# Research on the Security Level of $\boldsymbol{\mu}^{2}$ against Impossible Differential cryptanalysis 

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#### Abstract

In the year 2020, a new lightweight block cipher $\mu^{2}$ is proposed. It has both good software and hardware performance, and it is especially suitable for constrained resource environment. However, the security evaluation on $\mu^{2}$ against impossible differential cryptanalysis seems missing from the specification. To fill this gap, an impossible differential cryptanalysis on $\mu^{2}$ is proposed. In this paper, firstly, some cryptographic properties on $\mu^{2}$ are proposed. Then several longest 7 -round impossible differential distinguishers are constructed. Finally, an impossible differential cryptanalysis on $\mu^{2}$ reduced to 10 rounds is proposed based on the constructed distinguishers. The time complexity for the attack is about $2^{69.63} 10$-round $\mu^{2}$ encryptions, the data complexity is $O\left(2^{48}\right)$, and the memory complexity is $2^{63.57}$ Bytes. The reported result indicates that $\mu^{2}$ reduced to 10 rounds can't resist against impossible differential cryptanalysis.


Keywords: Cryptanalysis, Lightweight Block Cipher, $\mu^{2}$ Block Cipher, Impossible Differential cryptanalysis

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## 1. Introduction

With the rapid development of micro devices in IoT(Internet of Things), RFID(Radio Frequency Identification), and smart card, there is a great demand for lightweight block ciphers in both scenarios of IoT[1,2] and RFID[3,4]. These lightweight block ciphers can protect the sensitive information in the devices with constrained computing capability. For lightweight block ciphers, there are many advantages such as simple structure, efficiency in both software and hardware platforms. And the design for this kind of block ciphers has been a focus for recent several years. Many good lightweight block ciphers are presented such as LBlock[5], PRESENT[6], GIFT[7], Midori[8], SIMON and SPECK[9] etc. In the year 2019, NIST proposed a standardization project LWC (LightWeight Cryptography) to enhance the development for lightweight ciphers. The lightweight block cipher $\mu 2[10]$ is proposed in the year 2020. It has both good software and hardware performance, and it is especially suitable for constrained resource environment.

After the proposal of a new block cipher, various cryptanalytic methods should be considered to evaluate the security level thoroughly, such as differential cryptanalysis[11], linear cryptanalysis[12], integral cryptanalysis[13], impossible differential cryptanalysis[14,15], zero correlation linear cryptanalysis[16]. Due to the enormous workload, some cipher designers may miss some of the cryptanalytic methods, or estimate the security level roughly which needs cryptographers to fill the gap with more refined research.

Impossible differential cryptanalysis was independently put forward by Knudsen[14] and Biham[15]. So far, it is one of the most effective cryptanalytic techniques. The basic idea of impossible differential cryptanalysis is establishing an impossible differential distinguisher, then filter the wrong key candidates with this distinguisher until the correct key is recovered. The method of impossible differential cryptanalysis has been successfully applied to many block ciphers [17-21].

## Contributions

In this paper, the main target is to evaluate the security level on $\mu^{2}$ against impossible differential cryptanalysis.

- Firstly, according to the structure of $\mu^{2}$, the diffusion property for the key schedule and some cryptographic properties for the round function are illustrated.
- Secondly, with "miss-in-the-middle" technique and an automatic approach, several longest impossible differential distinguishers are established.
- Finally, on the basis of "early-abort" technique and constructed impossible differential distinguishers, a concrete key recovery impossible differential cryptanalysis on $\mu^{2}$ reduced to 10 rounds is proposed.
The organization for this paper is as follows. The notations used in this paper and a brief introduction on $\mu^{2}$ are illustrated in section 2. Section 3 proposes some properties on $\mu^{2}$ which will be used in later cryptanalysis. Some longest impossible differential distinguishers are explored in section 4. A key recovery attack on $\mu^{2}$ is proposed in section 5 and section 6 concludes the paper.


## 2. Preliminary

### 2.1 Notations

- $\quad P$ : represents a 64 -bit plaintext;
- $C$ : represents a 64 -bit ciphertext;
- $X^{i}$ : intermediate value before the $i$-th round, which is consist of four 16 -bit words $X^{i}=\left(X_{3}^{i}, X_{2}^{i}, X_{1}^{i}, X_{0}^{i}\right)$;
- $K$ : master key;
- key $^{i}: 80$-bit key register at the $i$-th round;
- $r k^{i}$ : round key for the $i$-th round, for each round key, $r k^{i}$ can be seperated into four 16-bit parts, i.e. $\left(r k_{3}^{i}, r k_{2}^{i}, r k_{1}^{i}, r k_{0}^{i}\right)$;
- $x \| z$ : represents the relationship of concatenation for vectors $x$ and $z$.


### 2.2 Brief Description on $\boldsymbol{\mu}^{2}$

The block size for lightweight block cipher $\mu^{2}$ is 64 bits, the key length is 80 bits and the cipher has 15 rounds.

## Round Function

The structure of $\mu^{2}$ is a Type-II GFS (GFS is short for generalized Feistel Structure, see Fig. 1, The $F$ function in the GFS takes a 4-round SPN (SPN is short for substitution permutation network) structure which is illustrated in Fig. 2. Sbox is presented in Table 1 and $\pi$ bitwise permutation is defined as:

$$
\pi\left[b_{15} b_{14} \cdots b_{1} b_{0}\right]=\left[b_{3} b_{6} b_{9} b_{12} b_{7} b_{10} b_{13} b_{0} b_{11} b_{14} b_{1} b_{4} b_{15} b_{2} b_{5} b_{8}\right]
$$



Fig. 1. Structure of $\mu^{2}$


Fig. 2. Structure of $F$ function
In Fig. 2, variable const $t_{i}$ is a constant which is related to round of the SPN structure, round and position of the $F$ function. Round keys in the bracket are for the right $F$ function.

Table 1. Sbox for $\mu^{2}$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S(x)$ | C | 5 | 6 | В | 9 | 0 | A | D | 3 | E | F | 8 | 4 | 7 | 1 | 2 |

## Key Schedule

The key schedule of $\mu^{2}$ is modified from a famous block cipher PRESENT, which can be summarized as below.

Firstly, the 80-bit register is initialized with the master key. Secondly, the key register is rotated by 61 bits on the left. Thirdly, substitute the 64th to 67th bits of the key register with the Sbox. Fourthly, XOR the 15th to the 18th bits with a four-bit round counter. Finally, the 64 most significant bits of the key register are extracted as the round key $R K$, which is segmented into 4 sub-round keys ( $r k_{3}, r k_{2}, r k_{1}, r k_{0}$,

The mathematical form of this key register updating progress can be illustrated as below.
(1) $\left[k_{79}, k_{78}, \cdots k_{1}, k_{0}\right]=\left[k_{18}, k_{17}, \cdots k_{20}, k_{19}\right]$
(2) $\left[k_{79}, k_{78}, k_{77}, k_{76}\right]=S\left[k_{79}, k_{78}, k_{77}, k_{76}\right]$
(3) $\left[k_{67}, k_{66}, k_{65}, k_{64}\right]=S\left[k_{67}, k_{66}, k_{65}, k_{64}\right]$
(4) $\left[k_{18}, k_{17}, k_{16}, k_{15}\right]=\left[k_{18}, k_{17}, k_{16}, k_{15}\right] \oplus$ round counter

## 3. Cryptographic Properties for $\mu^{2}$

Property 1: The key schedule of $\mu^{2}$ has limited diffusion property. Specifically speaking, at the 12th round, there are still four bits of the key register key ${ }^{12}$ which are only affected by one bit of the master key ( $18,17,16,15$ bits of key $^{12}$ and $67,66,65,64$ bits of master key $K$ have one-to-one correspondence, At the 10th round, 12 bits of the key register key ${ }^{10}$ are only affected by one bit of the master key. Even for the last round, half of the key register (40 bits) are only affected by four bits of the master key.

The number of master key bits which affect 10th, 12th and last round's key register are illustrated in Table 2 below. This property is critical for the phase of key recovery and it is the foundation to recover the master key bits rather than round key bits.

Table 2. Number of master key bits which affect the 15th, 12th and 10th round key register bits

| Bit position of $\mathrm{key}^{15}$ | $\begin{aligned} & 7-10,26-29 \\ & 45-52,64-71 \end{aligned}$ | 11-14,30-33,53-56,72-75 |  | $\begin{gathered} 0-6,15-25,34-44 \\ 57-63,76-79 \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number of master key bits correlated | 8 | 7 |  | 4 |  |
| Bit position of key $^{12}$ | 45-48, 64-67 | $\begin{aligned} & 7-10,72-78, \\ & 49-52,68-71 \end{aligned}$ | $\begin{aligned} & 0-6,11-14,19-25 \\ & 30-44,53-63,72-79 \end{aligned}$ |  | 15-18 |
| Number of master key bits correlated | 8 | 7 | 4 |  | 1 |
| Bit position of $\mathrm{key}^{10}$ | $\begin{aligned} & 7-10,26-29, \\ & 45-48,64-67 \end{aligned}$ | $\begin{gathered} 0-6,11-14,19-25,30-33 \\ 38-44,49-52,57-63,68-79 \end{gathered}$ |  | $\begin{gathered} 15-18,34-37, \\ 53-56 \end{gathered}$ |  |
| Number of master key bits correlated | 7 | 4 |  | 1 |  |

Property 2: For any nonzero input difference of the Sbox, there exist 6.4 possible output differences on average.

Through analyzing the Sbox, it can be shown that the number of possible output differences for any nonzero input difference ranges from four to eight. And the mathematical expectation for this value is 6.4 . If input difference zero is also taken into consideration, this mathematical expectation value is reduced to 6 .

Property 3: For any nonzero input difference of one SPN round of the $F$ function, there exist 1351 possible output differences on average.

For any nonzero input difference for one SPN round, they can be classified into four circumstances, i.e. one active Sbox to four active Sboxes. So in this circumstance, the number of output differences $N$ can be calculated into the following formula:

$$
N=\frac{C_{4}^{1} \cdot\left(2^{4}-1\right) \cdot 6 \cdot 4+C_{4}^{2} \cdot\left(2^{4}-1\right) 2 \cdot 6 \cdot 4^{2}+C_{4}^{3} \cdot\left(2^{4}-1\right) 3 \cdot 6 \cdot 4^{3}+C_{4}^{4} \cdot\left(2^{4}-1\right) 4 \cdot 6 \cdot 4^{4}}{2^{16}}=88529280 / 65536 \approx 1351
$$

To validate the correctness of Property 3, a simulation process is conducted. For each input difference, the number of output difference for one SPN round of the $F$ function ranges from 4 to 4096, which is averaged to 1351 . This result coincides with the theoretical analysis.

Property 4: For any nonzero input to output difference of the $F$ function, the highest probability is $38 / 65536$, and for each input difference, there exist 25779 possible output differences on average.

The difference distribution table (DDT) for the Sbox and $F$ function is calculated. After analyzing the DDT of the $F$ function, it is found that the highest differential probability is 38/65536. Table 3 shows four highest probability differential characters in the DDT of the $F$ function. After analyzing the difference distribution table of $F$ function, it can be found that for each input difference, the number of output difference for the $F$ function ranges from

23794 to 26023, which is averaged to 25779.
Table 3. Four highest probability differential character for the $F$ function

| Input Difference | Output Difference | Differential Probability |
| :---: | :---: | :---: |
| $0 \times 0 f d 0$ | $0 \times 2505$ | $38 / 65536$ |
| $0 \times d d 00$ | $0 \times 1850$ | $34 / 65536$ |
| $0 \times 007 d$ | $0 \times 5058$ | $32 / 65536$ |
| $0 \times 0 f 70$ | $0 \times 0185$ | $32 / 65536$ |

## 4. Impossible Differential Distinguishers for $\mu^{2}$

Using "miss-in-the-middle" technique, with an automatic approach, the possibility of constructing impossible differential distinguishers on $\mu^{2}$ block cipher is investigated. All the possibilities for the modes of input and output differences are exhaustively searched, two 7 -round and ten 6-round impossible differential distinguishers are found. Table 4 illustrates these concrete distinguishers.

Here, the first 7-round impossible differential distinguisher $(0, a, 0,0) \nrightarrow(0,0, h, 0)$ is taken as an example to illustrate the constructing process for the distinguisher. The detail of the structure for the distinguisher can be explained in Fig. 3 below.

Table 4. Some longest Impossible Differential Distinguishers for $\mu^{2}$

| Length | Impossible Differential Distinguishers |
| :---: | :---: |
| 7-round | $(0, a, 0,0) \nrightarrow(0,0, h, 0)$ |
|  | $(0,0,0, a) \nrightarrow(h, 0,0,0)$ |
| 6-round | $(0,0,0, a) \nrightarrow(0,0, h, 0)$ |
|  | $(0,0,0, a) \nrightarrow(0, h, 0,0)$ |
|  | $(0,0,0, a) \nrightarrow(0, h, y, 0)$ |
|  | $(0,0, a, 0) \nrightarrow(h, 0,0,0)$ |
|  | $(0,0, a, b) \nrightarrow(h, 0,0,0)$ |
|  | $(0, a, 0,0) \nrightarrow(0,0,0, h)$ |
|  | $(0, a, 0,0) \nrightarrow(h, 0,0,0)$ |
|  | $(0, a, 0,0) \nrightarrow(h, 0,0, y)$ |
|  | $(a, 0,0,0) \nrightarrow(0,0, h, 0)$ |
|  | $(a, b, 0,0) \nrightarrow(0,0, h, 0)$ |

In Fig. 3, " $a, b, c, h, y$, $z$ " represent 16 -bit nonzero differences and "?" represents the difference for the word is unknown, the word marked in red implies a contradiction.

## 5. Impossible Differential Cryptanalysis on $\boldsymbol{\mu}^{2}$

Based on the 7-round impossible differential constructed in section 4, a key recovery attack on $\mu^{2}$ reduced to 10 rounds is proposed with adding two rounds before and one round after the distinguisher (See Fig. 4, The key recovery attack is mainly composed of two stages: data collection stage and key recovery stage.

## Data Collection Stage:

On one hand, construct $2^{m}$ data sets. For each set, word $X_{1}^{0}$ of the plaintext $P=$ $\left(X_{3}^{0}, X_{2}^{0}, X_{1}^{0}, X_{0}^{0}\right)$ are fixed to the same value and other words are arbitrary. So there are 48 bits (i.e. $X_{3}^{0}, X_{2}^{0}, X_{0}^{0}$ ) can be any distinctive values and 16 bits (i.e. $X_{1}^{0}$ ) are fixed to a constant for a data set. That is to say, for each data set, there can be about $2^{95}$ plaintext pairs ( $C_{2^{6}}^{2} \approx 2^{95}$, It is noted that when choosing a plaintext pair, $\Delta X_{3}^{0}, \Delta X_{2}^{0}, \Delta X_{0}^{0}$ should be nonzero difference. So for each data set, there are about $\left[2^{16 *}\left(2^{16}-1\right) / 2\right]^{3} \approx 2^{93}$ pairs satisfying the constraint of the input. On the other hand, suppose the difference of the output for the 10th round is $\Delta C=$ $\left(\Delta X_{3}^{10}, \Delta X_{2}^{10}, \Delta X_{1}^{10}, \Delta X_{0}^{10}\right), \Delta X_{3}^{10}, \Delta X_{0}^{10}$ should be zero difference and $\Delta X_{2}^{10}, \Delta X_{1}^{10}$ should be nonzero difference. So after sieving the difference of the ciphertexts, there are totally $2^{m+61}$ $\left(2^{m+93} \cdot 2^{-32}\right)$ plaintext-cipher pairs left.


Fig. 3. Construction of the 7-round Impossible Differential for $\mu^{2}$

During the research, it is found that if $\Delta X_{0}^{0} \rightarrow \Delta X_{3}^{0}, \Delta X_{3}^{0} \rightarrow \Delta X_{2}^{0}$ and $\Delta X_{1}^{10} \rightarrow \Delta X_{0}^{10}$ be three possible input difference to output differences for $F$ function, the efficiency of sieving the wrong subkeys can be improved. This probability for the $F$ function can be derived through Property 4, i.e. $25779 / 65536 \approx 39.3 \% \approx 2^{-1.35}$. Through this further sieving, there are $2^{m+56.95}$ $\left(2^{m+61} \cdot 2^{-1.35 * 3}\right)$ plaintext-cipher pairs left.

## Key Recovery Stage:

In the key recovery stage, "early-abort technique" is utilized to reduce the complexity, i.e. guessing the subkey bits in its smallest unit.

The details of the attack for the key recovery phase are illustrated as following three steps.

Step 1. For each plaintext-ciphertext pair after data collection stage, guess the 16 bits for the first round key $r k_{3}^{1}$. If $\Delta F\left(\Delta X_{3}^{0}, r k_{3}^{1}, r k_{2}^{1}\right)=\Delta X_{2}^{0}$, save the $r k_{3}^{1}$. The round key $r k_{2}^{1}$ needn't to be guessed because it doesn't affect the difference. According to the key schedule, these 16 bits have one-to-one correspondence with 79 to 64 bits of the master key which is illustrated in Table 5 below. For the $F$ function, each nonzero input difference can lead to 25779 output difference on average, so after this step, there are $2^{m+42.3}$ plaintext-ciphertext pairs left.

In this step, $r k_{3}^{1}$ is split into four segments at first. However, as the $F$ function is a 4-round SPN structure which makes the confusion and diffusion of the input difference quite well, it is hard to split $r k_{3}^{1}$ and guess it step by step.

Table 5. Correspondence between $r k_{3}^{1}$ and master key bits

| Number of $r k_{3}^{1}$ bit | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Relevant master key bit | 79 | 78 | 77 | 76 | 75 | 74 | 73 | 72 |
| Number of $r k_{3}^{1}$ bit | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| Relevant master key bit | 71 | 70 | 69 | 68 | 67 | 66 | 65 | 64 |

Step 2. After step 1, guess the 16 bits for the 10 th round key $r k_{1}^{10}$. If $\Delta F\left(\Delta X_{2}^{10}, r k_{1}^{10}, r k_{0}^{10}\right)=\Delta X_{1}^{10}$, save the $r k_{1}^{10}$. Similarly, the round key $r k_{0}^{10}$ needn't to be guessed. According to the key schedule, the 16 bits of $r k_{1}^{10}$ corresponds with 41 to 59 bits of the master key bits, the detailed correspondence is illustrated in Table 6 below. Accordingly, 19 master key bits: $K[59-41]$ should be guessed to derive $r k_{1}^{10}$ [15-0]. Also, for the $F$ function, each nonzero input difference can lead to 25779 output difference on average, so after this step, there are $2^{m+27.65}$ plaintext-ciphertext pairs left.

## Differential of the

 Internal State

Fig. 4. Key Recovery Attack for $\mu^{2}$

In Fig. 4, " $a, b, c, h$, $u$ " represent 16 -bit nonzero difference, the state marked in red represents an impossible differential distinguisher.

The concrete relationship between the round key bits of $r k_{1}^{10}$ and master key bits are as follows:
(1) $r k_{1}^{10}[9-6]=S(K[52-49] \oplus 0111)$
(2) $r k_{1}^{10}[5-2]=K[48-45]$
(3) $r k_{1}^{10}[1,0]=S(K[44-41])$ (two most significant bits)
(4) $r k_{1}^{10}[12,11,10]=S(K[56-53] \oplus 0011)$ (three least significant bits)
 significant bits)

Table 6. Correspondence between $r k_{1}^{10}$ and master key bits

| Number of $r k_{1}^{10}$ bit | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 53 | 53 | 53 |  |  |  |  |  |
|  | 54 | 54 | 54 | 53 | 53 | 53 | 49 | 49 |
| Relevant master key bits | 55 | 55 | 55 | 54 | 54 | 54 | 50 | 50 |
|  | 57 | 56 | 56 | 57 | 57 | 55 | 55 | 55 |
| 51 | 51 |  |  |  |  |  |  |  |
|  | 58 | 58 | 58 | 56 | 56 | 56 | 52 | 52 |
|  | 59 | 59 | 59 |  |  |  |  |  |
| Number of $r k_{1}^{10}$ bit | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|  | 49 | 49 |  |  |  |  | 41 | 41 |
| Relevant master key bits | 50 | 50 | 48 | 47 | 46 | 45 | 42 | 42 |
|  | 51 | 51 |  |  |  |  | 43 | 43 |
|  | 52 | 52 |  |  |  |  | 44 |  |

Step 3. After step 2, at most 48 round key bits, i.e. $r k_{1}^{1}, r k_{0}^{1}$ and $r k_{1}^{2}$ need to be guessed to derive the difference at the boundary of the impossible differential distinguisher. For $r k_{1}^{1}$, extra 9 bits of the master key $K[40-32]$ should be guessed. As $r k_{0}^{1} \oplus r k_{1}^{2}$ can be regarded as a whole, so 16 bits $(K[31-29], K[28] \oplus K[63], K[27] \oplus K[62], K[26] \oplus K[61]$, $K[25] \oplus K[60], K[24-16])$ should be guessed to derive $r k_{0}^{1} \oplus r k_{1}^{2}$. If the output difference for the right $F$ function of the second round equals to $\Delta X_{0}^{1}$, i.e. $\Delta F\left(\Delta X_{1}^{1}, r k_{1}^{2}, r k_{0}^{2}\right)=\Delta X_{0}^{1}$, the output difference for the right branch of the second round equals to zero after Xoring $\Delta X_{0}^{1}$, and the probability for this process is $1 / 25779$. Iterate Step 1 to Step 3 until only one correct key is left.

After the steps above, through partial encryption (two rounds) and partial decryption (one round), if all the guessed key bits are correct, the 7 -round impossible differential distinguisher in the middle will never occur. If the guessed key bits are incorrect, the impossible differential distinguisher will emerge with a fixed probability. This is the rule to judge the incorrect key from right one.

Following is the detailed relationship between the involved round key bits and master key bits in Step 3.

As $r k_{1}^{1}[15-0]=K[47-32]$, according to Step 2, seven bits (i.e. $K[47-41]$ ) have already been guessed, so only nine master key bits (i.e. K[40-32]) should be guessed here. Then as $r k_{0}^{1}[15-0] \oplus r k_{1}^{2}[15-0]=K[31-16] \oplus K[66-51]$, twelve involved master key bits ( $K[59-51]$ and $K[66-64]$ ) have already been guessed, so extra 16 bits information of the master key is needed.

Complexities of the Attack: There are altogether 60 bits information of the key which should be guessed during the attack.

Next, the number of data sets needed for the attack is illustrated. After sieving all the plaintext-ciphertext pairs for $2^{m}$ data sets, there are about $\left(2^{60}-1\right) \cdot\left(1-\frac{1}{25779} 2^{2^{m+27.65}}\right.$ wrong
key candidates left. If $m=0$, as $\left(2^{60}-1\right) \cdot\left(1-\frac{1}{25779}\right)^{2^{27.65}} \approx 0$, that is to say, almost all the wrong key candidates can be eliminated with only one data set for the attack.

- Time complexity: about $2^{69.63} 10$-round encryption (the detail is illustrated in Table 7 below,
- Data complexity: $2^{48}$ plaintext-ciphertext pairs.
- Memory complexity: about $2^{56.95} \cdot 4 \cdot 8+2^{60} .8$ Bytes $\approx 2^{63.57}$ Bytes, which mainly depends on stored plaintext-ciphertext pairs and key candidates.

Table 7. Details of the time complexity for each step

| Step | Number of master key bits guessed | Round key bits guessed | Corresponding master key bits guessed | Complexity (unit: 1-round encryption) |
| :---: | :---: | :---: | :---: | :---: |
| Step 1 | 16 | $r k_{3}^{1}$ | K[79-64] | $2^{m+56.95} \cdot 2^{16} \approx 2^{m+72.95}$ |
| Step 2 | 19 | $r k_{1}^{10}$ | K[59-41] | $2^{m+42.3} 2^{19} \approx 2^{m+61.3}$ |
| Step 3 | 25 | $r k_{1}^{1}, r k_{0}^{1} \oplus r k_{1}^{2}$ | K[40-29], K[24-16], $K[28] \oplus K[63]$, $K[27] \oplus K[62]$, $K[26] \oplus K[61]$, $K[25] \oplus K[60]$ | $2^{m+27.65 .} 2^{25} \approx 2^{m+52.65}$ |
| Total Complexity |  |  | $\approx 2^{m+69.63} 10$-round encryption |  |

## 6. Conclusion

$\mu^{2}$ is a newly proposed lightweight block cipher in 2020. However, the security evaluation for $\mu^{2}$ against impossible differential cryptanalysis seems missing from the specification. To fill this gap, an impossible differential cryptanalysis on $\mu^{2}$ is proposed. Firstly, some cryptographic properties on $\mu^{2}$ are proposed. Secondly, with an automatic approach, several longest impossible differential distinguishers are proposed. Thirdly, based on one of the longest distinguishers, a concrete impossible differential cryptanalysis on $\mu^{2}$ is proposed. The result of this paper shows that $\mu^{2}$ reduced to 10 rounds can't resist against impossible differential cryptanalysis. To enhance the security level of $\mu^{2}$ against the reported attack, more complex key schedule should be considered to shorten the length of the key recovery phase. The security level of $\mu^{2}$ against other cryptanalytic methods should also be investigated which is left as a future work.

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