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REPRESENTATION OF SOLUTIONS OF A SYSTEM OF FIVE-ORDER NONLINEAR DIFFERENCE EQUATIONS

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Abstract. In this paper, we deal with the existence of solutions of the following system of nonlinear rational difference equations with order five

$$x_{n+1} = \frac{y_{n-3}x_{n-4}}{y_n(a + by_{n-3}x_{n-4})}, \quad y_{n+1} = \frac{x_{n-3}y_{n-4}}{x_n(c + dx_{n-3}y_{n-4})}, \quad n = 0, 1, \cdots ,$$

where parameters a, b, c and d are not executed at the same time and initial conditions $x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_0, y_{-4}, y_{-3}, y_{-2}, y_{-1}$ and y_0 are non zero real numbers.

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1. Introduction

The theory of difference equations developed greatly during the last twenty-five years of the twentieth century. The applications of the theory of difference equations is rapidly increasing to various fields such as numerical analysis, economics,biology, control theory, finite computer science and mathematics.
Thus, there is every reason for studying the theory of difference equations as a well deserved discipline.
The are many papers related to the difference equations systems for example, solvability of a systems of nonlinear difference equations of higher order .

$$x_n = \frac{x_{n-k}y_{n-k-l}}{y_{n-l}(a_n + b_nx_{n-k}y_{n-k-l})}, \quad y_n = \frac{y_{n-k}x_{n-k-l}}{x_{n-l}(\alpha_n + \beta_ny_{n-k}x_{n-k-l})}, \tag{1}$$

has been studied by Kara et al. in [21].
El-Dessoky et. al [10] has studied the following systems of difference equations

$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n(\pm 1 \pm x_{n-3}y_{n-4})}, \quad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n(\pm 1 \pm y_{n-3}x_{n-4})}. \tag{2}$$

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Stević et al [33] have got the solutions of the equation

$$x_n = \frac{x_{n-2}x_{n-k-2}}{x_{n-k}(a_n + b_n x_{n-2}x_{n-k-2})}, \tag{3}$$

Elsayed et al [8] found Periodicity and solutions for some systems of nonlinear rational difference equations

$$x_{n+1} = \frac{x_{n-2}y_{n-1}}{y_n(\pm 1 \pm x_{n-2}y_{n-1})}, \quad y_{n+1} = \frac{y_{n-2}x_{n-1}}{x_n(\pm 1 \pm y_{n-2}x_{n-1})}. \tag{4}$$

Yazlik and kara [36] gave the solution of the following systems of difference equations

$$x_n = \frac{x_{n-4}y_{n-5}}{y_{n-1}(a_n + b_n x_{n-2}y_{n-3}x_{n-4}y_{n-5})}, \quad y_n = \frac{y_{n-4}x_{n-5}}{x_{n-1}(\alpha_n + \beta_n y_{n-2}x_{n-3}y_{n-4}x_{n-5})}. \tag{5}$$

Similar nonlinear systems of rational difference equations were studied [7], [10], [30], [20],[21].

Motivated by the above mentioned papers in this paper , we show that we are able to express in a closed form the well defined solutions of the following system of difference equations

$$x_{n+1} = \frac{y_{n-3}x_{n-4}}{y_n(a + by_{n-3}x_{n-4})}, \quad y_{n+1} = \frac{x_{n-3}y_{n-4}}{x_n(c + dx_{n-3}y_{n-4})}.$$

where $n \in \mathbb{N}_0$, a, b, c, d and initial values$x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_0, y_{-4}, y_{-3}, y_{-2}, y_{-1}$ and y_0 are nonzero real numbers.

2. Main results

In this section, we investigate the solutions of the system of difference equations

$$x_{n+1} = \frac{y_{n-3}x_{n-4}}{y_n(a + by_{n-3}x_{n-4})}, \quad y_{n+1} = \frac{x_{n-3}y_{n-4}}{x_n(c + dx_{n-3}y_{n-4})}. \tag{6}$$

where $n \in \mathbb{N}_0$ and the initial conditions $x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_0, y_{-4}, y_{-3}, y_{-2}, y_{-1}$ and y_0 are arbitrary non zero real numbers.
So, the system (6) can be written as the following system

$$u_{n+1} = \frac{v_{n-3}}{a + bv_{n-3}}, \quad v_{n+1} = \frac{u_{n-3}}{c + du_{n-3}}, \tag{7}$$

Using the following change of variables

$$\begin{cases} u_n = x_n y_{n-1}, \\ v_n = y_n x_{n-1}. \end{cases} \tag{8}$$

2.1. Solutions of $u_{n+1} = \frac{v_n}{a+bv_n}, \quad v_{n+1} = \frac{u_n}{c+du_n}$. Here, to give a closed form for the well defined solutions of the system (7), We consider the system of two difference equations nonlinear first-order .

$$u_{n+1} = \frac{v_n}{a+bv_n}, \quad v_{n+1} = \frac{u_n}{c+du_n} \quad n \geq 0 \tag{9}$$

The system (9) can be written as the following equation

$$u_{n+1} = \frac{u_{n-1}}{ac+(ad+b)u_{n-1}}, \quad n \geq 1. \tag{10}$$

Let

$$u_n^{(j)} = u_{2n+j}, \quad n \in \mathbb{N}_0, j \in \{0,1\}. \tag{11}$$

Using notation (11), we can write (10) as

$$u_{n+1}^{(j)} = \frac{u_n^{(j)}}{ac+(ad+b)u_n^{(j)}}. \tag{12}$$

where $j \in \{0,1\}$.
Now consider the equation

$$\mathcal{W}_{n+1} = \frac{\mathcal{W}_n}{ac+(ad+b)\mathcal{W}_n}. \tag{13}$$

Using the change of variables

$$\mathcal{W}_n = \frac{1}{(ad+b)}\left(\mathcal{H}_n - ac\right). \tag{14}$$

we can write (13) as

$$\mathcal{H}_{n+1} = \frac{(ac+1)\mathcal{H}_n - ac}{\mathcal{H}_n}. \tag{15}$$

To obtain solutions of equation (15), let's review the following lemmas.

✓ **if** $a \neq \frac{1}{c}$

Lemma 2.1. *Consider the linear difference equation*

$$k_{n+1} - (ac+1)k_n + ack_{n-1} = 0, \quad n \in \mathbb{N}_0, \tag{16}$$

with initial conditions $k_{-1}, k_0 \in \mathbb{R}$. Thus all solutions of equation (16) *can be written in the following form*

$$k_n = \frac{1}{1-ac}\left[k_0\left(1-(ac)^{(n+1)}\right) - ack_{-1}\left(1-(ac)^n\right)\right]. \tag{17}$$

Proof of Lemma 2.1. Thus we have the equation

$$k_{n+1} - (1+ac)k_n + ack_{n-1} = 0. \tag{18}$$

(the homogeneous linear second order difference equation with constant coefficients), where k_0 and $k_{-1} \in \mathbb{R}$, is usually solved by using the characteristic roots $\lambda_1 = ac$ et $\lambda_2 = 1$ of the characteristic polynomial $P(\lambda) = (\lambda^2 - (1+ac)\lambda + ac$, and the formulas of general solution is

$$k_n = c_1 + c_2(ac)^n.$$

Using the initial conditions k_0 and k_{-1}, with some calculations we get

$$\begin{aligned} c_1 &= \frac{k_0 - k_{-1}ac}{1-ac} \\ c_2 &= \frac{ac(k_{-1} - k_0)}{1-ac}. \end{aligned}$$

and the formulas of the general solution is (18) is

$$k_n = \frac{1}{1-ac}\left[k_0\left(1-(ac)^{(n+1)}\right) - ack_{-1}\left(1-(ac)^n\right)\right]. \tag{19}$$

□

✓ **if** $a = \frac{1}{c}$

Lemma 2.2. *Consider the linear difference equation*

$$k_{n+1} - 2k_n + k_{n-1} = 0, \quad n \in \mathbb{N}_0, \tag{20}$$

with initial conditions $k_{-1}, k_0 \in \mathbb{R}$. *Then all solutions of equation* (20) *will be written under the form*

$$k_n = k_0(n+1) - k_{-1}n. \tag{21}$$

Proof of Lemma 2.2. Thus we have the equation

$$k_{n+1} - 2k_n + k_{n-1} = 0. \tag{22}$$

(the homogeneous linear second order difference equation with constant coefficients), where k_0 and $k_{-1} \in \mathbb{R}$, is usually solved by using the characteristic roots $\lambda_1 = \lambda_2 = 1$ of the characteristic polynomial $P(\lambda) = (\lambda - 1)^2$, and the formulas of general solution is

$$k_n = c_1 + c_2 n.$$

Using the initial conditions k_0 and k_{-1}, with some calculations we get

$$\begin{aligned} c_1 &= k_0 \\ c_2 &= k_0 - k_{-1}. \end{aligned}$$

And, the general solution of equation (22) obtained is :

$$k_n = k_0(n+1) - k_{-1}n. \tag{23}$$

□

Through an analytical approach. We put

$$\mathcal{H}_n = \frac{k_n}{k_{n-1}.} \tag{24}$$

which reduces equation (15) to the following one

$$k_{n+1} = (ac+1)k_n - ack_{n-1}. \tag{25}$$

So, from Lemma (2.1) and Lemma (2.2) we get

✓ **if** $a \neq \frac{1}{c}$

$$k_n = \frac{1}{1-ac}\left[k_0\left(1-(ac)^{n+1}\right) - ack_{-1}\left(1-(ac)^n\right)\right]. \tag{26}$$

✓ **if** $a = \frac{1}{c}$

$$k_n = k_0(n+1) - k_{-1}n. \tag{27}$$

By substituting the formulas obtained in (24),(26) and (27) into the equation (15) the general solution becomes:

✓ **if** $a \neq \frac{1}{c}$

$$\mathcal{H}_n = \frac{ac(1-(ac)^n) - \mathcal{H}_0(1-(ac)^{n+1})}{ac(1-(ac)^{n-1}) - \mathcal{H}_0(1-(ac)^n)}.$$

✓ **if** $a = \frac{1}{c}$

$$\mathcal{H}_n = \frac{n - \mathcal{H}_0(n+1)}{(n-1) - \mathcal{H}_0 n}.$$

From all above mentioned we see that the following theorem holds .

Theorem 2.3. *Let* $\{\mathcal{W}_n\}_{n\geq 0}$ *be a solution of* (13). *Then, for* $n = 2, 3, \ldots,$

$$\textbf{\textit{if}}\ a \neq \tfrac{1}{c} \qquad \mathcal{W}_n = \frac{\mathcal{W}_0}{(ac)^n + (ad+b)\mathcal{W}_0\sum_{r=0}^{n-1}(ac)^r}.$$

$$\textbf{\textit{if}}\ a = \tfrac{1}{c} \qquad \mathcal{W}_n = \frac{\mathcal{W}_0}{1 + (ad+b)\mathcal{W}_0 n}.$$

With the initial condition $w_0 \in \mathbb{R} - \mathbb{G}_1$*, with* $\mathbb{G}_1$ *is the Forbidden Set of system* (13) *given by*

$$\mathbb{G}_1 = \bigcup_{n=-1}^{\infty}\left\{\mathcal{W}_0 : (ac)^n + (ad+b)\mathcal{W}_0\sum_{r=0}^{n-1}(ac)^r = 0 \ \textbf{\textit{or}}\ 1 - (ad+b)\mathcal{W}_0 n = 0\right\}.$$

From Theorem (2.3), the solution of equation (12) is given by these formulas

$$\begin{aligned} &\textbf{if}\ a \neq \tfrac{1}{c} \quad u_n^{(j)} = \frac{u_0^{(j)}}{(ac)^n + (ad+b)u_0^{(j)}\sum_{r=0}^{n-1}(ac)^r}. \\ &\textbf{if}\ a = \tfrac{1}{c} \quad u_n^{(j)} = \frac{u_0^{(j)}}{1 + (ad+b)u_0^{(j)} n}. \end{aligned} \qquad n \in \mathbb{N}_0, j = \{0, 1\}. \tag{28}$$

From theorem(2.3) , and formula (11) It is easy to obtain the following corollary.

Corollary 2.4. *Let $\{u_n\}_{n\geq 0}$ be a solution of* (11). *Then*

$$\textbf{\textit{if}}\ a \neq \tfrac{1}{c} \quad u_{2n+j} = \frac{u_j}{(ac)^n + b(a+1)u_j\displaystyle\sum_{r=0}^{n-1}(ac)^r}. \quad n \in \mathbb{N}_0, j = \{0,1\}.$$

$$\textbf{\textit{if}}\ a = \tfrac{1}{c} \quad u_{2n+j} = \frac{u_j}{1+(ad+b)u_j n}.$$

where $j \in \{0,1\}$ and $x_j \in \mathbb{R} - G_i$, with G_j is the Forbidden set of equation (12) *given by*

$$G_j = \bigcup_{n=0}^{\infty}\left\{(x_0, x_{-1}) : (ac)^n + (ad+b)u_j\sum_{r=0}^{n-1}(ac)^r = 0, \ \textbf{\textit{or}}\ 1+(ad+b)u_j n = 0\right\}.$$

Corollary 2.5. *Let $\{u_n\}_{n\geq 0}$ be a solution of* (9). *Then*

$\textbf{\textit{if}}\ a \neq \frac{1}{c}$

$$u_{2n} = \frac{u_0}{(ac)^n + (ad+b)u_0\displaystyle\sum_{r=0}^{n-1}(ac)^r},$$

$$u_{2n+1} = \frac{v_0}{a^{n+1}c^n + v_0\left(ad\displaystyle\sum_{r=0}^{n-1}(ac)^r + b\sum_{r=0}^{n}(ac)^r\right)},$$

$$v_{2n} = \frac{v_0}{(ac)^n + (bc+d)v_0\displaystyle\sum_{r=0}^{n-1}(ac)^r},$$

$$v_{2n+1} = \frac{u_0}{a^n c^{n+1} + u_0\left(bc\displaystyle\sum_{r=0}^{n-1}(ac)^r + d\sum_{r=0}^{n}(ac)^r\right)}.$$

$\textbf{\textit{if}}\ a = \frac{1}{c}$

$$u_{2n} = \frac{u_0}{1+(ad+b)nu_0},$$

$$u_{2n+1} = \frac{v_0}{a+((ad+b)n+b)v_0},$$

$$v_{2n} = \frac{v_0}{1+(bc+d)nv_0},$$

$$v_{2n+1} = \frac{u_0}{c+((bc+d)n+d)u_0}.$$

where $n \in \mathbb{N}_0$, u_0 and $v_0 \in \mathbb{R} - G_2$, with G_2 is the Forbidden set of equation (12).

Proof of Corollary 2.5. Let $\{u_n, v_n\}_{n\geq -1}$ be a solution of system (11), so $\{u_n\}_{n\geq -1}$ is a solution of equation (12). Then,

✓ **if** $a \neq \frac{1}{c}$
Let
$$u_{2n+1} = \frac{u_1}{(ac)^n + (ad+b)u_1 \sum_{r=0}^{n-1} (ac)^r},$$
And $u_1 = \frac{v_0}{a+bv_0}$, so
$$\begin{aligned} u_{2n+1} &= \frac{u_1}{(ac)^n + (ad+b)u_1 \sum_{r=0}^{n-1} (ac)^r} = \frac{v_0}{(ac)^n(a+bv_0) + (ad+b)v_0 \sum_{r=0}^{n-1} (ac)^r} \\ &= \frac{v_0}{a^{n+1}c^n + \left[b(ac)^n + (ad+b) \sum_{r=0}^{n-1} (ac)^r \right] v_0} \\ &= \frac{v_0}{a^{n+1}c^n + \left[ad \sum_{r=0}^{n-1} (ac)^r + b \sum_{r=0}^{n} (ac)^r \right] v_0}. \end{aligned}$$

✓ **if** $a = \frac{1}{c}$
Let
$$u_{2n+1} = \frac{u_1}{1 + (ad+b)nu_1},$$
et $u_1 = \frac{v_0}{a+bv_0}$, so
$$\begin{aligned} u_{2n+1} &= \frac{u_1}{1 + (ad+b)nu_1} = \frac{u_1}{a + bv_0 + (ad+b)nv_0} \\ &= \frac{v_0}{a + ((ad+b)n + b)v_0}. \end{aligned}$$

In the same way, and using these formulas
$$v_{2n} = \frac{u_{2n-1}}{a + bu_{2n-1}} \quad \textbf{and} \quad v_{2n+1} = \frac{u_{2n}}{a + bu_{2n}}$$
we obtain

if $a \neq \frac{1}{c}$
$$v_{2n} = \frac{v_0}{(ac)^n + (bc+d)v_0 \sum_{r=0}^{n-1} (ac)^r},$$
$$v_{2n+1} = \frac{u_0}{a^n c^{n+1} + u_0 \left(bc \sum_{r=0}^{n-1} (ac)^r + d \sum_{r=0}^{n} (ac)^r \right)}.$$

if $a = \frac{1}{c}$
$$v_{2n} = \frac{v_0}{1 + (bc+d)nv_0},$$
$$v_{2n+1} = \frac{u_0}{c + ((bc+d)n + d)u_0}.$$

□

2.2. Solutions of $u_{n+1} = \frac{v_{n-3}}{a+bv_{n-3}}$, $v_{n+1} = \frac{u_{n-3}}{c+du_{n-3}}$. In this section, we discuss the solution of the system (7) by using an appropriate transformation reducing this system to the system of first-order difference equations (9).

Analysis of the form of system. The initial values with the smallest indexes are u_{-3} and v_{-3}. By using (7) with $n = 0$, we obtain the values of u_1 and v_1 as follows

$$u_1 = \frac{v_{-3}}{a + bv_{-3}}, \quad v_1 = \frac{u_{-3}}{c + du_{-3}}.$$

Having the u_1 and v_1 values, by using (7) with $n = 2$ we get the values of u_3 and v_3 values

$$u_3 = \frac{v_{-1}}{a + bv_{-1}}, \quad v_3 = \frac{u_{-1}}{c + du_{-1}}.$$

With u_3 and v_3 values, and by using the formula (7) with $n = 4$, , we can obtain u_5 and v_5 values

$$\begin{aligned} u_5 &= \frac{v_1}{a + bv_1}, & v_5 &= \frac{u_1}{c + du_1}. \\ &\vdots & &\vdots \\ u_{4m+1} &= \frac{v_{4m-1}}{a + bv_{4m-1}}, & v_{4m+1} &= \frac{u_{4m-1}}{c + du_{4m-1}}. \end{aligned}$$

In the same way, it is shown that the initial values u_{-i} and v_{-i}, for fixed i, with $i \in \{0, 1, 2, 3\}$, determine all the values of the sequences $(u_{4(m+1)-i})_m$ and $(v_{4(m+1)-i})_m$. Also we have

$$\begin{cases} u_{4(m+1)-i} &= \dfrac{v_{4m-i}}{a + bv_{4m-i}}, \\ \\ v_{4(m+1)-i} &= \dfrac{u_{4m-i}}{c + du_{4m-i}}. \end{cases} \tag{29}$$

Let

$$\begin{cases} u_n^{(i)} = u_{4n-i}, \\ v_n^{(i)} = v_{4n-i}. \end{cases} \quad i \in \{0, 1, 2, 3\}. \tag{30}$$

Using notation (30), we can write (7) as

$$u_{n+1}^{(i)} = \frac{v_n^{(i)}}{a + bv_n^{(i)}}, \quad v_{n+1}^{(i)} = \frac{u_n^{(i)}}{c + du_n^{(i)}}.$$

From all above mentioned we see that the following theorem holds.

Theorem 2.6. *Let $\{u_n, v_n\}_{n \geq -3}$ be a solution of (7). Then, for $n = -3, -2, \ldots$,*

- ***if*** $a \neq \frac{1}{c}$

$$\begin{cases} u_{8n-3} = \dfrac{u_{-3}}{(ac)^n + (ad+b)u_{-3}\sum\limits_{r=0}^{n-1}(ac)^r}, \\ u_{8n-2} = \dfrac{u_{-2}}{(ac)^n + (ad+b)u_{-2}\sum\limits_{r=0}^{n-1}(ac)^r}, \\ u_{8n-1} = \dfrac{u_{-1}}{(ac)^n + (ad+b)u_{-1}\sum\limits_{r=0}^{n-1}(ac)^r}, \\ u_{8n} = \dfrac{u_{0}}{(ac)^n + (ad+b)u_{0}\sum\limits_{r=0}^{n-1}(ac)^r}, \end{cases} \quad \begin{cases} u_{8n+1} = \dfrac{v_{-3}}{a^{n+1}c^n + v_{-3}\left(ad\sum\limits_{r=0}^{n-1}(ac)^r + b\sum\limits_{r=0}^{n}(ac)^r\right)}, \\ u_{8n+2} = \dfrac{v_{-2}}{a^{n+1}c^n + v_{-2}\left(ad\sum\limits_{r=0}^{n-1}(ac)^r + b\sum\limits_{r=0}^{n}(ac)^r\right)}, \\ u_{8n+3} = \dfrac{v_{-1}}{a^{n+1}c^n + v_{-1}\left(ad\sum\limits_{r=0}^{n-1}(ac)^r + b\sum\limits_{r=0}^{n}(ac)^r\right)}, \\ u_{8n+4} = \dfrac{v_{0}}{a^{n+1}c^n + v_{0}\left(ad\sum\limits_{r=0}^{n-1}(ac)^r + b\sum\limits_{r=0}^{n}(ac)^r\right)}. \end{cases}$$

$$\begin{cases} v_{8n-3} = \dfrac{v_{-3}}{(ac)^n + (bc+d)v_{-3}\sum\limits_{r=0}^{n-1}(ac)^r}, \\ v_{8n-2} = \dfrac{v_{-2}}{(ac)^n + (bc+d)v_{-2}\sum\limits_{r=0}^{n-1}(ac)^r}, \\ v_{8n-1} = \dfrac{v_{-1}}{(ac)^n + (bc+d)v_{-1}\sum\limits_{r=0}^{n-1}(ac)^r}, \\ v_{8n} = \dfrac{v_{0}}{(ac)^n + (bc+d)v_{0}\sum\limits_{r=0}^{n-1}(ac)^r}, \end{cases} \quad \begin{cases} v_{8n+1} = \dfrac{u_{-3}}{a^{n}c^{n+1} + u_{-3}\left(bc\sum\limits_{r=0}^{n-1}(ac)^r + d\sum\limits_{r=0}^{n}(ac)^r\right)}, \\ v_{8n+2} = \dfrac{u_{-2}}{a^{n}c^{n+1} + u_{-2}\left(bc\sum\limits_{r=0}^{n-1}(ac)^r + d\sum\limits_{r=0}^{n}(ac)^r\right)}, \\ v_{8n+3} = \dfrac{u_{-1}}{a^{n}c^{n+1} + u_{-1}\left(bc\sum\limits_{r=0}^{n-1}(ac)^r + d\sum\limits_{r=0}^{n}(ac)^r\right)}, \\ v_{8n+4} = \dfrac{u_{0}}{a^{n}c^{n+1} + u_{0}\left(bc\sum\limits_{r=0}^{n-1}(ac)^r + d\sum\limits_{r=0}^{n}(ac)^r\right)}. \end{cases}$$

- ***if*** $a = \frac{1}{c}$

$$\begin{cases} u_{8n-3} = \dfrac{u_{-3}}{1+(ad+b)nu_{-3}}, \\ u_{8n-2} = \dfrac{u_{-2}}{1+(ad+b)nu_{-2}}, \\ u_{8n-1} = \dfrac{u_{-1}}{1+(ad+b)nu_{-1}}, \\ u_{8n} = \dfrac{u_{0}}{1+(ad+b)nu_{0}}, \end{cases} \quad \begin{cases} u_{8n+1} = \dfrac{v_{-3}}{a+((ad+b)n+b)v_{-3}}, \\ u_{8n+2} = \dfrac{v_{-2}}{a+((ad+b)n+b)v_{-2}}, \\ u_{8n+3} = \dfrac{v_{-1}}{a+((ad+b)n+b)v_{-1}}, \\ u_{8n+4} = \dfrac{v_{0}}{a+((ad+b)n+b)v_{0}}. \end{cases}$$

$$\left\{\begin{array}{l} v_{8n-3}=\dfrac{v_{-3}}{1+(bc+d)nv_{-3}},\\ \\ v_{8n-2}=\dfrac{v_{-2}}{1+(bc+d)nv_{-2}},\\ \\ v_{8n-1}=\dfrac{v_{-1}}{1+(bc+d)nv_{-1}},\\ \\ v_{8n}=\dfrac{v_{0}}{1+(bc+d)nv_{0}}, \end{array}\right. \qquad \left\{\begin{array}{l} v_{8n+1}=\dfrac{u_{-3}}{c+((bc+d)n+d)u_{-3}},\\ \\ v_{8n+2}=\dfrac{u_{-2}}{c+((bc+d)n+d)u_{-2}},\\ \\ v_{8n+3}=\dfrac{u_{-1}}{c+((bc+d)n+d)u_{-1}},\\ \\ v_{8n+4}=\dfrac{u_{0}}{c+((bc+d)n+d)u_{0}}. \end{array}\right.$$

where $n\in\mathbb{N}_0,\ u_{-3},u_{-2},u_{-1},u_0,v_{-3},v_{-2},v_{-1}$ *and* $v_0\in\mathbb{R}-G_3$*, with* G_3 *is the Forbidden set of system* (7)*.*

2.3. Solutions of $x_{n+1}=\frac{y_{n-3}x_{n-4}}{y_n(a+by_{n-3}x_{n-4})},\quad y_{n+1}=\frac{x_{n-3}y_{n-4}}{x_n(c+dx_{n-3}y_{n-4})}$**.** Let

$$x_n = \frac{u_n}{y_{n-1}}, \tag{31}$$

$$y_n = \frac{v_n}{x_{n-1}}, \tag{32}$$

Using (32) in (31), we obtain

$$x_{8n}=\frac{u_{8n}u_{8n-2}u_{8n-4}u_{8n-6}}{v_{8n-1}v_{8n-3}v_{8n-5}v_{8n-7}}x_{8n-8}. \tag{33}$$

Using (31) in (32), we obtain

$$y_{8n}=\frac{v_{8n}v_{8n-2}v_{8n-4}v_{8n-6}}{u_{8n-1}u_{8n-3}u_{8n-5}u_{8n-7}}y_{8n-8}. \tag{34}$$

For $n\in\mathbb{N}$

Multiplying obtained qualities from (33) and (34) from 1 to n, respectively, it follows that

$$x_{8n} = x_0\prod_{i=0}^{n-1}\left(\frac{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}\right), \tag{35}$$

$$y_{8n} = y_0\prod_{i=0}^{n-1}\left(\frac{v_{8i}v_{8i-2}v_{8i-4}v_{8i-6}}{u_{8i-1}u_{8i-3}u_{8i-5}u_{8i-7}}\right). \tag{36}$$

By employing the (35) and (36) in (31) and (32), we obtain

$$x_{8n-1}=\frac{v_{8n}}{y_{8n}}=\frac{v_{8n}}{y_0}\prod_{i=0}^{n-1}\left(\frac{u_{8i-1}u_{8i-3}u_{8i-5}u_{8i-7}}{v_{8i}v_{8i-2}v_{8i-4}v_{8i-6}}\right),$$

hence, we have

$$x_{8n-1}=\frac{v_{8n}}{y_0}\prod_{i=0}^{n-1}\left(\frac{u_{8i-1}u_{8i-3}u_{8i-5}u_{8i-7}}{v_{8i}v_{8i-2}v_{8i-4}v_{8i-6}}\right). \tag{37}$$

$$y_{8n-1}=\frac{u_{8n}}{x_{8n}}=\frac{u_{8n}}{x_0}\prod_{i=0}^{n-1}\left(\frac{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}\right),$$

hence, we have

$$y_{8n-1}=\frac{u_{8n}}{x_0}\prod_{i=0}^{n-1}\left(\frac{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}\right). \tag{38}$$

Using the equalities (37) and (38) in (31) and (32), we obtain

$$x_{8n-2}=\frac{v_{8n-1}}{y_{8n-1}}=x_0\frac{v_{8n-1}}{u_{8n}}\prod_{i=0}^{n-1}\left(\frac{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}\right),$$

hence, we have

$$x_{8n-2}=x_0\frac{v_{8n-1}}{u_{8n}}\prod_{i=0}^{n-1}\left(\frac{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}\right). \tag{39}$$

And

$$y_{8n-2}=\frac{u_{8n-1}}{x_{8n-1}}=y_0\frac{u_{8n-1}}{v_{8n}}\prod_{i=0}^{n-1}\left(\frac{v_{8i}v_{8i-2}v_{8i-4}v_{8i-6}}{u_{8i-1}u_{8i-3}u_{8i-5}u_{8i-7}}\right),$$

so,we have

$$y_{8n-2}=y_0\frac{u_{8n-1}}{v_{8n}}\prod_{i=0}^{n-1}\left(\frac{v_{8i}v_{8i-2}v_{8i-4}v_{8i-6}}{u_{8i-1}u_{8i-3}u_{8i-5}u_{8i-7}}\right). \tag{40}$$

By employing the (39) and (40) in (31) and (32), we obtain

$$x_{8n-3}=\frac{v_{8n-2}}{y_{8n-2}}=\frac{v_{8n-2}v_{8n}}{y_0u_{8n-1}}\prod_{i=0}^{n-1}\left(\frac{u_{8i-1}u_{8i-3}u_{8i-5}u_{8i-7}}{v_{8i}v_{8i-2}v_{8i-4}v_{8i-6}}\right),$$

hence, we have

$$x_{8n-3}=\frac{v_{8n-2}v_{8n}}{y_0u_{8n-1}}\prod_{i=0}^{n-1}\left(\frac{u_{8i-1}u_{8i-3}u_{8i-5}u_{8i-7}}{v_{8i}v_{8i-2}v_{8i-4}v_{8i-6}}\right). \tag{41}$$

And

$$y_{8n-3}=\frac{u_{8n-2}}{x_{8n-2}}=\frac{u_{8n-2}u_{8n}}{v_{8n-1}x_0}\prod_{i=0}^{n-1}\left(\frac{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}\right),$$

so,we have

$$y_{8n-3}=\frac{u_{8n-2}u_{8n}}{v_{8n-1}x_0}\prod_{i=0}^{n-1}\left(\frac{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}\right). \tag{42}$$

Using the equalities (41) and (42) in (31) and (32), we obtain

$$x_{8n-4}=\frac{v_{8n-3}}{y_{8n-3}}=x_0\frac{v_{8n-1}v_{8n-3}}{u_{8n-2}u_{8n}}\prod_{i=0}^{n-1}\left(\frac{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}\right),$$

hence, we have

$$x_{8n-4}=x_0\frac{v_{8n-3}v_{8n-1}}{u_{8n-2}u_{8n}}\prod_{i=0}^{n-1}\left(\frac{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}\right). \tag{43}$$

And

$$y_{8n-4}=\frac{u_{8n-3}}{x_{8n-3}}=y_0\frac{u_{8n-3}u_{8n-1}}{v_{8n-2}v_{8n}}\prod_{i=0}^{n-1}\left(\frac{v_{8i}v_{8i-2}v_{8i-4}v_{8i-6}}{u_{8i-1}u_{8i-3}u_{8i-5}u_{8i-7}}\right),$$

so,we have

$$y_{8n-4}=y_0\frac{u_{8n-3}u_{8n-1}}{v_{8n-2}v_{8n}}\prod_{i=0}^{n-1}\left(\frac{v_{8i}v_{8i-2}v_{8i-4}v_{8i-6}}{u_{8i-1}u_{8i-3}u_{8i-5}u_{8i-7}}\right). \tag{44}$$

Using the equalities (35) and (36) in (31) and (32), we obtain

$$x_{8n+1}=\frac{u_{8n+1}}{y_{8n}}=\frac{u_{8n+1}}{y_0}\prod_{i=0}^{n-1}\left(\frac{u_{8i-1}u_{8i-3}u_{8i-5}u_{8i-7}}{v_{8i}v_{8i-2}v_{8i-4}v_{8i-6}}\right),$$

hence, we have

$$x_{8n+1}=\frac{u_{8n+1}}{y_0}\prod_{i=0}^{n-1}\left(\frac{u_{8i-1}u_{8i-3}u_{8i-5}u_{8i-7}}{v_{8i}v_{8i-2}v_{8i-4}v_{8i-6}}\right). \tag{45}$$

And

$$y_{8n+1}=\frac{v_{8n+1}}{x_{8n}}=\frac{v_{8n+1}}{x_0}\prod_{i=0}^{n-1}\left(\frac{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}\right),$$

so,we have

$$y_{8n+1}=\frac{v_{8n+1}}{x_0}\prod_{i=0}^{n-1}\left(\frac{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}\right). \tag{46}$$

Using the equalities (45) and (46) in (31) and (32), we obtain

$$x_{8n+2}=\frac{u_{8n+2}}{y_{8n+1}}=x_0\frac{u_{8n+2}}{v_{8n+1}}\prod_{i=0}^{n-1}\left(\frac{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}\right),$$

hence, we have

$$x_{8n+2}=x_0\frac{u_{8n+2}}{v_{8n+1}}\prod_{i=0}^{n-1}\left(\frac{u_{8i}u_{8i-2}u_{8i-4}u_{8i-6}}{v_{8i-1}v_{8i-3}v_{8i-5}v_{8i-7}}\right). \tag{47}$$

And

$$y_{8n+2}=\frac{v_{8n+2}}{x_{8n+1}}=y_0\frac{v_{8n+2}}{u_{8n+1}}\prod_{i=0}^{n-1}\left(\frac{v_{8i}v_{8i-2}v_{8i-4}v_{8i-6}}{u_{8i-1}u_{8i-3}u_{8i-5}u_{8i-7}}\right),$$

so,we have

$$y_{8n+2} = y_0 \frac{v_{8n+2}}{u_{8n+1}} \prod_{i=0}^{n-1} \left(\frac{v_{8i} v_{8i-2} v_{8i-4} v_{8i-6}}{u_{8i-1} u_{8i-3} u_{8i-5} u_{8i-7}} \right). \tag{48}$$

Using the equalities (47) and (48) in (31) and (32), we obtain

$$x_{8n+3} = \frac{u_{8n+3}}{y_{8n+2}} = \frac{u_{8n+3} u_{8n+1}}{y_0 v_{8n+2}} \prod_{i=0}^{n-1} \left(\frac{u_{8i-1} u_{8i-3} u_{8i-5} u_{8i-7}}{v_{8i} v_{8i-2} v_{8i-4} v_{8i-6}} \right),$$

hence, we have

$$x_{8n+3} = \frac{u_{8n+3} u_{8n+1}}{y_0 v_{8n+2}} \prod_{i=0}^{n-1} \left(\frac{u_{8i-1} u_{8i-3} u_{8i-5} u_{8i-7}}{v_{8i} v_{8i-2} v_{8i-4} v_{8i-6}} \right). \tag{49}$$

And

$$y_{8n+3} = \frac{v_{8n+3}}{x_{8n+2}} = \frac{v_{8n+3} v_{8n+1}}{x_0 u_{8n+2}} \prod_{i=0}^{n-1} \left(\frac{v_{8i-1} v_{8i-3} v_{8i-5} v_{8i-7}}{u_{8i} u_{8i-2} u_{8i-4} u_{8i-6}} \right),$$

so,we have

$$y_{8n+3} = \frac{v_{8n+3} v_{8n+1}}{x_0 u_{8n+2}} \prod_{i=0}^{n-1} \left(\frac{v_{8i-1} v_{8i-3} v_{8i-5} v_{8i-7}}{u_{8i} u_{8i-2} u_{8i-4} u_{8i-6}} \right). \tag{50}$$

Using relationships the theorem (2.6) we conclude the following :

- **if** $a \neq \frac{1}{c}$

$$\begin{cases} u_{8n-3} = \dfrac{u_{-3}}{(ac)^n + (ad+b)u_{-3} \displaystyle\sum_{r=0}^{n-1} (ac)^r}, \\[2ex] u_{8n-2} = \dfrac{u_{-2}}{(ac)^n + (ad+b)u_{-2} \displaystyle\sum_{r=0}^{n-1} (ac)^r}, \\[2ex] u_{8n-1} = \dfrac{u_{-1}}{(ac)^n + (ad+b)u_{-1} \displaystyle\sum_{r=0}^{n-1} (ac)^r}, \\[2ex] u_{8n} = \dfrac{u_{0}}{(ac)^n + (ad+b)u_{0} \displaystyle\sum_{r=0}^{n-1} (ac)^r}, \end{cases}$$

$$\begin{cases} u_{8n-7} = \dfrac{v_{-3}}{a^n c^{n-1} + v_{-3}\left(ad\displaystyle\sum_{r=0}^{n-2}(ac)^r + b\sum_{r=0}^{n-1}(ac)^r\right)}, \\[2ex] u_{8n-6} = \dfrac{v_{-2}}{a^n c^{n-1} + v_{-2}\left(ad\displaystyle\sum_{r=0}^{n-2}(ac)^r + b\sum_{r=0}^{n-1}(ac)^r\right)}, \\[2ex] u_{8n-5} = \dfrac{v_{-1}}{a^n c^{n-1} + v_{-1}\left(ad\displaystyle\sum_{r=0}^{n-2}(ac)^r + b\sum_{r=0}^{n-1}(ac)^r\right)}, \\[2ex] u_{8n-4} = \dfrac{v_{0}}{a^n c^{n-1} + v_{0}\left(ad\displaystyle\sum_{r=0}^{n-2}(ac)^r + b\sum_{r=0}^{n-1}(ac)^r\right)}. \end{cases}$$

$$\begin{cases} v_{8n-3} = \dfrac{v_{-3}}{(ac)^n + (bc+d)v_{-3}\displaystyle\sum_{r=0}^{n-1}(ac)^r}, \\[2ex] v_{8n-2} = \dfrac{v_{-2}}{(ac)^n + (bc+d)v_{-2}\displaystyle\sum_{r=0}^{n-1}(ac)^r}, \\[2ex] v_{8n-1} = \dfrac{v_{-1}}{(ac)^n + (bc+d)v_{-1}\displaystyle\sum_{r=0}^{n-1}(ac)^r}, \\[2ex] v_{8n} = \dfrac{v_{0}}{(ac)^n + (bc+d)v_{0}\displaystyle\sum_{r=0}^{n-1}(ac)^r}, \end{cases}$$

$$\begin{cases} v_{8n-7} = \dfrac{u_{-3}}{a^{n-1} c^{n} + u_{-3}\left(bc\displaystyle\sum_{r=0}^{n-2}(ac)^r + d\sum_{r=0}^{n-1}(ac)^r\right)}, \\[2ex] v_{8n-6} = \dfrac{u_{-2}}{a^{n-1} c^{n} + u_{-2}\left(bc\displaystyle\sum_{r=0}^{n-2}(ac)^r + d\sum_{r=0}^{n-1}(ac)^r\right)}, \\[2ex] v_{8n-5} = \dfrac{u_{-1}}{a^{n-1} c^{n} + u_{-1}\left(bc\displaystyle\sum_{r=0}^{n-2}(ac)^r + d\sum_{r=0}^{n-1}(ac)^r\right)}, \\[2ex] v_{8n-4} = \dfrac{u_{0}}{a^{n-1} c^{n} + u_{0}\left(bc\displaystyle\sum_{r=0}^{n-2}(ac)^r + d\sum_{r=0}^{n-1}(ac)^r\right)}. \end{cases}$$

- **if** $a = \frac{1}{c}$

$$\begin{cases} u_{8n-3} = \dfrac{u_{-3}}{1+(ad+b)nu_{-3}}, \\ u_{8n-2} = \dfrac{u_{-2}}{1+(ad+b)nu_{-2}}, \\ u_{8n-1} = \dfrac{u_{-1}}{1+(ad+b)nu_{-1}}, \\ u_{8n} = \dfrac{u_{0}}{1+(ad+b)nu_{0}}, \end{cases} \quad \begin{cases} u_{8n-7} = \dfrac{v_{-3}}{a+((ad+b)n-ad)v_{-3}}, \\ u_{8n-6} = \dfrac{v_{-2}}{a+((ad+b)n-ad)v_{-2}}, \\ u_{8n-5} = \dfrac{v_{-1}}{a+((ad+b)n-ad)v_{-1}}, \\ u_{8n-4} = \dfrac{v_{0}}{a+((ad+b)n-ad)v_{0}}. \end{cases}$$

$$\begin{cases} v_{8n-3} = \dfrac{v_{-3}}{1+(bc+d)nv_{-3}}, \\ v_{8n-2} = \dfrac{v_{-2}}{1+(bc+d)nv_{-2}}, \\ v_{8n-1} = \dfrac{v_{-1}}{1+(bc+d)nv_{-1}}, \\ v_{8n} = \dfrac{v_{0}}{1+(bc+d)nv_{0}}, \end{cases} \quad \begin{cases} v_{8n-7} = \dfrac{u_{-3}}{c+((bc+d)n-bc)u_{-3}}, \\ v_{8n-6} = \dfrac{u_{-2}}{c+((bc+d)n-bc)u_{-2}}, \\ v_{8n-5} = \dfrac{u_{-1}}{c+((bc+d)n-bc)u_{-1}}, \\ v_{8n-4} = \dfrac{u_{0}}{c+((bc+d)n-bc)u_{0}}. \end{cases}$$

From all above mentioned and

$$u_{-1} = x_{-1}y_{-2}, \quad u_0 = x_0y_{-1}, \quad v_{-1} = y_{-1}x_{-2}, \quad v_0 = y_0x_{-1}. \tag{51}$$

$$u_{-2} = x_{-2}y_{-3}, \quad u_{-3} = x_{-3}y_{-4}, \quad v_{-2} = y_{-2}x_{-3}, \quad v_{-3} = y_{-3}x_{-4}. \tag{52}$$

we see that the following result holds.

Theorem 2.7. *Let $\{x_n, y_n\}_{n\geq -1}$ be a solution of (6). Then, for $n = 0, 1, 2, 3, \ldots,$*

- ***if*** $a \neq \frac{1}{c} \quad (a \in \mathbb{R} - \{\frac{1}{c}\})$

$$x_{8n-4} = \frac{x_0^n y_0^n}{x_{-4}^{n-1} y_{-4}^n} \frac{\left((ac)^n + x_{-2}y_{-3}(ad+b)\sum_{r=0}^{n-1}(ac)^r\right)\left((ac)^n + x_0y_{-1}(ad+b)\sum_{r=0}^{n-1}(ac)^r\right)}{\left((ac)^n + y_{-3}x_{-4}(bc+d)\sum_{r=0}^{n-1}(ac)^r\right)\left((ac)^n + y_{-1}x_{-2}(bc+d)\sum_{r=0}^{n-1}(ac)^r\right)} \prod_{i=0}^{n-1} \phi_i.$$

$$x_{8n-3} = \frac{x_{-3}x_{-4}^n y_{-4}^n}{x_0^n y_0^n} \frac{\left((ac)^n + x_{-1}y_{-2}(ad+b)\sum_{r=0}^{n-1}(ac)^r\right)}{\left((ac)^n + y_{-2}x_{-3}(bc+d)\sum_{r=0}^{n-1}(ac)^r\right)\left((ac)^n + y_0x_{-1}(bc+d)\sum_{r=0}^{n-1}(ac)^r\right)} \prod_{i=0}^{n-1} \psi_i.$$

$$x_{8n-2}=\frac{x_{-2}x_0^ny_0^n}{x_{-4}^ny_{-4}^n}\frac{\left((ac)^n+x_0y_{-1}(ad+b)\sum\limits_{r=0}^{n-1}(ac)^r\right)}{\left((ac)^n+y_{-1}x_{-2}(bc+d)\sum\limits_{r=0}^{n-1}(ac)^r\right)}\prod_{i=0}^{n-1}\phi_i.$$

$$x_{8n-1}=\frac{x_{-4}^ny_{-4}^n}{x_0^ny_0^n}\frac{x_{-1}}{\left((ac)^n+y_0x_{-1}(bc+d)\sum\limits_{r=0}^{n-1}(ac)^r\right)}\prod_{i=0}^{n-1}\psi_i.$$

$$x_{8n}=\frac{x_0^{n+1}y_0^n}{x_{-4}^ny_{-4}^n}\prod_{i=0}^{n-1}\phi_i.$$

$$x_{8n+1}=\frac{x_{-4}^{n+1}y_{-4}^n}{x_0^ny_0^{n+1}}\frac{y_{-3}}{a^{n+1}c^n+y_{-3}x_{-4}\left(ad\sum\limits_{r=0}^{n-1}(ac)^r+b\sum\limits_{r=0}^{n}(ac)^r\right)}\prod_{i=0}^{n-1}\psi_i.$$

$$x_{8n+2}=\frac{y_{-2}x_0^{n+1}y_0^n}{x_{-4}^ny_{-4}^{n+1}}\frac{\left(a^nc^{n+1}+x_{-3}y_{-4}\left(bc\sum\limits_{r=0}^{n-1}(ac)^r+d\sum\limits_{r=0}^{n}(ac)^r\right)\right)}{\left(a^{n+1}c^n+y_{-2}x_{-3}\left(ad\sum\limits_{r=0}^{n-1}(ac)^r+b\sum\limits_{r=0}^{n}(ac)^r\right)\right)}\prod_{i=0}^{n-1}\phi_i.$$

$$\begin{aligned}x_{8n+3}&\\ =&\frac{\left(\frac{y_{-1}x_{-4}^{n+1}y_{-4}^n}{x_0^ny_0^{n+1}}\right)\left(a^nc^{n+1}+x_{-2}y_{-3}\left(bc\sum\limits_{r=0}^{n-1}(ac)^r+d\sum\limits_{r=0}^{n}(ac)^r\right)\right)}{\left(a^{n+1}c^n+y_{-1}x_{-2}\left(ad\sum\limits_{r=0}^{n-1}(ac)^r+b\sum\limits_{r=0}^{n}(ac)^r\right)\right)}\\ &\times\frac{\prod_{i=0}^{n-1}\psi_i}{\left(a^{n+1}c^n+y_{-3}x_{-4}\left(ad\sum\limits_{r=0}^{n-1}(ac)^r+b\sum\limits_{r=0}^{n}(ac)^r\right)\right)}.\end{aligned}$$

Where

$$\phi_i = \frac{\left((ac)^i + y_{-1}x_{-2}(bc+d)\sum_{r=0}^{i-1}(ac)^r\right)\left((ac)^i + y_{-3}x_{-4}(bc+d)\sum_{r=0}^{i-1}(ac)^r\right)}{\left((ac)^n + x_0y_{-1}(ad+b)\sum_{r=0}^{i-1}(ac)^r\right)\left((ac)^i + x_{-2}y_{-3}(ad+b)\sum_{r=0}^{i-1}(ac)^r\right)} \times \frac{\left(a^{i-1}c^n + x_{-1}y_{-2}\left(bc\sum_{r=0}^{i-2}(ac)^r + d\sum_{r=0}^{i-1}(ac)^r\right)\right)}{\left(a^ic^{i-1} + y_0x_{-1}\left(ad\sum_{r=0}^{i-2}(ac)^r + b\sum_{r=0}^{i-1}(ac)^r\right)\right)} \times \frac{\left(a^{i-1}c^i + x_{-3}y_{-4}\left(bc\sum_{r=0}^{i-2}(ac)^r + d\sum_{r=0}^{i-1}(ac)^r\right)\right)}{\left(a^ic^{i-1} + y_{-2}x_{-3}\left(ad\sum_{r=0}^{i-2}(ac)^r + b\sum_{r=0}^{i-1}(ac)^r\right)\right)},$$

$$\psi_i = \frac{\left((ac)^i + y_0x_{-1}(bc+d)\sum_{r=0}^{i-1}(ac)^r\right)\left((ac)^i + y_{-2}x_{-3}(bc+d)\sum_{r=0}^{i-1}(ac)^r\right)}{\left((ac)^i + x_{-1}y_{-2}(ad+b)\sum_{r=0}^{i-1}(ac)^r\right)\left((ac)^i + x_{-3}y_{-4}(ad+b)\sum_{r=0}^{i-1}(ac)^r\right)} \times \frac{\left(a^{i-1}c^n + x_0y_{-1}\left(bc\sum_{r=0}^{i-2}(ac)^r + d\sum_{r=0}^{i-1}(ac)^r\right)\right)}{\left(a^ic^{i-1} + y_{-1}x_{-2}\left(ad\sum_{r=0}^{i-2}(ac)^r + b\sum_{r=0}^{i-1}(ac)^r\right)\right)} \times \frac{\left(a^{i-1}c^i + x_{-2}y_{-3}\left(bc\sum_{r=0}^{i-2}(ac)^r + d\sum_{r=0}^{i-1}(ac)^r\right)\right)}{\left(a^ic^{i-1} + y_{-3}x_{-4}\left(ad\sum_{r=0}^{i-2}(ac)^r + b\sum_{r=0}^{i-1}(ac)^r\right)\right)}.$$

Or

$$y_{8n-4} = \frac{x_0^ny_0^n}{x_{-4}^ny_{-4}^{n-1}} \frac{\left((ac)^n + y_{-2}x_{-3}(bc+d)\sum_{r=0}^{n-1}(ac)^r\right)\left((ac)^n + y_0x_{-1}(bc+d)\sum_{r=0}^{n-1}(ac)^r\right)}{\left((ac)^n + x_{-3}y_{-4}(ad+b)\sum_{r=0}^{n-1}(ac)^r\right)\left((ac)^n + (ad+b)x_{-1}y_{-2}\sum_{r=0}^{n-1}(ac)^r\right)} \prod_{i=0}^{n-1}\chi_i.$$

$$y_{8n-3} = \frac{y_{-3}x_{-4}^ny_{-4}^n}{x_0^ny_0^n} \frac{\left((ac)^n + y_{-1}x_{-2}(bc+d)\sum_{r=0}^{n-1}(ac)^r\right)}{\left((ac)^n + x_{-2}y_{-3}(ad+b)\sum_{r=0}^{n-1}(ac)^r\right)\left((ac)^n + x_0y_{-1}(ad+b)\sum_{r=0}^{n-1}(ac)^r\right)} \prod_{i=0}^{n-1}\xi_i.$$

$$y_{8n-2}=\frac{y_{-2}x_0^n y_0^n}{x_{-4}^n y_{-4}^n}\frac{\left((ac)^n+y_0x_{-1}(bc+d)\sum_{r=0}^{n-1}(ac)^r\right)}{\left((ac)^n+x_{-1}y_{-2}(ad+b)\sum_{r=0}^{n-1}(ac)^r\right)}\prod_{i=0}^{n-1}\chi_i.$$

$$y_{8n-1}=\frac{x_{-4}^n y_{-4}^n}{x_0^n y_0^n}\frac{y_{-1}}{\left((ac)^n+x_0y_{-1}(ad+b)\sum_{r=0}^{n-1}(ac)^r\right)}\prod_{i=0}^{n-1}\xi_i.$$

$$y_{8n} \quad = \quad \frac{x_0^n y_0^{n+1}}{x_{-4}^n y_{-4}^n}\prod_{i=0}^{n-1}\chi_i.$$

$$y_{8n+1}=\frac{x_{-4}^n y_{-4}^{n+1}}{x_0^{n+1} y_0^n}\frac{x_{-3}}{\left(a^nc^{n+1}+x_{-3}y_{-4}\left(bc\sum_{r=0}^{n-1}(ac)^r+d\sum_{r=0}^{n}(ac)^r\right)\right)}\prod_{i=0}^{n-1}\xi_i.$$

$$y_{8n+2}=\frac{x_{-2}x_0^n y_0^{n+1}}{x_{-4}^{n+1} y_{-4}^n}\frac{\left(a^{n+1}c^n+y_{-3}x_{-4}\left(ad\sum_{r=0}^{n-1}(ac)^r+b\sum_{r=0}^{n}(ac)^r\right)\right)}{\left(a^nc^{n+1}+x_{-2}y_{-3}\left(bc\sum_{r=0}^{n-1}(ac)^r+d\sum_{r=0}^{n}(ac)^r\right)\right)}\prod_{i=0}^{n-1}\chi_i.$$

$$y_{8n+3}=\frac{\left(\frac{x_{-1}x_{-4}^n y_{-4}^{n+1}}{x_0^{n+1}y_0^n}\right)\left(a^{n+1}c^n+y_{-2}x_{-3}\left(ad\sum_{r=0}^{n-1}(ac)^r+b\sum_{r=0}^{n}(ac)^r\right)\right)}{\left(a^nc^{n+1}+x_{-1}y_{-2}\left(bc\sum_{r=0}^{n-1}(ac)^r+d\sum_{r=0}^{n}(ac)^r\right)\right)}$$
$$\times\frac{\prod_{i=0}^{n-1}\xi_i}{\left(a^nc^{n+1}+x_{-3}y_{-4}\left(bc\sum_{r=0}^{n-1}(ac)^r+d\sum_{r=0}^{n}(ac)^r\right)\right)}.$$

Where

$$\chi_i = \frac{\left((ac)^i+x_{-1}y_{-2}(ad+b)\sum_{r=0}^{i-1}(ac)^r\right)\left((ac)^i+x_{-3}y_{-4}(ad+b)\sum_{r=0}^{i-1}(ac)^r\right)}{\left((ac)^i+y_0x_{-1}(bc+d)\sum_{r=0}^{i-1}(ac)^r\right)\left((ac)^i+y_{-2}x_{-3}(bc+d)\sum_{r=0}^{i-1}(ac)^r\right)}\times$$
$$\frac{\left(a^ic^{i-1}+y_{-1}x_{-2}\left(ad\sum_{r=0}^{i-2}(ac)^r+b\sum_{r=0}^{i-1}(ac)^r\right)\right)}{\left(a^{i-1}c^i+x_0y_{-1}\left(bc\sum_{r=0}^{i-2}(ac)^r+d\sum_{r=0}^{i-1}(ac)^r\right)\right)}\times$$
$$\frac{\left(a^ic^{i-1}+y_{-3}x_{-4}\left(ad\sum_{r=0}^{i-2}(ac)^r+b\sum_{r=0}^{i-1}(ac)^r\right)\right)}{\left(a^{i-1}c^i+x_{-2}y_{-3}\left(bc\sum_{r=0}^{i-2}(ac)^r+d\sum_{r=0}^{i-1}(ac)^r\right)\right)},$$

$$\xi_i = \frac{\left((ac)^i + x_0 y_{-1}(ad+b)\sum_{r=0}^{i-1}(ac)^r\right)\left((ac)^i + x_{-2}y_{-3}(ad+b)\sum_{r=0}^{i-1}(ac)^r\right)}{\left((ac)^i + y_{-1}x_{-2}(bc+d)\sum_{r=0}^{i-1}(ac)^r\right)\left((ac)^i + y_{-3}x_{-4}(bc+d)\sum_{r=0}^{i-1}(ac)^r\right)} \times \frac{\left(a^i c^{i-1} + y_0 x_{-1}\left(ad\sum_{r=0}^{i-2}(ac)^r + b\sum_{r=0}^{i-1}(ac)^r\right)\right)}{\left(a^{i-1}c^i + x_{-1}y_{-2}\left(bc\sum_{r=0}^{i-2}(ac)^r + d\sum_{r=0}^{i-1}(ac)^r\right)\right)} \times \frac{\left(a^i c^{i-1} + y_{-2}x_{-3}\left(ad\sum_{r=0}^{i-2}(ac)^r + b\sum_{r=0}^{i-1}(ac)^r\right)\right)}{\left(a^{i-1}c^i + x_{-3}y_{-4}\left(bc\sum_{r=0}^{i-2}(ac)^r + d\sum_{r=0}^{i-1}(ac)^r\right)\right)}.$$

- ***if*** $a = \frac{1}{c}$.

$$x_{8n-4} = \frac{x_0^n y_0^n}{x_{-4}^{n-1} y_{-4}^n} \frac{(1 + x_{-2}y_{-3}(ad+b)n)(1 + x_0 y_{-1}(ad+b)n)}{(1 + y_{-3}x_{-4}(bc+d)n)(1 + y_{-1}x_{-2}(bc+d)n)} \prod_{i=0}^{n-1} \phi_i.$$

$$x_{8n-3} = \frac{x_{-3}x_{-4}^n y_{-4}^n}{x_0^n y_0^n} \frac{(1 + x_{-1}y_{-2}(ad+b)n)}{(1 + y_{-2}x_{-3}(bc+d)n)(1 + (bc+d)nv_0)} \prod_{i=0}^{n-1} \psi_i.$$

$$x_{8n-2} = \frac{x_{-2}x_0^n y_0^n}{x_{-4}^n y_{-4}^n} \frac{(1 + x_0 y_{-1}(ad+b)n)}{(1 + y_{-1}x_{-2}(bc+d)n)} \prod_{i=0}^{n-1} \phi_i.$$

$$x_{8n-1} = \frac{x_{-4}^n y_{-4}^n}{x_0^n y_0^n} \frac{x_{-1}}{(1 + y_0 x_{-1}(bc+d)n)} \prod_{i=0}^{n-1} \psi_i.$$

$$x_{8n} = \frac{x_0^{n+1} y_0^n}{x_{-4}^n y_{-4}^n} \prod_{i=0}^{n-1} \phi_i.$$

$$x_{8n+1} = \frac{x_{-4}^{n+1} y_{-4}^n}{x_0^n y_0^{n+1}} \frac{y_{-3}}{(a + y_{-3}x_{-4}((ad+b)n+b))} \prod_{i=0}^{n-1} \psi_i.$$

$$x_{8n+2} = \frac{y_{-2}x_0^{n+1} y_0^n}{x_{-4}^n y_{-4}^{n+1}} \frac{(c + x_{-3}y_{-4}((bc+d)n+d))}{(a + y_{-2}x_{-3}((ad+b)n+b))} \prod_{i=0}^{n-1} \phi_i.$$

$$x_{8n+3} = \frac{y_{-1}x_{-4}^{n+1} y_{-4}^n}{x_0^n y_0^{n+1}} \frac{(c + x_{-2}y_{-3}((bc+d)n+d))}{(a + y_{-1}x_{-2}((ad+b)n+b))(a + y_{-3}x_{-4}((ad+b)n+b))} \prod_{i=0}^{n-1} \psi_i.$$

Where

$$\phi_i = \frac{(1 + y_{-1}x_{-2}(bc+d)i)(1 + y_{-3}x_{-4}(bc+d)i)}{(1 + x_0 y_{-1}(ad+b)i)(1 + x_{-2}y_{-3}(ad+b)i)} \times \frac{(c + x_{-1}y_{-2}((bc+d)i - bc))(c + x_{-3}y_{-4}((bc+d)i - bc))}{(a + y_0 x_{-1}((ad+b)i - ad))(a + y_{-2}x_{-3}((ad+b)i - ad))}.$$

$$\psi_i = \frac{(1+y_0x_{-1}(bc+d)i)\,(1+y_{-2}x_{-3}(bc+d)i)}{(1+x_{-1}y_{-2}(ad+b)i)\,(1+x_{-3}y_{-4}(ad+b)i)} \times \frac{(c+x_0y_{-1}((bc+d)i-bc))\,(c+x_{-2}y_{-3}((bc+d)i-bc))}{(a+y_{-1}x_{-2}((ad+b)i-ad))\,(a+y_{-3}x_{-4}((ad+b)i-ad))}.$$

Or

$$y_{8n-4} = \frac{x_0^n y_0^n}{x_{-4}^n y_{-4}^{n-1}} \frac{(1+y_{-2}x_{-3}(bc+d)n)\,(1+y_0x_{-1}(bc+d)n)}{(1+x_{-3}y_{-4}(ad+b)n)\,(1+x_{-1}y_{-2}(ad+b)n)} \prod_{i=0}^{n-1} \chi_i.$$

$$y_{8n-3} = \frac{y_{-3}x_{-4}^n y_{-4}^n}{x_0^n y_0^n} \frac{(1+y_{-1}x_{-2}(bc+d)n)}{(1+x_{-2}y_{-3}(ad+b)n)\,(1+x_0y_{-1}(ad+b)n)} \prod_{i=0}^{n-1} \xi_i.$$

$$y_{8n-2} = \frac{y_{-2}x_0^n y_0^n}{x_{-4}^n y_{-4}^n} \frac{(1+y_0x_{-1}(bc+d)n)}{(1+x_{-1}y_{-2}(ad+b)n)} \prod_{i=0}^{n-1} \chi_i.$$

$$y_{8n-1} = \frac{x_{-4}^n y_{-4}^n}{x_0^n y_0^n} \frac{y_{-1}}{(1+x_0y_{-1}(ad+b)n)} \prod_{i=0}^{n-1} \xi_i.$$

$$y_{8n} = \frac{x_0^n y_0^{n+1}}{x_{-4}^n y_{-4}^n} \prod_{i=0}^{n-1} \chi_i.$$

$$y_{8n+1} = \frac{x_{-4}^n y_{-4}^{n+1}}{x_0^{n+1} y_0^n} \frac{x_{-3}}{(c+x_{-3}y_{-4}((bc+d)n+d))} \prod_{i=0}^{n-1} \xi_i.$$

$$y_{8n+2} = \frac{x_{-2}x_0^n y_0^{n+1}}{x_{-4}^{n+1} y_{-4}^n} \frac{(a+y_{-3}x_{-4}((ad+b)n+b))}{(c+x_{-2}y_{-3}((bc+d)n+d))} \prod_{i=0}^{n-1} \chi_i.$$

$$y_{8n+3} = \frac{x_{-1}x_{-4}^n y_{-4}^{n+1}}{x_0^{n+1} y_0^n} \frac{(a+y_{-2}x_{-3}((ad+b)n+b))}{(c+x_{-1}y_{-2}((bc+d)n+d))\,(c+x_{-3}y_{-4}((bc+d)n+d))} \prod_{i=0}^{n-1} \xi_i.$$

Where

$$\chi_i = \frac{(1+x_{-1}y_{-2}(ad+b)i)\,(1+x_{-3}y_{-4}(ad+b)i)}{(1+y_0x_{-1}(bc+d)i)\,(1+y_{-2}x_{-3}(bc+d)i)} \times \frac{(a+y_{-1}x_{-2}((ad+b)i-ad))\,(a+y_{-3}x_{-4}((ad+b)i-ad))}{(c+x_0y_{-1}((bc+d)i-bc))\,(c+x_{-2}y_{-3}((bc+d)i-bc))}.$$

$$\xi_i = \frac{(1+x_0y_{-1}(ad+b)i)\,(1+x_{-2}y_{-3}(ad+b)i)}{(1+y_{-1}x_{-2}(bc+d)i)\,(1+y_{-3}x_{-4}(bc+d)i)} \times \frac{(a+y_0x_{-1}((ad+b)i-ad))\,(a+y_{-2}x_{-3}((ad+b)i-ad))}{(c+x_{-1}y_{-2}((bc+d)i-bc))\,(c+x_{-3}y_{-4}((bc+d)i-bc))}.$$

3. Conclusion

In this study, we mainly obtained solutions to the rational difference equations system.

$$x_{n+1} = \frac{y_{n-3}x_{n-4}}{y_n(a + by_{n-3}x_{n-4})}, \quad y_{n+1} = \frac{x_{n-3}y_{n-4}}{x_n(c + dx_{n-3}y_{n-4})}, \quad n = 0, 1, \cdots,$$

where parameters a, b, c and d are executed separately and initial conditions x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_0, y_{-4}, y_{-3}, y_{-2}, y_{-1} and y_0 are non zero real numbers.

4. Future works

The results in this paper can be extended to the following system of difference equations

$$x_{n+1} = \frac{y_{n-k}x_{n-(k+1)}}{y_n(a + by_{n-k}x_{n-(k+1)})}, \quad y_{n+1} = \frac{x_{n-k}y_{n-(k+1)}}{x_n(c + dx_{n-k}y_{n-(k+1)})}, \quad n = 0, 1, \cdots,$$

where $k \in \mathbb{N}$ the initial conditions x_{-j} and y_{-j} are non zero real numbers, $j = \overline{0, (k+1)}$.

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