of centralizers is also known as multipliers (see [13]). We call such map T a centralizer, in case T is both left as well as right centralizer. Following Zalar [14], if $k \in R$, then $l_k(w) = kw$ is a left centralizer and $r_k(w) = wk$ is a right centralizer for a fixed integer k. A map $T : R \to R$ on a ring R having identity element is a left (right) centralizer if and only if its of the form $l_k(w)$ $\big(r_k(w)\big)$.

A remarkable contribution to the study of centralizers and their properties on prime and semiprime ring has been done like [1, 6, 7, 9, 13]. Molnar [8] in 1995 proved that: if R is a 2-torsion free prime ring and $T : R \to R$ is an additive map such that $T(wzw) = T(w)zw$ for each w, z in R, then T is a left (right) centralizer. Let us consider a map $T : R \to R$ of an arbitrary ring is a centralizer if it satisfies the relation $T(wzw) = wT(z)w$, for each w, z in R. We can think that what will happen, if we draw our attention for converse situation of the last relation. By considering the converse situation, Vukman [10] came across an affirmative answer for semiprime rings. More precisely, author in [10] established that: let R be a semiprime and 2-torsionfree ring and let $T : R \to R$ be an additive mapping. Suppose that $T(wzw) = wT(z)w$, holds for every w and z in R. In this case T is a centralizer.

Several generalization of previous result has obtained by number of mathematician. An implicit idea found through Theorem 2.3.2 in [2], if R is semiprime ring with extended centroid $\mathcal{C}$ and $T : R \to R$ is a left and right centralizer, then there exists an element $\lambda \in \mathcal{C}$ such that $T(x) = \lambda x$ for all $x \in R$. Later, Vukman and Kosi-Ulbl [11] established a result by taking an algebraic equation $3T(xyx) = T(x)yx + xT(y)x + xyT(x)$. In fact they proved that a mapping $T : R \to R$ is additive and is of the form $T(x) = \lambda x$ if it satisfies the algebraic equation $3T(zwz) = T(z)wz + zT(w)z + zwT(z)$ for every w, z in R and $\lambda \in \mathcal{C}$, where R is a semiprime ring possesssing 2-torsionfreeness and $\mathcal{C}$ noted for the extended centroid. Inspired by the above literature review, we present our ideas to generalize the notion of centralizer in virtue of some identities mentioned in abstract. Likewise we examine such identities in the setting of ring with involution. To build the proof of our main theorems, we need below mentioned lemmas:

Lemma 1.1 ([3, Lemma 1]). *Let R be a $m!$-torsion free semiprime ring. Suppose that $y_1, y_2, ..., y_m \in R$ satisfy $\sum_{i=1}^{m} \alpha^i y_i = 0$ for $\alpha = 1, 2, ..., m$. Then $y_i = 0$ for all i.*

Lemma 1.2 ([9, Theorem 1]). *Let R be a 2 torsion free semiprime ring and $T : R \to R$ be an additive mapping satisfying the condition $2T(x^2) = T(x)x + xT(x)$ for all $x \in R$, then T is a left and right centralizer on R.*

2. Main results

We begin our investigation with the following problems:

Theorem 2.1. *Let $n \geq 1$ be a fixed integer and R be any $n!$-torsion free semiprime ring. If $T : R \to R$ is an additive mapping which satisfies $T(x^n y^n + y^n x^n) = T(x^n)y^n + y^n T(x^n)$ for every x, y in R, then T is a left and right centralizer on R.*

Proof. We proceed with the condition

$$T(x^n y^n + y^n x^n) = T(x^n)y^n + y^n T(x^n) \text{ for every } x, y \in R. \tag{1}$$

Put e in place of x in (1) to perceive

$$2T(y^n) = T(e)y^n + y^n T(e) \text{ for every } y \text{ in } R. \tag{2}$$

Reinstate (2) by writing $y + e$ for y to find

$$\sum_{i=0}^{n} {}^nC_i[2T(y^{n-i}) - T(e)y^{n-i} - y^{n-i}T(e)] = 0 \text{ for every } y \text{ in } R.$$

Substituting ky in place of y, we find

$$\sum_{i=0}^{n} k^{n-i}\ {}^nC_i[2T(y^{n-i}) - T(e)y^{n-i} - y^{n-i}T(e)] = 0 \text{ for every } y \text{ in } R.$$

Make use of Lemma 1.1, which turn into

$${}^nC_i[2T(y^{n-i}) - T(e)y^{n-i} - y^{n-i}T(e)] = 0 \text{ for every } y \text{ in } R, \text{ and } i = 1, 2, 3 ... n-1.$$

In certain, take $i = n - 1$, we obtain

$$n[2T(y) - T(e)y - yT(e)] = 0 \text{ for every } y \text{ in } R. \text{ and for all } i = 1, 2, ... n-1.$$

The condition of n-torsion on R yields that

$$2T(y) = T(e)y + yT(e) \text{ for every } y \text{ in } R. \tag{3}$$

Next, replace x by $x + e$ in (1) to find

$$\begin{aligned}
&{}^nC_0[T(x^n y^n + y^n x^n) - T(x^n)y^n - y^n T(x^n)] \\
&+{}^nC_1[T(x^{n-1}y^n + y^n x^{n-1}) - T(x^{n-1})y^n - y^n T(x^{n-1})] \\
&+{}^nC_2[T(x^{n-2}y^n + y^n x^{n-2}) - T(x^{n-2})y^n - y^n T(x^{n-2})] + ... \\
&+{}^nC_{n-1}[T(xy^n + y^n x) - xT(y^n) - y^n T(x)] \\
&+{}^nC_n[T(2y^n) - T(e)y^n - y^n T(e)] = 0.
\end{aligned}$$

Comparing (1) and (2), which yields that

$$\begin{aligned}
&{}^nC_1[T(x^{n-1}y^n + y^n x^{n-1}) - T(x^{n-1})y^n - y^n T(x^{n-1})] \\
&+{}^nC_2[T(x^{n-2}y^n + y^n x^{n-2}) - T(x^{n-2})y^n \\
&-y^n T(x^{n-2})] + ... + {}^nC_{n-1}[T(xy^n + y^n x) \\
&-xT(y^n) - y^n T(x)] = 0.
\end{aligned}$$

Firstly substitute kx for x in the last equation and secondly make use of Lemma 1.1 to acquire

$$^{n}C_{i}k^{n-i}[T(x^{n-i}y^{n}+y^{n}x^{n-i}) \quad - \quad T(x^{n-i})y^{n}-y^{n}T(x^{n-i})]=0$$

for all $i=1,2,...,n-1$. In certain put $i=n-1$ and utilizing n-torsion freeness of R, we find that

$$T(xy^{n}+y^{n}x)=T(x)y^{n}+y^{n}T(x) \text{ for every } x,y \text{ in } R. \tag{4}$$

Again, restore y by $y+e$ in the above expression, and utilizing (3) and (4) to observe

$$\begin{aligned}&^{n}C_{1}[T(xy^{n-1}+y^{n-1}x)-T(x)y^{n-1}-xT(y^{n-1})]\\&+^{n}C_{2}[T(xy^{n-2}+y^{n-2}x)-T(x)y^{n-2}-xT(y^{n-2})]\\&+...+{}^{n}C_{n-1}[T(xy+yx)-T(x)y-xT(y)]=0.\end{aligned}$$

Replacing y by ky to arrive at

$$\sum_{i=1}^{n-i}{}^{n}C_{i}[T(xy^{n-i}+y^{n-i}x)-T(x)y^{n-i}-xT(y^{n-i})]=0 \text{ for every } x,y \text{ in } R.$$

Using the backward process and similar steps, we arrive at $T(xy+yx)=T(x)y+xT(y)$ for all $x,y\in R$. Replacing y by x and applying Lemma 1.2, we conclude the result.

□

Theorem 2.2. *Let $n\geq 1$ be a fixed integer and R be any $n!$-torsion free semiprime ring. If $T:R\to R$ is an additive mapping which satisfies $2T(x^{n}y^{n})=T(x^{n})y^{n}+y^{n}T(x^{n})$ for all $x,y\in R$, then T is a left and right centralizer on R.*

Proof. Given that

$$2T(x^{n}y^{n})=T(x^{n})y^{n}+y^{n}T(x^{n}) \text{ for every } x,y \text{ belongs to } R. \tag{5}$$

Reinstate the above expression by substituting e for x to obtain

$$2T(y^{n})=T(e)y^{n}+y^{n}T(e) \text{ for every } y \text{ in } R. \tag{6}$$

Replace y by $y+e$ in (6) to look

$$\sum_{i=0}^{n}{}^{n}C_{i}[2T(y^{n-i})-T(e)y^{n-i}-y^{n-i}T(e)]=0 \text{ for every } y \text{ belongs to } R.$$

Replacing y by ky, we obtain

$$\sum_{i=0}^{n}k^{n-i}\ {}^{n}C_{i}[2T(y^{n-i})-T(e)y^{n-i}-y^{n-i}T(e)]=0 \text{ for every } y \text{ in } R.$$

Make use of Lemma 1.1, we find

${}^nC_i[2T(y^{n-i})-T(e)y^{n-i}-y^{n-i}T(e)] = 0$ for all $y \in R$ and for all $i = 1, 2, ...n-1$.

Certainly take $i = n-1$ to obtain

$$ {}^nC_{n-1}[2T(y) - T(e)y - yT(e)] = 0 \text{ for every } y \text{ in } R. $$

As given the condition n-torsion free on R, therefore

$$ 2T(y) = T(e)y + yT(e) \text{ for every } y \text{ in } R. \tag{7} $$

Next, replace x by $x + e$ and utilizing (5) and (6), we obtain

$$ \begin{aligned} &{}^nC_1[2T(x^{n-1}y^n) - T(x^{n-1})y^n - y^nT(x^{n-1})] \\ &+{}^nC_2[2T(x^{n-2}y^n) - T(x^{n-2})y^n - y^nT(x^{n-2})] + ... \\ &+{}^nC_{n-1}[2T(xy^n) - T(x)y^n - y^nT(x)] = 0. \end{aligned} $$

Replace x by kx to obtain

$$ \begin{aligned} &{}^nC_1k^{n-1}[2T(x^{n-1}y^n) - T(x^{n-1})y^n - y^nT(x^{n-1})] \\ &+{}^nC_2k^{n-2}[2T(x^{n-2}y^n) - T(x^{n-2})y^n - y^nT(x^{n-2})] + ... \\ &+{}^nC_{n-1}k[2T(xy^n) - xT(y^n) - y^nT(x)] = 0. \end{aligned} $$

Applying the same arguments, we find that

$$ n[2T(xy^n) - T(x)y^n - y^nT(x)] = 0 \text{ for all } x, y \in R. $$

Torsion free condition of R implies that

$$ 2T(xy^n) = T(x)y^n + y^nT(x) \text{ for every } y, x \text{ in } R. \tag{8} $$

Putting $y + e$ in place of y to find

$$ \begin{aligned} &{}^nC_1[2T(xy^{n-1}) - T(x)y^{n-1} - xT(y^{n-1})] \\ &+{}^nC_2[2T(xy^{n-2}) - T(x)y^{n-2} - xT(y^{n-2})] \\ &+... + {}^nC_{n-1}[2T(xy) - T(x)y - xT(y)] = 0. \end{aligned} $$

Rewrite the above equation by changing ky for y, to get

$$ \sum_{r=1}^{n-i} {}^nC_i[2T(xy^{n-i}) - T(x)y^{n-i} - xT(y^{n-i})] = 0 \text{ for every } x, y \text{ in } R. $$

Applying the previous similar line of reasoning, we observe that $2T(xy) = T(x)y + xT(y)$ for all $x, y \in R$. Replacing y by x and utilize Lemma 1.2, we can derive the conclusion.

□

The upcoming example indicate that **Theorem 2.1** and **Theorem 2.2** are not appropriate in arbitrary rings:

Example 2.3. *Let* $R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in 2\mathbb{Z}_8 \right\}$ *is a ring under matrix addition and matrix multiplication, where* $\mathbb{Z}_8$ *denotes the ring of integers addition and multiplication modulo 8. Define mapping* $T : R \to R$ *by* $T\left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \right\} = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$ *for all* $a, b, c \in 2\mathbb{Z}_8$. *It is clear that* T *satisfy the identities (1) and (5) and* R *is neither a semiprime ring possessing* 2*-torsion freeness nor* T *is a centralizer on* R.

Now we restrict our attention for the involution ring. An additive mapping $* : R \to R$ is known as involution if it satisfies $(xy)^* = y^*x^*$ and $(x^*)^* = x$ for each x and y in R. We say R an involution ring (also known as $*$-ring) if R is appointed with $*$ map. An additive mapping $T : R \to R$ is termed as a left (correspondingly right)$*$-centralizer if $T(wz) = T(w)z^*$ $\big($correspondingly $T(wz) = w^*T(z)\big)$ holds for each w, z inside R and T denotes a left (correspondingly right) Jordan $*$-centralizer if for every w in R, $T(w^2) = T(w)w^*$ $\big($correspondingly $T(w^2) = w^*T(w)\big)$. If T is both left and right Jordan $*$-centralizer of R, then it is frequently termed as Jordan $*$-centralizer of R. Zalar [14] established the fact any Jordan left or Jordan right centralizer on a semiprime ring possessing 2-torsion freeness is a centralizer (both left and right). Later, Vukman [9] has proved that an additive mapping $T : R \to R$ is a centralizer (both sided left and right) if T satisfies an algebraic equation $2T(w^2) = T(w)w + wT(w)$ for each $w \in R$, where R is a ring having 2-torsion free condition and semiprimeness. If $T : R \to R$ is both sided Jordan $*$-centralizers, then obviously T satisfies $2T(w^nz^n) = T(w^n)(z^*)^n + (z^*)^nT(w^n)$ and $T(w^nz^n + z^nw^n) = T(w^n)(z^*)^n + (z^*)^nT(w^n)$ for each w and z in R, but the reverse approach is not generally true. The solution to this problem by parallel approach include in this paper. In fact, it is shown that an additive map T on a $n!$-torsion free semiprime $*$-ring R satisfying $2T(w^nz^n) = T(w^n)(z^*)^n + (z^*)^nT(w^n)$ or $T(w^nz^n + z^nw^n) = T(w^n)(z^*)^n + (z^*)^nT(w^n)$ for every z and w in R, is a $*$-centralizer of R. To fix our main conclusion, we must putout below lemma.

Lemma 2.4 ([1, Corollary 2.1]). *Let* R *be a 2 torsion free semiprime ring with involution* $*$ *and* $T : R \to R$ *be an additive mapping satisfying the condition* $2T(x^2) = T(x)x^* + x^*T(x)$ *for all* $x \in R$, *then* T *is a* $*$*-centralizer on* R.

Next, we start with the below written theorem:

Theorem 2.5. *Let* $n \geq 1$ *be a fixed integer and* R *be any* $n!$*-torsion free semiprime ring with involution* $*$. *If* $T : R \to R$ *is an additive mapping which satisfies* $2T(x^ny^n) = T(x^n)(y^*)^n + (y^*)^nT(x^n)$ *for every* $x, y \in R$, *then* T *is a left and right* $*$*-centralizer on* R.

Proof. Given that

$$2T(x^ny^n) = T(x^n)(y^*)^n + (y^*)^nT(x^n) \text{ for every } x, y \in R. \tag{9}$$

Putting e for x in (9) to find out

$$2T(y^n) = T(e)(y^*)^n + (y^*)^n T(e) \text{ for every } y \text{ in } R. \tag{10}$$

Replace y by $ky + e$ in the above equation and use the fact that $e^* = e^*e = (ee^*)^* = (e^*)^* = e$ to get

$$\sum_{i=0}^{n} k^{n-i}\, {}^nC_i[2T(y^{n-i}) - T(e)(y^*)^{n-i} - (y^*)^{n-i}T(e)] = 0 \text{ for every } y \text{ in } R.$$

Using Lemma 1.1, we find

$${}^nC_i[2T(y^{n-i}) - T(e)(y^*)^{n-i} - (y^*)^{n-i}T(e)] = 0 \text{ for every } y \text{ in } R$$

and for all $i = 1, 2, ...n-1$. Take $i = n-1$ in certain to observe

$${}^nC_{n-1}[2T(y) - T(e)y^* - y^*T(e)] = 0 \text{ for every } y \text{ in } R.$$

The n-torsion free condition gives us that

$$2T(y) = T(e)y^* + y^*T(e) \text{ for every } y \text{ in } R. \tag{11}$$

Next, replace x by $x + e$ in (9) and using (9) and (10), we have

$$\begin{aligned} {}^nC_1[2T(x^{n-1}y^n) \;\; &- \;\; T(x^{n-1})(y^*)^n - (y^*)^nT(x^{n-1})] \\ &+ \;\; {}^nC_2[2T(x^{n-2}y^n) - T(x^{n-2})(y^*)^n - (y^*)^nT(x^{n-2})] + ... \\ &+ \;\; {}^nC_{n-1}[2T(xy^n) - T(x)(y^*)^n - (y^*)^nT(x)] = 0. \end{aligned}$$

Putting x by kx, applying the same arguments, we arrive at

$$n[2T(xy^n) \;\; - \;\; T(x)(y^*)^n - (y^*)^nT(x)] = 0.$$

The condition n-torsion freeness of R allow us to find

$$2T(xy^n) = T(x)(y^*)^n + (y^*)^nT(x) \text{ for every } y, x \text{ in } R. \tag{12}$$

Again, repeating the same process for y, we obtain

$$\sum_{r=1}^{n-i} {}^nC_i[2T(xy^{n-i}) - T(x)(y^*)^{n-i} - (y^*)^{n-i}T(x)] = 0 \text{ for every } y, x \text{ in } R.$$

By utilizing parallel arguments as above to have $2T(xy) = T(x)y^* + y^*T(y)$ for every x, y in R. Replacing y by x and making use of Lemma 2.4, we conclude our result.

$\square$

Theorem 2.6. *Let $n \geq 1$ be a fixed integer and R be any $n!$-torsion free semiprime ring with involution $*$. If $T : R \to R$ is an additive mapping which satisfies*

$$T(x^ny^n + y^nx^n) = T(x^n)(y^*)^n + (y^*)^nT(x^n) \textit{ for every } x, y \textit{ in } R, \tag{13}$$

then T is a left and right $*$-centralizer on R.

Proof. Define a mapping $S : R \to R$ so that $S(x) = T(x^*)$ for each x belongs to R. It is clear that S is an additive mapping. Now, consider

$$\begin{aligned} S(x^n y^n + y^n x^n) &= T((x^n y^n + y^n x^n)^*) \\ &= T\big[(x^*)^n (y^*)^n + (y^*)^n (x^*)^n\big] \\ &= T(x^*)^n y^n + y^n T(x^*)^n \\ &= S(x^n) y^n + y^n S(x)^n \text{ for each } x, y \text{ in } R. \end{aligned}$$

Using Theorem 2.1, we notice that S is a centralizer on R. Hence, $S(x^2) = S(x)x = xS(x)$ for each x in R. This intimate that $T(x^*)^2 = T(x^*)x = xT(x^*)$ for each x in R. Now, substituting x in place of x^* and y in place of y^* respectively and applying Lemma 2.4, we reached our conclusion.

□

Example 2.7. *Let* $R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in 2\mathbb{Z}_8 \right\}$ *is a ring with involution* $* : R \to R$ *by* $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}^* = \begin{pmatrix} c & -b \\ 0 & a \end{pmatrix}$ *for every* $a, b, c \in 2\mathbb{Z}_8$ *under matrix addition and matrix multiplication, where* $\mathbb{Z}_8$ *denotes the ring of integers addition and multiplication modulo* 8. *Define mapping* $T : R \to R$ *by* $T\left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \right\} = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$ *for every* $a, b, c \in 2\mathbb{Z}_8$. *It is clear that* T *satisfying the identities (9) and (13) and* R *is neither a semiprime ring having* 2*-torsion freeness nor* T *act as a centralizer on* R.

We conclude our investigation with the fact that the mentioned restriction in the hypothesis of Theorem 2.5 and Theorem 2.6 are mandatory.

Acknowledgements : The authors of the paper extend their sincere gratitude to the Deanship of Scientific Research at the Islamic University of Madinah for the support provided to the Post-Publishing Program 1.

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