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ANTI-HYBRID INTERIOR IDEALS IN ORDERED SEMIGROUPS

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Abstract. The main theme of this present paper is to study ordered semigroups in the context of anti-hybrid interior ideals. The notion of anti-hybrid interior ideals in ordered semigroups is introduced. We prove that the concepts of ideals and interior coincide in some particular classes of ordered semigroups; regular, intra-regular, and semisimple. Finally, the characterization of semisimple ordered semigroups in terms of anti-hybrid interior ideals is considered.

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1. Introduction

The concept of fuzzy set is first introduced by Zadeh [31]. Kuroki [17, 18] applied fuzzy set theory to semigroups. Kehayopulu and Tsingelis [9, 10] applied fuzzy set theory to ordered semigroups. Many researchers interested class of ordered semigroups that the ideals and interior ideals coincide. In [11], Kehayopulu proved that in regular and in intra-regular poe-semigroups the ideal elements and the interior elements coincide. Several authors applied fuzzy sets to ordered semigroups. Kehayopulu and Tsingelis [10] proved that in regular and in intra-regular ordered semigroups the fuzzy ideals and the fuzzy interior ideals coincide. Shabir and Khan [29] proved that in regular, intra-regular and semisimple ordered semigroups intuitionistic fuzzy ideals and intuitionistic fuzzy interior ideals coincide. Khan and Shabir [16] proved that in regular and in intra-regular ordered semigroups the (α, β)-fuzzy ideals and the (α, β)-fuzzy interior ideals coincide. Khan et al. [13] proved that in regular, intra-regular and semisimple ordered semigroups the $(\epsilon, \epsilon \vee q)$-fuzzy ideals and $(\epsilon, \epsilon \vee q)$-fuzzy

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interior ideals coincide. Some generalizations of fuzzy sets were applied to investigate algebraic properties of semigroups by Khan et al. [12] and Khan et al. [14]. Khan et al. [15] showed that in regular and in intra-regular ordered semigroups the fuzzy soft ideals and the fuzzy soft interior ideals coincide.

Molodtsov [23] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. The notion of soft sets introduced in different algebraic structures was applied and studied by several authors; Aktas and Cagman [1] for soft groups, Feng et al. [7] for soft semirings and Naz et al. [26, 27] for soft semihypergroups. Song et al. [30] initiated the study of int-soft semigroups, int-soft left (resp. right) ideals, and int-soft products. Dudek and Jun [3] introduced and characterized the concept of a soft interior ideal of semigroups. In [24], Muhiuddin and Mahboob gave notions and introduced int-soft left (right) ideals, int-soft interior ideals and int-soft bi-ideals of ordered semigroups over the soft sets.

As a parallel circuit of fuzzy sets and soft sets, Jun et al. [8] was first introduced the notion of hybrid structures. A hybrid structure in S over U is defined to be a mapping $\widetilde{f}_\lambda := (\widetilde{f}, \lambda) : S \to \mathcal{P}(U) \times I, x \mapsto (\widetilde{f}(x), \lambda(x))$, where $\widetilde{f} : S \to \mathcal{P}(U)$ and $\lambda : S \to I$ are mappings such that I is the unit interval, that is, $I = [0, 1]$, S a set of parameters and $\mathcal{P}(U)$ denote the power set of an initial universe set U. The authors applied this concept to BCK/BCI-algebras and linear spaces. They introduced the concepts of a hybrid subalgebra, a hybrid field and a hybrid linear space.

The hybrid structure can be applied in many areas including mathematics, statistics, computer science, electrical instruments, industrial operations, business, engineering, social decisions, etc. Anis et al. [2] applied the notion of hybrid structure to semigroups. Elavarasan et al. [4] introduced the notion of hybrid generalized bi-ideal of a semigroup and characterizations of regular and left quasiregular semigroups in terms of hybrid generalized bi-ideals. Elavarasan and Jun [5] established some equivalent conditions for semigroups to be regular and intra-regular, in terms of hybrid ideals and hybrid bi-ideals. Modules over semirings and near-rings were also studied in terms of hybrid structures in [6] and [25].

The algebraic theory of hybrid structures has been extensively studied by many authors. In [21], Mekwian and Lekkoksung was first applied hybrid structures to ordered semigroups. They characterized regular ordered semigroups by using their hybrid ideals. Later, Sarasit et al. [28] was first introduced anti-hybrid ideals in an ordered semigroup and studied some properties of anti-hybrid ideals. Several researchers interest to classify ordered semigroups in terms of their anti-hybrid ideals, for example, Linesawat et al. [20], Mekwian et al. [22], and Linesawat and Lekkoksung [19].

In this present paper is to study ordered semigroups in the context of anti-hybrid interior ideals. In this paper, we introduce the concept of anti-hybrid interior ideals in an ordered semigroup. We prove that in regular, in intra-regular and in semisimple ordered semigroups the concepts of anti-hybrid interior ideals and anti-hybrid ideals coincide. Finally, we characterize semisimple ordered semigroups in terms of anti-hybrid interior ideals.

2. Preliminary

In this section, we will recall the basic terms and definitions from the ordered semigroup theory and the hybrid structure theory that we will use later in this paper.

A *groupoid* is an algebra $(S; \cdot)$ consisting of a nonempty set S together with a (binary) operation $\cdot$ on S. A *semigroup* $(S; \cdot)$ is a groupoid in which the operation $\cdot$ is associative, that is, $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ for all $x, y, z \in S$.

Definition 2.1. The structure $(S; \cdot, \leq)$ is called an *ordered semigroup* if the following conditions are satisfied:

(1) $(S; \cdot)$ is a semigroup.
(2) $(S; \leq)$ is a partially ordered set.
(3) For every $a, b, c \in S$ if $a \leq b$, then $a \cdot c \leq b \cdot c$ and $c \cdot a \leq c \cdot b$.

For simplicity, we denoted an ordered semigroup $(S; \cdot, \leq)$ by its carrier set as a bold letter $\mathbf{S}$ and if $a, b \in S$, we will instead of $a \cdot b$ by ab.

For $K \subseteq S$, we denote

$$(K] := \{a \in S : a \leq k \text{ for some } k \in K\}.$$

Let A and B be two nonempty subsets of S. Then we define

$$AB := \{ab : a \in A \text{ and } b \in B\}.$$

Let S be an ordered semigroup. A nonempty subset A of S is called a *subsemigroup* of S [9, 10] if $AA \subseteq A$.

Definition 2.2 ([9, 10])**.** Let $\mathbf{S}$ be an ordered semigroup. A nonempty subset A of S is called a *left (resp. right) ideal* of S if it satisfies

(1) $SA \subseteq A$ (resp. $AS \subseteq A$).
(2) For $x, y \in S$, if $x \leq y$ and $y \in A$, then $x \in A$.

A nonempty subset I of S is called *ideal* if it is both a left and a right ideal of S.

Definition 2.3 ([10])**.** Let $\mathbf{S}$ be an ordered semigroup. A nonempty subset A of S is called an *interior ideal* of S if it satisfied

(1) $SAS \subseteq A$.
(2) For $x, y \in S$, if $x \leq y$ and $y \in A$, then $x \in A$.

In what follows, let I be the unit interval, i.e., $I = [0, 1]$, S a set of parameters and $\mathcal{P}(U)$ denote the power set of an initial universe set U.

Definition 2.4 ([2])**.** A hybrid structure in S over U is defined to be a mapping

$$\widetilde{f}_\lambda := (\widetilde{f}, \lambda) : S \to \mathcal{P}(U) \times I, x \mapsto (\widetilde{f}(x), \lambda(x)),$$

where

$$\widetilde{f} : S \to \mathcal{P}(U) \text{ and } \lambda : S \to I$$

are mappings.

Let us denote by $H(S)$ the set of all hybrid structures in S over U. We define an order $\ll$ on $H(S)$ as follows: For all $\widetilde{f}_\lambda, \widetilde{g}_\gamma \in H(S)$,

$$\widetilde{f}_\lambda \ll \widetilde{g}_\gamma \Leftrightarrow \widetilde{f} \sqsubseteq \widetilde{g} \text{ and } \lambda \succeq \gamma,$$

where $\widetilde{f} \sqsubseteq \widetilde{g}$ means that $\widetilde{f}(x) \subseteq \widetilde{g}(x)$ and $\lambda \succeq \gamma$ means that $\lambda(x) \geq \gamma(x)$ for all $x \in S$ and $\widetilde{f}_\lambda = \widetilde{g}_\gamma$ if $\widetilde{f}_\lambda \ll \widetilde{g}_\gamma$ and $\widetilde{g}_\alpha \ll \widetilde{f}_\lambda$.

Definition 2.5 ([2])**.** Let $\widetilde{f}_\lambda$ and $\widetilde{g}_\gamma$ be hybrid structures in S over U. Then the hybrid union of $\widetilde{f}_\lambda$ and $\widetilde{g}_\gamma$ is denoted by $\widetilde{f}_\lambda \Cup \widetilde{g}_\gamma$ and is defined to be a hybrid structure

$$\widetilde{f}_\lambda \Cup \widetilde{g}_\gamma := (\widetilde{f} \cup \widetilde{g}, \lambda \wedge \gamma) : S \to \mathcal{P}(U) \times I, x \mapsto ((\widetilde{f} \cup \widetilde{g})(x), (\lambda \wedge \gamma)(x)),$$

where

$$(\widetilde{f} \cup \widetilde{g})(x) := \widetilde{f}(x) \cup \widetilde{g}(x) \text{ and } (\lambda \wedge \gamma)(x) := \min\{\lambda(x), \gamma(x)\}.$$

We denote $\widetilde{S}_S$ the hybrid structure in S over U and is defined to be a hybrid structure

$$\widetilde{S}_S := (\widetilde{S}, S) : S \to \mathcal{P}(U) \times I : x \mapsto (\widetilde{S}(x), S(x)),$$

where

$$\widetilde{S}(x) := \emptyset \text{ and } S(x) := 1.$$

Let $a \in S$. Then, we set

$$\mathbf{S}_a := \{(x, y) \in S \times S : a \leq xy\}.$$

Definition 2.6 ([28])**.** Let $\widetilde{f}_\lambda$ and $\widetilde{g}_\gamma$ be hybrid structures in S over U. Then the hybrid products of $\widetilde{f}_\lambda$ and $\widetilde{g}_\gamma$ is denoted by $\widetilde{f}_\lambda \otimes \widetilde{g}_\gamma$ and is defined to be a hybrid structure

$$\widetilde{f}_\lambda \otimes \widetilde{g}_\gamma := (\widetilde{f} \odot \widetilde{g}, \lambda \circ \gamma) : S \to \mathcal{P}(U) \times I, x \mapsto ((\widetilde{f} \odot \widetilde{g})(x), (\lambda \circ \gamma)(x)),$$

where

$$(\widetilde{f} \odot \widetilde{g})(x) = \begin{cases} \bigcap\limits_{(a,b) \in \mathbf{S}_x} \left[\widetilde{f}(a) \cup \widetilde{g}(b)\right] & \text{if } \mathbf{S}_x \neq \emptyset, \\ U & \text{otherwise,} \end{cases}$$

and

$$(\lambda \circ \gamma)(x) = \begin{cases} \bigvee\limits_{(a,b) \in \mathbf{S}_x} \{\min\{\lambda(a), \gamma(b)\}\} & \text{if } \mathbf{S}_x \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases}$$

By Definition 2.6, it is easy to verify that $(H(S);\otimes)$ is a semigroup.

Let $A \subseteq S$. We denote $\chi_{A^c}(\widetilde{S}_S)$ the characteristic hybrid structure of complement of A in S over U and is defined to be a hybrid structure

$$\chi_{A^c}(\widetilde{S}_S) := (\chi_{A^c}(\widetilde{S}), \chi_{A^c}(S)) : S \to \mathcal{P}(U) \times I, x \mapsto (\chi_{A^c}(\widetilde{S})(x), \chi_{A^c}(S)(x)),$$

where

$$\chi_{A^c}(\widetilde{S})(x) = \begin{cases} \emptyset & \text{if } x \in A, \\ U & \text{otherwise,} \end{cases}$$

and

$$\chi_{A^c}(S)(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{otherwise.} \end{cases}$$

We set $\chi_{A^c}(\widetilde{S}_S) := \widetilde{S}_S$ in the case that $A = S$.

Remark 2.1. Let $\mathbf{S}$ be an ordered semigroup and A a nonempty subset of S. Suppose that one of the following conditions holds:

(1) $\chi_{A^c}(\widetilde{S})(x) \subseteq \chi_{A^c}(\widetilde{S})(y)$;
(2) $\chi_{A^c}(S)(x) \geq \chi_{A^c}(S)(y)$;

for all $x, y \in S$. We have that $x \in A$ if $y \in A$.

Remark 2.2. Let $\mathbf{S}$ be an ordered semigroup and A, B be nonempty subsets of S. Suppose that one of the following conditions holds:

(1) $\chi_{B^c}(\widetilde{S})(x) \subseteq \chi_{A^c}(\widetilde{S})(x)$;
(2) $\chi_{B^c}(S)(x) \geq \chi_{A^c}(S)(x)$;

for all $x \in S$. We have that $A \subseteq B$ whenever $x \in A$.

3. Main results

In this main section, we discuss the coincidence relations of anti-hybrid ideals and anti-hybrid interior ideals. Finally, we characterize semisimple ordered semigroups in terms of anti-hybrid interior ideals.

Definition 3.1 ([20])**.** Let $\mathbf{S}$ be an ordered semigroup. A hybrid structure $\widetilde{f}_\lambda$ in S over U is called an *anti-hybrid left (resp. right) ideal* in S over U if the following statements are satisfied: For every $x, y \in S$,

(1) $\widetilde{f}(xy) \subseteq \widetilde{f}(y)$ (resp. $\widetilde{f}(xy) \subseteq \widetilde{f}(x)$).
(2) $\lambda(xy) \geq \lambda(y)$ (resp. $\lambda(xy) \geq \lambda(x)$).
(3) If $x \leq y$, then $\widetilde{f}(x) \subseteq \widetilde{f}(y)$ and $\lambda(x) \geq \lambda(y)$.

A hybrid structure in S over U is called an *anti-hybrid ideal* in S over U if it is both an anti-hybrid left and an anti-hybrid right ideal in S over U.

Example 3.2. Let $S = \{a, b, c\}$. We define a binary operation $\circ$ and a binary relation on S as follows:

$\circ$	a	b	c
a	a	a	a
b	a	a	a
c	c	c	c

and $\leq := \{(a, c), (b, c)\} \cup \Delta_S$, where $\Delta_S := \{(x, x) : x \in S\}$. Then $\mathbf{S} := (S; \circ, \leq)$ is an ordered semigroup. Let $U = \mathbb{N}$. Define a hybrid structure $\widetilde{f}_\lambda$ in S over U as follows:

$$\widetilde{f}(x) = \begin{cases} 4\mathbb{N} & \text{if } x = a \\ 2\mathbb{N} & \text{if } x = b \\ \mathbb{N} & \text{if } x = c \end{cases} \quad \text{and} \quad \lambda(x) = \begin{cases} 0.8 & \text{if } x = a \\ 0.7 & \text{if } x = b \\ 0.2 & \text{if } x = c \end{cases}$$

Then $\widetilde{f}_\lambda$ is an anti-hybrid right ideal in S over U. We can see that it is not an anti-hybrid left ideal in S over U since $\widetilde{f}(ca) = \widetilde{f}(c) = \mathbb{N} \not\subseteq 4\mathbb{N} = \widetilde{f}(a)$.

Example 3.3. Let $S = \{a, b, c\}$. We define a binary operation $\circ$ and a binary relation on S as follows:

$\circ$	a	b	c
a	a	a	a
b	a	a	a
c	a	b	c

and $\leq := \{(a, b)\} \cup \Delta_S$, where $\Delta_S := \{(x, x) : x \in S\}$. Then $\mathbf{S} := (S; \circ, \leq)$ is an ordered semigroup. Let $U = \mathbb{N}$. Define a hybrid structure $\widetilde{f}_\lambda$ in S over U as follows:

$$\widetilde{f}(x) = \begin{cases} 4\mathbb{N} & \text{if } x = a \\ \mathbb{N} & \text{if } x = b \\ 2\mathbb{N} & \text{if } x = c \end{cases} \quad \text{and} \quad \lambda(x) = \begin{cases} 0.8 & \text{if } x = a \\ 0.2 & \text{if } x = b \\ 0.7 & \text{if } x = c \end{cases}$$

Then $\widetilde{f}_\lambda$ is an anti-hybrid left ideal in S over U. We can see that it is not an anti-hybrid right ideal in S over U since $\widetilde{f}(cb) = \widetilde{f}(b) = \mathbb{N} \not\subseteq 2\mathbb{N} = \widetilde{f}(c)$.

Definition 3.4. Let $\mathbf{S}$ be an ordered semigroup. A hybrid structure $\widetilde{f}_\lambda$ in S over U is called an *anti-hybrid interior ideal* in S over U if the following statements are satisfied: For every $x, y, z \in S$,

(1) $\widetilde{f}(xyz) \subseteq \widetilde{f}(y)$.
(2) $\lambda(xyz) \geq \lambda(y)$.
(3) If $x \leq y$, then $\widetilde{f}(x) \subseteq \widetilde{f}(y)$ and $\lambda(x) \geq \lambda(y)$.

Example 3.5. Let $S = \{a, b, c, d\}$. We define a binary operation $\circ$ and a binary relation on S as follows:

$\circ$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	b	a
d	a	a	b	b

and $\leq := \{(a,b),(a,d)\} \cup \Delta_S$, where $\Delta_S := \{(x,x) \mid x \in S\}$. Then $\mathbf{S} := (S; \circ, \leq)$ is an ordered semigroup. Let $U = \{1, 2, 3\}$. Define a hybrid structure $\widetilde{f}_\lambda$ in S over U as follows:

$$\widetilde{f}(x) = \begin{cases} \emptyset & \text{if } x = a \\ \{1\} & \text{if } x = b \\ \{1,2\} & \text{if } x = c \\ U & \text{if } x = d \end{cases} \quad \text{and} \quad \lambda(x) = \begin{cases} 1 & \text{if } x = a \\ 0.8 & \text{if } x = b \\ 0.5 & \text{if } x = c \\ 0.1 & \text{if } x = d \end{cases}$$

Then $\widetilde{f}_\lambda$ is an anti-hybrid interior ideal in S over U.

Proposition 3.6. *Let $\mathbf{S}$ be an ordered semigroup and A, B subsets of S. Then the following conditions are equivalent:*

(1) $A \subseteq B$.

(2) $\chi_{B^c}(\widetilde{S}_S) \ll \chi_{A^c}(\widetilde{S}_S)$.

Proof. (1)$\Rightarrow$(2). Let $x \in B$. Then, we obtain that $\chi_{B^c}(\widetilde{S})(x) = \emptyset \subseteq \chi_{A^c}(\widetilde{S})(x)$. This implies that $\chi_{B^c}(\widetilde{S}) \sqsubseteq \chi_{A^c}(\widetilde{S})$, and $\chi_{B^c}(S)(x) = 1 \geq \chi_{A^c}(S)(x)$. This implies that $\chi_{B^c}(S) \succeq \chi_{A^c}(S)$. If $x \notin B$, then, since $A \subseteq B$, thus $x \notin A$ and we obtain $\chi_{B^c}(\widetilde{S})(x) = U = \chi_{A^c}(\widetilde{S})(x)$, This implies that $\chi_{B^c}(\widetilde{S}) \sqsubseteq \chi_{A^c}(\widetilde{S})$, and $\chi_{B^c}(S)(x) = 0 = \chi_{A^c}(S)(x)$, This implies that $\chi_{B^c}(S) \succeq \chi_{A^c}(S)$. Altogether, we have that $\chi_{B^c}(\widetilde{S}_S) \ll \chi_{A^c}(\widetilde{S}_S)$.

(2)$\Rightarrow$(1). Let $x \in A$. Then, since $\chi_{B^c}(\widetilde{S}_S) \ll \chi_{A^c}(\widetilde{S}_S)$, by Remark 2.2, we obtain $A \subseteq B$.

$\square$

A nonempty hybrid structure $\widetilde{f}_\lambda$ is mean that for $x \in S$ we have that $\widetilde{f}(x) \neq \emptyset$ and $\lambda(x) \neq 0$ for all $x \in S$. The following proposition shows that the intersection of anti-hybrid interior ideals in S over U is also an anti-hybrid interior ideal in S over U.

Proposition 3.7. *Let $\mathbf{S}$ be an ordered semigroup and $\{(\widetilde{f}_i)_{\lambda_i} : i \in I\}$ is a family of anti-hybrid interior ideals in S over U. Then a nonempty hybrid structure $\bigcap_{i \in I} (\widetilde{f}_i)_{\lambda_i}$ is an anti-hybrid interior ideal in S over U, where $\bigcap_{i \in I} (\widetilde{f}_i)_{\lambda_i}$ is defined by*

$$\bigcap_{i \in I} (\widetilde{f}_i)_{\lambda_i} := \left(\bigcap_{i \in I} \widetilde{f}_i, \bigvee_{i \in I} \lambda_i \right) : S \to \mathcal{P}(U) \times I, x \mapsto \left(\left(\bigcap_{i \in I} \widetilde{f}_i \right)(x), \left(\bigvee_{i \in I} \lambda_i \right)(x) \right),$$

where

$$\left(\bigcap_{i\in I}\widetilde{f}_i\right)(x) := \bigcap_{i\in I}\widetilde{f}_i(x) \quad \text{and} \quad \left(\bigvee_{i\in I}\lambda_i\right)(x) := \bigvee_{i\in I}\lambda_i(x).$$

Proof. Let $x, y, z \in S$. Then we obtain

$$\left(\bigcap_{i\in I}\widetilde{f}_i\right)(xyz) = \bigcap_{i\in I}\widetilde{f}_i(xyz) \subseteq \bigcap_{i\in I}\widetilde{f}_i(y) = \left(\bigcap_{i\in I}\widetilde{f}_i\right)(y),$$

and

$$\left(\bigvee_{i\in I}\lambda_i\right)(xyz) = \bigvee_{i\in I}\lambda_i(xyz) \geq \bigvee_{i\in I}\lambda_i(y) = \left(\bigvee_{i\in I}\lambda_i\right)(y).$$

Let $x, y \in S$ be such that $x \leq y$. Then we have that

$$\left(\bigcap_{i\in I}\widetilde{f}_i\right)(x) = \bigcap_{i\in I}\widetilde{f}_i(x) \subseteq \bigcap_{i\in I}\widetilde{f}_i(y) = \left(\bigcap_{i\in I}\widetilde{f}_i\right)(y),$$

and

$$\left(\bigvee_{i\in I}\lambda_i\right)(x) = \bigvee_{i\in I}\lambda_i(x) \geq \bigvee_{i\in I}\lambda_i(y) = \left(\bigvee_{i\in I}\lambda_i\right)(y).$$

Therefore $\bigcap_{i\in I}(\widetilde{f}_i)_{\lambda_i}$ is an anti-hybrid interior ideal in S over U. □

Proposition 3.8. *Let* $\mathbf{S}$ *be an ordered semigroup. Then every anti-hybrid ideal in S over U is an anti-hybrid interior ideal in S over U.*

Proof. Let $\widetilde{f}_\lambda$ be an anti-hybrid ideal in S over U and $x, y, z \in S$. Then we obtain

$$\widetilde{f}(xyz) = \widetilde{f}((xy)z) \subseteq \widetilde{f}(xy) \subseteq \widetilde{f}(y),$$

and

$$\lambda(xyz) = \lambda((xy)z) \geq \lambda(xy) \geq \lambda(y).$$

Hence $\widetilde{f}_\lambda$ is an anti-hybrid interior ideal in S over U. □

The converse of Proposition 3.8, in general, is not true as the following example.

Example 3.9. Let $S = \{a, b, c, d\}$. Define a binary operation $*$ on S as following table:

$*$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	b	b
d	a	a	b	b

We define an order relation $\leq$ on S as follows:

$$\leq := \{(a,a),(b,b),(c,c),(d,d),(a,b),(a,c),(a,d),(b,d),(c,d)\}.$$

Then, $(S;*,\leq)$ is an ordered semigroup. Let $U=\{1,2,3\}$. Then, we define hybrid structure $\widetilde{f}_\lambda$ in S over U as follows:

$$\widetilde{f}(x) := \begin{cases} \emptyset & \text{if } x=a, \\ U & \text{if } x=b, \\ \{1\} & \text{if } x=c, \\ \{1,2\} & \text{if } x=d, \end{cases}$$

and

$$\lambda(x) := \begin{cases} 1 & \text{if } x=a, \\ 0 & \text{if } x=b, \\ 0.8 & \text{if } x=c, \\ 0.5 & \text{if } x=d. \end{cases}$$

It is easy to see that $\widetilde{f}_\lambda$ is an anti-hybrid interior ideal in S over U but it is not an anti-hybrid ideal in S over U because of $\widetilde{f}(c*d) \nsubseteq \widetilde{f}(d)$ and also $\lambda(c*d) \ngeq \lambda(d)$.

An ordered semigroup $\mathbf{S}$ is *regular* [9] if for each $a\in S$ there exists $x\in S$ such that $a\leq axa$.

We show that in regular ordered semigroups the concepts of anti-hybrid interior ideals and anti-hybrid ideals coincide as the following theorems.

Theorem 3.10. *Let $\mathbf{S}$ be a regular ordered semigroup. Then every anti-hybrid interior ideal in S over U is an anti-hybrid ideal in S over U.*

Proof. Let $\widetilde{f}_\lambda$ be an anti-hybrid interior ideal in S over U and $a,b\in S$. Then since $\mathbf{S}$ is a regular ordered semigroup, there exists $x\in S$ such that $a\leq axa$ and then $ab\leq axab$, we obtain

$$\widetilde{f}(ab) \subseteq \widetilde{f}(axab) = \widetilde{f}((ax)ab) \subseteq \widetilde{f}(a),$$

and

$$\lambda(ab) \geq \lambda(axab) = \lambda((ax)ab) \geq \lambda(a).$$

Thus $\widetilde{f}_\lambda$ is an anti-hybrid right ideal in S over U. Similarly, we can prove that $\widetilde{f}_\lambda$ is an anti-hybrid left ideal in S over U. Therefore $\widetilde{f}_\lambda$ is an anti-hybrid ideal in S over U. □

Combining Proposition 3.8 and Theorem 3.10 we have the following theorem.

Theorem 3.11. *In regular ordered semigroup the concepts of anti-hybrid ideals and anti-hybrid interior ideals coincide.*

An ordered semigroup $\mathbf{S}$ is *intra-regular* [9] if for each $a \in S$ there exist $x, y \in S$ such that $a \leq xa^2y$.

As in the previous theorems, in the next theorems, the concept of anti-hybrid interior ideals and anti-hybrid ideals coincide in an intra-regular ordered semigroup as follows.

Theorem 3.12. *Let* $\mathbf{S}$ *be an intra-regular ordered semigroup. Then every anti-hybrid interior ideal in* S *over* U *is an anti-hybrid ideal in* S *over* U.

Proof. Let $\widetilde{f}_\lambda$ be an anti-hybrid interior ideal in S over U and $a, b \in S$. Then since $\mathbf{S}$ is an intra-regular ordered semigroup, there exist $x, y \in S$ such that $a \leq xa^2y$ and then $ab \leq xa^2yb = (xa)a(yb)$, we obtain

$$\widetilde{f}(ab) \subseteq \widetilde{f}(xa^2yb) = \widetilde{f}((xa)a(yb)) \subseteq \widetilde{f}(a),$$

and

$$\lambda(ab) \geq \lambda(xa^2yb) = \lambda((xa)a(yb)) \geq \lambda(a).$$

Thus $\widetilde{f}_\lambda$ is an anti-hybrid right ideal in S over U. Similarly, we can prove that $\widetilde{f}_\lambda$ is an anti-hybrid left ideal in S over U. Therefore $\widetilde{f}_\lambda$ is an anti-hybrid ideal in S over U. □

Combining Proposition 3.8 and Theorem 3.12 we have the following theorem.

Theorem 3.13. *In intra-regular ordered semigroup the concepts of anti-hybrid ideals and anti-hybrid interior ideals coincide.*

An ordered semigroup $\mathbf{S}$ is *semisimple* [9] if for each $a \in S$ there exist $x, y, z \in S$ such that $a \leq xayaz$.

In semisimple ordered semigroup the concepts of anti-hybrid interior ideals and anti-hybrid ideals coincide as well as in regular and intra-regular ordered semigroups as the following theorems.

Theorem 3.14. *Let* $\mathbf{S}$ *be a semisimple ordered semigroup. Then every anti-hybrid interior ideal in* S *over* U *is an anti-hybrid ideal in* S *over* U.

Proof. Let $\widetilde{f}_\lambda$ be an anti-hybrid interior ideal in S over U and $a, b \in S$. Then since $\mathbf{S}$ is a semisimple ordered semigroup, there exist $x, y, z \in S$ such that $a \leq xayaz$ and then $ab \leq xayazab = (xay)a(zab)$, we obtain

$$\widetilde{f}(ab) \subseteq \widetilde{f}(xayazab) = \widetilde{f}((xay)a(zab)) \subseteq \widetilde{f}(a),$$

and

$$\lambda(ab) \geq \lambda(xayazab) = \lambda((xay)a(zab)) \geq \lambda(a).$$

Thus $\widetilde{f}_\lambda$ is an anti-hybrid right ideal in S over U. Similarly, we can prove that $\widetilde{f}_\lambda$ is an anti-hybrid left ideal in S over U. Therefore $\widetilde{f}_\lambda$ is an anti-hybrid ideal in S over U. □

Combining Proposition 3.8 and Theorem 3.14 we have the following theorem.

Theorem 3.15. *In semisimple ordered semigroup the concepts of anti-hybrid ideals and anti-hybrid interior ideals coincide.*

Lemma 3.16 ([19])**.** *Let* $\mathbf{S}$ *be an ordered semigroup and* A, B *nonempty subsets of* S*. Then the following conditions hold.*

(1) $\chi_{A^c}(\widetilde{S}_S) \Cup \chi_{B^c}(\widetilde{S}_S) = \chi_{A^c \cup B^c}(\widetilde{S}_S)$.
(2) $\chi_{A^c}(\widetilde{S}_S) \otimes \chi_{B^c}(\widetilde{S}_S) = \chi_{(A^c B^c]}(\widetilde{S}_S)$.

Lemma 3.17. *Let* $\mathbf{S}$ *be an ordered semigroup and* $\widetilde{f}_\lambda$ *a hybrid structure in* S *over* U*. If* $\widetilde{f}_\lambda$ *is an anti-hybrid interior ideal in* S *over* U*, then* $\widetilde{f}_\lambda \ll \widetilde{S}_S \otimes \widetilde{f}_\lambda \otimes \widetilde{S}_S$*.*

Proof. Let $\widetilde{f}_\lambda$ be an anti-hybrid interior ideal in S over U and $x \in S$. If $\mathbf{S}_x = \emptyset$, then we obtain that $\widetilde{f}(x) \subseteq U = (\widetilde{S} \odot \widetilde{f} \odot \widetilde{S})(x)$, this means that $\widetilde{f} \sqsubseteq \widetilde{S} \odot \widetilde{f} \odot \widetilde{S}$ and $\lambda(x) \geq 0 = (S \circ \lambda \circ S)(x)$, this means that $\lambda \succeq S \circ \lambda \circ S$. If $\mathbf{S}_x \neq \emptyset$, then we obtain that

$$\begin{aligned}
(\widetilde{S} \odot \widetilde{f} \odot \widetilde{S})(x) &= \bigcap_{(y,z) \in S_x} \left[\widetilde{S}(y) \cup (\widetilde{f} \odot \widetilde{S})(z) \right] \\
&= \bigcap_{(y,z) \in \mathbf{S}_x} \left[\bigcap_{(u,v) \in \mathbf{S}_z} \left(\widetilde{f}(u) \cup \widetilde{S}(v) \right) \right] \\
&= \bigcap_{(y,z) \in \mathbf{S}_x} \left[\bigcap_{(u,v) \in \mathbf{S}_z} \widetilde{f}(u) \right] \\
&= \bigcap_{(y,uv) \in \mathbf{S}_x} \left[\widetilde{f}(u) \right] \\
&\supseteq \bigcap_{(y,uv) \in \mathbf{S}_x} \left[\widetilde{f}(yuv) \right] \\
&\supseteq \bigcap_{(y,uv) \in \mathbf{S}_x} \left[\widetilde{f}(x) \right] \\
&= \widetilde{f}(x).
\end{aligned}$$

This means that $\widetilde{f} \sqsubseteq \widetilde{S} \odot \widetilde{f} \odot \widetilde{S}$, and we obtain

$$\begin{aligned}
(S \circ \lambda \circ S)(x) &= \bigvee_{(y,z) \in \mathbf{S}_x} \{\min\{S(y), (\lambda \circ S)(z)\}\} \\
&= \bigvee_{(y,z) \in \mathbf{S}_x} \{ \bigvee_{(u,v) \in \mathbf{S}_z} \{\min\{\lambda(u), S(v)\}\}\} \\
&= \bigvee_{(y,z) \in \mathbf{S}_x} \left[\bigvee_{(u,v) \in \mathbf{S}_z} \lambda(u) \right] \\
&= \bigvee_{(y,uv) \in \mathbf{S}_x} [\lambda(u)]
\end{aligned}$$

$$\begin{aligned} &\leq \bigvee_{(y,uv)\in \mathbf{S}_x} [\lambda(yuv)] \\ &\leq \bigvee_{(y,uv)\in \mathbf{S}_x} [\lambda(x)] \\ &= \lambda(x). \end{aligned}$$

This means that $\lambda \succeq S \circ \lambda \circ S$. Therefore $\widetilde{f}_\lambda \ll \widetilde{S}_S \otimes \widetilde{f}_\lambda \otimes \widetilde{S}_S$. □

We now characterize interior ideals of an ordered semigroup in terms of anti-hybrid interior ideals as the following lemma.

Lemma 3.18. *Let* $\mathbf{S}$ *be an ordered semigroup and* A *a nonempty subset of* S. *Then the following conditions are equivalent.*

(1) *A is an interior ideal of* $\mathbf{S}$.
(2) *$\chi_{A^c}(\widetilde{S}_S)$ is an anti-hybrid interior ideal in S over U.*

Proof. (1)⇒(2). Let A be an interior ideal of $\mathbf{S}$ and $x, y, a \in S$. If $a \in A$, then $xay \in A$ since A is an interior ideal of $\mathbf{S}$ and we obtain $\chi_{A^c}(\widetilde{S})(xay) = \emptyset = \chi_{A^c}(\widetilde{S})(a)$, and $\chi_{A^c}(S)(xay) = 1 = \chi_{A^c}(S)(a)$. If $a \notin A$, then we obtain $\chi_{A^c}(\widetilde{S})(xay) \subseteq U = \chi_{A^c}(\widetilde{S})(a)$, and $\chi_{A^c}(S)(xay) \geq 0 = \chi_{A^c}(S)(a)$. Let $x, y \in S$ be such that $x \leq y$. If $y \in A$, then $x \in A$, since A is an interior ideal of $\mathbf{S}$ and we obtain $\chi_{A^c}(\widetilde{S})(x) = \emptyset = \chi_{A^c}(\widetilde{S})(y)$ and $\chi_{A^c}(S)(x) = 1 = \chi_{A^c}(S)(y)$. If $a \notin A$, then we obtain $\chi_{A^c}(\widetilde{S})(x) \subseteq U = \chi_{A^c}(\widetilde{S})(y)$ and $\chi_{A^c}(S)(x) \geq 0 = \chi_{A^c}(S)(y)$. It completed to prove that $\chi_{A^c}(\widetilde{S}_S)$ is an anti-hybrid interior ideal in S over U.

(2)⇒(1). Let $\chi_{A^c}(\widetilde{S}_S)$ is an anti-hybrid interior ideal in S over U. By Lemma 3.16 and Lemma 3.17, we obtain

$$\begin{aligned} \chi_{A^c}(\widetilde{S}_S) &\ll \widetilde{S}_S \otimes \chi_{A^c} \otimes \widetilde{S}_S \\ &= \chi_{S^c}(\widetilde{S}_S) \otimes \chi_{A^c} \otimes \chi_{S^c}(\widetilde{S}_S) \\ &= \chi_{(S^cA^cS^c]}(\widetilde{S}_S) \\ &= \chi_{(SAS]^c}(\widetilde{S}_S). \end{aligned}$$

By Proposition 3.3, it follows that $(SAS] \subseteq A$, we obtain that $SAS \subseteq A$. Let $x, y \in S$ be such that $x \leq y$. If $y \in A$, then, since $\chi_A(\widetilde{S}_S)$ is an anti-hybrid interior ideal in S over U, by Remark 2.1, we have that $x \in A$. Therefore A is an interior ideal of $\mathbf{S}$. □

Lemma 3.19. *Let* $\mathbf{S}$ *be a semisimple ordered semigroup and* $\widetilde{f}_\lambda, \widetilde{g}_\alpha$ *anti-hybrid interior ideals in S over U. Then* $\widetilde{f}_\lambda \Cup \widetilde{g}_\alpha \ll \widetilde{f}_\lambda \otimes \widetilde{g}_\alpha$.

Proof. Let $\mathbf{S}$ be a semisimple ordered semigroup and $\widetilde{f}_\lambda, \widetilde{g}_\alpha$ anti-hybrid interior ideals in S over U. Since $\mathbf{S}$ is semisimple, so by Theorem 3.14, $\widetilde{f}_\lambda, \widetilde{g}_\alpha$ are anti-hybrid ideals in S over U. Let $a \in S$. Then since $\mathbf{S}$ is semisimple, there exist

$x, y, z \in S$ such that $a \leq xayaz$. This implies that $\mathbf{S}_a \neq \emptyset$ and we obtain

$$\begin{aligned}
(\widetilde{f} \odot \widetilde{g})(a) &= \bigcap_{(p,q)\in \mathbf{S}_a} \left[\widetilde{f}(p) \cup \widetilde{g}(q)\right] \\
&\supseteq \bigcap_{(p,q)\in \mathbf{S}_a} \left[\widetilde{f}(pq) \cup \widetilde{g}(pq)\right] \\
&\supseteq \bigcap_{(p,q)\in \mathbf{S}_a} \left[\widetilde{f}(a) \cup \widetilde{g}(a)\right] \\
&= \widetilde{f}(a) \cup \widetilde{g}(a) \\
&= (\widetilde{f} \cup \widetilde{g})(a),
\end{aligned}$$

thus $\widetilde{f} \cup \widetilde{g} \sqsubseteq \widetilde{f} \odot \widetilde{g}$, and

$$\begin{aligned}
(\lambda \circ \alpha)(a) &= \bigvee_{(p,q)\in \mathbf{S}_a} \{\min\{\lambda(p), \alpha(q)\}\} \\
&\leq \bigvee_{(p,q)\in \mathbf{S}_a} \{\min\{\lambda(pq), \alpha(pq)\}\} \\
&\leq \bigvee_{(p,q)\in \mathbf{S}_a} \{\min\{\lambda(a), \alpha(a)\}\} \\
&= \min\{\lambda(a), \alpha(a)\} \\
&= (\lambda \wedge \alpha)(a),
\end{aligned}$$

thus $\lambda \wedge \alpha \succeq \lambda \circ \alpha$. Therefore $\widetilde{f}_\lambda \Cup \widetilde{g}_\alpha \ll \widetilde{f}_\lambda \otimes \widetilde{g}_\alpha$. □

Corollary 3.20. *Let* $\mathbf{S}$ *be a semisimple ordered semigroup and* $\widetilde{f}_\lambda$ *an anti-hybrid interior ideal in* S *over* U*. Then* $\widetilde{f}_\lambda \ll \widetilde{f}_\lambda \otimes \widetilde{f}_\lambda$.

Lemma 3.21 ([11])**.** *Let* $\mathbf{S}$ *be an ordered semigroup. Then the following conditions are equivalent.*

(1) $\mathbf{S}$ *is semisimple.*
(2) $A \cap B = (AB]$ *for every ideals* A *and* B *of* $\mathbf{S}$.

By Lemma 3.21, we obtain the following corollary.

Corollary 3.22. *Let* $\mathbf{S}$ *be an ordered semigroup. Then the following conditions are equivalent.*

(1) $\mathbf{S}$ *is semisimple.*
(2) $A = (A^2]$ *for every ideal* A *of* $\mathbf{S}$.

Now, we are characterizing the semisimple ordered semigroups by using some properties of anti-hybrid interior ideals.

Theorem 3.23. *Let* $\mathbf{S}$ *be an ordered semigroup. Then the following conditions are equivalent.*

(1) $\mathbf{S}$ *is semisimple.*

(2) $\widetilde{f}_\lambda \otimes \widetilde{g}_\alpha = \widetilde{f}_\lambda \Cup \widetilde{g}_\alpha$ *for every anti-hybrid interior ideals* $\widetilde{f}_\lambda$ *and* $\widetilde{g}_\alpha$ *in* S *over* U.

Proof. (1)⇒(2). Let $\widetilde{f}_\lambda$ and $\widetilde{g}_\alpha$ be anti-hybrid interior ideals in S over U. Since $\mathbf{S}$ is a semisimple ordered semigroup, so by Theorem 3.14, $\widetilde{g}_\alpha$ is an anti-hybrid ideal in S over U. Let $a \in S$. Then since $\mathbf{S}$ is semisimple, there exist $x, y, z \in S$ such that $a \leq xayaz = (xay)(az)$. This implies that $\mathbf{S}_a \neq \emptyset$, we obtain that

$$\begin{aligned}
(\widetilde{f} \odot \widetilde{g})(a) &= \bigcap_{(p,q) \in \mathbf{S}_a} \left[\widetilde{f}(p) \cup \widetilde{g}(q)\right] \\
&\subseteq \widetilde{f}(xay) \cup \widetilde{g}(az) \\
&\subseteq \widetilde{f}(a) \cup \widetilde{g}(a) \\
&= (\widetilde{f} \cup \widetilde{g})(a).
\end{aligned}$$

This implies that $\widetilde{f} \otimes \widetilde{g} \sqsubseteq \widetilde{f} \cup \widetilde{g}$ and

$$\begin{aligned}
(\lambda \circ \alpha)(a) &= \bigvee_{(p,q) \in \mathbf{S}_a} \{\min\{\lambda(p), \alpha(q)\}\} \\
&\geq \min\{\lambda(xay), \alpha(az)\} \\
&\geq \min\{\lambda(a), \alpha(a)\} \\
&= (\lambda \wedge \alpha)(a).
\end{aligned}$$

This implies that $\lambda \circ \alpha \succeq \lambda \wedge \alpha$. Therefore $\widetilde{f}_\lambda \otimes \widetilde{g}_\alpha \ll \widetilde{f}_\lambda \Cup \widetilde{g}_\alpha$. On the other hand by Lemma 3.19, we obtain $\widetilde{f}_\lambda \otimes \widetilde{g}_\alpha = \widetilde{f}_\lambda \Cup \widetilde{g}_\alpha$.

(2)⇒(1). Let A and B be ideals of $\mathbf{S}$. Then A and B are interior ideals of $\mathbf{S}$ and by Lemma 3.18, we obtain $\chi_{A^c}(\widetilde{S}_S)$ and $\chi_{B^c}(\widetilde{S}_S)$ are anti-hybrid interior ideals in S over U and then

$$\begin{aligned}
\chi_{(AB]^c}(\widetilde{S}_S) &= \chi_{(A^cB^c]}(\widetilde{S}_S) \\
&= \chi_{A^c}(\widetilde{S}_S) \otimes \chi_{B^c}(\widetilde{S}_S) \\
&= \chi_{A^c}(\widetilde{S}_S) \Cup \chi_{B^c}(\widetilde{S}_S) \\
&= \chi_{A^c \cup B^c}(\widetilde{S}_S) \\
&= \chi_{(A \cap B)^c}(\widetilde{S}_S).
\end{aligned}$$

This implies that $(A \cap B)^c = (AB]^c$, and then $A \cap B = (AB]$. By Lemma 3.21, we obtain $\mathbf{S}$ is semisimple. □

By the Theorem 3.23, we obtain the following corollary.

Corollary 3.24. *Let* $\mathbf{S}$ *be an ordered semigroup. Then the following conditions are equivalent.*

(1) $\mathbf{S}$ *is semisimple.*

(2) $\widetilde{f}_\lambda = \widetilde{f}_\lambda \otimes \widetilde{f}_\lambda$ *for every anti-hybrid interior ideal* $\widetilde{f}_\lambda$ *in* S *over* U.

4. Conclusions

In this present paper, we introduced the concept of an anti-hybrid interior ideal in an ordered semigroup. Furthermore, we proved that in regular, intra-regular, and semisimple ordered semigroups, the concepts of anti-hybrid interior ideals and anti-hybrid ideals coincide. Finally, we characterized semisimple ordered semigroups in terms of anti-hybrid interior ideals. The notions presented in this paper can be applied to the theory of hyperstructures, ordered hyperstructures, semirings, hemirings, groups, BCI/BCK algebras, etc.

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