

Performance Analysis for Weaker Channel User in Non-Uniform Source SSC NOMA with Novel BTS

Kyuhyuk Chung

Professor, Department of Software Science, Dankook University, Korea
khchung@dankook.ac.kr

Abstract

Recently, to improve the performance of the strongest channel gain user in non-orthogonal multiple access (NOMA) with a non-uniform source and symmetric superposition coding (SSC), a novel bit-to-symbol (BTS) mapping have been proposed. However, only the performance of the user with the stronger channel gain was analyzed. Thus, we compare the bit-error rate (BER) of the new BTS scheme with that of uniform sources, especially for the user with weakest channel gain. First, we show that the performance of the novel BTS scheme for the user with weakest channel gain also improves, compared to that of the uniform sources. Furthermore, the signal-to-noise (SNR) gain of the new BTS scheme over the uniform sources is calculated. As a consequence, the novel BTS scheme would improve the performance of the user with weakest channel gain as well as that with the stronger channel gain for SSC NOMA with a non-uniform source.

Keywords: NOMA, SSC, Lempel-Ziv coding, Superposition coding, Power allocation.

1. Introduction

Lately, a novel bit-to-symbol (BTS) mapping [1] for non-orthogonal multiple access (NOMA) [2-4] with a non-uniform source and symmetric superposition coding (SSC) [5] has been proposed, in order to improve the bit-error rate (BER) for the strongest channel gain user. Unipodal binary pulse amplitude modulation (2PAM) was studied for NOMA [6]. Asymmetric 2PAM non-SIC NOMA was investigated in [7]. Quadrature correlated superposition was proposed for NOMA [8]. Low-correlated superposition coding was studied in [9]. For now, a non-uniform source is usually produced by Lempel-Ziv coding [10]. In this paper, we compare the BER of the new BTS scheme with that of uniform sources, especially for the user with weakest channel gain. First, we show that the BER of the novel BTS scheme for the weakest channel gain user also improves greatly, compared to that of the uniform sources. Furthermore, the signal-to-noise (SNR) gain of the new BTS scheme over the uniform sources is demonstrated numerically.

The remainder of this paper is organized as follows. In Section 2, the system model is described. The BER expression for the novel BTS scheme is presented in Section 3. The numerical results are discussed in Section 4. Finally, Section 5 concludes the paper.

The main contributions of the paper are summarized as follows:

- We compare the BER of the new BTS scheme with that of uniform sources, especially for the user with weakest channel gain.
- First, we show that the performance of the novel BTS scheme for the user with weakest channel gain also improves greatly, compared to that of the uniform sources.
- Furthermore, the SNR gain of the new BTS scheme over the uniform sources is calculated, i.e., about 4 dB, at the BER of 10^{-4} .

2. System and Channel Model

There exist a base station and two users in a downlink cellular NOMA network. The complex channel coefficient is h_m , $m=1,2$, between the m th user and base station, with $|h_1| \geq |h_2|$. The superimposed signal $x = \sqrt{P_A \alpha} s_1 + \sqrt{P_A (1-\alpha)} s_2$ is transmitted to the m th user from the base station. Given the total transmitted power P of x , P_A is the average total allocated power. s_m is the signal for the m th user with the unit power, and α is the power allocation coefficient. The signal r_m received at the m th user is expressed as follows:

$$r_m = |h_m| x + n_m, \tag{1}$$

where $n_m \sim N(0, N_0 / 2)$ is additive white Gaussian noise (AWGN). It is assumed that the information bits are $b_1, b_2 \in \{0,1\}$ for the user-1 and user-2. A joint probability mass function (PMF) $P(b_1, b_2)$ is given by [11]

		$P(b_2)$		
	$P(b_1, b_2)$	$P(b_2 = 0) = \frac{1}{2}$	$P(b_2 = 1) = \frac{1}{2}$	
$P(b_1)$	$P(b_1 = 0) = 2\delta_{0,0}$	$P(b_1 = 0, b_2 = 0) = \delta_{0,0}$	$P(b_1 = 0, b_2 = 1) = \delta_{0,1} = \delta_{0,0}$	(2)
	$P(b_1 = 1) = 1 - 2\delta_{0,0}$	$P(b_1 = 1, b_2 = 0) = \delta_{1,0} = \frac{1}{2} - \delta_{0,0}$	$P(b_1 = 1, b_2 = 1) = \delta_{1,1} = \frac{1}{2} - \delta_{0,0}$	

Note that the source of the first user becomes the uniform one when $\delta_{0,0} = \frac{1}{4}$; hence, the source is non-uniformly distributed when $\delta_{0,0} > \frac{1}{4}$. We use the binary phase shift keying (BPSK) modulation, $s_1, s_2 \in \{+1, -1\}$. The existing BTS scheme is given as [5]

$$\begin{cases} s_1(b_1 = 0, b_2 = 0) = +1 \\ s_1(b_1 = 1, b_2 = 0) = -1 \end{cases} \quad \begin{cases} s_1(b_1 = 0, b_2 = 1) = -1 \\ s_1(b_1 = 1, b_2 = 1) = +1 \end{cases} \tag{3}$$

$$\begin{cases} s_2(b_2 = 0) = +1 \\ s_2(b_2 = 1) = -1 \end{cases}$$

and a novel BTS for SSC schemes is given by [1]

$$\begin{cases} s_1(b_1 = 0, b_2 = 0) = -1 \\ s_1(b_1 = 1, b_2 = 0) = +1 \end{cases} \quad \begin{cases} s_1(b_1 = 0, b_2 = 1) = +1 \\ s_1(b_1 = 1, b_2 = 1) = -1 \end{cases} \quad (4)$$

$$\begin{cases} s_2(b_2 = 0) = +1 \\ s_2(b_2 = 1) = -1 \end{cases}.$$

3. BER of Weaker Channel Gain User for Novel BTS in Non-Uniform Source SSC NOMA

Before we start this section, we should mention the main difference with [1]; this paper focuses on the performance analysis of the weaker channel gain user, even though the assumptions, such as $\delta_{0,0} > \frac{1}{4}$, are same as those in [1]. In this section, we present the BER expression of the weaker channel gain user for the novel BTS in non-uniform source SSC NOMA. Based on [1], the total average power of the novel BTS scheme is given by

$$P_A^{(\text{novel BTS})} = \frac{P}{1 + 2\sqrt{\alpha}\sqrt{(1-\alpha)}4\left(\frac{1}{4} - \delta_{0,0}\right)}. \quad (5)$$

And the BER of the weaker channel gain user for non-uniform source SSC NOMA is given by [11]

$$P_2^{(\text{optimal MAP receiver: non-uniform})} = P(b_1 = 0 | b_2 = 0)F\left(\frac{\Sigma_2 P(\sqrt{(1-\alpha)} + \sqrt{\alpha})^2}{N_0}\right) + P(b_1 = 1 | b_2 = 0)F\left(\frac{\Sigma_2 P(\sqrt{(1-\alpha)} - \sqrt{\alpha})^2}{N_0}\right). \quad (6)$$

where $\Sigma_2 = \mathbb{E}[|h_2|^2]$,

$$P(b_1 | b_2 = 0) = \frac{\delta_{b_1,0}}{P(b_2 = 0)}, \quad (7)$$

and

$$F(\gamma_b) = \frac{1}{2} \left(1 - \sqrt{\frac{\gamma_b}{1 + \gamma_b}} \right). \quad (8)$$

Thus, we obtain the exact analytical BER expression of the weaker channel gain user for the novel BTS is given by

$$\begin{aligned}
 P_2^{(\text{novel BTS})} = & \\
 P(b_1 = 0 | b_2 = 0) & F \left(\frac{\Sigma_2 P(\sqrt{(1-\alpha)} + \sqrt{\alpha})^2}{\left(1 + 2\sqrt{\alpha}\sqrt{(1-\alpha)}4\left(\frac{1}{4} - \delta_{0,0}\right)\right) N_0} \right) \\
 + P(b_1 = 1 | b_2 = 0) & F \left(\frac{\Sigma_2 P(\sqrt{(1-\alpha)} - \sqrt{\alpha})^2}{\left(1 + 2\sqrt{\alpha}\sqrt{(1-\alpha)}4\left(\frac{1}{4} - \delta_{0,0}\right)\right) N_0} \right).
 \end{aligned} \tag{9}$$

Based on the aforementioned equation, the BER improvement is limited for a near-uniform source when the BTS scheme is used in SSC NOMA systems. Thus, in the next section, in order to investigate the performance improvement of the novel BTS scheme, we choose the highly non-uniform source, i.e., $\delta_{0,0} \gg \frac{1}{4}$.

4. Numerical Results and Discussions

It is assumed that $\Sigma_1 = \mathbb{E}[|h_1|^2] = 1.8$ and $\Sigma_2 = \mathbb{E}[|h_2|^2] = 0.2$. With the noise power normalized as unit power, the transmitted total SNR is $P/N_0 = 40$ dB and $\delta_{0,0} = \frac{23}{48}$. In order to compare the BER of the novel BTS scheme to that of the uniform sources, the BERs are depicted for the novel BTS scheme and the uniform sources, in Fig. 1.

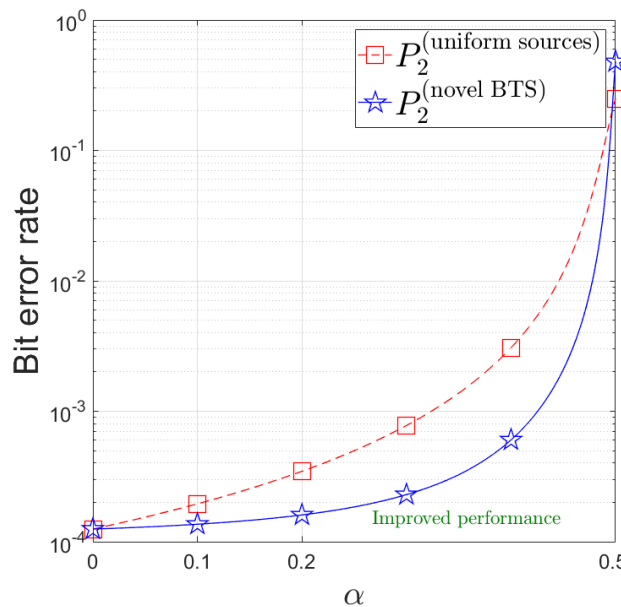


Figure 1. Comparison of BERs of SSC NOMA with uniform sources or novel BTS for a non-uniform source.

As shown in Fig. 1, the BER of the novel BTS scheme improves greatly, compared to that of the uniform sources. Remark that there are no increases of complexity and latency with the novel BTS scheme. In addition,

to simulate the various levels of the transmission power, the SNRs are given as $5000 < (1-\alpha)P/N_0 < 10000$.

Second, to show the superiority of the novel BTS scheme over the uniform sources, we depict the BERs versus the SNR, $0 \leq P/N_0 \leq 50$ (dB), with the fixed power allocation, $\alpha = 0.2$, in Fig. 2.

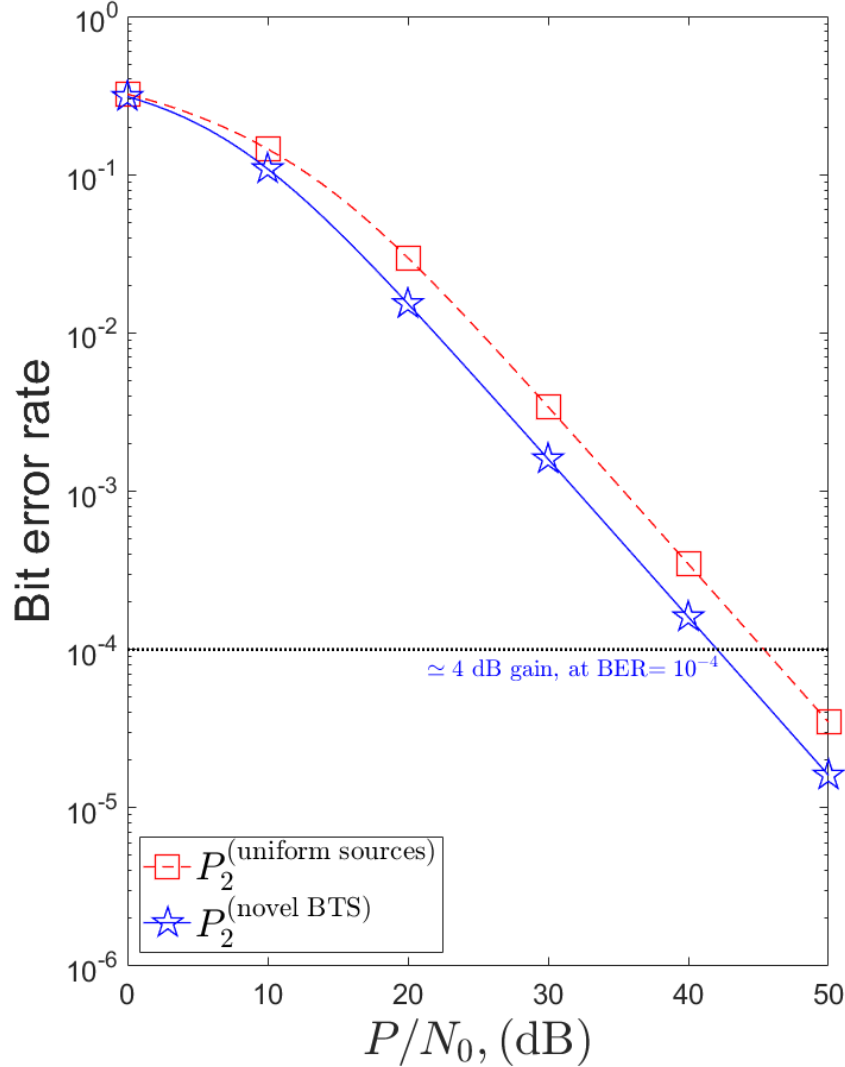


Figure 2. Comparison of BERs of SSC NOMA with uniform sources or novel BTS for a non-uniform source, with varying SNR P/N_0 .

As shown in Fig. 2, the BER of the novel BTS scheme is better than that of the uniform sources, by about 4 dB, at the BER of 10^{-4} . Remark an increase of complexity is not required for the novel BTS scheme.

5. Conclusion

In the paper, we analyzed the performance of the user with weakest channel gain for SSC NOMA with a non-uniform source. For this, we compared the BER of the new BTS scheme with that of uniform sources, especially for the user with weakest channel gain. First, we showed that the BER performance of the novel

BTS scheme for the user with weakest channel gain also improves, compared to that of the uniform sources. Furthermore, the SNR gain of the new BTS scheme over the uniform sources is calculated, i.e., about 4 dB. In result, we concluded that the novel BTS scheme improves the BER performance of the user with weakest channel gain as well as that with the stronger channel gain for SSC NOMA with a non-uniform source.

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