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ERRATUM TO "STATIC AND RELATED CRITICAL SPACES WITH HARMONIC CURVATURE AND THREE RICCI EIGENVALUES" [J. KOREAN MATH. SOC. 57 (2020), NO. 6, PP. 1435–1449]

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Abstract. In this erratum, we offer a correction to [J. Korean Math. Soc. 57 (2020), No. 6, pp. 1435–1449]. Theorem 1 in the original paper has three assertions (i)-(iii), but we add (iv) after having clarified the argument.

Theorem 1 in [2] has three assertions (i)-(iii), but it needs one more assertion (iv) as below;

(iv) (V, g) is isometric to a domain in the Riemannian product of two Einstein manifolds (N_1^k, g_1) and (N_2^{n-k}, g_2) with Ricci tensors satisfying $r_{g_1} = \frac{(k-1)R}{k(n-1)} g_1$ and $r_{g_2} = \frac{R}{n-1} g_2$. Moreover, $x\frac{R}{n-1} + y(R) = 0$ and f satisfies $f'' = -\frac{R}{k(n-1)}(f + x)$. (V, g) has exactly two distinct Ricci eigenvalues at each point.

In the proof of Theorem 1 in p. 1447 of [2], non-existence of any space in the $(p, n-p)$ case was wrongly claimed. On the contrary, we shall get the assertion (iv) in Theorem 1 out of the $(p, n-p)$ case, in which we can assume that the Ricci eigenvalues satisfy $\lambda_1 = \cdots = \lambda_k \neq \lambda_{k+1} = \cdots = \lambda_n$. For the proof, the formulas (4)$\sim$(12) and the equality $\lambda_1 = -b_2 + \frac{R}{n-1}$ still hold. Now (13) should become $ap' + \frac{(n-k)}{c_n + v} = 0$. With this, we follow the subsequent argument to get $z = 0$ and $\frac{R}{n-1} = kb_2$. As λ_n is a constant, by 16.12(i) in [1], (V, g) is isometric to the Riemannian product of two Einstein manifolds (N_1^k, g_1) and (N_2^{n-k}, g_2) with Ricci tensors $r_{g_1} = \lambda_1 g_1$ and $r_{g_2} = \lambda_n g_2$. One can get $\lambda_1 = \frac{(k-1)R}{k(n-1)}$ and $\lambda_n = \frac{R}{n-1}$. From $z = 0$ and formula (9), $f'' = -\frac{R}{k(n-1)}(f + x)$. So we get the assertion (iv). Clearly, $(N_1, g_1, f + x)$ is a static space and following Example 1 of [3], $(V, g, f + x)$ belongs to the case (i) in the abstract of [2].

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