

**MODELING POLITICAL AND ECONOMIC RELATIONS
BETWEEN NORWAY AND RUSSIA:
A BEHAVIORAL GAME THEORY APPROACH**

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ABSTRACT. From the past until now, political and economic relations among countries have been one of the most important issues among analysts and numerous studies have tried to analyze these relations from different theoretical perspectives. The dynamic system of games has introduced a new modeling method in the game theory. In this study, we use behavioral models (level- k) along with the dynamic system in games to model rational agent behavior. As an application, we study Russia- Norway economic and political relations (1970-2019). The dynamic system in games along with behavioral games theory can be used to predict the players behavior in the future.

1. INTRODUCTION

Game theory seeks to find some kind of optimal and mathematically sensible strategy for running a game. In this theory, players consider the Nash equilibrium strategy, which is a poor predictor of human behavior in the real world [15, 16, 31]. The behavioral game theory seeks to solve the problem of selecting strategically involved players by analyzing the players behavior over a period of time or an experiment [9]. In this theory, behavioral models (cognitive hierarchy [9], quantitative hierarchy [30], and level- k [10, 11, 17]) are used to predict the players behavior in games. Players randomly start with a simple nonstrategic behavior (level-0), and then the reason for a fixed number of iterations about responses to that starting point [32]. The level-0 model makes predictions at higher levels. Higher-level players respond strategically to the behavior of lower level players (based on level-0).

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Nonstrategic players do not model other players while strategic players model other players [8, 32, 31].

In recent years, the number of international differences has increased, which can lead to dangerous political games. However, countries prefer to resolve their differences through negotiation and peaceful cooperation. This is the only solution that is beneficial for them [2, 14]. For this purpose, new modeling methods are used in a game theory called the dynamic system of strategic games [2, 12]. The game dynamics system is a dynamic model of 2×2 games and a combination of dynamic and static interactive positions. In this system, new properties of games such as action maker game, strategy-maker game and the pair of rational actions are presented and the dynamics of players behavior are studied using these properties. Action maker game is of two groups: action maker game of order (2, 2), and action maker game of order (2, 1). In addition, the games that are not action-maker itself are classified into two groups: games that have pair of rational actions or that do not have pair of rational actions [1, 12, 14]. In a dynamic system with the strategic play, rational players and strategic environment are considered. Players in this system move simultaneously and step by step and design new games. In fact, players who use the produced game conditions, action-maker, and pair of rational actions decide what moves they make in their interests and what games they design. The system ends when players are reluctant to move and design a new game. Also in this system, players can choose from strategies and a pair of rational actions that benefits the most based on their ability and future conditions, using the available information and according to their rationality and strategic preferences. Therefore, players can agree to choose the next move and choose a move that benefits the group or choose a move based on personal interests[12].

In this regard, relations between Norway and Russia since 1905 up to now have had many ups and downs. The two countries neighborhood highlights the long-standing bond between the two states. Norwegians and Russians communicated with each other even before formalizing bilateral relations in the trade known as Pomor. Informal trade routes were established in the High North, and Russian traders traveled to Norway to trade goods, including fish. This long tradition has created a strong bond between the two countries in the High North that still exists today. Several factors have influenced Norwegian-Russian relations over time. Norwegian and Russian political officials see improving relations and building interdependence based on mutual interests of both parties in long term. Therefore, the expansion of relations

between the two countries is more a function of long-term strategies than a short-term tactic and a tool in foreign policy. The study of relations between the two countries indicates that the relations between the two countries have fluctuated over the years, from cooperation and interdependence to tension and conflict. Therefore, the analysis of relations between the two countries is of particular importance and complexity [28, 20, 24]. In the following, we model and examine the political games between the two countries with dynamic system of strategic games and various level-0 behavioral models. For this purpose, we divide this time interval into eight periods, and in each period, we examine static games with complete information. In each period, each player, based on his strategic and tactical preferences and that of his opponent, considers one of the behavioral models of Maxmax, Minmax, and Maxmin and enters a new game by selecting the appropriate rational action profile. We also model economic relations between Norway and Russia using this dynamic system and behavioral models. We divide this time interval into 5 periods and in each period, we consider static games with complete information. The dynamic system of strategic games and behavioral models show that both countries are in line with the strategic preferences and behavioral models to increase national interests and expand cooperation.

2. MODELING POLITICAL RELATIONS

2.1. Cold war era. Norway and Russia have had the productive and positive relations after formalizing bilateral relations in 1905. During the Cold War, security concerns were considered in both countries. As a member of NATO and cross-border cooperation, Norway followed a reassurance policy to allay Soviet concerns about Western aggression and reduce tensions. Once World War II ended, Finnmark was liberated by the Soviet forces in the northernmost region of Norway after several years of German occupation. The Soviet forces behavior in Finnmark was an important trust-building element in neighboring relations that we have seen throughout the history. Diplomatic talks on the Barents Sea borderlines also began in the early 1970s. Therefore, in general, the cooperation between the two countries, which led to more profit, has been dominated by non-cooperation. As a result, the two countries entered the Prisoners Dilemma game g_1 . Norway is assumed as row player (player 1) and Russia (Soviet Union relations) is assumed as column player (player 2). The players set of actions includes cooperation (${}_1C$) and defect (${}_1D$). In this

game, the two countries have two actions: either to stabilize relations and cooperate (${}_1C$) or to follow their future interests and adopt non-cooperation (${}_1D$). In g_1 , dominant action ${}_1D_1$ is defect and dominated action ${}_1C_1$ is cooperation for player 1. Also, this game is the producer of a dominant action of defect ${}_1D_2$ and dominated action of cooperation ${}_1C_2$ for player 2. In other words, game g_1 is action maker of order (2, 2). The game Nash equilibrium is (${}_1D, {}_1D$). The pair of actions (${}_1C, {}_1C$) is Pareto dominant game g_1 . Thus, the pair of the rational actions of this game is (${}_1C, {}_1C$) $_{1,2}$ and (${}_1D, {}_1D$) $_{1,2}$. Both countries continuing the game can cooperate by choosing pair of actions (${}_1C, {}_1C$) $_{1,2}$ or make defect by choosing the dominant strategy of non-cooperation or a pair of actions (${}_1D, {}_1D$) $_{1,2}$. In the first period, consider the behavioral model of Maxmin g_1 game.

$$a_{i,1}^* = \arg \max_{a'_i \in A_i} \min_{a_{-i} \in A_{-i}} u_i(a'_i, a_{-i})$$

Where $a_{i,1}^*$ represent the selected action of the i player in game g_1 . player 1 with the Maxmin behavioral model chooses the ${}_1C$ cooperat strategy.

$$\begin{cases} u_1({}_1C, {}_1C) = 3 \\ u_1({}_1D, {}_1C) = 4 \Rightarrow \min\{{}_1C, {}_1D\} = u_1({}_1C, {}_1C) = 3, \end{cases}$$

$$\begin{cases} u_1({}_1C, {}_1D) = 1 \\ u_1({}_1D, {}_1D) = 2 \Rightarrow \min\{{}_1C, {}_1D\} = u_1({}_1C, {}_1D) = 1 \\ \max\{u_1({}_1C, {}_1C), u_1({}_1C, {}_1D)\} = \{3, 1\} = 3 \Rightarrow a_{1,1}^* = {}_1C \end{cases}$$

Similarly, player 2 with the Maxmin behavioral model chooses the ${}_1C$ cooperat strategy.

$$\begin{cases} u_2({}_1C, {}_1C) = 3 \\ u_2({}_1C, {}_1D) = 4 \Rightarrow \min\{{}_1C, {}_1D\} = u_2({}_1C, {}_1C) = 3, \end{cases}$$

$$\begin{cases} u_2({}_1D, {}_1C) = 1 \\ u_2({}_1D, {}_1D) = 3 \Rightarrow \min\{{}_1C, {}_1D\} = u_2({}_1D, {}_1C) = 1 \\ \max\{u_2({}_1C, {}_1C), u_2({}_1D, {}_1C)\} = \{3, 1\} = 3 \Rightarrow a_{2,1}^* = {}_1C \end{cases}$$

Both players want to have their maximum gains, and according to the players' behavioral model, the best decision is to choose a pair of rational actions (${}_1C, {}_1C$) $_{1,2}$. Once the Cold War ended, Norway sought to strengthen multilateral relations in the North, including engagement with Russia and international law relations. Norway significantly tried to establish the Barents Sea Polar European Council in 1993. The council focused on reducing the importance of security issues, reducing military tensions, and increasing trust and prosperity through cross-border economic,

cultural, environmental, health, and educational activities. So, Russia has had better relations with Norway than many Western European countries in two decades of the collapse of the Soviet Union. It can be said that Norway and Russia have been at peace for more than 1000 years. In this game, two countries with rational actions pair $({}_1C, {}_1C)_{1,2}$ entered Mixed Harmony game g_2 . In this game, Norway has two actions: changing its approach to building trust, reducing military tension, strengthening cooperation in creating a polar European Council (${}_2C$), or thinking about its interests and non-cooperation (${}_2D$). Russia has two actions in the game, either following a cooperation-seeking policy and developing relations (${}_2C$) or following non-cooperation (${}_2D$). In g_2 , the dominant action ${}_2C_1$ is cooperation and the dominated action ${}_2D_1$ is defect for player 1. Also, this game produces a dominant action of cooperation ${}_2C_2$ and dominated action of defect ${}_2D_2$ for player 2. In other words, game g_2 is action maker of order (2, 2). In g_2 game, the behavioral model of the players is Maxmax. The player who chooses the rational action profile with the Maxmax behavioral model tends to achieve his maximum gain.

$$a_{i,2}^* = \arg \max_{a'_i \in A_i} \max_{a_{-i} \in A_{-i}} u_i(a'_i, a_{-i})$$

Where $a_{i,2}^*$ represent the selected action of the i player in game g_2 . With the Maxmax behavioral model, the action selected by player 1 and player 2 is as follows:

$$\begin{cases} u_1({}_2C, {}_2C) = 4 \\ u_1({}_2D, {}_2C) = 1 \Rightarrow \max\{{}_2C, {}_2D\} = u_1({}_2C, {}_2C) = 4, \end{cases}$$

$$\begin{cases} u_1({}_2C, {}_2D) = 3 \\ u_1({}_2D, {}_2D) = 2 \Rightarrow \max\{{}_2C, {}_2D\} = u_1({}_2C, {}_2D) = 3 \\ \max\{u_1({}_2C, {}_2C), u_1({}_2C, {}_2D)\} = \{4, 3\} = 4 \Rightarrow a_{1,2}^* = {}_2C \end{cases}$$

In this game, with the Maxmax behavioral model, the best decision for the Norway (player 1) is to build trust ${}_2C$.

$$\begin{cases} u_2({}_2C, {}_2C) = 4 \\ u_2({}_2C, {}_2D) = 1 \Rightarrow \max\{{}_2C, {}_2D\} = u_2({}_2C, {}_2C) = 4 \end{cases}$$

$$\begin{cases} u_2({}_2D, {}_2C) = 3 \\ u_2({}_2D, {}_2D) = 2 \Rightarrow \max\{{}_2C, {}_2D\} = u_2({}_2D, {}_2C) = 3 \end{cases}$$

$$\max\{u_2({}_2C, {}_2C), u_2({}_2D, {}_2C)\} = \{4, 3\} = 4 \Rightarrow a_{2,2}^* = {}_2C$$

Therefore, Russia follows cooperation-seeking policy and develops relations with a behavioral model Maxmax. The game g_2 Nash equilibrium and the pair of the rational actions of this game is $({}_2C, {}_2C)_{1,2}$.

2.2. Towards cooperation. There are several examples of cooperation and bilateral agreements between the two countries. The year 2010 was a turning point in relations between the two countries, when Norway and Russia signed the Maritime Restriction and Cooperation Agreement in the Barents Sea and the Arctic Ocean, resolving an important issue in Norwegian-Russian relations. After 40 years of negotiations, the parties signed the agreement on September 15, 2010. In short, the treaty defines the border between Norway and Russia in the Barents Sea and the North Ocean. The agreement also includes agreements to continue cooperation in fisheries and the exploitation of potential hydrocarbon reserves. The same holds true for the Maritime Accident Agreement, signed by Norway and Russia in 1990. The Maritime Accident Agreement aims to prevent unintended accidents when the two parties ships and military aircraft operate outside their borders. The agreement also includes the cooperation of the Coast Guard, including search and rescue missions. The agreement was of great political value, and once decades of conflict ended, when many observers saw the North Pole as a target for intensifying geopolitical rivalry, the two main Arctic states sent a clear message to the world that any dispute in the North Pole can be resolved peacefully under international law. The domestic reaction to the agreement was quite different in the two countries. In Norway, the majority saw it as a great step forward. In fact, the country had resolved its last and largest territorial dispute with its neighbors. However, fishers in northern Norway believed that the agreement could worsen the fishing situation in the region due to increased oil and gas activity and weaker protection of fisheries. On the other hand, the agreement faced considerable doubt. The fishermen, strongly supported by the Communist faction in the Duma, emerged as Russias strongest opponents as they announced that they had lost access to the rich fishing regions in the western part of the formerly disputed area, which is now a part of the Norwegian waters.

The dominant action of the Norwegian cooperation in the game g_2 ends in a game of harmony g_3 . In this game, Norway has two actions: to sign a maritime restrictions agreement to develop relations with Russia and stabilize cooperation (${}_3C$) or non-cooperation (${}_3D$). On the other hand, Russia may cooperate in coordinating and developing relations with Norway (${}_3C$) or may not cooperate (${}_3D$). In g_3 , the dominant action ${}_3C_1$ is cooperation and dominated action ${}_3D_1$ is defect for player 1. Also, this game produces a dominant action of cooperation ${}_3C_2$ and dominated action of defect ${}_3D_2$ for player 2. In other words, game g_2 is action maker of order (2, 2). The game Nash equilibrium and the pair of the rational actions of this game

is $({}_3C, {}_3C)_{1,2}$. Both players choose the $({}_3C)$ action with the Maxmax behavioral model.

The pair of Russian rational actions $({}_2C, {}_2C)_{1,2}$ in the game g_2 leads to the coordination game g_4 . In this game, Russia has two actions: to sign a maritime restrictions agreement to develop more relations with Norway $({}_4C)$ or non-cooperation $({}_4D)$. On the other hand, Norway has two options: either remove the existing barriers and facilitate cooperation $({}_4C)$ or adopt non-cooperation $({}_4D)$. The game Nash equilibria are $({}_4C, {}_4C)$ and $({}_4D, {}_4D)$. Game g_4 is action maker of order $(2, 0)$. The players pairs of rational actions are $({}_4C, {}_4C)_{1,2}$ and $({}_4D, {}_4D)_{1,2}$. Both players choose the $({}_4C)$ action with the Maxmax behavioral model.

In the official talks between Russia and Norway in 2012, the High North was introduced as a region for peaceful cooperation. Norway considers stability and peace in the Arctic regions and good neighboring relations with Russia to depend on a partnership with NATO and the United States in the region. On the other hand, Russia sees it as a threat to continual peaceful development in the North Pole and a way to militarize Western countries. Russia sees the permanent deployment of American submarines on the coast of Norway and American flights along the Russian-Norwegian border as a threat to its country. Therefore, Russia takes action regarding military modernization and defense capabilities in the North Pole to manage resources, keep sovereignty and prepare for any possible threat, indicating the Russian governments growing ambitions in 2013. Based on rationality of players and strategic preferences, players selecting pairs of rational actions $({}_4C, {}_4C)_{1,2}$ and $({}_3C, {}_3C)_{1,2}$ end in Stag Hunt game g_5 . In this game, the two countries change their policy to develop cooperation $({}_5C)$ or choose non-cooperation $({}_5D)$. g_5 is action maker of order $(2, 0)$. The players pairs of rational actions are $({}_5C, {}_5C)_{1,2}$ and $({}_5D, {}_5D)_{1,2}$. In this game, players are at a crossroads, and the best case scenario with the Maxmax behavioral model is the choice of $({}_5C)$ action. In this period, the best decision is the pair of rational action $({}_5C, {}_5C)_{1,2}$.

2.3. Creating tension and reducing relationships. On March 18, 2014, Russian and Crimean leaders signed an agreement in Moscow that formalized the Russian annexation of Crimea after three months of violent conflict in Ukraine and the Crimean Peninsula. The Russian annexation of Crimea was a direct threat to a set of liberal values and norms (common to Norway and the West), such as respect for international law and human rights, democracy, freedom of the press, political

transparency, and multilateral cooperation. The Russian annexation of Crimea has been the Russias attempt to gain power regardless of motives, and there are conflicting arguments about it. The West (Western European countries, NATO, and the United States) argues that Russia has tremendous ambitions and seeks power to dominate the international arena. Russia argues that the expansion of NATO and the European Union, and Russias neighboring countries forced Russia to take a decisive action to protect its strategic national interests, such as keeping sea communication lines for the Russian Navy in Crimea. Defense structure realism argues that it was not wise for the West to expand its scope of influence in the vicinity of Russia and that the Russian annexation of Crimea would be a threat to both parties. In reality, military and economic capabilities are the main tools of power, and ways to balance power between states usually increase their military power by investing, strengthening, and reassuring military allies, increasing preventive measures, and so on. In 2015, the Norwegian government proposed an increase in its defense budget to improve the military capability of its country, aiming at increasing the operational capabilities of the Norwegian Armed Forces. In addition, the Norwegian government has offered to purchase new F-35 fighters to replace worn-out F-16 fighters in the future. On June 16, 2016, the Norwegian government presented a strong and sustainable long-term defense plan to improve the readiness level and strength of the Norwegian military, combat power in the High North and the regions near the Russian border, increase the Allies military presence, and strengthen NATOs collective defense capability. The stoppage of all planned military activities with Russia by the end of 2015, the imposition of sanctions against Russian trade, and, based on the EU sanctions, are other measures taken by the Norwegian government against Russia since 2014. The sanctions include financial sanctions, a ban on imports from Crimea, an arms embargo, and Russias oil industry restrictions. The Norwegian government sees the sanctions as a direct response to Russias actions in Ukraine and its violation of international law. Immediately after imposition of the EU, US, and Norwegian sanctions against Russia, Vladimir Putin, the Russian President imposed sanctions to prevent the import of Western food and agricultural products into Russia. Russias sanctions include 28 countries in the European Union, the United States, Norway, Canada, and Australia among which Norway was most affected by the sanctions. The sudden trip of Ragozin, Russian Deputy Prime Minister and Development Officer of the High North from the Archipelagoes of the Norwegian Sea (Svalbard) to the Arctic Ocean in April 2015, increased tension

between Russia and Norway. During the trip, Ragozin stressed the importance of increasing Russias presence in the North Pole, calling the North Pole as belonging to Russia. In response, Norway strengthened sanctions against Russia. Therefore, it can be said that the Norwegian-Russian tit for tat behavior is a measure of the balance of power. Since governments prefer soft power and diplomacy to military action, Norway has decided to cooperate with Russia in areas such as the Coast Guard, border guards, search and rescue operations, and the Maritime Accidents Agreement. Therefore, it seems that little emphasis has been placed on bilateral cooperation and trust-building measures in this period. Norways behavior toward the issues ended in Alibi game g_6 . In this game, Norway has two actions: it is silent in the face of the developments and tries to solve problems (${}_6C$), or it seeks to confront Russia and gain an opportunity to show off (${}_6D$). On the other hand, Russia has two actions: resolve the disputes (${}_6C$) or continue its policies regarding the regional issues, regardless of Norways concerns (${}_6D$). This game produces a dominant action of defect ${}_6D_2$ and dominated action of ${}_6C_2$ for player 2. In other words, game g_6 is action maker of order (2, 1). The game Nash equilibrium or the pair of the rational actions of this game is $({}_6D, {}_6D)_{1,2}$. In g_6 game, the behavioral model of the players is Minmax. The player who chooses the rational action profile with the Minmax behavioral model and technically to his detriment will make more profit in the future while considering the tactical and strategic preferences. In this game, with the Minmax behavioral model, the best decision for player 1 and player 2 is (${}_6D$). Rational action pair $({}_5C, {}_5C)_{1,2}$ in the game g_5 results in the Bully game g_7 . Russia has two actions in this game: either it considers Norways interests and coordinates its regional policies with Norway and prevents the escalation of tensions (${}_7C$), or it only seeks its own interests and follows its own policy (${}_7D$). Norway also has two actions: either to stay close to the West and insist on disputes that may lead to more tensions (${}_7C$) or to choose a policy of non- cooperation and tendency towards the West (${}_7D$). g_7 is strategy maker of order (2, 1). Nash equilibrium of game and the only pair of rational action for both players is $({}_7C, {}_7D)_{1,2}$. In this game, with the Maxmax behavioral model, the best decision for the Russia (player 2) is ${}_2D$.

2.4. Future prospects. In 2018, the largest NATO's measure in Norway caused anger and concern in Russia. The military exercise was designed to show defense capability to train NATO military forces to assist each of the allies during an enemy

attack. The military exercise, which took place near Russia, caused Moscow to express its anger and dissatisfaction with its holding, calling it an aggressive maneuver. NATO insists that the exercise is not related to Russia, despite tensions between the West and the East, including the Russian annexation of Crimea and the US threat to withdraw from the Nuclear Non-Proliferation Treaty. In addition, once Norway announced in 2018 that the number of the US and Norwegian Marine Corps would increase from 330 to 700 and it was deployed in the North, Russia conducted a military exercise the next day with 36 Russian warships near the Norwegian border. Norway has stopped all bilateral military activities and cooperation with Russia and instead, seeks closer cooperation with the United States as its closest ally. All of these factors indicate that Norway is strengthening its military and preventive measures to encounter the potential threat posed by Russia. Therefore, the political relations between the two countries have entered into a kind of conflict due to these measures.

Based on the players rationality and strategic preferences, players selection of pairs of rational actions $({}_6D, {}_6D)_{1,2}$ and $({}_7C, {}_7D)_{1,2}$ ends to game Chicken g_8 . In this game, the two countries have two actions: continuing cooperation $({}_8C)$ or reducing relations and non-cooperation $({}_8D)$. The game Nash equilibrium is $({}_8C, {}_8D)$ and $({}_8D, {}_8C)$. g_8 is action maker of order $(2, 0)$. In other words, this game is not action maker for players. Both players choose the $({}_8C)$ action with the maxmin behavioral model. The players pairs of rational actions are $({}_8C, {}_8C)_{1,2}$, $({}_8C, {}_8D)_2$ and $({}_8D, {}_8C)_1$. At this level, according to the players behavioral model, the only pair of rational actions for both players is $({}_8C, {}_8C)_{1,2}$. The dynamic system of game with strategic games between Norway and Russia is represented by graphs in Figure1.

The history of the system is as follows:

$$\begin{aligned}
 H = & \left\{ \emptyset, \{g_1, ({}_1C, {}_1C)_{1,2}\}, \{({}_1C, {}_1C)_{1,2}, \{g_2, ({}_2C, {}_2C)_1, ({}_2C, {}_2C)_2\}\}, \right. \\
 & \{({}_2C, {}_2C)_1, \{g_3, ({}_3C, {}_3C)_{1,2}\}\}, \\
 & \{({}_2C, {}_2C)_2, \{g_4, ({}_4C, {}_4C)_{1,2}\}\}, \\
 & \{({}_3C, {}_3C)_{1,2}, ({}_4C, {}_4C)_{1,2}, \{g_5, ({}_5C, {}_5C)_{1,2}, ({}_5C, {}_5C)_{1,2}\}\}, \\
 & \{g_5, ({}_5C)_{1,2}, \{g_6, ({}_6D, {}_6D)_{1,2}\}\}, \{({}_5C, {}_5C)_{1,2}, \{g_7, {}_7D_2\}\}, \\
 & \left. \{({}_6D, {}_6D)_{1,2}, {}_7D_2, \{g_8\}\}\right\}.
 \end{aligned}$$

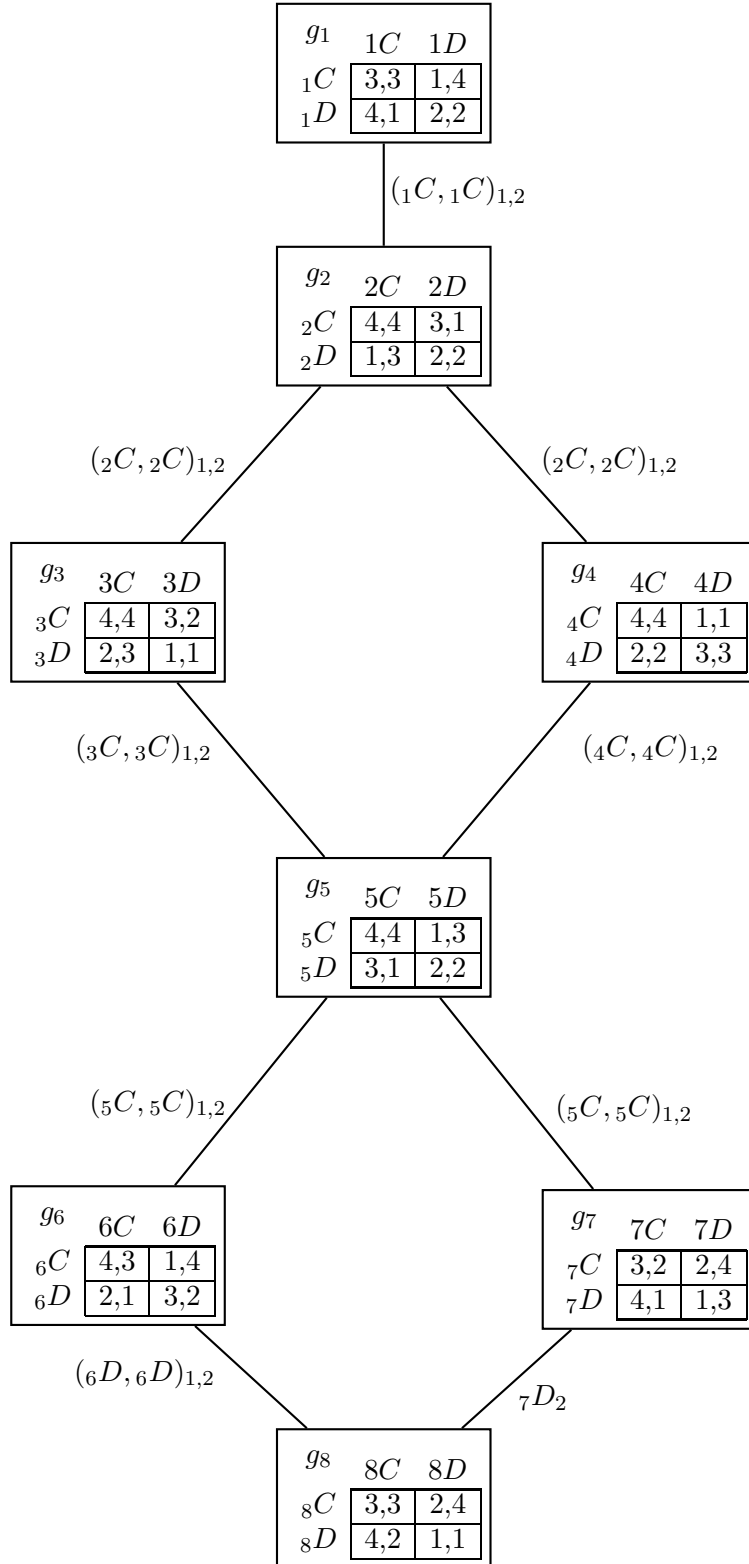


Figure 1. Modeling political relations between Norway and Russia

Therefore, it can be said that the policies followed by governments are taken from the other party's actions. Norway views Moscow's actions in Ukraine as a threat and prioritizes building a strong NATO in the North Pole. In response to Norway's actions, Moscow increases military and defense activities in the North Pole. All these measures may affect the future policy of the two countries. This weakness of not having the dominant action in game Chicken causes the players to prefer strategic preferences over tactical preferences. It seems better for both countries to fall short, which requires cooperation.

3. MODELING ECONOMIC RELATIONS

Norway and Russia are among the largest suppliers of oil and natural gas in the world, which is economically dependent on its exports. Both countries consider the Arctic suitable for significant oil and gas explorations. The Arctic is a strategic national priority for Norway and Russia, which affects how they are involved in exploitation of oil and natural gas resources and brings political and economic benefits for both countries [19]. Norway has gained access to oil and gas resources since the mid-1970s. The most important part of the Norwegian economy is the oil and gas industry and the related technologies. Based on the experience of the Middle East, Norway has taken a different path to use the oil resources and its revenues, so that the economic structure of this country has been the least dependent on oil revenues, and its economy passes a natural and normal way. The Norwegian government not only saved revenue from oil and gas sales but also invested in it. The Norwegian government created a fund called the Oil Fund in 1990. The value of this fund has always been growing rapidly since its establishment. A more significant part of Norway's revenue from non-oil products exports came from oil, and a small portion was related to the oil revenues.

The first formal talks between the Soviet Union/Russia and Norway about the Barents Sea and the Arctic Ocean began in 1974 and continued regularly (this area disputed by two parties has strained relations between the two countries). In 1976, the two countries signed a long-term fisheries cooperation agreement and established the Norwegian-Russian Joint Fisheries Commission, which set the quota for fishing in the Barents Sea. In the early 2000s, economic cooperation and activities such as fishing, maritime transport, and offshore oil exploration increased in the Barents Sea and the Arctic. The development of Norwegian-Russian relations is based

on developing national interests. Therefore, the Norwegian-Russian Chamber of Commerce was established in 2003, the founders of which are Russian and Norwegian companies and organizations. This trend continued with several agreements in trade, investment, and fishery in the coming years. Russia is one of the largest producers of oil on land, and the Russians need Norwegian experience in oil production in those areas to produce oil in the High North, and the Norwegians were given offers from the Russians in this regard [25].

During this period, Norway and Russia have turned to cooperation instead of non-cooperation and want to take advantage of cooperation instead of confrontation due to the geographical neighborhood of the two countries and economic and trade relations. Thus, Norway and Russia entered the Prisoners Dilemma, g_1 (Figure 2). Both countries cooperate (${}_1C$) or do not cooperate (${}_1D$) to gain more profit in this game. The game Nash equilibrium is $({}_1D, {}_1D)$. In g_1 , dominant action ${}_1D_1$ is defect and dominated action ${}_1C_1$ is cooperation for player 1 (Norway). Also, this game produces a dominant action of defect ${}_1D_2$ and dominated action of cooperation ${}_1C_2$ for player 2 (Russia). In other words, game g_1 is the action maker of order $(2, 2)$. The pair of actions $({}_1C, {}_1C)$ is Pareto dominant of game g_1 . Thus, the pair of the rational actions of this game is $({}_1C, {}_1C)_{1,2}$ and $({}_1D, {}_1D)_{1,2}$. The behavioral model of both players in g_1 is Maxmin, with both players choosing $({}_1C)$ action with this behavioral model. Depending on the tactical preferences of the players seeking to reach an agreement, in the g_1 game, they choose the rational action $({}_1C, {}_1C)_{1,2}$.

After signing the "Maritime Delimitation and Cooperation in the Barents Sea and the Arctic Ocean" in 2010, the two countries insisted on continuing cooperation in fishery and hydrocarbon reserves, and the Russian National Oil Company received a license to use the region. Once Russias accession to the World Trade Organization was accepted in 2011, it has created more motivation and confidence for Norway.

The World Bank named the Russian economy profitable in 2013. Based on rationality of players and strategic preferences, players selecting pairs of rational action $({}_1C, {}_1C)_{1,2}$ results in Coordination game g_2 . In this game, Norway has two actions: to trust Russia (${}_2C$) to increase cooperation or not to cooperate (${}_2D$) and not to trust Russia. On the other hand, Russia has two options: either to increase its economic interests and benefit from its geographical position in developing cooperation with Norway (${}_2C$) or to avoid expanding economic cooperation (${}_2D$). The game Nash equilibrium is $({}_2C, {}_2C)$ and $({}_2D, {}_2D)$. g_2 is action maker of order $(2, 0)$. The players pairs of rational actions are $({}_2C, {}_2C)_{1,2}$ and $({}_2D, {}_2D)_{1,2}$. Both players choose

the $({}_2C)$ action with the Maxmax behavioral model or choose the $({}_2D)$ action with the Minmax behavioral model.

Norway and Russias economies are dependent on oil and gas production and exports, and the sudden drop in oil price in 2014 has affected the economies of the two countries. As oil and gas fields declined, Norway invested in maintaining its oil revenues. Therefore, the National Center for Enhanced Oil Recovery (EOR) was established in 2014, emphasizing its strengthening. Following the Annexation of Crimea to the Russian Federation in 2014 and eastern Ukraine and Russias intervention in it and the violation of international law, the Norwegian government, the European Union, and the United States decided to suspend all military and civilian cooperation and impose sanctions against trade with Russia.

These sanctions include financial sanctions, a ban on goods importation from Crimea, arms embargoes, and restrictions on Russian industry. The final goal of these sanctions is to force Russia to return Crimea to its original owners and adhere to international law. It also prevents Russia from gaining access to financial resources to strengthen its military power leading to the devaluation of the Russian ruble and rising fear of the Russian financial crisis. Immediately after the imposition of the sanctions, Russian President Vladimir Putin responded and imposed sanctions on food imports from the European Union, the United States, and Norway, among which Norway was the most affected by these sanctions. Based on rationality of players and strategic preferences, players selecting pairs of rational action $({}_2D, {}_2D)_{1,2}$ results in Conflict game g_3 . In this game, the two countries have two actions: to continue economic cooperation regardless of political tensions $({}_3C)$ or reduce economic relations and interactions and non-cooperation $({}_3D)$. The game Nash equilibrium is $({}_3D, {}_3D)$. This game also produces the dominant action of non-cooperation and the dominated action of cooperation for two players. Game g_3 is the action maker of order $(2, 2)$. Both players choose the $({}_3D)$ action with the Minmax behavioral model. Thus, the pair of the rational action of this game is $({}_3D, {}_3D)_{1,2}$.

In addition to the Ukraine crisis, Russian oil and gas transit from Russia to the European Union got into trouble, forcing Russia to diversify its export routes. On the other hand, global changes in energy markets and the EUs tendency to reduce its dependence on Russian energy have led Rosneft and Gazprom to change their strategies and techniques. Therefore, the Russian companies, supported by the authorities, undertook to supply large quantities of oil and gas to China and undertook

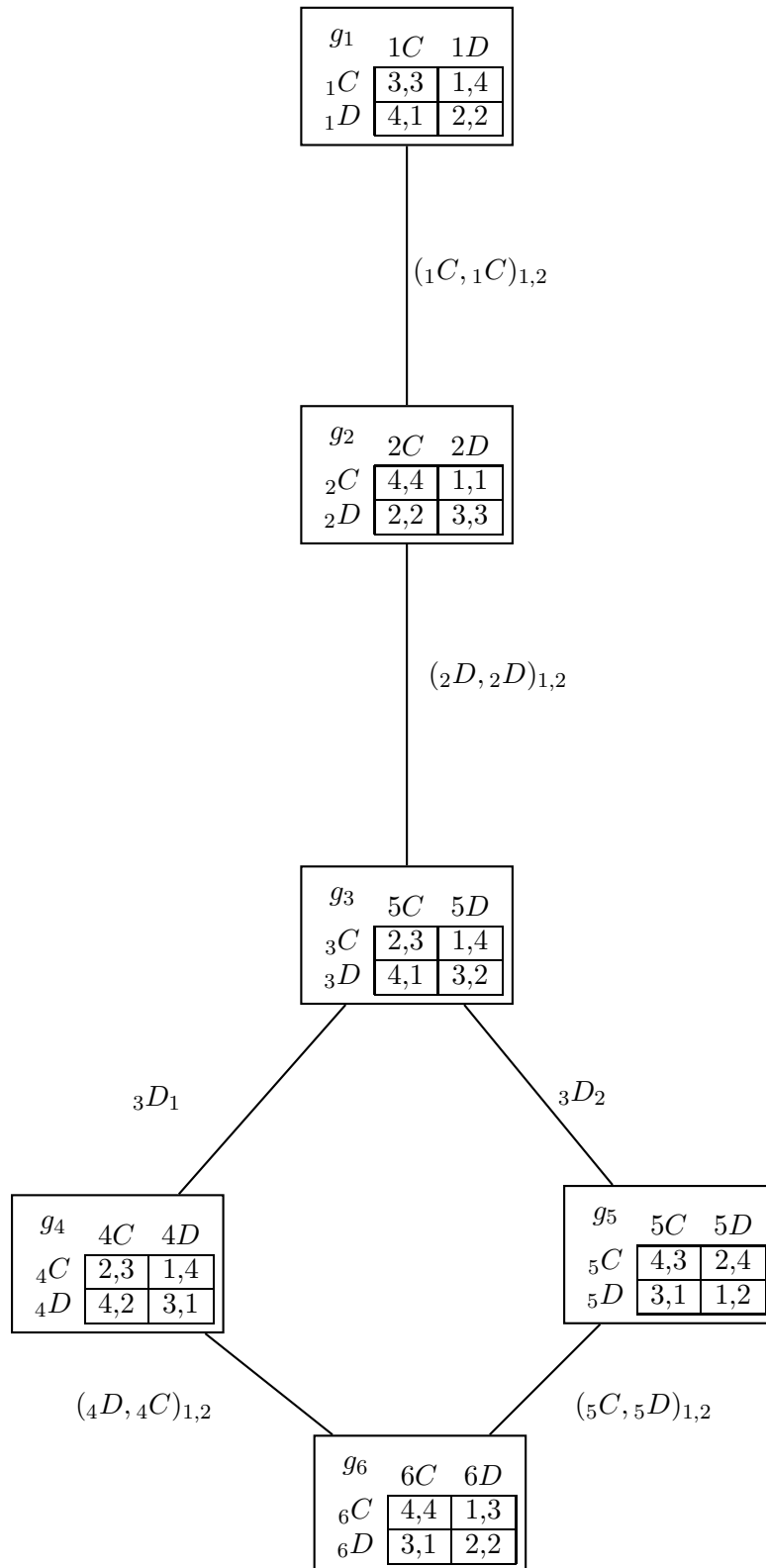


Figure 2. Modeling economic relations between Norway and Russia

several memoranda of understanding and joint venture projects with Chinese companies. Sanctions against Russia and reducing oil prices have caused problems for the Russian economy, but Russia insists on keeping military power while its economy has been damaged. Russias mutual sanctions on Norwegian food and agricultural products caused an economic loss on Norway. To compensate for the loss, Norway exported its seafood to Belarus. On the other hand, after the Annexation of Crimea to the Russian Federation, Norways defense expenditure has increased up to now. In fact, it can be argued that a possible Russian threat led Norway to increase its defense expenditure to strengthen its military power and security. Meanwhile, Norway has kept its cooperation with Russia in the Barents Sea in civilian areas such as anti-illegal fishing measures in Arctic waters, environmental management, search and rescue services, and public interaction. Russias interests are enhanced by bilateral and multilateral agreements, and cooperation favors both countries. Therefore, due to suspicion and political distrust, these trust-building measures contribute to the regions stability [24].

Under the circumstances, Norway took a non-cooperation strategy and reduced dependence on Russia. This Norwegian behavior led to the Bully Game, g_4 . In this game, Norway has two actions: cooperating with Russia (${}_4C$) or expanding relations with its neighbors and reducing dependence on Russia(${}_4D$). On the other hand, Russia has two options: either to keep the cooperation level (${}_4C$) or to reduce the trading volume(${}_4D$). The game Nash equilibrium is $({}_4D, {}_4C)$. This game produces dominant action of defect ${}_4D_1$ and dominated action of cooperation ${}_4C_1$ for player 1. This game is not an action-maker for player 2. Game g_4 is action maker of order (2,1). In this step, the Maxmax behavioral model is the best for player 2. Player 1 chooses the (${}_4D$) action with the Minmax behavioral model. Thus, the pair of the rational action of this game is $({}_4D, {}_4C)_{1,2}$.

According to the tactical preferences of player 2, action ${}_3D_2$ results in Hegemony game, g_5 . In this game, Russia has two actions: either it continues to cooperate (${}_5C$) or, based on the coming developments, it seeks to change its approach and keep its superiority by expanding cooperation with its other allies (${}_5D$). On the contrary, Norway has two options: prioritizing continued cooperation (${}_5C$) and gradually changing its approach over time or adopting a non-cooperation strategy (${}_5D$). The games Nash equilibrium is $({}_5C, {}_5D)$. This game produces dominant action of cooperation ${}_5C_1$ and dominated action of defect ${}_5D_1$ for player 1. Thus, this game produces dominant action of defect ${}_5D_2$ and dominated action of cooperation ${}_5C_2$

for player 2. Game g_5 is action maker of order $(2, 2)$. Player 1 chooses the $({}_5C)$ action with the Minmax behavioral model. Player 2 chooses the $({}_5D)$ action with the Maxmax behavioral model. Thus, the pair of the rational action of this game is $({}_5C, {}_5D)_{1,2}$.

Norway and Russia Relations have deteriorated in recent years after the Ukraine crisis, and NATO and Norway focus on deterrence of Russia. Although Norway adheres to its NATO commitments, it has preferred to cooperate with Russia in other civilian areas, and Russia must reduce geopolitical tensions to improve its economic situation. Considering the changes in the region, the strategy is that the two countries should try to increase cooperation before the period, instead of non-cooperation. Perhaps, the two countries will benefit more through economic cooperation.

Based on players rationality and strategic preferences result in game Stag Hunt g_6 players by selecting pairs of rational actions $({}_4D, {}_4C)_{1,2}$ and $({}_5C, {}_5D)_{1,2}$. In this game, the two countries have two actions: we put aside conflicts to increase national and regional interests, cooperate $({}_6C)$, or only seek to increase our interests and prevent reestablishment of relations and expansion of cooperation $({}_6D)$. The games Nash equilibrium are $({}_6C, {}_6C)$ and $({}_6D, {}_6D)$. g_6 is action maker of order $(2, 0)$. The players pairs of rational actions are $({}_6C, {}_6C)_{1,2}$ and $({}_6D, {}_6D)_{1,2}$. In this game, both players are at a crossroads, and the best case with the Maxmax behavioral model is the choice of $({}_6C)$ action. In this period, the best decision is the pair of rational actions $({}_6C, {}_6C)_{1,2}$. The history of the system is as follows:

$$H = \left\{ \emptyset, \{g_1, ({}_1C, {}_1C)_{1,2}\}, \{(1C, 1C)_{1,2}, \{g_2, ({}_2C, {}_2C)_{1,2}\}\}, \right. \\ \left. \{(2C, 2C)_{1,2}, \{g_3, {}_3D_1, {}_3D_2\}\}, \right. \\ \left. \{{}_3D_1, \{g_4, ({}_4D, {}_4C)_{1,2}\}\}, \right. \\ \left. \{{}_3D_2, \{g_5, ({}_5C, {}_5D)_{1,2}\}\}, \right. \\ \left. \{(4D, 4C)_{1,2}, ({}_5C, {}_5D)_{1,2}, \{g_6\}\} \right\}.$$

4. CONCLUSION

We used a dynamic system in games along with the behavioral game theory to study players behavior in the present study. In the dynamics system of games, players make decisions using game conditions, actions, pairs of rational actions, and design games and move in the direction of their interests. On the other hand,

thoughtful players strategically select the optimal outcomes in the behavioral game theory. In this theory, the behavioral models (cognitive hierarchy, quantitative cognitive hierarchy and level- k) are used to predict the players behavior in games. In this study, we analyze Russia-Norway economic and political relations (1970-2019) using the dynamic system of strategic games and various level-0 behavioral models, based on historical events by dividing them into different periods. In each period, we examine static games with complete information. Each player, based on his and his opponents strategic and tactical preferences, considers one of the behavioral models of Maxmax, Minmax, and Maxmin and enters a new game by selecting the appropriate rational action profile. Thus, the dynamic system in games along with behavioral games theory can be used to analyze the players behavior over a period of time and predict the players behavior in the future.

REFERENCES

1. N. Asghari & M. Eshaghi Gordji: North Korea Nuclear Crisis; Policies and Strategies. *The Pure and Applied Mathematics* **26** (2019), 133-156.
2. N. Asghari, M. Eshaghi Gordji & A. Ghaffari: Modelling US-China relationship: A game theory perspective. *International Journal of Nonlinear Analysis and Applications* **12** (2021), no. 2, 1371-1382.
3. GH. Askari & M. Eshaghi Gordji: Decision making: rational choice or hyperrational choice. *Statistics, Optimization Information Computing* **8** (2020), no. 2, 583-589.
4. GH. Askari, M. Eshaghi Gordji & C. Park: The behavioral model and game theory. *Palgrave Communications* **5** (2019), no. 1, 1-8.
5. GH. Askari, M. Eshaghi Gordji, S. Shabani & J.A. Filipe: Game Theory and Trade Tensions between Advanced Economies. *European Research Studies*, **23** (2020), 50-65.
6. S.J. Brams: *Theory of Moves*. Cambridge, Cambridge University Press, (1994).
7. B. Bruns: Visualizing the Topology of 2×2 Games: From Prisoners Dilemma to Win-Win. In *International Conference on Game Theory*, Stony Brook, NY, (2011), 11-15.
8. C.F. Camerer: *Behavioral game theory: Experiments in strategic interaction*. Princeton university press, (2011).
9. C.F. Camerer, T.H. Ho & J.K. Chong: A cognitive hierarchy model of games. *The Quarterly Journal of Economics* **119** (2004), no. 3, 861-898.
10. M. Costa-Gomes, V.P. Crawford & B. Broseta: Cognition and behavior in normal-form games. An experimental study. *Econometrica* **69** (2001), no. 5, 1193-1235.
11. V.P. Crawford, M. Costa-Gomes & N. Iriberri: Strategic thinking. *Levines Working Paper Archive* (2010).

12. M. Eshaghi Gordji & GH. Askari: Dynamic system of strategic games. *Int. J. Nonlinear Anal. Appl.* **9** (2018), no. 1, 83-98.
13. M. Eshaghi Gordji & GH. Askari: Hyper-Rational Choice and Economic Behaviour. *Advances in Mathematical Finance and Applications* **3** (2018), no. 1, 69-76.
14. M. Eshaghi Gordji, N. Asghari & M. De la Sen: Modeling Political and Economic Relations between Russia and Turkey. A Game Theory Approach. *IJCSNS.* **19** (2019), no. 2, 163.
15. J.K. Goeree & C.A. Holt: Ten little treasures of game theory and ten intuitive contradictions. *American Economic Review* **91** (2001), no. 5, 1402-1422.
16. J.H. Kagel & A.E. Roth: *The handbook of experimental economics. Volume 2*, Princeton university press, (2020).
17. R. Nagel: Unraveling in guessing games: An experimental study. *The American economic review* **85** (1995), no. 5, 1313-1326.
18. O. Morgenstern & J. Von Neumann: *Theory of games and economic behavior*. Princeton university press, (1953).
19. M. Morgunova: Why is exploitation of Arctic offshore oil and natural gas resources ongoing? A multi-level perspective on the cases of Norway and Russia. *The Polar Journal* **10** (2020), no. 1, 64-81.
20. I. Overland & A. Krivorotov: Norwegian-Russian political relations and Barents oil and gas developments. (2015).
21. A. Rapoport: A taxonomy of 2×2 games. *General systems* **11** (1966), 203-214.
22. A. Rapoport, M. Guyer & G. Gordon: *The 2×2 games*. Michigan university press, (1976).
23. D. Robinson & D. Goforth: *The topology of the 2×2 games: a new periodic table*. Psychology Press, (2005).
24. J. Sandberg: *The Beast to the East: Norways Russia-Policy After Crimea-Status Quo or a Shift in Policy?* (2018)
25. I. Skvortsova, R. Latyshev & M. Oskolkova: Cluster as a form of international cooperation in the development of the Arctic region in the framework of the international innovation system. *MATEC web of conferences. Vol. 239*. EDP Sciences, (2018).
26. N. Van Long: Dynamic games in the economics of natural resources: a survey. *Dynamic Games and Applications* **1** (2011), no. 1, 115-148.
27. J.B. Webb: *Game theory: decisions, interaction and Evolution*. Springer Science & Business Media, (2007).
28. J.M. Wilhelmsen & K.L. Gjerde: *Norway and Russia in the Arctic: New Cold War contamination?* (2018).
29. J.R. Wright & K. Leyton-Brown: Level-0 meta-models for predicting human behavior in games. In *Proceedings of the fifteenth ACM conference on Economics and computation* (2014), 857-874.

30. J.R. Wright & K. Leyton-Brown: Predicting human behavior in unrepeated, simultaneous-move games. *Games and Economic Behavior* **106** (2017), 16-37.
31. J.R. Wright & K. Leyton-Brown: Level-0 models for predicting human behavior in games. *Journal of Artificial Intelligence Research* **64** (2019), 357-383.
32. J.R. Wright & K. Leyton-Brown: A formal separation between strategic and nonstrategic behavior. In *Proceedings of the 21st ACM Conference on Economics and Computation* (2020), 535-536.
33. D. Yeung & O. Petrosian: Infinite horizon dynamic games: a new approach via information updating. *International Game Theory Review* **19** (2017), no. 04, 1750026.

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