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OPTIMAL CONTROL STRATEGY TO COMBAT THE SPREAD OF COVID-19 IN ABSENCE OF EFFECTIVE VACCINE

M.H.A. BISWAS*, M.S. KHATUN, M.A. ISLAM, S. MANDAL, A.K. PAUL AND A. ALI

Abstract. Many regions of the world are now facing the second wave of boomed cases of COVID-19. This time, the second wave of this highly infectious disease (COVID-19) is becoming more devastating. To control the existing situation, more mass testing, and tracing of COVID-19 positive individuals are required. Furthermore, practicing to wear a face mask and maintenance of physical distancing are strongly recommended for everyone. Taking all these into consideration, an optimal control problem has been reformulated in terms of nonlinear ordinary differential equations in this paper. The aim of this study is to explore the control strategy of coronavirus-2 disease (COVID-19) and thus, minimize the number of symptomatic, asymptomatic and infected individuals as well as cost of the controls measures. The optimal control model has been analyzed analytically with the help of the necessary conditions of very well-known Pontryagin's maximum principle. Numerical simulations of the optimal control problem are also performed to illustrate the results.

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1. Introduction

At present, COVID-19 is the main concern of the whole world and the second wave of the spreading of the infection has been already started. It is a blood vessel disease caused by SARS-CoV-2 which was emerged in Wuhan, China in December 2019 [34, 35, 42]. Several infectious diseases like COVID-19 which taken millions of lives such as Bubonic plague (Europe and West Asia in 541-542 [19, 33, 40], Europe, Asia and North Africa in 1346-1353 [2], Italy in 1629-1656

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[23, 42]), Smallpox (Japan in 735-737 [29, 44], Mexico in 1519-1520 [1]), Influenza (England in 1775-1776 [38]), Influenza A (H2N2, H3N2) (Worldwide in 1957-1970 [37]) Cholera (Asia, Europe and North America in 1817-1837, Middle East in 1863-1875 [23]), Typhus (Russia in 1918-1922 [36]), HIV/AIDS (Worldwide in 1981 to present [45]), Ebola (Worldwide in 2013-2016 [22]) came in several time. Since COVID-19 is one of the most infectious diseases, older people with diabetes, cardiovascular disease, cancer, and chronic respiratory disease are mostly in a dangerous region. The virus may be transmitted to others through the sneezes, coughs, saliva, even respiratory secretions of an infected individual. Therefore, separation, awareness, and self-protection are the best efficacious ways to control the spread of COVID-19 until an effective vaccine come.

The infected individuals having no symptoms spread the disease from human to human. Therefore, the identification of the infected individuals is very important to separate them from uninfected individuals. The infection can also spread through the household waste of infected individuals [17]. In this case, mass testing is crying need to identify the infected individuals and it can control the transmission of the infection in the mass community. But it is not possible to identify all the infected patients among more than 7 billion people. A clinical report presented that COVID-19 positive patients with lower immune systems lose the capacity of smelling but not for the patients with a higher immune system [14]. In this case, the spread of COVID-19 can be controlled by maintaining physical distancing (at least 6 feet) and wearing a virus protectable mask especially a nose mask. Even in the hospital, proper physical distancing and virus protecting mask should have to be ensured, otherwise, all the hospitalized uninfected patients may be infected widely [21]. However, after identifying the COVID-19 positive patients, proper treatments should be served as soon as possible according to the symptoms. Besides, in order to promote the immune system, one may take immune-boosting foods and some vitamins because the immunized individuals have less possibility to be infected by COVID-19 [3].

The mechanisms of spreading infectious diseases were described briefly at different times by Biswas et al. [4, 5, 6, 7, 8, 9, 10, 11, 12]. In these papers, the spreading of infectious diseases and the control strategies were well-described through several mathematical models and optimal control techniques. In the pandemic situation of COVID-19, several research models on the spreading and controlling of COVID-19 were developed. Kucharski et al. [30] developed a mathematical model on early transmission and the control of the spread of COVID-19. Lin et al. [32] developed a conceptual model for the outbreak of COVID-19 in Wuhan considering the behaviors of individuals and the actions of the government, whereas Wu et al. [47] proposed and analyzed briefly a mathematical model on the domestic and international spread of COVID-19. Kabir et al. [24] developed nonlinear mathematical modeling to describe the dispersal effect to moderate the infection of COVID-19 in Bangladesh. Lin et al. [31] showed that the infection of COVID-19 can transmit easily at lower temperatures and hardly spread at higher temperatures. Considering in mind, he developed an optimal control model considering relative humidity as the control measures to control the transmission of COVID-19. Readers can also follow (Rowe, et al. [41]; Eckardt et al. [18]; Zhang et al. [49]; Zhao et al. [50, 51]; Chen et al. [15]; Xu et al. [48]; Khatun and Biswas [25, 26, 27]) for more information about the spreading causes and controlling techniques of COVID-19 as well as very recent implementation of models and optimal control techniques.

The spreading of infection of COVID-19 is continuously going on and will remain until an effective vaccine will come in hand. In such a situation, we have to aware of the factors by which the infection is spreading mostly, and have also to adopt more testing to identify more COVID-19 infected individuals. To come out from such a pandemic situation, mass awareness, more testing, and proper treatment should be implemented. Therefore, we developed an optimal control strategy approaching a nine compartmental nonlinear mathematical model considering three control variables. Mass testing and tracing or identification of the COVID-19 positive patients, maintenance of physical distancing and wearing face mask, and effective treatment for corresponding complications and taking immune-boosting foods and drugs are the most effectual ways to defend COVID-19 at this stage. We have introduced all these things in the formulated model. The model has been analyzed both analytically and numerically. Our aim is to employ the control strategies so that the number of symptomatic, asymptomatic and infected individuals is minimized at the minimum cost of the optimal controls and eventually, control the outbreak of COVID-19 among populations.

2. Optimal Control Model of COVID-19

Coronavirus-2 disease (COVOD-19) is exceedingly transmissible having no specific drugs that can cure the infection. Although vaccines for this disease are under development, several of these vaccines are still in the human testing phase [46]. The WHO assures that when a safe and efficacious vaccine is discovered, they will help to provide these vaccines to all countries in the world. In that case they will prioritize the most vulnerable people in the world. Until then, we have to fight with COVID-19 by adopting some measures like maintenance of physical distancing and identification of COVID-19 positive patients (mass testing). Since COVID-19 is a highly infectious disease, maintenance of physical distancing is strictly recommended as much as possible to avoid unexpected infections. Further, the more people will be tested, the more COVID-19 positive carriers will be identified. That is why people are being highly suggested to come under test. However, once COVID-19 is infected, doctors suggest treatment considering the symptoms like fever, cough, and diarrhoea to control complications and give our body time to heal. So, taking all these scenarios into consideration, we have introduced three control measures $(u_1(t), u_2(t), u_3(t))$ in our previously developed mathematical model [13]. In our prior study, we formulated a nine compartmental model of COVID-19 showing the impact of symptomatic and asymptomatic individuals in the outbreak of this novel coronavirus disease through the following set of nonlinear ordinary differential equations:

$$\begin{cases} \frac{dS}{dt} = \Delta + \rho\, S\,(t)\, Q\,(t) - (\alpha\, E\,(t) + \varphi\, I\,(t))\, S\,(t) - \mu\, S\,(t) \\ \frac{dE}{dt} = \alpha\, S\,(t)\, E\,(t) - (\beta_1 + \beta_2 + \gamma_3)\, E\,(t) - \mu\, E\,(t) \\ \frac{dQ}{dt} = \gamma_3\, E\,(t) - \rho\, S\,(t)\, Q\,(t) - \gamma_4\, Q\,(t) - \mu\, Q\,(t) \\ \frac{dM}{dt} = \beta_1\, E\,(t) + \gamma_4\, Q\,(t) - \gamma_1\, M\,(t) - \mu\, M\,(t) \\ \frac{dA}{dt} = \beta_2\, E\,(t) - \gamma_2\, A\,(t) - \mu\, A\,(t) \\ \frac{dI}{dt} = \gamma_1\, M\,(t) + \gamma_2\, A\,(t) + \varphi\, S\,(t)\, I\,(t) - (\delta + \psi_1 + \psi_2 + \mu)\, I\,(t) \\ \frac{dH}{dt} = \delta\, I\,(t) - (\lambda_1 + \lambda_2 + \mu)\, H\,(t) \\ \frac{dR}{dt} = \lambda_2\, H\,(t) + \psi_1\, I\,(t) - \mu\, R\,(t) \\ \frac{dD}{dt} = \lambda_1\, H\,(t) + \psi_2\, I\,(t) \end{cases} \tag{1}$$

with initial conditions, $S(0) = S_0$, $E(0) = E_0$, $Q(0) = Q_0$, $M(0) = M_0$, $A(0) = A_0$, $I(0) = I_0$, $H(0) = H_0$, $R(0) = R_0$, $D(0) = D_0$.

In model (1), Δ represents the source rate of susceptible individuals and $\rho\, S\,(t)\, Q\,(t)$ is the latent term of the individuals.$\beta_1 E(t)$ and $\beta_1 E(t)$ are the infection terms of the symptomatic and asymptomatic individuals. $\gamma_1 M(t)$, $\gamma_2 A(t)$ and $\varphi\, S\,(t)\, I\,(t)$ are the terms which represent the probabilities of transmission of infections from symptomatic, asymptomatic and susceptible individuals respectively. $\gamma_3 E(t)$ denotes the quarantined term of exposed individuals. The quarantined individuals move to the susceptible and symptomatic individual compartment by terms $\rho\, S\,(t)\, Q\,(t)$ and $\gamma_4 Q(t)$ respectively. $\delta\, I\,(t)$ is the term at which the infected individuals become hospitalized. $\lambda_1\, H\,(t)$ and $\psi_2\, I\,(t)$ are the death terms of hospitalized individuals and infected individuals respectively. $\lambda_2\, H\,(t)$ represents the recovery term of hospitalized individuals and $\psi_1\, I\,(t)$ is the recovery term infected individuals for self-immunity system. The terms $\mu S(t)$, $\mu E(t)$, $\mu Q(t)$, $\mu M(t)$, $\mu A(t)$, $\mu I(t)$, $\mu H(t)$ and $\mu R(t)$ are the natural deaths of susceptible, exposed, quarantined, symptomatic, asymptomatic, infected, hospitalized and recovered individuals respectively.

Herein, we have proposed an optimal control problem based on this mathematical model (1). The following Figure 1 shows the flow chart of the optimal control system:

Taking the Figure 1 into consideration, a dynamical system can be re-constructed in terms of the following set of nonlinear ordinary differential equations (ODEs):

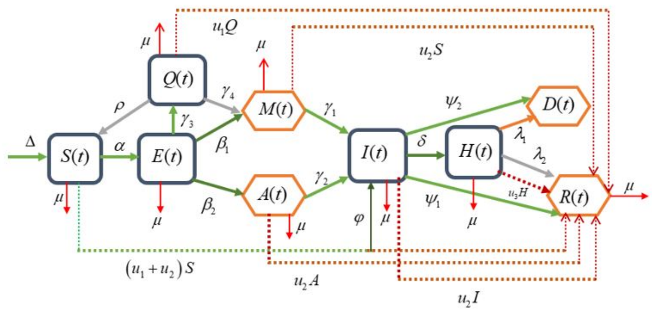

FIGURE 1. Schematic diagram of optimal control system of COVID-19. In the diagram $u_1(t)$, $u_2(t)$, and $u_3(t)$ (shown by orange color with dotted lines) represent the mass testing, physical distancing and treatment control measures respectively.

$$\begin{cases} \frac{dS}{dt} = \Delta + \rho\, S\,(t)\, Q\,(t) - (\alpha\, E\,(t) + \varphi\, I\,(t))\, S\,(t) - \mu\, S\,(t) - (u_1(t) + u_2(t))S(t) \\ \frac{dE}{dt} = \alpha\, S\,(t)\, E\,(t) - (\beta_1 + \beta_2 + \gamma_3)\, E\,(t) - \mu\, E\,(t) \\ \frac{dQ}{dt} = \gamma_3\, E\,(t) - \rho\, S\,(t)\, Q\,(t) - \gamma_4\, Q\,(t) - \mu\, Q\,(t) - u_1 Q(t) \\ \frac{dM}{dt} = \beta_1\, E\,(t) + \gamma_4\, Q\,(t) - \gamma_1\, M\,(t) - \mu\, M\,(t) - u_2(t)M(t) \\ \frac{dA}{dt} = \beta_2\, E\,(t) - \gamma_2\, A\,(t) - \mu\, A\,(t) - u_2(t)A(t) \\ \frac{dI}{dt} = \gamma_1\, M\,(t) + \gamma_2\, A\,(t) + \varphi\, S\,(t)\, I\,(t) - (\delta + \psi_1 + \psi_2 + \mu)\, I\,(t) - u_2(t)I(t) \\ \frac{dH}{dt} = \delta\, I\,(t) - (\lambda_1 + \lambda_2 + \mu)\, H\,(t) - u_3(t)H(t) \\ \frac{dR}{dt} = \lambda_2\, H\,(t) + \psi_1\, I\,(t) - \mu\, R\,(t) + u_1(t)(S(t) + Q(t)) + u_2(S(t) \\ \qquad + M(t) + A(t) + I(t)) + u_3 H(t) \\ \frac{dD}{dt} = \lambda_1\, H\,(t) + \psi_2\, I\,(t) \end{cases} \tag{2}$$

with initial conditions, $S(0) = S_0$, $E(0) = E_0$, $Q(0) = Q_0$, $M(0) = M_0$, $A(0) = A_0$, $I(0) = I_0$, $H(0) = H_0$, $R(0) = R_0$, $D(0) = D_0$.

The exhibited model (2) is an optimal control model and the set of control variables $(u_1(t), u_2(t), u_3(t)) \in U$ is Lebesgue measurable, where

$$U = \{(u_1(t), u_2(t), u_3(t)) : 0 \leq a_i \leq u_i \leq b_i \leq 1\}, \forall t \in [0, t_f].$$

Considering these three control variables, the cost functional of the problem is given by

$$\text{Minimize } J(u) = \int_0^{t_f} (M(t) + A(t) + I(t) + \frac{C_1}{2}{u_1}^2 + \frac{C_2}{2}{u_2}^2 + \frac{C_3}{2}{u_3}^2)dt \tag{3}$$

where C_1, C_2 and C_3 are the weight parameters of the cost functional. This indicates that we aim to minimize the average number of asymptomatic populations, symptomatic populations and infected populations of the model as well as the cost of the three control measures. Hence, with the help of cost functional

(3), we can reformulate model (2) as an optimal control problem in Lagrange form:

$$(P_{OC})\left\{\begin{array}{l}\text{Minimize } J(x,u)=\int_0^{t_f} L(t,x(t),u(t))dt\\ \text{Subject to}\\ \dot{x}=f(x(t))+g(x(t))u(t), \forall t\in[0,t_f]\\ u(t)\in U, \forall t\in[0,t_f]\\ x(0)=x_0\end{array}\right\} \tag{4}$$

where, $x(t)=\begin{bmatrix} S(t)\\ E(t)\\ Q(t)\\ M(t)\\ A(t)\\ I(t)\\ H(t)\\ R(t)\\ D(t)\end{bmatrix}$, $g(x)=\begin{bmatrix} -S & -S & 0\\ 0 & 0 & 0\\ -Q & 0 & 0\\ 0 & M & 0\\ 0 & -A & 0\\ 0 & -I & 0\\ 0 & 0 & -H\\ S+Q & S+M+A+I & H\\ 0 & 0 & 0\end{bmatrix}$,

$$f(x)=\begin{bmatrix} \Delta+\rho\, S\,(t)\, Q\,(t)-(\alpha\, E\,(t)+\varphi\, I\,(t))\, S\,(t)-\mu\, S\,(t)\\ \alpha\, S\,(t)\, E\,(t)-(\beta_1+\beta_2+\gamma_3)\, E\,(t)-\mu\, E\,(t)\\ \gamma_3\, E\,(t)-\rho\, S\,(t)\, Q\,(t)-\gamma_4\, Q\,(t)-\mu\, Q\,(t)\\ \beta_1\, E\,(t)+\gamma_4\, Q\,(t)-\gamma_1\, M\,(t)-\mu\, M\,(t)\\ \beta_2\, E\,(t)-\gamma_2\, A\,(t)-\mu\, A\,(t)\\ \gamma_1\, M\,(t)+\gamma_2\, A\,(t)+\varphi\, S\,(t)\, I\,(t)-(\delta+\psi_1+\psi_2+\mu)\, I\,(t)\\ \delta\, I\,(t)-(\lambda_1+\lambda_2+\mu)\, H\,(t)\\ \lambda_2\, H\,(t)+\psi_1\, I\,(t)-\mu\, R\,(t)\\ \lambda_1\, H\,(t)+\psi_2\, I\,(t)\end{bmatrix},$$

$u(t)=\begin{bmatrix} u_1(t)\\ u_2(t)\\ u_3(t)\end{bmatrix}$ and the integrand of the cost functional is denoted by

$L(t,x,u)=M(t)+A(t)+I(t)+\frac{C_1}{2}{u_1}^2+\frac{C_2}{2}{u_2}^2+\frac{C_3}{2}{u_3}^2$.

3. Characterization of Optimal Control

In the cost functional (3), Pontryagin's Maximum Principle is applied to the Hamiltonian (H) in order to attain the necessary conditions for the optimal control problem (4). According to Pontryagin's Maximum Principle, the standard Hamiltonian function H with respect to $(u_1(t),u_2(t),u_3(t))$ can be defined as follows [20]:

$H(t, x(t), u(t), p(t), \lambda) = \langle p(t), f(x(t)) + g(x(t))u(t)\rangle - \lambda L(x(t), u(t)), \lambda \in \mathbb{R}$
where, $p(t) = (p_S, p_E, p_Q, p_M, p_A, p_I, p_H, p_R, p_D) \in \Re^9$ denotes the adjoint variables.

Let us assume that the pair (x^*, u^*) is the optimal solution of the above optimal control problem (4). Then, the maximum principle states the existence of a scalar parameter $\lambda_0 \geq 0$, an absolutely continuous function $p(t)$, such that the following conditions are satisfied:

(i) $\max\{|p(t)| : t \in [0, t_f]\} + \lambda > 0$;
(ii) $p^{'}(t) = \lambda L_x[t] - \langle p[t], f_x[t] + g_x[t]u^*(t)\rangle$;
(iii) $p(t) = (0, 0, 0, 0, 0, 0, 0, 0, 0)$;
(iv) $H(x^*(t), u^*(t), p(t)) = \max_u\{H(x^*(t), u^*(t), p(t))\}$, here $a_1 \leq u_1 \leq b_1$, $a_2 \leq u_2 \leq b_2, a_3 \leq u_3 \leq b_3$,
where time argument $[t]$ denotes the evaluation along with the optimal solution.

We write $p(t) = (p_S, p_E, p_Q, p_M, p_A, p_I, p_H, p_R, p_D)$, $x^*(t) = (S^*, E^*, Q^*, M^*, A^*, I^*, H^*, R^*, D^*)$, and $u^*(t) = ({u_1}^*, {u_2}^*, {u_3}^*)$. Then, from equation (ii) adjoint equations in normal form ($i.e. \lambda = 1$) are explicitly given by

$p_S{}^{'} = \alpha p_S E^* - \rho p_S Q^* + \varphi p_S I^* + \mu p_S + p_S({u_1}^* + {u_2}^*) - \alpha p_E E^* + \rho p_Q Q^* - \varphi p_I I^* - p_R({u_1}^* + {u_2}^*)$
$= (\alpha E^* - \rho Q^* + \varphi I^* + \mu + {u_1}^* + {u_2}^*)p_S - \alpha p_E E^* + \rho p_Q Q^* - \varphi p_I I^* - p_R({u_1}^* + {u_2}^*)$
$p_E{}^{'} = \alpha p_S S^* - \alpha p_E S^* + p_E(\beta_1 + \beta_2 + \gamma_3) + \mu p_E - \gamma_3 p_Q - \beta_1 p_M - \beta_2 p_A$
$= \alpha p_S S^* + p_E(\beta_1 + \beta_2 + \gamma_3 - \alpha S^*) + \mu p_E - \gamma_3 p_Q - \beta_1 p_M - \beta_2 p_A$
$p_Q{}^{'} = \rho p_Q S^* - \rho p_S S^* + \gamma_4 p_Q + \mu p_Q - \gamma_4 p_M - {u_1}^* p_Q$
$= (\rho S^* + \gamma_4 + \mu - {u_1}^*)p_Q - \rho p_S S^* - \gamma_4 p_M$
$p_M{}^{'} = 1 + \gamma_1 p_M + \mu p_M - \gamma_1 p_I + {u_2}^* p_M$
$= 1 + (\gamma_1 + \mu + {u_2}^*)p_M - \gamma_1 p_I$
$p_A{}^{'} = 1 + \gamma_2 p_A + \mu p_A - \gamma_2 p_I + {u_2}^* p_A$
$= 1 + (\gamma_2 + \mu + {u_2}^*)p_A - \gamma_2 p_I$
$p_I{}^{'} = 1 + \varphi p_S S^* - \varphi p_I S^* + p_I(\delta + \psi_1 + \psi_2) + \mu p_I - \delta p_H - \psi_1 p_M - \psi_2 p_D + {u_2}^* p_I$
$= 1 + \varphi p_S S^* + p_I(\delta + \psi_1 + \psi_2 + \mu + {u_2}^* - \varphi S^*) - \delta p_H - \psi_1 p_M - \psi_2 p_D$
$p_H{}^{'} = p_S(\lambda_1 + \lambda_2 + \mu + {u_3}^*) - \lambda_2 p_R - \lambda_1 p_D$
$p_R{}^{'} = \mu p_R$ with transversality condition $p_i(t_f) = 0, i = 1, 2, 3, 4, 5, 6, 7, 8, 9.$

3.1. Existence of the Optimal Controls. In the present model, (x^*, u^*) is the optimal pair where x^* denotes the state variables and u^* represents control variables. So, in order to prove the existence of the optimal control, we have to show the existence of the state as well as the existence of the control variables [20].

3.1.1. Existence of the State Variables. The state equations in model (1) with the initial conditions can be written in the following form

$$S^{'} = \Delta + \rho\, S\,(t)\, Q\,(t) - (\alpha\, E\,(t) + \varphi\, I\,(t))\, S\,(t) - \mu\, S\,(t) + (0)\, M\,(t) + (0)\, A\,(t) + (0)\, I\,(t) + (0)\, H\,(t) + (0)\, R\,(t) + (0)\, D\,(t) \quad (5)$$
$$E^{'} = \alpha\, S\,(t)\, E\,(t) - (\beta_1 + \beta_2 + \gamma_3)\, E\,(t) - \mu\, E\,(t) + (0)\, Q\,(t) + (0)\, M\,(t)$$

$$+ (0)\,A\,(t) + (0)\,I\,(t) + (0)\,H\,(t) + (0)\,R\,(t) + (0)\,D\,(t)$$

$$Q^{'} = (0)\,S\,(t) + \gamma_3\,E\,(t) - \rho\,S\,(t)\,Q\,(t) - \gamma_4\,Q\,(t) - \mu\,Q\,(t) + (0)\,M\,(t)$$

$$+ (0)\,A\,(t) + (0)\,I\,(t) + (0)\,H\,(t) + (0)\,R\,(t) + (0)\,D\,(t)$$

$$M^{'} = (0)\,S\,(t) + \beta_1\,E\,(t) + \gamma_4\,Q\,(t) - \gamma_1\,M\,(t) - \mu\,M\,(t) + (0)\,A\,(t)$$

$$+ (0)\,I\,(t) + (0)\,H\,(t) + (0)\,R\,(t) + (0)\,D\,(t)$$

$$A^{'} = (0)\,S\,(t) + \beta_2\,E\,(t) + (0)\,Q\,(t) + (0)\,M\,(t) - \gamma_2\,A\,(t) - \mu\,A\,(t)$$

$$+ (0)\,I\,(t) + (0)\,H\,(t) + (0)\,R\,(t) + (0)\,D\,(t)$$

$$I^{'} = \gamma_1\,M\,(t) + (0)\,E\,(t) + (0)\,Q\,(t) + \gamma_2\,A\,(t) + \varphi\,S\,(t)\,I\,(t)$$

$$- (\delta + \psi_1 + \psi_2 + \mu)\,I\,(t) + (0)\,H\,(t) + (0)\,R\,(t) + (0)\,D\,(t)$$

$$H^{'} = (0)\,S\,(t) + (0)\,E\,(t) + (0)\,Q\,(t) + (0)\,M\,(t) + (0)\,A\,(t) + \delta\,I\,(t)$$

$$- (\lambda_1 + \lambda_2 + \mu)\,H\,(t) + (0)\,R\,(t) + (0)\,D\,(t)$$

$$R^{'} = (0)\,S\,(t) + (0)\,E\,(t) + (0)\,Q\,(t) + (0)\,M\,(t) + (0)\,A\,(t) + \lambda_2\,H\,(t)$$

$$+ \psi_1\,I\,(t) - \mu\,R\,(t) + (0)\,D\,(t)$$

$$D^{'} = (0)\,S\,(t) + (0)\,E\,(t) + (0)\,Q\,(t) + (0)\,M\,(t) + (0)\,A\,(t) + \lambda_1\,H\,(t)$$

$$+ \psi_2\,I\,(t) + (0)\,R\,(t) + (0)\,D\,(t)\,.$$

Let $N(t) = S(t) + E(t) + Q(t) + M(t) + A(t) + I(t) + H(t) + R(t) + D(t)$
So that,

$$N^{'}(t) = S^{'}(t) + E^{'}(t) + Q^{'}(t) + M^{'}(t)A^{'}(t)I^{'}(t) + H^{'}(t) + R^{'}(t) + D^{'}(t) \quad (6)$$

Now from all the above equations in (5) and equation (6), we can write
$N^{'}(t) = \Delta - \mu N(t) + (\gamma_1 - \gamma_3)M(t)$
$\implies N^{'}(t) \leq \Delta - \mu N(t)$
$\therefore N(t) \leq \frac{\Delta}{\mu} + (N_0 - \frac{\Delta}{\mu})e^{-\mu t}$.
Here, $N(t)$ is the total number of populations.

So we have, $N(t) \leq \frac{\Delta}{\mu} + (N_0 - \frac{\Delta}{\mu})e^{-\mu t} = V_1 \in \mathbb{R}_+$and $\lim_{t\to\infty} \sup N(t) \leq V_1$ which gives, $(S(t), E(t), Q(t), M(t), A(t), I(t), H(t), R(t), D(t)) \leq V_1,$ as $t \to \infty$. Then, we can rewrite equation (5) in the following form:

$$\phi_t = G\phi + F(\phi) \quad (7)$$

where $\phi = \begin{bmatrix} S(t) \\ E(t) \\ Q(t) \\ M(t) \\ A(t) \\ I(t) \\ H(t) \\ R(t) \\ D(t) \end{bmatrix}$, $\phi_t = \begin{bmatrix} S'(t) \\ E'(t) \\ Q'(t) \\ M'(t) \\ A'(t) \\ I'(t) \\ H'(t) \\ R'(t) \\ D'(t) \end{bmatrix}$, $F(\phi) = \begin{bmatrix} \rho SQ - (\alpha E + \varphi I)S \\ \alpha SE \\ -\rho SQ \\ 0 \\ 0 \\ \varphi IS \\ 0 \\ 0 \\ 0 \end{bmatrix}$, and

$$G = \begin{bmatrix} -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_3 & k_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_1 & \gamma_4 & k_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & k_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_1 & \gamma_2 & k_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta & k_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_1 & \lambda_2 & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_2 & \lambda_1 & 0 & 0 \end{bmatrix}$$

where, $k_2 = -(\beta_1+\beta_2+\gamma_3+\mu)$, $k_3 = -(\gamma_4+\mu)$, $k_4 = -(\gamma_1+\mu)$, $k_5 = -(\gamma_2+\mu)$, $k_6 = -(\delta + \psi_1 + \psi_2 + \mu)$, $k_7 = -(\lambda_1 + \lambda_2 + \mu)$.

Now,

$$F(\phi_1) - F(\phi_2) = \begin{bmatrix} \rho S_1 Q_1 - (\alpha E_1 + \varphi I_1)S_1 \\ \alpha S_1 E_1 \\ -\rho S_1 Q_1 \\ 0 \\ 0 \\ \varphi I_1 S_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \rho S_2 Q_2 - (\alpha E_2 + \varphi I_2)S_2 \\ \alpha S_2 E_2 \\ -\rho S_2 Q_2 \\ 0 \\ 0 \\ \varphi I_2 S_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

Equation (7) is a non-linear form with a bounded co-efficient.
We consider $D(\phi) = \phi_t = G\phi + F(\phi)$. For the existence of optimal control and

optimality system, the boundedness of solution of the system for finite time is needed and we assume for $u \in U$, there exists a bounded solution.

$|F(\phi_1) - F(\phi_2)| = |\rho(S_1Q_1 - S_2Q_2) + \alpha(E_2S_2 - E_1S_1) + \phi(I_2S_2 - I_1S_1) +$
$|\alpha(E_1S_1 - E_2S_2) + \rho(S_1Q_1 - S_2Q_2) + \phi(I_1S_1 - I_2S_2)|$
$\leq 2\rho|(S_1Q_1 - S_2Q_2)| + 2\alpha|(E_2S_2 - E_1S_1|) + 2\phi|(I_2S_2 - I_1S_1)|$
$\leq M|\phi_1 - \phi_2|$
where, M is a constant. Also, we get
$|D(\phi_1) - D(\phi_2)| \leq ||B|||\phi_1 - \phi_2| + M|\phi_1 - \phi_2| \leq V|\phi_1 - \phi_2|$.
where, $V = max(M, ||B||) < \infty$.
Thus, it follows that the function D is uniformly Lipschitz continuous.
From the definition of the control $U(t)$ and the restriction on S, E, Q, M, A, I and $D > 0$, we see that asolution of the system (7) exists.

3.1.2. Existence of the Control Variables. Now, by applying Pontryagin's Maximum Principle [39] we have the following Theorem 3.1 and by proving the theorem, we show the existence of controls.

Theorem 3.1. *There exists optimal control* $(u_1{}^*, u_2{}^*, u_3{}^*)$ *that minimizes the performance index* $J(x, u)$ *over* U *given by*

$$u_1{}^* = \max_{[0,t_f]}\left\{0, min\left(1, \frac{(P_S - P_R)S^* + P_QQ^*}{C_1}\right)\right\},$$

$$u_2{}^* = \max_{[0,t_f]}\left\{0, min\left(1, \frac{(P_S-P_R)S^*+(P_M-P_R)M^*+(P_A-P_R)A^*+(P_I-P_R)I^*}{C_2}\right)\right\}$$

$$and\ u_3{}^* = \max_{[0,t_f]}\left\{0, min\left(1, \frac{(P_H - P_R)H^*}{C_3}\right)\right\}$$

Proof. According to optimality conditions, we have

$$\frac{\partial H}{\partial u_1^*} = C_1u_1^* - P_SS^* - P_QQ^* + P_RS^* = 0 \implies u_1^* = \frac{(P_S - P_R)S^* + P_QQ^*}{C_1} = \overline{u_1}$$

$$\frac{\partial H}{\partial u_2^*} = C_2u_2^* - P_SS^* - P_MM^* - P_AA^* - P_II^* + P_R(S^* + M^* + A^* + I^*) = 0$$

$$\implies u_2^* = \frac{(P_S - P_R)S^* + (P_M - P_R)M^*(P_A - P_R)A^*(P_I - P_R)I^*}{C_2} = \overline{u_2}$$

$$\frac{\partial H}{\partial u_3^*} = C_3u_3^* - P_HH^* - P_RH^* = 0 \implies u_3^* = \frac{(P_H - P_R)H^*}{C_3} = \overline{u_3}$$

According to the property of U, the three controls $(u_1^*, u_2{}^*, u_3{}^*)$ are bounded with upper bound 1 and lower bound 0.

$$u_1^*(t) = \begin{cases} 0 & \text{if } u_1^* \leq 0 \\ \frac{(P_S-P_R)S^*+P_QQ^*}{C_1} & \text{if } 0 < u_1^* \leq 1 \\ 1 & \text{if } u_1^* \geq 1 \end{cases}$$

This can be written in compact form as
$u_1^*(t) = \max_{[0,t_f]} \left\{0, min\left(1, \frac{(P_S-P_R)S^*+P_QQ^*}{C_1}\right)\right\}.$

Similarly,

$$u_2^*(t) = \begin{cases} 0 & \text{if } u_2^* \leq 0 \\ \frac{(P_S-P_R)S^*+(P_M-P_R)M^*(P_A-P_R)A^*(P_I-P_R)I^*}{C_2} & \text{if } 0 < u_2^* \leq 1 \\ 1 & \text{if } u_2^* \geq 1 \end{cases}$$

In the same way, this can be written in compact form as
$u_2^*(t) =$
$\max_{[0,t_f]} \left\{0, min\left(1, \frac{(P_S-P_R)S^*+(P_M-P_R)M^*+(P_A-P_R)A^*+(P_I-P_R)I^*}{C_2}\right)\right\}$
Again,

$$u_3^*(t) = \begin{cases} 0 & \text{if } u_3^* \leq 0 \\ \frac{(P_H-P_R)H^*}{C_3} & \text{if } 0 < u_3^* \leq 1 \\ 1 & \text{if } u_3^* \geq 1 \end{cases}$$

which can be written in compact form as

$$u_3^*(t) = \max_{[0,t_f]} \left\{0, min\left(1, \frac{(P_H-P_R)H^*}{C_3}\right)\right\}.$$

Thus, we get the optimal solutions as $(u_1^*, u_2^*, u_3^*) =$

$$\begin{pmatrix} \max_{[0,t_f]} \left\{0, min\left(1, \frac{(P_S-P_R)S^*+P_QQ^*}{C_1}\right)\right\} \\ \max_{[0,t_f]} \left\{0, min\left(1, \frac{(P_S-P_R)S^*+(P_M-P_R)M^*+(P_A-P_R)A^*+(P_I-P_R)I^*}{C_2}\right)\right\} \\ \max_{[0,t_f]} \left\{0, min\left(1, \frac{(P_H-P_R)H^*}{C_3}\right)\right\} \end{pmatrix}$$

This completes the proof of the Theorem 3.1. □

4. Numerical Simulations

In this section, we have used Open-OCL [28] solver to perform numerical simulations of the optimal control model (1) in MATLAB programming language. In order to carry out numerical solutions of the model, we use a set of parameter values which are shown in Table 1. We perform numerical simulations to compare the results of our model with the real data obtained from several reports specifically Worldometer [16].

We use a set of suitable parameter values. The description of all the parameters with the estimated values used in the model (2) is presented in Table 1. We have considered the initial conditions $S_0 = 100 \times 10^5$, $E_0 = 50 \times 10^5$, $Q_0 = 60 \times 10^5$, $M_0 = 70 \times 10^4$, $A_0 = 40 \times 10^4$, $I_0 = 60 \times 10^4$ $H_0 = 20 \times 10^4$, $R_0 = 40 \times 10^4$ and $D_0 = 5 \times 10^3$. Firstly, we simulate the model (2) considering the initial values and all other parameters that are shown in Table 1.

TABLE 1. Description and estimation of the parameters of model (1)

Symbols	Descriptions	Values	Units	Sources
Δ	Recruitment rate of the susceptible individuals	0.0185	day^{-1}	[20, 22]
α	Exposed rate of the individuals	0.153	day^{-1}	[22]
β_1	Effective rate of exposed becoming symptomatic	0.138	Dimensionless	[22]
β_2	Effective rate of exposed becoming asymptomatic	0.013	Dimensionless	[46]
γ_1	Probability of being infected from symptomatic	0.025	day^{-1}	[48]
γ_2	Probability of being infected from asymptomatic	0.015	day^{-1}	[6]
φ	Probability of being infected from susceptible	1.56	day^{-1}	[6]
γ_3	Quarantined rate from exposed individuals	0.25	Dimensionless	[16, 17]
γ_4	Effective rate from quarantined to symptomatic	0.02	day^{-1}	[16, 17]
ρ	Effective rate from quarantined to susceptible	0.025	day^{-1}	[20, 22]
δ	Hospitalized rate of the infected individuals	0.4127	day^{-1}	[46, 48]
λ_1	Death rate of the hospitalized individuals	0.0427	day^{-1}	[46, 48]
λ_2	Rate of recovery from hospitalized individuals	0.8971	day^{-1}	[6]
ψ_1	Effective recovery rare using self-immunity system	0.5887	day^{-1}	[46, 48]
ψ_2	Death rate of the infected individuals	0.0412	day^{-1}	[48]
μ	Natural death rate	0.0078	day^{-1}	[48]

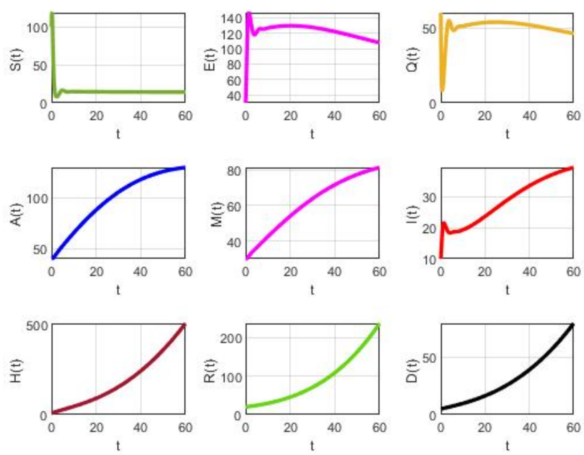

FIGURE 2. Numerical simulation of the model (2) when no control measures are applied to the system.

Also, we have performed the numerical simulations for time interval $t \in [0, 120]$ for 120 days. For convenient, on t-axis, we consider 1 unit is equivalent to 6 days in Figures 2-5 and 1 unit is equivalent to 2 days in Figures 6-11. We have considered three control variables: $u_1(t)$ as the control of mass testing and tracing or identifying the COVID-19 positive patients, $u_2(t)$ as the control of maintaining physical distancing and $u_3(t)$ as the control of effective treatment for corresponding symptoms. The result of simulation of the combined classes without optimal control model (2) is presented in Figure 2.

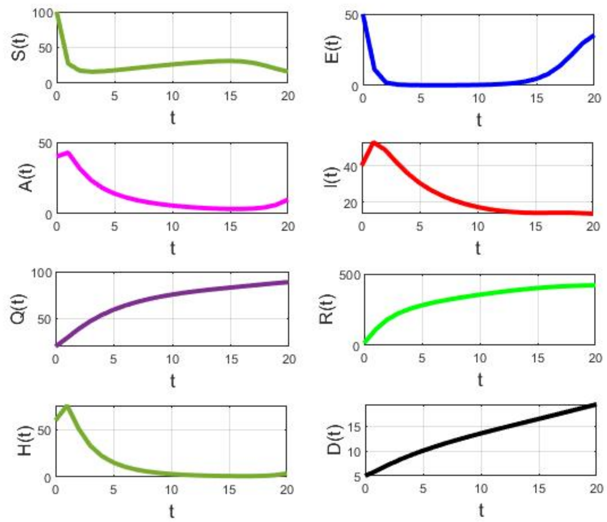

FIGURE 3. Numerical simulation of susceptible, symptomatic, asymptomatic, infected, hospitalized and recovered individuals when both the three control measures are activated.

We observe from Figure 2 that when no optimal controls are applied to the system then symptomatic, asymptomatic, infected and hospitalized individuals are increased. As a result, the number of death people is also increased due to this pandemic outbreak. Considering the optimal controls into account, the behavior of the state trajectories is simulated in the Figure 3 and the control trajectories in Figure 4.

We observe from Figures 3 and 4 that when optimal controls are applied to the system then infected individuals are decreased whereas recovered individual is increased tremendously till the end of the period, at the same time the overall cost of all the controls u_1, u_2, u_3 are minimized. The model (2) shows the significant result for considering the asymptomatic individuals which have remarkably influenced on the spread of COVID-19. After employing the optimal controls (i.e. mass testing and tracing among the individuals and maintaining the physical distancing as well as wearing mask, the asymptomatic individuals are decreased surprisingly as a result recovered individuals are also increased from this pandemic outbreak. At the same time the cost of the management of mass testing system and cost of buying mask is minimized. On the other hand, the cost of the control u_3 (i.e. treatment for corresponding symptoms and taking immune boosting foods and drugs to develop self-immunity system) is maximum

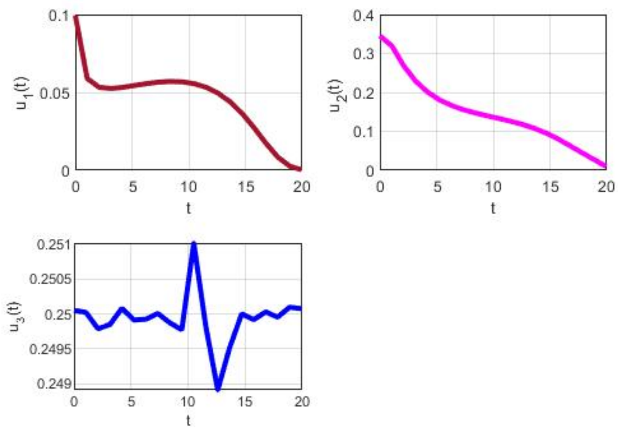

Figure 4. Illustration of the three control measures.

at the beginning of the period and then decreased after day by day that is our objective to minimize the cost of the controls.

Now, we solve the model numerically for the classes of susceptible, infected and recovered individuals to show how the changes in these states due to applying mass testing control. The result in this case is presented in Figure 5.

Figure 5 shows the state trajectories of susceptible, asymptomatic, infected and recovered individuals in the present of mass testing and tracing (u_1) and maintaining physical distancing (u_2) as optimal control. We have observed that from the beginning period the number of infected people is increasing but after performing mass testing and tracing, it leads to quick identification of COVID-19 and immediate isolation to prevent spread. It is also helped to trace anyone who came into contact with infected individuals and quickly quarantined. We also investigate that the infected population is extremely decreased due to decay of asymptomatic individuals. An asymptomatic positive patient does not exhibit the symptoms of COVID-19 outbreak but can transmit the virus to others susceptible individuals rapidly. As a result, the symptomatic and infected population is extensively diminished and recovered individuals are increased that World Health Organization (WHO) has highlighted the eventual significant of mass testing. They suggest that three things are important: tracing positive patient, identifying their household and identifying the people who are contacted and quarantining them minimum fourteen days. The cost of the management of tracing software or mass testing system is the largest value at the beginning period and then it is remained constant. After implementing the tracing and testing, the cost of the control (u_1) is decreased till at the end of the period. Due to managing the cost of maintaining physical distancing and wearing mask,

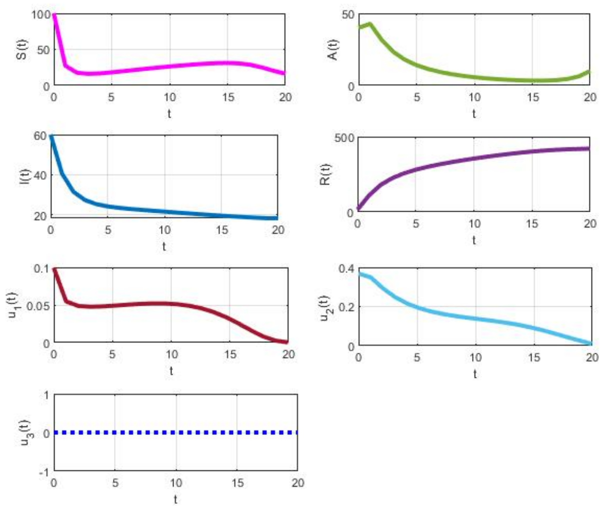

FIGURE 5. The asymptomatic and infected individuals are significantly decreased and recovered individuals are increased due to maintain physical distance and applying mass testing $(u_1 \neq 0, u_2 \neq 0, u_3 = 0)$ as optimal control.

the cost of the control (u_2) is initially boomed but gradually reduced day by day.

Then we run the program for the infected and recovered individuals to show the effect of quarantined individuals, keeping the parameters value same as before. The result is shown in Figure 6.

In Figure 6, we see the variation of symptomatic, infected and recovered individuals with time due to applying mass testing (u_1) as optimal control. It is easy to understand that the infected individual is significantly decreased due to increase of quarantined rate. As a result, recovered individuals are increased extensively. Thus, after the implementation of mass testing or tracing (u_1) and isolated the COVID-19 positive patient, the symptomatic individuals are extensively decreased as a result infected individuals are also decreased and recovered individuals are increased surprisingly.

Now, we solve the program to see the variation for the susceptible class after using optimal control (u_1). The result is shown in Figure 7.

Figure 7 shows the optimal trajectories of the susceptible individuals with time. We observe that the susceptible individuals are extremely decreased due to apply the optimal control mass testing and tracing to the community. Next, we solve the model to see the variation for the symptomatic individuals after

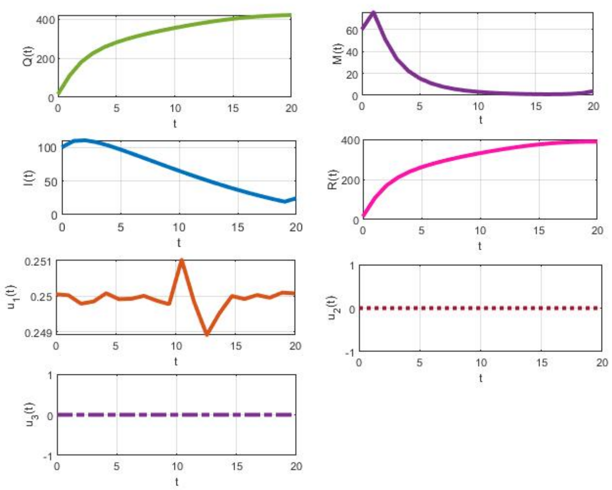

FIGURE 6. Dynamics of quarantined, symptomatic, infected and recovered individuals where the symptomatic individuals are significantly decreased as a result recovered individuals are increased surprisingly due to applying optimal controls u_1 whereas $u_2 = 0, u_3 = 0$.

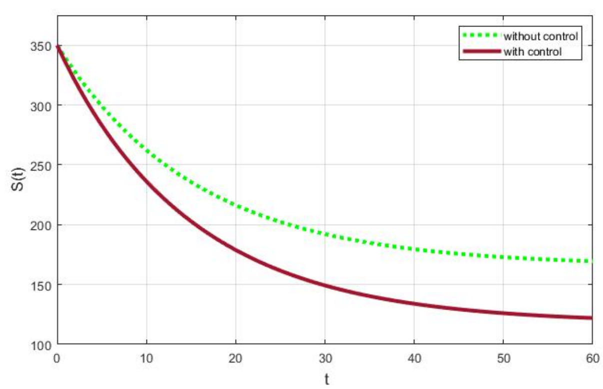

FIGURE 7. The susceptible individuals are dramatically decreased due to apply the optimal control mass testing and tracing (u_1) .

using optimal control (u_2), keeping the parameters value same as before. The result is presented in Figure 8.

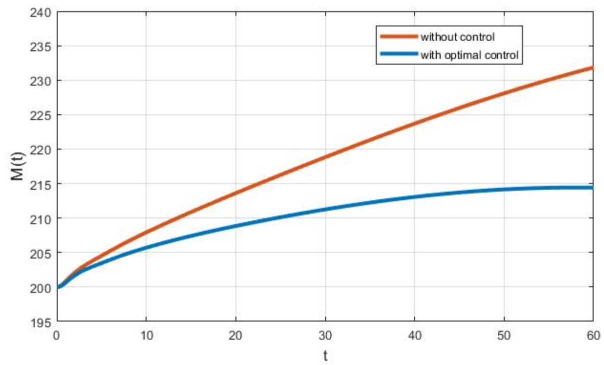

Figure 8. Variation of symptomatic individuals by using optimal control (u_2) where the symptomatic individuals is significantly decreased due to maintaining social distancing.

Figure 8 represents the variation of symptomatic individuals with time due to maintain physical and social distance and other effects are not considered. We observe that the symptomatic population is extremely decreased after applying (u_2) as optimal control. Because, we all are known that coronavirus is primarily spread between people during close contact, often via small droplets produced by coughing, sneezing, or talking. These small droplets have transmitted when a person is in close contact (within 1 m) with someone who has respiratory symptoms. Thus, the symptomatic population is significantly decreased as a result recovered individuals increased due to maintain physical and social distance at least one meter from COVID-19 positive people as a protective measure.

Now, we run the program to see the variation for the asymptomatic class after using optimal control u_1 and u_2. The result is shown in Figure 9.

We understand easily from Figure 9 that the variation of asymptomatic individuals with time due to apply mass testing and maintaining physical and social distance. We observe that the asymptomatic population is extremely decreased after applying u_1 and u_2 as optimal control. Because an asymptomatic positive patient does not exhibit the symptoms of COVID-19 outbreak but can transmit the virus to others rapidly. Thus, after identifying the asymptomatic individuals through mass testing and tracing who've contacted with them and maintaining physical and social distance, the asymptomatic population is extensively decreased as a result infected population is also decreased and recovered individuals are increased tremendously.

Finally, we run the program to see the variation for the infected individuals after using optimal control u_2, keeping the parameters value same as before. The result in this case is given in Figure 10.

From Figure 10, we observe the variation of infected individuals with time due to maintain physical and social distance and other effects are not considered.

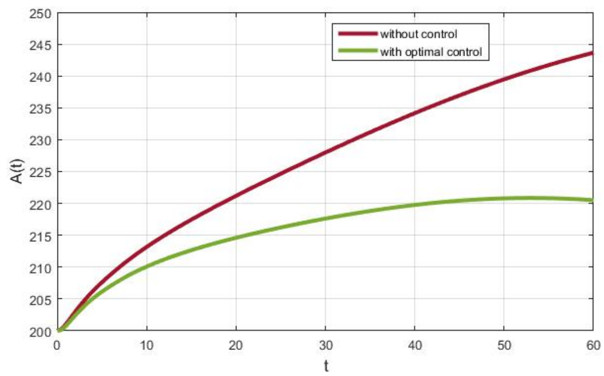

FIGURE 9. Variation of asymptomatic individuals by using optimal control where the asymptomatic population is significantly decreased due to apply mass testing and maintaining social distancing ($u_1 and u_2$).

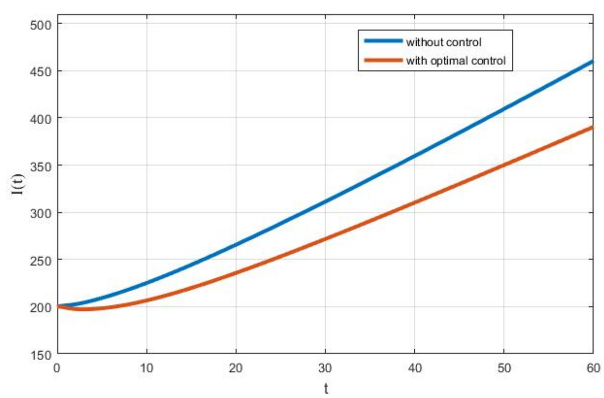

FIGURE 10. Variation of infected individuals by using optimal control where the infected population is significantly decreased due to maintaining social distancing.

At present, there are no specific vaccines are invented for COVID-19. If we controlled and reduced the symptomatic and asymptomatic population from this pandemic situation through maintaining physical distance as well as social distance from the sick people who has respiratory symptoms and spread small droplets produced by coughing, sneezing or talking as a preventive measure then the infected population is significantly decreased as a result number of recovered individuals also increased surprisingly from this outbreak.

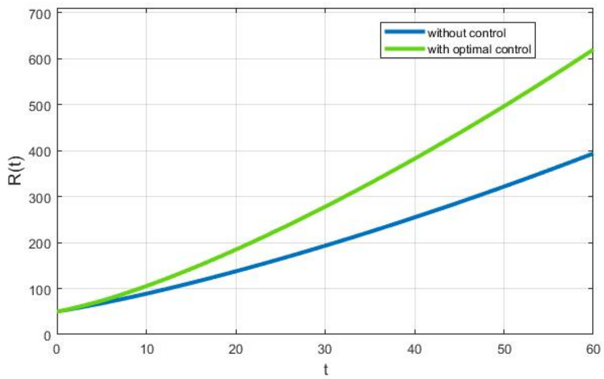

FIGURE 11. Variation of recovered individuals by using optimal control where the recovered individuals are increased surprisingly due to apply the vaccine or treatment (u_3) as optimal control.

Next, we solve the model for the class of hospitalized and recovered individuals to show how the change occurs in the recovered individuals using treatment as optimal control. The result in this case is given in Figure 11.

From Figure 11, it is observed that the hospitalized population is decreased due to applying treatment based on the patient's clinical condition and developing self-immunity system. There are no specific vaccines or medicines for COVID-19 still nowadays, so, to reduce the infected individuals from this outbreak and increased recovered individuals, self-immunity system must be developed for all the population of a community and the symptoms can be treated and treatment through clinical trials. For that reason, it is mandatory for each of the individuals to develop a strong immune system through indoor and outdoor activities, by trying muscle strength training, by eating a diet high in fruits and vegetables with minimizing the consumption of red and processed meats.

5. Conclusions

In this contribution, a mathematical model on transmission dynamics of COVID-19 is presented introducing three control variables such as mass testing and tracing of COVID-19 positive patients, maintenance of physical distancing and effective treatment for corresponding symptoms with taking immune boosting foods and drugs to develop self-immunity system. We have used Pontryagin maximum principle for the existence of the state variables, objective functional and characterization of the optimal control to minimize the number of symptomatic, asymptomatic and infected individuals with minimum cost of the controls. The major findings of this study are given below:

- When no preventive control measure is applied, symptomatic, asymptomatic and infected individuals are continuously increased due to this highly infectious pandemic disease.
- After implementation of the optimal control $u_1(t)$ (mass testing and tracing) in the community, the susceptible, exposed and infected population is extensively decreased as a result recovered individuals are increased. Moreover, WHO has encouraged to all of the people for mass testing and tracing of COVID-19 positive patients, at the same time identifying their household and quarantined them.
- When the optimal control $u_2(t)$ (maintenance of physical distancing and wearing face mask) is applied, the infected individuals are significantly decreased due to increase of maintaining physical distancing and wearing face mask with high quarantined rate. As a result, recovered individuals are increased extensively. Since an asymptomatic (but COVID-19 positive patient) individual does not show the symptoms of corona transmission but can spread the virus to others that leads to increase the infected individuals rapidly. Hence after using the optimal control $u_2(t)$, the asymptomatic individuals are extensively decreased as a result the infected individuals are also decreased so that recovered individuals are increased quickly.

After execution the optimal control $u_3(t)$ (efficacious treatment for corresponding symptoms), the hospitalized individuals are decreased with the increase of recovered individuals. In this pandemic situation, it is mandatory for each of the individuals to develop a strong immune system through indoor and outdoor activities, by trying muscle strength training, by eating fruits and vegetables. Our study (the dynamics of the optimal control in Figures 2-4) also ensure that the overall cost of the optimal control is minimized with the reduction of infected individuals.

Conflicts of Interest : The authors declare that there are no conflicts of interest regarding the publication of this manuscript.

Data Availability : The data used to support the findings of this study are included within the article.

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M.H.A. Biswas received M.Sc. from University of Chittagong, M.Phil. from University of Rajshahi, Bangladesh and Ph.D. at University of Porto, Portugal. Since 2002 he has been at Khulna University, Bangladesh and He is currently a Professor at the same university. His research interests include optimal control, dynamic optimization, mathematical modeling, mathematical biology and epidemiology of infectious diseases. Prof. Biswas has been elected as the Vice-president of Bangladesh Mathematical Society for the year 2022-2023. Professor Biswas has been serving as the founding President of Bangladesh Society for Mathematical Biology since 2020.

Mathematics Discipline, Khulna University, Khulna-9208, Bangladesh.
e-mail: mhabiswas@yahoo.com

M.S. Khatun received B.Sc. (Honors) in Mathematics from Khulna University, Bangladesh, and M.Sc. in Applied Mathematics from the same University. Her research interests are optimal control, mathematical modeling, mathematical biology and epidemiology of infectious diseases.

Mathematics Discipline, Khulna University, Khulna-9208, Bangladesh.
e-mail: shantajsku@gmail.com

M.A. Islam received B.Sc. (Honors) in Mathematics from Khulna University, Bangladesh, and M.Sc. in Applied Mathematics from the same University. His research interests include optimal control, mathematical modeling, mathematical biology and epidemiology of infectious diseases.

Mathematics Discipline, Khulna University, Khulna-9208, Bangladesh.
e-mail: anarul1996@gmail.com

S. Mandal received B.Sc. (Honors) in Mathematics from Bangabandhu Sheikh Mujibur Rahman Science and Technology University, Gopalganj, Bangladesh, and M.Sc. in Applied Mathematics from the same University. His research interests include optimal control,

mathematical modeling, mathematical ecology, environmental modeling, climate change and epidemiology of infectious diseases.

Department of Mathematics, Bangabandhu Sheikh Mujibur Rahman Science and Technology University, Gopalganj-8100, Bangladesh.
e-mail: sajibmandal1997@gmail.com

A.K. Paul received B.Sc. (Honors) in Mathematics from Khulna University, Bangladesh, and he is currently pursuing M.Sc. in Applied Mathematics from the same University. His research interests include optimal control, mathematical modeling, mathematical biology and epidemiology of infectious diseases.

Mathematics Discipline, Khulna University, Khulna-9208, Bangladesh.
e-mail: arindam017@gmail.com

A. Ali is currently an associate professor at Lawrence Technological University, Southfield, Michigan, USA. His research interests are Smart Manufacturing and Lean Systems, modeling, simulation and optimization. Dr. Ali is serving as the Executive Director of IEOM Society International.

Department of Mechanical, Robotics and Industrial Engineering, Lawrence Technological University, Southfield, Michigan, MI 48075, USA.
e-mail: sali@ltu.edu