J. Appl. Math. & Informatics Vol. **40***(2022), No. 3 - 4, pp. 531 - 543*
https://doi.org/10.14317/jami.2022.531

ON THE PROPERTIES OF POSITIVE BOOLEAN DEPENDENCIES BY GROUPS IN THE DATABASE MODEL OF BLOCK FORM†

TRINH NGOC TRUC*, TRINH DINH THANG, TRAN MINH TUYEN

ABSTRACT. The article proposed a new type of data relationship: Positive Boolean dependencies by groups on block and slice in the database model of block form, where instead of considering value pairs, we consider a group of p values ($p \geq 2$). From this new concept, the article stated and demonstrated the equivalence of the three types of deduction, namely: deduction by logic, deduction by block with groups, deduction by block has no more than p elements with groups. Operations on blocks or slices performed for index attributes on blocks, the properties related to this new concept as theorem the equivalen of the three types of deduction, closure of set positive Boolean dependencies by groups and representation and tight representation set of positive Boolean dependencies by groups when the block degenerated into relation are true in the relational database model and also stated and proven in this paper.

AMS Mathematics Subject Classification : 65F10, 65F50.
Key words and phrases : Positive Boolean dependencies by groups, Boolean dependencies, block schemes, block.

1. Introduction

Currently, there are many models of database that are interested in reseach, one of them is the relational database model proposed by E.Codd [1]. However, in practice wwith dynamic data, this model is still limited.There are many research directions to extend relational data models, such as: data cube [2-3], multidimensional data model [4-6], data warehouse [7-10],.... and the database model of block form [11].

Received June 19, 2021. Revised January 3, 2022. Accepted January 5, 2022. *Corresponding author.

†This work was supported by the research grant of the Vietnam Women's Academy and University Hanoi Pedagogical University 2 of Vietnam.

In fact, not all data dependencies are limited to comparing two attribute values by equality. General positive Boolean dependencies and multi - valued positive Boolean dependencies have expandee the comparision of pairs of comparing each group of values each time. For example, positive Boolean dependency in groups as a natural extension of the general pasitive Boolean dependency class, where instead of considering pairs, we consider a group of p values, $(p \geq 2)$. With this new dependency class, researchers mainly consider them on relational database model. This paper introduces some research results of positive Boolean dependencies by groups on blocks and slices in the database model of block form.

2. THE DATABASE MODEL OF BLOCK FORM

2.1. The block, slice of the block.

Denfinition 2.1 [11]

Let $R = (id; A_1, A_2, ...A_n)$ is a finite set og elements, where id is non-empty finite index set, $A_i, (i = 1...n)$ is the attribute. Each attribute $A_i, (i = 1...n)$ there is a corresponding value domain dom (A_i). A block r on R, denoted $r(R)$ consists of a finite number of elements that each element is a family of mappings from the index set id to the value domain of the attributes $A_i, (i = 1...n)$. In other words: $v \in r(R) \Leftrightarrow v = v^i : id \to dom(A_i)_{i=1...n}$.

Then, we denote the block as $r(R)$ or $r(id; A_1, A_2, ..., A_n)$, sometimes we simply denoted r.

Denfinition 2.2 [11]

Let $R = (id; A_1, A_2, ...A_n), r(R)$ is a block over R. For each $x \in id$ we denoted $r(R_x)$ is block with $R_x = (x; A_1, A_2, ..., A_n)$ such that:

$v_x \in r(R_x) \Leftrightarrow v_x = \{v_x^i = v^i|\}_{i=1...n}$, where $v \in r(R)$,
$v = \{v^i : id \to dom(A_i)\}_{i=1...n}$, then $r(R_x)$ is called a slice of $r(R)$ at x.

2.2. Function dependence on the block. From now on for simplicity we use the notation:

$v^{(i)} = (v; A_i); id^{(i)} = v^(i)|v \in id.$

and $v^{(i)}(v \in id, i = 1...n)$ is called an index attribute of schema
$R = (id; A_1, A_2, ..., A_n)$

Denfinition 2.3 [11]

Let $R = (id; A_1, A_2, ...A_n), r(R)$ is a block, $X, Y \subseteq \bigcup\limits_{i=1}^{n} id^{(i)}$, $X \to Y$ is a notation of functional dependency. A block r satisfies $X \to Y$ if:

$\forall v_1, v_2 \in r$ such that $v_1(X) = v_2(X)$ then $v_1(Y) = v_2(Y)$.

Denfinition 2.4 [11]

Let block scheme $\alpha = (R, F), R = (id; A_1, A_2, ...A_n), F$ is the set of functional dependencies over R. Then, the closure of F denoted F^+ is defined as folows:

$$F^+ = \{X \to Y | F \Rightarrow X \to Y\}$$

.

The block r satisfies $u^{(m)} \to v^{(k)}$ if $\forall t_1, t_2 \in r$ such that $t_1(u^{(m)}) = t_2(u^{(m)})$ then $t_1(v^{(k)}) = t_2(v^{(k)})$, where: $t_1(u^{(m)}) = t_1(u; A_m), t_2(u^{(m)}) = t_2(u; A_m)$; $t_1(v^{(k)}) = t_1(v; A_k); t_2(v^{(k)}) = t_2(v; A_k)$.

3. POSITIVE BOOLEAN FORMULAS

3.1. Boolean formulas.

Denfinition 3.1 [12]

Let $U = \{x_1, x_2, ..., x_n\}$ is a finite set of Boolean variables, B is the set of Boolean values, $B = 0; 1$. Then the boolean formualas (CTB) also known as formulas are constructed as follows:

(i) Each value in B is a CTB.

(ii) Each variable in U is a CTB.

(iii) If u is a Boolean formula then (u) is a CTB.

(iv) If u and v are CTB then u and $u \to v$ are CTB.

(v) Only formulas created by rules from (i) - (iv) are CTB.

Then, we denote $L(U)$ as a set of CTB building on $U = \{x_1, x_2, ..., x_n\}$ and $B = \{0; 1\}$.

Denfinition 3.2 [12]

Each vector of elements $v = \{v_1, v_2, ..., v_n\}$ in space $B^n = B \times B \times ... \times B$ is called a value assignment. Thus, with each CTB $g \in L(U)$ we have $g(v) = g(v_1, v_2, ..., v_n)$ is the value of formula g for v value assignments.

For simplicity, we understand the symbol $P \subseteq U$ at the same time performing for the following subjects:

- An attribute set in U.
- A set of logical variables in U.
- A Boolean fomula is the logical union of variables in P.

Further, if $P = B_1, B_2, ..., B_n \subseteq U$ then we denoted.

$\wedge P = B_1 \wedge B_2 \wedge ... \wedge B_n$ called the associational form.

$\vee P = B_1 \vee B_2 \vee ... \vee B_n$ called the recruitmental form.

For finite set $G = \{f_1, f_2, ..., f_m\}$ in $L(U)$, we consider G as a formatted formula $G = f_1 \wedge f_2 \wedge ... \wedge f_m$. Then we have: $G(v) = f_1(v) \wedge f_2(v) \wedge ... \wedge f_m(v)$.

3.2. Table of values and truth tables. With each formula f on U, table of values for f, denote that V_f contains $n + 1$ colums, with the first n columns containing the values of the variables in U, and the last column contains the value of f for each values signment of the corresponding row. Thus, the value table contains 2^n row, n is the element number of U.

Denfinition 3.3 [13]

The truth table of f, denoted T_f is the set of assignments v such that $f(v)$ receive value 1:

$$T_f = \{v \in B^n | f(v) = 1\}$$

Then, the truth table T_F of finite sets of formulas F on U, is the intersection of the truth tables of each member formula in F.

$$T_F = \bigcap_{f \in F} T_f.$$

We have: $v \in T_F$ necessary and sufficient are $\forall F : f(v) = 1$.

3.3. Logical deduction.

Denfinition 3.4 [14]

Let f, g two CTB, we say fomula f derives fomula g and denoted $f \models g$ if $T_f \subseteq T_g$. We say f and g are two equivalent formulas, denoted $f \equiv g$ if $T_f = T_g$.

With F, G in $L(U)$ we say F derives G, denoted $F \models G$ if $T_F \subseteq T_G$. Moreover, we say F and G are equivalents, denoted $F \equiv G$ if $T_F = T_G$.

3.4. Positive Boolean formula.

Denfinition 3.5 [15]

Formula $f \in L(U)$ is called apositive Boolean formula (CTBD) if $f(e) = 1$ with the unit value assignment:$e = (1, 1, ..., 1)$, we denoted $P(u)$ is the set of all positive Boolean formulas on U.

4. RESEARCH RESULTS

4.1. The truth block by groups of the data block.

Denfinition 4.1

Let block scheme $\alpha = (R, F), R = (id; A_1, A_2, ..., A_n), r(R)$ is a block over R, we convention that each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n$) contains at least p elements. Then, with each value domain d_i, we consider the mapping $\beta_i : (d_i)^p \to B, (p \geq 2)$ satisfies the following properties:

(i) Reflectivity: $\forall a \in (d_i)^p : \beta_i(a) = 1$, if in a contains at least two identical components.

(ii) Commutation: $\forall a \in (d_i)^p : \beta_i(a) = \beta_i(a')$, where a' is permutation of a.

(iii) Sufficiency: $\exists a \in (d_i)^p : \beta_i(a) = 0$.

Thus, we see the mapping β_i is an evaluation on a group containing p $(p \geq 2)$ values of d_i satisfying reflection and commutative properties. Equality relation is a separate case of this relation.

Example 4.1

Let $R = (\{1, 2\}, A_1, A_2, A_3, A_4)$, then $U = 1^{(1)}, 1^{(2)}, 1^{(3)}, 1^{(4)}, 2^{(1)}, 2^{(2)}, 2^{(3)}, 2^{(4)}$; r is a block over R with elements are: u_1, u_2, u_3, and u_4:

- $u_1(1^{(1)}) = 1; u_1(1^{(2)}) = 0; u_1(1^{(3)}) = 2; u_1(1^{(4)}) = e;$
  $u_1(2^{(1)}) = 2; u_1(2^{(2)}) = 0; u_1(2^{(3)}) = 4; u_1(2^{(4)}) = g.$
- $u_2(1^{(1)}) = 2; u_2(1^{(2)}) = 1; u_2(1^{(3)}) = 4; u_2(1^{(4)}) = g;$
  $u_2(2^{(1)}) = 2; u_2(2^{(2)}) = 1; u_2(2^{(3)}) = 6; u_2(2^{(4)}) = h.$
- $u_3(1^{(1)}) = 3; u_3(1^{(2)}) = 0; u_3(1^{(3)}) = 6; u_3(1^{(4)}) = h;$
  $u_3(2^{(1)}) = 3; u_2(2^{(2)}) = 2; u_3(2^{(3)}) = 6; u_3(2^{(4)}) = e.$
- $u_4(1^{(1)}) = 4; u_4(1^{(2)}) = 1; u_4(1^{(3)}) = 8; u_4(1^{(4)}) = h;$
  $u_4(2^{(1)}) = 4; u_4(2^{(2)}) = 3; u_4(2^{(3)}) = 8; u_4(2^{(4)} = q.$

Then, with $p = 3, B = \{0; 1\}$, we define the mappings: $\beta_i : (d_i)^p \to B, (p \geq 2)$ as flollows: $\forall a \in d_i^{(3)} : \beta_i(a) = 1$ if a has at least two odentical components, ortherwise $\beta_i(a) = 0$.

We have: $\beta_1(1, 2, 3) = 0, \beta_2(0, 1, 0) = 1, \beta_3(2, 4, 6) = 0, \beta_4(e, g, h) = 0$.

Denfinition 4.2

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n,$) contains at least $p(p \geq 2)$ elements, β_i is an evaluation on groups containing $p(p \geq 2)$ values of$x^{(i)}, x \in id, 1 \leq i \leq n)$. For each group of p elements: $u_1, u_2, ..., u_p$ arbitrary (not necessarily distinguish) on the block, we call $\beta(u_1, u_2, ..., u_p)$ is the value assignment: $\beta(u_1, u_2, ..., u_p) = (t_{x1}, t_{x2}, ..., t_{xn})$ with $t_{xi} = \beta_i(u_1.x^{(i)}, ..., u_p.x^{(i)})$, $x \in id, 1 \leq i \leq n$.

Then, for each block r, we denote the truth block by groups of r as T_r^p:

$$T_r^p = \beta(u_1, u_2, ..., u_p)|u_j \in r, 1 \leq j \leq n.$$

From the definition we see the truth block by groups of block r is a binary block.

In the case $id = \{x\}$, then the block degenerates into a relation and the concept of the truth block by groups of the block becomes the concept of truth table groups relation in the relational data model. In other words, the truth block by groups of a block is to espand the concept of the truth table by groups of relation in the relational data model.

Example 4.2:

Let $R = (\{1, 2\}, A_1, A_2, A_3, A_4)$, then $U = 1^{(1)}, 1^{(2)}, 1^{(3)}, 1^{(4)}, 2^{(1)}, 2^{(2)}, 2^{(3)}, 2^{(4)}$, r is a block over R with elements are: u_1, u_2, u_3, and u_4, p and the mappings: $\beta_i : d_i^{(3)} \to B, (1 \leq i \leq 4)$ as shown in example 4.1.

Then, the truth block by by groups of a block r is determined as follows:

-$\beta(\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}) = \begin{pmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \end{pmatrix}$,

$t_{11} = \beta_1(u_1.1^{(1)}, u_2.1^{(1)}, u_3.1^{(1)}) = \beta_1(1, 2, 3) = 0;$

$t_{12} = \beta_2(u_1.1^{(2)}, u_2.1^{(2)}, u_3.1^{(2)}) = \beta_2(0, 1, 0) = 1.$

$t_{13} = \beta_3(u_1.1^{(3)}, u_3.1^{(3)}, u_3.1^{(3)}) = \beta_3(2, 4, 6) = 0;$

$t_{14} = \beta_4(u_1.1^{(4)}, u_2.1^{(4)}, u_3.1^{(4)}) = \beta_4(e, g, h) = 0.$

$t_{21} = \beta_1(u_1.2^{(1)}, u_2.2^{(1)}, u_3.2^{(1)}) = \beta_1(2, 2, 3) = 1;$

$t_{22} = \beta_2(u_1.2^{(1)}, u_2.2^{(2)}, u_3.2^{(2)}) = \beta_2(0, 1, 2) = 0.$

$t_{23} = \beta_3(u_1.2^{(3)}, u_2.2^{(3)}, u_3.2^{(3)}) = \beta_3(4, 6, 6) = 1;$

$t_{24} = \beta_4(u_1.2^{(4)}, u_2.2^{(4)}, u_3.2^{(4)}) = \beta_4(g, h, e) = 0.$

$\Rightarrow \beta(\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix}$

Similarly we can identify elements of the truth block by groups of block r: $\beta(u_1, u_2, u_4), \beta(u_2, u_3, u_4), \beta(u_2, u_3, u_3)...$

4.2. The positive Boolean dependencies by groups of a data block.

Denfinition 4.3

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n,$) contains at least p $(p \geq 2)$ elements, β_i is an evaluation on groups containing p $(p \geq 2)$ values of $x^{(i)}, x \in id, 1 \leq i \leq n)$. We call each positive Boolean formula in $P(U)$ with $U = \bigcup\limits_{i=1}^{n} id^{(i)}$ is a positive Boolean dependency by groups.

We say block r is satisfying the positive Boolean dependency by groups (PTB-DTNB) f and denoted $r^p(f)$ if $T_r^p \subseteq T_f$.

The block r is satisfying set of positive Boolean dependency by groups F and denoted $r^p(F)$ if r satisfies all PTBDTNB f in F:

$$r^p(F) \Leftrightarrow \forall f \in F : r^p(f) \Leftrightarrow T_r^p \subseteq T_F.$$

If $r^p(F)$ then we say PTBDTNB f is right in the block r.

Proposition 4.1

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on $R, U = \bigcup\limits_{i=1}^{n} id^{(i)}$. Then:

i) If r is satisfying the positive Boolean dependency by groups $f : r^p(f)$ then $r_x^p(f_x), \forall x \in id$.

ii) If r is satisfying set of positive Boolean dependency by groups $F : r^p(F)$ then $r_x^p(F_x), \forall x \in id$.

Proof:

i) Under the assumption we have:

$r^p(f) \Rightarrow T_r^p \subseteq T_f \Rightarrow T_{rx}^p = (T_r^p)_x \subseteq (T_f)_x = T_{fx}^p, \forall x \in id$.

So we have $T_{rx}^p \subseteq T_{fx}^p, \forall x \in id$.

ii) Under the assumption:

$r^p(F) \Rightarrow T_r^p \subseteq T_F \Rightarrow T_{rx}^p = (T_r^p)_x \subseteq (T_F)_x = T_{Fx}^p, \forall x \in id$.

Proposition 4.2

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on $R, U = \bigcup\limits_{i=1}^{n} id^{(i)}, f = \bigcup\limits_{x \in id} f_x$. Then:

i) If $r_x^p(f_x), \forall x \in id$ then r is satisfying the positive Boolean dependency by groups $f : r^p(f)$.

ii) If $r_x^p(F_x), \forall x \in id$ then r is satisfying set of positive Boolean dependency by groups $F : r^p(F)$.

Proof:

i) Under the assumption we have:

$$r_x^p(f_x), \forall x \in id \Rightarrow T_{rx}^p \subseteq T_{fx}, \forall x \in id \Rightarrow (T_r^p)_x \subseteq (T_f)_x, \forall x \in id.$$

So we have $T_r^p \subseteq T_f^p \Rightarrow r^p(f)$.

$\Rightarrow r$ is satisfying the positive Boolean dependency by groups f.

ii) Under the assumption:

$r_x^p(F_x), \forall x \in id \Rightarrow T_{rx}^p \subseteq T_{F_x}, \forall x \in id \Rightarrow (T^p)_x \subseteq (T^F)_x \forall x \in id$.

So we have $T_r^p \subseteq T_F \Rightarrow r^p(F)$.

$\Rightarrow r$ is satisfying set of positive Boolean dependency by groups F.

From the proposition 4.1 and 4.2 we have the following necessary and suffcient conditions:

Theorem 4.1

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on $R, U = \bigcup_{i=1}^{n} id^{(i)}, f = \bigcup_{x \in id} f_x$.

Then:

i) $r_x^p(f_x), \forall x \in id \Leftrightarrow r$ is satisfying the positive Boolean dependency by groups $f : r^p(f)$.

ii) $r_x^p(F_x), \forall x \in id \Leftrightarrow r$ is satisfying set of positive Boolean dependency by groups $F : r^p(F)$.

For the set PTBDTNB F and PTBDTNB f, we say:

- F deduced f by the block with groups and denoted:

$F \vdash^p f$ if: $\forall r : r^p(F) \Rightarrow r^p(f)$.

- F deduced f by the block with groups, block contains no more than p elements and denoted $F \vdash_p^p f$ if: $\forall r_p : r^p(F) \Rightarrow r_p^p(f)$.

We have the following equivalent theorem:

Theorem 4.2

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n$) contains at least p $(p \geq 2)$ elements, β_i is an evaluation on groups containing p $(p \geq 2)$ values of $x^{(i)}, x \in id, 1 \leq i \leq n,$) the set PTBDTNB F and PTBDTNB f. Then the following three propositions are equivalent:

(i) $F \vDash f$ (deduction by logic),

(ii) $F \vdash^p f$ (deduction in groups by block),

(iii) $F \vdash_p^p f$ (deduction in groups by block has no more than $p$elements).

Proof:

(i) $\Rightarrow$ (ii): Under the assumption we have: $F \vDash f \Rightarrow T_F \subseteq T_f$ (1).

Let r be an arbitrary block anf $r^p(F)$, then by definition: $T_r^p \subseteq T_F$ (2).

From (1) and (2) we infer: $T_r^p \subseteq T_f$, so we have: $r^p(f)$.

(ii) $\Rightarrow$ (iii): Obviously, because inference by the block has no more than p elements is the special case of inference by block.

(iii) $\Rightarrow$ (i): Suppose $t = (t_x^{(1)}, t_x^{(2)}, ..., t_x^{(n)})_{x \in id}, t \in T_F$, we need proof: $t \in T_f$.

Indeed, if $t = e$ then we have $t \in T_f$ because as we know f is a positive Boolean formula.

If $t \neq e$, we built the block r including p elements as follows:

According to the properties of the mapping:

$\beta_i : (d_i)^{(p)} \rightarrow B$ then $\exists a_i = (a_{i1}, a_{i2}, ..., a_{ip}), \beta_i(a_i) = 0$.

Then, with each value domain d_i of the index attribute $x^{(i)}$ in $U = \bigcup_{i=1}^{n} id^{(i)}$, we take an eletment $a_{ij} \in \{a_{i1}, a_{i2}, ..., a_{ip}\}$.

If $t_{xi} = 1$ then we fill in the column of the index attribute $x^{(i)}$ of block r with p equal value and equal to a_{ij}. If $t_{xi} = 0$ then we fill in the column of the index attribute $x^{(i)}$ of block r with p value $a_{i1}, a_{i2}, ..., a_{ip}$.

By the way build that r block, we have: $T_r^p = \{e, t\} \subseteq T_F$ with e is the unit value assigment. Thus r is a block with p elements and satisfying PTBDTNB F. Under the assumption that if r satisfies F then r will satisfy f, this means: $T_r^p = \{e, t\} \subseteq T_f$, infer: $t \in T_f$. □

Consequence 4.1

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \le i \le n$) contains at least p $(p \ge 2)$ elements, β_i is an evaluation on groups containing p $(p \ge 2)$ values of $x^{(i)}, x \in id, 1 \le i \le n$) the set PTBDTNB F and PTBDTNB f. Then on r_x the following three propositions are equivalent:

(i) $F \vDash f_x$ (deduction by logic),

(ii) $F \vdash^p f_x$ (deduction in groups by slice r_x),

(iii) $F \vdash_p^p f_x$ (deduction in groups by slice r_{px} has no more than $p$elements).

In the case $id = \{x\}$, then the block degenerates into a relation and the above equivalence theorem becomes the equivalent theorem in the relational data model. Speccifically, we have the following consequences.

Consequence 4.2

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \le i \le n,$) contains at least p $(p \ge 2)$ elements, β_i is an evaluation on groups containing p $(p \ge 2)$ values of $x^{(i)}, x \in id, 1 \le i \le n$) the set PTBDTNB F and PTBDTNB f. Then, if $id = \{x\}$, then the block r degenerates into a relation and in the relational data model the following three propositions are equivalent:

(i) $F \vDash f$ (deduction by logic),

(ii) $F \vdash^p f$ (deduction in groups by relation),

(iii) $F \vdash_p^p f$ (deduction in groups by relation has no more than $p$elements).

Definition 4.4

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \le i \le n$) contains at least p $(p \ge 2)$ elements, β_i is an evaluation on groups containing p $(p \ge 2)$ values of $x^{(i)}, x \in id, 1 \le i \le n$). Σ is the set PTBDTNB considered on groups with p elements. Then the closure Σ^+ of the set PTBDTNB Σ is determined as flollows:

$$\Sigma^+ = \{f | f \in P(U), \Sigma \models f\} = \{f | f \in P(U), T_\Sigma \subseteq T_f\}$$

Definition 4.5

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \le i \le n$) contains at least p $(p \ge 2)$ elements, β_i is an evaluation on groups containing p $(p \ge 2)$ values of $x^{(i)}, x \in id, 1 \le i \le n$).Then we denote NBD($r$) is the set of PTBDTNB satisfying in block r, meaning: NBD $(r) = \{f | f \in P(U), r^p(f)\}$.

Theorem 4.3

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \le i \le n$) contains at least

p $(p \geq 2)$ elements, β_i is an evaluation on groups containing p $(p \geq 2)$ values of $x^{(i)}, x \in id, 1 \leq i \leq n)$. Then we have: $(NBD(r))^+ = NBD(r)$.

Proof:

By definiton, we have:

$$(NBD(r))^+ = \{f|f \in P(U), NBD(r) \models f\} = \{f|f \in P(U), T_{NBD(r)} \subseteq T_f\}.$$

So inferred: $(NBD(r))^+ \supseteq NBD(r)$. (3)

on the other hand, suppose we have:

$g \in (NBD(r))^+$, we need proof $g \in NBD(r)$.

Indeed, under the assumption:

$g \in (NBD(r))^+ = \{f|f \in P(U), T_{NBD(r)} \subseteq T_f\} \Rightarrow g \in P(U), T_{NBD(r)} \subseteq T_g$.

Which according to the definition of $NBD(r)$ we have:

$T_r^p \subseteq T_{NBD}(r) \Rightarrow T_r^p \subseteq T_g$ (according to the transitive property) $\Rightarrow r$ satisfies PTBDTNB g.

From there we have: $g \in (NBD(r) \Rightarrow (NBD(r))^+ \subseteq NBD(r)$. (4)

From (3) and (4) we infer: $(NBD(r))^+ = NBD(r)$ $\Box$

Consequence 4.3

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n)$ contains at least p $(p \geq 2)$ elements, β_i is an evaluation on groups containing p $(p \geq 2)$ values of $x^{(i)}, x \in id, 1 \leq i \leq n$). Then on r_x we have:

$$(NBD(r_x))^+ = NBD(r_x), \forall x \in id.$$

Consequence 4.4

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n)$ contains at least p $(p \geq 2)$ elements, β_i is an evaluation on groups containing p $(p \geq 2)$ values of $x^{(i)}, x \in id, 1 \leq i \leq n)$. Then, if $id = \{x\}$, then the block r degenerates into a relation and in the relational data model we have: $(NBD(r))^+ = NBD(r)$.

Theorem 4.4

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n,$) contains at least $p(p \geq 2)$ elements, β_i is an evaluation on groups containing $p(p \geq 2)$ values of $x^{(i)}, x \in id, 1 \leq i \leq n$). Then we have: $T_r^p = T_{NBD(r)}$.

Proof:

By definiton of the set PTBDTNB $NBD(r)$, we have:

if $f \in NBD(r) \Rightarrow$ block r satisfies PTBDTNB $f \Rightarrow T_r^p \subseteq T_f$.

According to the properties of the Boolean formula and the truth block, from the truth block T_r^p, we find a Boolean formula h such that: $T_h = T_r^p$. On the other hand, because $e \in T_r^p = T_h \Rightarrow h$ is a positive Boolean formula.

From the equality $T_r^p = T_h$, we deuce the block r satises PTBDTNB h, mean: $h \in NBD(r)$.

So infer: $NBD(r) \models h$. Hence we have:

$T_{NBD(r)} \subseteq T_h = T_r^p \Rightarrow T_{NBD(r)} \subseteq T_r^p$ (5)

From the definition of $NBD(r)$ we have: $T_r^p \subseteq T_{NBD(r)}$ (6)

From (5) and (6) we infer: $T_r^p = T_{NBD(r)}$ $\square$

Consequence 4.5

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n$) contains at least p $(p \geq 2)$ elements, β_i is an evaluation on groups containing p $(p \geq 2)$ values of $x^{(i)}, x \in id, 1 \leq i \leq n)$. Then on r_x we have: $T_{rx}^p = T_{NBD(rx)}, \forall x \in id$.

Consequence 4.6

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n$) contains at least p $(p \geq 2)$ elements, β_i is an evaluation on groups containing p $(p \geq 2)$ values of $x^{(i)}, x \in id, 1 \leq i \leq n)$. Then, if $id = \{x\}$, then the block r degenerates into a relation and in the relational data model we have: $T_r^p = T_{NBD(r)}$.

Definition 4.6

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n$) contains at least p $(p \geq 2)$ elements, β_i is an evaluation on groups containing p $(p \geq 2)$ values of $x^{(i)}, x \in id, 1 \leq i \leq n)$.

We say block is representation by groups set PTBDTNB Σ if $NDB(r) \supseteq \Sigma^+$ and block r is tight representation by groups set PTBDTNB Σ if $NDB(r) = \Sigma^+$.

If block r is tight representation set PTBDTNB Σ then we say r is the Armstrong block of the set PTBDTNB Σ. We denoted $\Sigma_x = \Sigma \cap \bigcup\limits_{i=1}^{n} x^{(i)}$ is the subset PTBDTNB Σ on r_x.

Theorem 4.5

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n$) contains at least p $(p \geq 2)$ elements, β_i is an evaluation on groups containing p $(p \geq 2)$ values of $x^{(i)}, x \in id, 1 \leq i \leq n)$. Then block r is tight representation by groups set PTBDTNB Σ if and only if $T_r^p = T_\Sigma$.

Proof:

Use the results of proposition 4.3 and proposition 4.4, we have:

$(NBD(r))^+ = NBD(r)$ and $T_r^p = T_{NBD(r)}$.

Consequence 4.7

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n$) contains at least p $(p \geq 2)$ elements, β_i is an evaluation on groups containing p $(p \geq 2)$ values of $x^{(i)}, x \in id, 1 \leq i \leq n)$, $\Sigma = \bigcup\limits_{x \in id} \Sigma_x$. Then r_x is tight representation by groups set PTBDTNB Σ if and only if $T_r^p = T_{\Sigma x}, \forall x \in id$.

Consequence 4.8

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n$) contains at least p $(p \geq 2)$ elements, β_i is an evaluation on groups containing p $(p \geq 2)$

values of $x^{(i)}, x \in id, 1 \leq i \leq n$). Then, if $id = \{x\}$ then the block degenerates into a relation and in the relational data model we have: relation r is tight representation by groups set PTBDTNB Σ if and only if $T_r^p = T_\Sigma$.

Theorem 4.6

Let $R = (id; A_1, A_2, ..., A_n), r(R)$ is a block on R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}, x \in id, 1 \leq i \leq n$) contains at least p ($p \geq 2$) elements, β_i is an evaluation on groups containing p ($p \geq 2$) values of $x^{(i)}, x \in id, 1 \leq i \leq n$),$\Sigma = \bigcup\limits_{x \in id} \Sigma_x$.

Then block r is tight representation by groups set PTBDTNB Σ if and only if r_x is tight representation by groups set PTBDTNB $\Sigma_x, \forall x \in id$.

Proof:

$\Rightarrow$) Suppose r is tight representation by groups set PTBDTNB Σ we need proof r_x is tight representation by groups set PTBDTNB $\Sigma_x, \forall x \in id$.

Indeed, under the assumption we have: r is tight representation by groups set PTBDTNB Σ, using the results of theorem 4.5 we have: $T_r^p = T_\Sigma$.

Thence inferred: $(T_r^p)_x = (T_\Sigma)_x, \forall x \in id$.

Which we have: $T_{rx}^p = (T_r^p)_x = (T_\Sigma)_x = T_{\Sigma x}, \forall x \in id \Rightarrow T_{rx}^p = T_{\Sigma x}, \forall x \in id$.

$\Leftarrow$) Suppose r_x is tight representation by groups set PTBDTNB $\Sigma_x, \forall x \in id$ we need proof r is tight representation by groups set Σ.

Indeed, under the assumption r_x is tight representation by groups set $\Sigma_x, \forall x \in id \Rightarrow T_{rx}^p = T_{\Sigma x}, \forall x \in id$.

Inferred: $(T_r^p)_x = T_{rx}^p = (T_\Sigma)_x, \forall x \in id \Rightarrow (T_r^p)_x = (T_\Sigma)_x$.

Which we have: $T_r = \bigcup\limits_{x \in id} (T_r^p)_x, T_{\Sigma,m} = \bigcup\limits_{x \in id} (T_\Sigma)_x \Rightarrow T_r^p = T_\Sigma$.

So r is tight representation by groups set Σ $\square$

5. CONCLUSION

The research results presented in the article have shown some properties of positive boolean dependencies by groups. The equivalence of the three types of deduction still holds true for positive boolean dependencies by groups on block and slice. The necessary and sufficient condition for a block is tight representation by groups set Σ, a slice is tight representation by groups set Σ_x. In addition, if $id = \{x\}$ then the block degenerated into a relation and the results found for positive boolean dependencies by groups on the block are still true on the relation in the relational data model.

The study of new types of logical dependencies and their relationships in the database model of block forms is still of great interest. Further research results will also contribute to enrich the design theoory for the database model of block form.

References

1. D. Maier, *The Theory of Relational Databases*, Computer Science Press, Rockville, 1983.

2. Apostolos Benisis, *Business Process Management: A Data Cube To Analyze Business Process Simulation Data For Decision Making Paperback*, VDM Verlag Dr. Müller, 2010.
3. Maurizio Rafanelli, M.Rafanelli, *Multidimensional Databases: Problems and Solutions Hardcover*, Idea Group Publishing, 2003.
4. Omar Boussaïd and Doulkifli Boukraâ. *Multidimensional Modeling of Complex Data*, Encyclopedia of Data Warehousing and Mining, Second Edition, edited by John Wang, IGI Global, 2009.
5. Haiping Lu, Konstantinos N. Plataniotis, Anastasios Venetsanopoulos, *Multilinear Subspace Learning: Dimensionality Reduction of Multidimensional Data* (Chapman & Hall/Crc Machine Learning & Pattern Recognition) 1 edition, 2013.
6. R. Agrawal, A. Gupta, and S. Sarawagi, *Modeling Multidimensional Databases*, In Proc. of 13 th, Int. Conf. on Data Engineering (ICDE) IEEE press, 1997, 232-243.
7. R. Kimball and M. Ross, *The Data Warehouse Toolkit: The Complete Guide to Dimensional Modeling. 2ed.* J. Wiley and Sons, Canada, 2002.
8. Ralph Kimball, Margy Ross, Warren Thornthwaite, Joy Mundy, Bob Becker, *The Data Warehouse Lifecycle Toolkit Paperback*, J. Wiley and Sons, Canada, 2008.
9. G. Garani, G.K. Adam and D. Ventzas, *Temporal data warehouse logical modelling*, Int. J. Data Mining, Modelling and Management **8** (2016), 144–159.
10. Krish Krishnan, *Data Warehousing in the Age of Big Data*, The Morgan Kaufmann Series on Business Intelligence, Morgan Kaufmann; 1 edition, USA, 2013.
11. Nguyen Xuan Huy, Trinh Dinh Thang, *The database model of blocks form*, Journal of Informatics and Cybernetics, **14** (1998), 52-60.
12. Nguyen Xuan Huy, *Logical dependencies in the database*, Statistical Publisher, HaNoi, 2006.
13. Trinh Dinh Thang, Tran Minh Tuyen, *Logic dependencies in the database model of block forms*, Scientific and Technical publishing House, HaNoi, 2020.
14. Trinh Dinh Thang, Tran Minh Tuyen, *The translation of block scheme and the present problem of the closure, key in the database model of block forms*, Proceedings of the National Conference XIII "Some the selected issues of Information Technology and Communication", 2010, 276-286.
15. Trinh Dinh Thang, Tran Minh Tuyen, *Closed mapping and translation of block schemes*,. Proceedings of the National Conference on the VI for Fundamental and Applied Information Technology Research, 2013, 677-683.
16. Trinh Dinh Thang, Tran Minh Tuyen, Trinh Ngoc Truc, *Deductive formula in the database model of block forms*, Proceedings of the National Conference on the VIII for Fundamental and Applied Information Technology Research, 2015, 103-110, DOI: 10.15625/vap.2015.000143.

Trinh Ngoc Truc received Trinh Ngoc Truc from Vietnam Women's Academy. He is a postgraduate. His research interests include the mathematical theory of Applied Mathematics and Computer Science.

Infomation Technology Institute, Vietnam Women's Academy.
e-mail: `trinhngoctruc@gmail.com`

Trinh Dinh Thang received Trinh Dinh Thang from Hanoi Pedagogical University 2 of Vietnam. He is currently a professor at Hanoi Pedagogical University 2 since 2000. His research interests are the mathematical theory of Applied Mathematics and Computer Science.

Infomation Technology Institute, Hanoi Pedagogical University 2 of Vietnam.
e-mail: `thangdhsp2@hpu2.edu.vn`

Tran Minh Tuyen received Tran Minh Tuyen from University Union Vietnam. She is currently a Ph.D. at University Union Vietnam since 2015. She research interests are the mathematical theory of Applied Mathematics and Computer Science.

Department of Infomation Technology, University union Vietnam.
e-mail: tuyentm@dhcd.edu.vn