

# V608 CASSIOPEIAE: A W UMA-TYPE ECLIPSING BINARY WITH TWO POSSIBLE CIRCUMBINARY COMPANIONS

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**Abstract:** We present the photometric properties of V608 Cas from detailed studies of light curves and eclipse timings. The light curve synthesis indicates that the eclipsing pair is an overcontact binary with parameters of  $\Delta T = 155$  K,  $q = 0.328$ , and  $f = 26\%$ . We detected the third light  $\ell_3$ , which corresponds to about 8% and 5% of the total systemic light in  $V$  and  $R$  bands, respectively. Including our 6 timing measurements, a total of 38 times of minimum light were used for a period study. It was found that the orbital period of V608 Cas has varied in some combination of an upward parabola and two periodic variations. The continuous period increase with a rate of  $+3.99 \times 10^{-7}$  d yr $^{-1}$  can be interpreted as a mass transfer from the secondary component to the primary star at a rate of  $1.51 \times 10^{-7} M_{\odot}$  yr $^{-1}$ . The periods and semi-amplitudes of the two periodic variations are about  $P_3 = 16.0$  yr and  $P_4 = 26.3$  yr, and  $K_3 = 0.0341$  d and  $K_4 = 0.0305$  d, respectively. The most likely explanation of both cycles is a pair of light-traveling time effects operated by the possible presence of third and fourth components with estimated masses of  $M_3 = 2.20 M_{\odot}$  and  $M_4 = 1.27 M_{\odot}$  in eccentric orbits of  $e_3 = 0.66$  and  $e_4 = 0.52$ . Because the contribution of  $\ell_3$  is very low compared to the estimated masses of two circumbinary objects, they can be inferred as very faint compact objects.

**Key words:** binaries: close — binaries: eclipsing — stars: individual: V608 Cassiopeiae — techniques: photometric

## 1. INTRODUCTION

W UMa-type eclipsing binaries are generally composed of two dwarfs with similar temperature and luminosity. Both components are in contact by filling their Roche lobes, and have a common envelope. The contact binaries have evolved through angular momentum loss by magnetic braking in a detached stage, and it is thought that circumbinary object(s) may have played an important role physically and dynamically in this process (Bradstreet & Guinan 1994; Pribulla & Rucinski 2006). In reality, it can be the basis for supporting this scenario that the circumbinary objects have been found in many W UMa-type contact binary systems (D'Angelo et al. 2006; Pribulla & Rucinski 2006; Tokovinin et al. 2006; Rucinski et al. 2007; Eggleton & Tokovinin 2008; Eggleton 2009; Rappaport et al. 2013). Photometrically, circumbinary objects can be found from the periodic changes of the times of minima in the eclipse timing diagram and the detection of a third light ( $\ell_3$ ) in the light curves. To present the specific characteristics such as the orbital elements, the estimated mass, and the light contribution, a precise analysis of observational data accumulated through long-term monitoring is required.

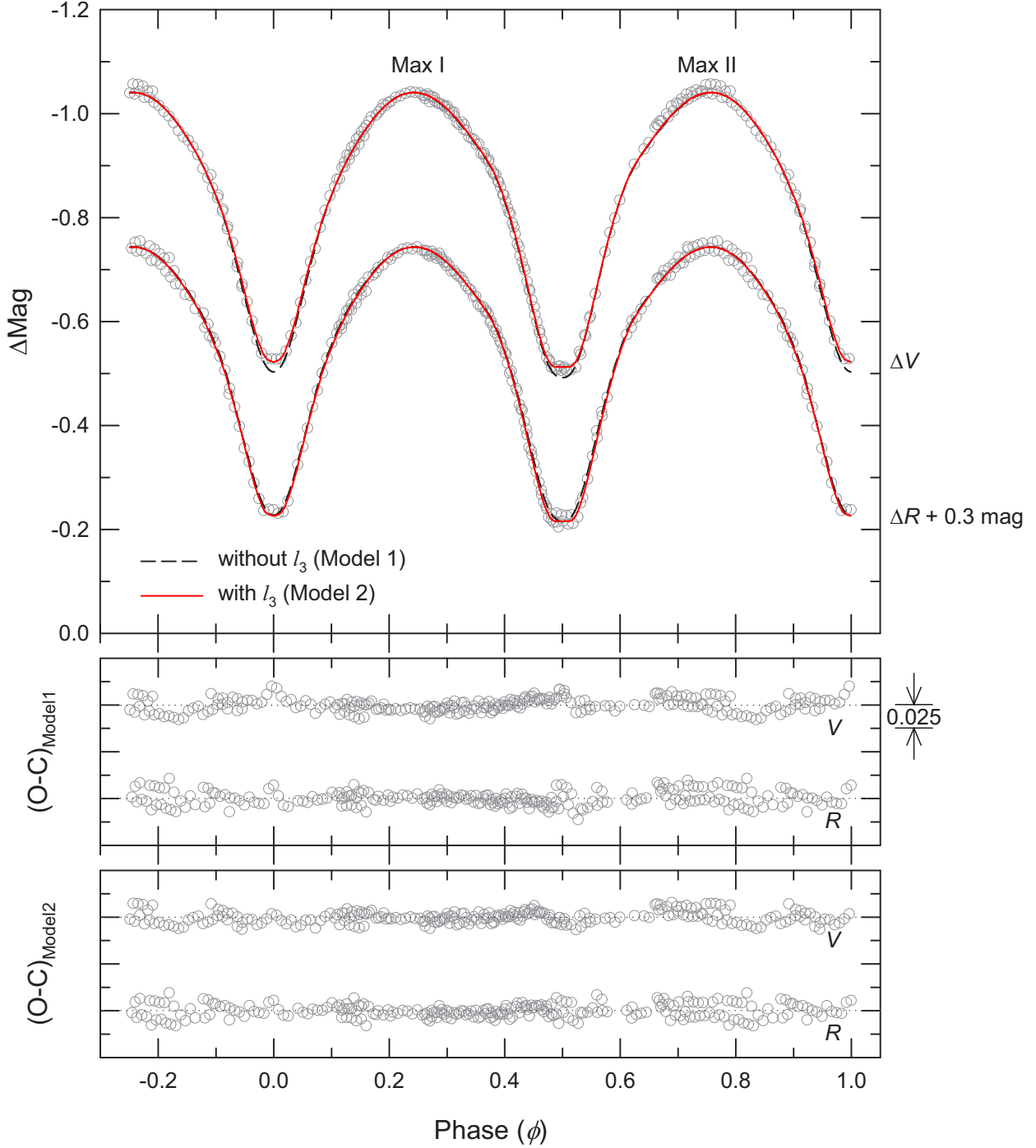
V608 Cas (GSC 4320-1035, 2MASS J02242609+7123066, Gaia DR2 545350624848618496) was discovered by Hübel (1976) as a variable star with an orbital period of 0.47 days and an amplitude of about 1.00 mag. The unfiltered light curve by Blättler & Diethelm (2001) showed the typical light variation of a W UMa-type bi-

nary with an eclipse depths of about 0.55 mag, and the most suitable orbital period for their observational data was given as 0.38 days. Liu et al. (2016) estimated the fundamental parameters of V608 Cas through an analysis of their  $VRI$  multi-band light curves. They also found a continuous increase of the orbital period at a rate of  $+6.49 \times 10^{-7}$  d yr $^{-1}$  from the eclipse timing  $O-C$  diagram, and the possibility of the presence of a sinusoidal variation was mentioned from the residuals of the quadratic term. Panpiboon et al. (2018) investigated the evolutionary status of this system through their absolute parameters.

The aim of this study is to present the physical properties of V608 Cas through our new binary modeling, and to examine the characteristics of the orbital period change newly discovered through continuous monitoring of the times of minima.

## 2. CCD PHOTOMETRIC OBSERVATIONS

New CCD photometric observations of V608 Cas were made on ten nights from November 2018 to February 2021, using an FLI 4K CCD camera and  $VR$  filters attached to the 61-cm reflector at Sobaeksan Optical Astronomy Observatory (SOAO) in Korea. The observations of the first season (2018) were carried out to secure complete multi-band light curves, and the others were conducted to collect additional eclipse timings. Time-series CCD frames were processed with the IRAF/CCDRED package to correct for flat, bias, and dark images, and aperture photometry for the stars was accomplished with the IRAF/PHOT package.



**Figure 1.** Light curves of V608 Cas in  $VR$  bandpasses as defined by individual observations. The dashed and solid curves were computed without and with  $l_3$  adjustment, respectively. The middle and bottom panels show the light residuals from the solutions of Model 1 and Model 2 listed in Table 1, respectively.

Among the stars with a color index similar to V608 Cas in the observing field, GSC 4320-0549 and GSC 4319-1090, which were stable without variations in brightness during our observing runs, were selected as comparison (C) and check stars (K), respectively.

The standard deviations of the  $(K-C)$  differences ( $1\sigma$ ) are about  $\pm 0.005$  mag in both bandpasses. As a result, we obtained a total of 411 individual points (208 in  $V$ , 203 in  $R$ ) from the 2018 season observations and they are plotted in Figure 1.

**Table 1**  
Binary parameters of V608 Cas.

Parameter	Model 1		Model 2	
	Primary	Secondary	Primary	Secondary
Photometric solutions:				
$T_0$ (HJD)	2,457,989.49765(6)		2,457,989.49765(5)	
$P_{\text{orb}}$ (day)	0.38040268(5)		0.38040268(4)	
$q$	0.317(2)		0.328(2)	
$i$ (deg)	78.5(1)		80.7(1)	
$T$ (K)	5222(324) <sup>a</sup>	5332(331)	5222(324) <sup>a</sup>	5377(334)
$\Omega$	2.462(5)	2.462(5)	2.475(3)	2.475(3)
$L_1/(L_1+L_2)_V$	0.7138(8)	0.2862	0.6966(9)	0.3034
$L_1/(L_1+L_2)_R$	0.7182(6)	0.2818	0.7030(9)	0.2970
$l_{3V}$	...		0.079(5)	
$l_{3R}$	...		0.049(5)	
$r$ (pole) <sup>b</sup>	0.4600(12)	0.2742(17)	0.4594(9)	0.2792(17)
$r$ (side) <sup>b</sup>	0.4961(16)	0.2869(21)	0.4956(12)	0.2925(21)
$r$ (back) <sup>b</sup>	0.5245(22)	0.3270(40)	0.5252(17)	0.3348(42)
$r$ (volume) <sup>c</sup>	0.4928(17)	0.2952(26)	0.4927(13)	0.3013(26)
$\sum W(O - C)^2$	0.00329		0.00299	
Absolute parameters:				
$M$ ( $M_\odot$ )	...	...	0.88(9)	0.29(3)
$R$ ( $R_\odot$ )	...	...	1.15(4)	0.70(2)
$L$ ( $L_\odot$ )	...	...	0.88(27)	0.37(12)
$a$ ( $R_\odot$ )	...		2.33(8)	

<sup>a</sup> Fixed parameter.

<sup>b</sup> Values of radii determined in each direction relative to semimajor axis ( $a$ ).

<sup>c</sup> Mean volume radius calculated from the tables of Mochnicki (1984).

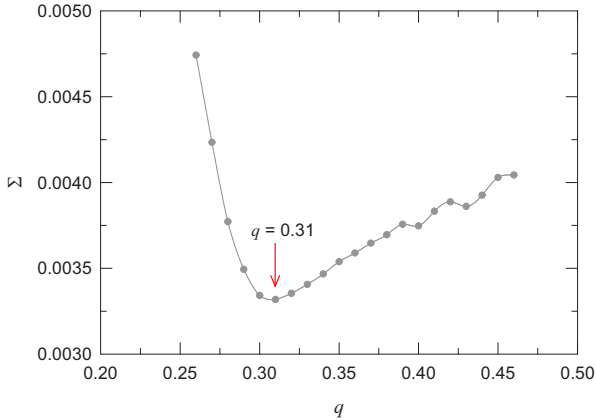
### 3. LIGHT CURVE SYNTHESIS

To obtain the photometric solutions of V608 Cas, we analyzed our  $VR$  light curves. The binary modeling was performed using the 2003 version of the Wilson-Devinney synthesis code (Wilson & Devinney 1971; van Hamme & Wilson 2003), which adequately represents the geometric and photometric distortions of the components of a binary system.<sup>1</sup> Among the modes supported by this code, mode 3 (contact binary) was adopted for synthesis. Our analyses were separately performed in two cases: the light curves were solved with basic parameters (hereafter Model 1), and were analyzed by including an additional component in basic parameters (hereafter Model 2). This process is detailed in the next paragraph. In the modeling, it was found that the less massive component was completely occulted by the heavier and larger component at the secondary minimum. Therefore, we refer to the more massive component as the primary star for this system and fixed its temperature to be  $T_1 = 5222 \pm 324$  K taken from the *Gaia* DR2 (Gaia Collaboration et al. 2018), where the error indicates the typical uncertainty of this catalogue. The gravity-darkening exponents ( $g$ ) and the bolometric albedos ( $A$ ) were adopted to be 0.32 and 0.5, respectively (Lucy 1967; Rucinski 1969), because

they should have convective atmospheres, according to the temperature of both components presented in Table 1. The linear bolometric ( $X, Y$ ) and monochromatic ( $x, y$ ) limb-darkening coefficients were initialized from the values of van Hamme (1993). A reliable mass ratio ( $q$ ) of V608 Cas through spectroscopic observations has not yet been reported. Therefore, we used the  $q$ -search method to find a suitable photometric mass ratio of the system. In this process, the orbital inclination ( $i$ ), the temperature of the secondary star ( $T_2$ ), the dimensionless surface equipotential ( $\Omega_1 = \Omega_2$ ), and the monochromatic luminosity of the primary star ( $L_1$ ) were considered as adjustment parameters. The  $q$ -search result is shown in Figure 2, where the arrow indicates the global minimum of the weighted sum of squared residuals ( $\Sigma$ ) achieved at  $q = 0.31$ . We included the  $q$  value as a adjustable parameter in subsequent analyses.

The light curves of Liu et al. (2016) and Panpi-boon et al. (2018) displayed the O’Connell effect in which Max II ( $\phi = 0.75$ ) was brighter than Max I ( $\phi = 0.25$ ), presumably due to dark spots on the massive primary star. However, as shown in Figure 1, ours did not show difference between Max I and Max II. First, we analyzed the light curves with only basic parameters of  $i$ ,  $T_2$ ,  $\Omega$ ,  $q$ , and  $L_1$ . The solution is listed in the Model 1 column of Table 1, and are presented as dashed curves in the top panel of Figure 1. The corresponding light residuals are plotted in the middle panel.

<sup>1</sup>More information is available at: <ftp://ftp.astro.ufl.edu/pub/wilson/lcdc2003/ebdoc2003.2feb2004.pdf>. gz



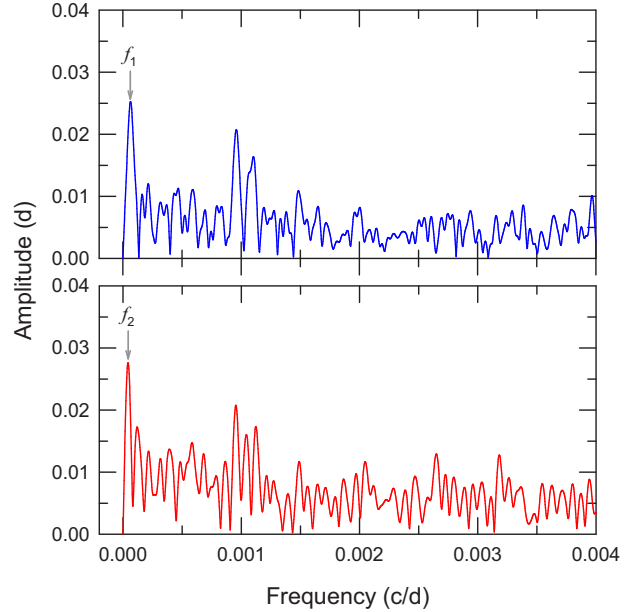
**Figure 2.** Behavior of  $\Sigma$  (the weighted sum of squared residuals) as a function of mass ratio  $q$ , indicating a minimum value at  $q = 0.31$  for V608 Cas.

As shown in this figure, the model light curves do not fit well both of the eclipse minima, especially  $V$  light curve. In this case, the discordance of the eclipse depths may be caused by either a third light ( $\ell_3$ ) dependent on the bandpasses (e.g. Ogloza et al. 1998 for SW Lyn; Lee et al. 2006 for XZ Leo; Lee et al. 2014 for V407 Peg; Tian & Zhu 2019 for V723 Per), or time-varying spot activity on the components (e.g. Kang et al. 2002 for TY UMa; Qian & Yang 2005 for FG Hya). From the eclipse timing analyses, we detected the possible presence of circumbinary objects around V608 Cas (see Section 4). Thus, the light curves were analyzed by including the  $\ell_3$  in basic parameters ( $i$ ,  $T_2$ ,  $\Omega$ ,  $q$ , and  $L_1$ ). The results are given in the Model 2 column of Table 1. The synthetic solid curves are presented in the top panel of Figure 1, and the light residuals from this solution are plotted in the bottom panel. From these results, we can see that Model 2 fits the observations better than Model 1 and gives a smaller value of  $\Sigma$ . On the other hand, we could not obtain satisfactory results for our light curves with the applicable spot models. Thus, we adopted the Model 2 as an optimal model for the contact binary system.

Our photometric solutions indicate that V608 Cas is an overcontact binary with parameters of  $i = 80^\circ.7$ , temperature difference ( $\Delta T$ ) = 155 K,  $q = 0.328$ , and fill-out factor ( $f$ ) = 26%. The  $\ell_3$  values contribute to 7.9% in  $V$ , and 4.9% in  $R$  of the total systemic light. The absolute parameters for each component were calculated by applying the parameters of Model 2 to the equation of Mardirossian et al. (1980). The mass of the primary star was estimated from the Pecaut & Mamajek (2013) relation between temperature and mass. The results are listed in the lower part of Table 1.

#### 4. ORBITAL PERIOD STUDY

From our observations, a total of six new times of minimum light were determined with the method of Kwee & van Woerden (1956). Then, 32 timings were collected from Kreiner et al. (2001) database and recent



**Figure 3.** Periodogram from the PERIOD04 formalism for the timing residuals from the quadratic ephemeris. Two frequencies of  $f_1 = 0.0000650$  cycle  $d^{-1}$  and  $f_2 = 0.0000426$  cycle  $d^{-1}$  were detected by a pre-whitening process, as indicated by the arrows in the upper and lower panels. These frequencies in turn correspond to periods of 16.0 yr and 24.4 yr, respectively.

literature. These timings are listed in Table 2. For the period analysis, the weights for the timings were derived from the inverse squares of the standard deviations ( $\pm 0.0007$  d) of the full  $O-C$  residuals.

The orbital period of V608 Cas shows a tendency to increase continuously in the form of an upward parabola. A parabolic variation can be represented as  $C = T_0 + PE + AE^2$  by adding a quadratic term ( $AE^2$ ) to a linear ephemeris ( $T_0 + PE$ ). Therefore, we fitted all times of minimum light to this quadratic ephemeris. Interestingly, sine-like variations were seen in the timing residuals. Using the PERIOD04 (Lenz & Breger 2005) program, which can extract individual frequencies from the multi-periodic content of an astronomical time series containing gaps, we checked whether the residuals actually represent periodic variations. As a result, as shown in Figure 3, the two dominant frequencies were sequentially detected at  $f_1 = 0.0000650$  cycle  $d^{-1}$  and  $f_2 = 0.0000426$  cycle  $d^{-1}$  corresponding to about 16.0 yr and 24.4 yr, respectively. The periodic variations suggest light-traveling time (LTT) effects operated by the presence of additional components orbiting the eclipsing pair. Thus, the timing data were fitted to the following quadratic *plus* two-LTT ephemeris:

$$C = T_0 + PE + AE^2 + \tau_3 + \tau_4 \quad (1)$$

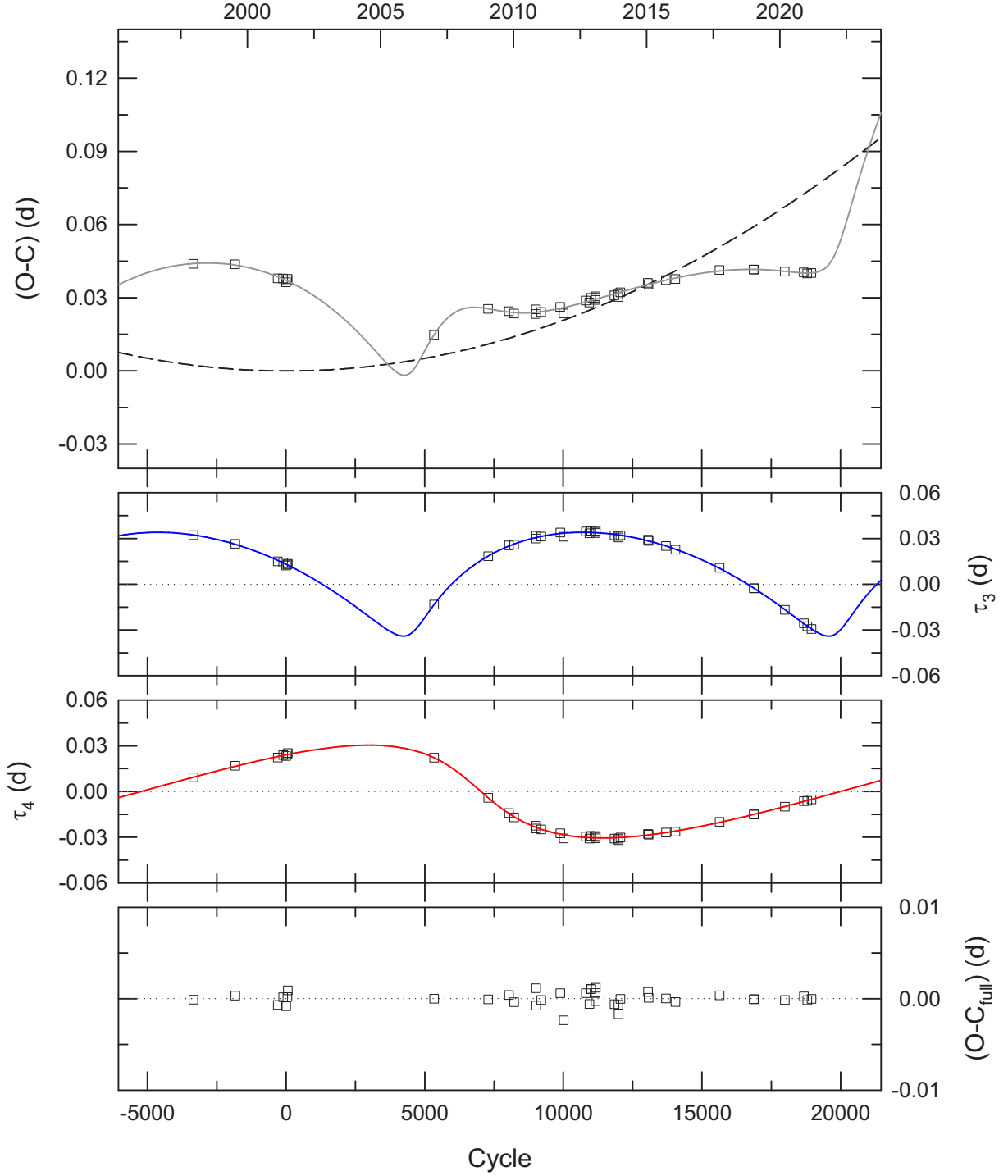
where  $\tau_3$  and  $\tau_4$  are the LTT due to circumbinary objects in the system (Irwin 1952, 1959), and each include five parameters ( $a_{12} \sin i_{3,4}$ ,  $e$ ,  $\omega$ ,  $n$  and  $T$ ). The

**Table 2**  
 CCD timings of the minimum light for V608 Cas.

HJD (2,400,000+)	Error	Epoch	$O-C_{\text{full}}$	Min	References
50,769.64		-3343.5	-0.000109	II	Cook (1999)
51,343.668		-1834.5	+0.000336	II	Liu et al. (2016, Paschke)
51,926.4397	$\pm 0.0009$	-302.5	-0.000671	II	Blättler & Diethelm (2001)
52,001.3790	$\pm 0.0008$	-105.5	+0.000167	II	Blättler & Diethelm (2001)
52,041.5100	$\pm 0.0008$	0.0	-0.000820	I	Blättler & Diethelm (2001)
52,058.4387	$\pm 0.0007$	44.5	+0.000178	II	Blättler & Diethelm (2001)
52,065.4768	$\pm 0.0009$	63.0	+0.000919	I	Blättler & Diethelm (2001)
54,071.3191	$\pm 0.0024$	5336.0	+0.000003	I	Hübscher & Walter (2007)
54,812.7353	$\pm 0.0005$	7285.0	-0.000062	I	Diethelm (2009)
55,100.8897	$\pm 0.0006$	8042.5	+0.000406	II	Diethelm (2010)
55,170.6927	$\pm 0.0002$	8226.0	-0.000370	I	Nelson (2010)
55,473.3031	$\pm 0.0016$	9021.5	-0.000745	II	Hübscher (2011)
55,473.4952	$\pm 0.0021$	9022.0	+0.001153	I	Hübscher (2011)
55,542.7275	$\pm 0.0002$	9204.0	-0.000131	I	Diethelm (2011)
55,804.4469	$\pm 0.0006$	9892.0	+0.000618	I	Hübscher & Lehmann (2012)
55,850.8534	$\pm 0.0006$	10014.0	-0.002357	I	Diethelm (2012)
56,155.5615	$\pm 0.0002$	10815.0	+0.000606	I	Honkova et al. (2013)
56,203.8719	$\pm 0.0003$	10942.0	-0.000578	I	Diethelm (2013)
56,226.3175	$\pm 0.0005$	11001.0	+0.001055	I	Liu et al. (2016)
56,226.1273	$\pm 0.0003$	11000.5	+0.001059	II	Liu et al. (2016)
56,288.1322	$\pm 0.0004$	11163.5	-0.000259	II	Liu et al. (2016)
56,289.0841	$\pm 0.0005$	11166.0	+0.000625	I	Liu et al. (2016)
56,290.0357	$\pm 0.0004$	11168.5	+0.001210	II	Liu et al. (2016)
56,549.2808	$\pm 0.0003$	11850.0	-0.000590	I	Liu et al. (2016)
56,602.1572	$\pm 0.0004$	11989.0	-0.000667	I	Liu et al. (2016)
56,604.0582		11994.0	-0.001699	I	Liu et al. (2016)
56,629.1667	$\pm 0.0002$	12060.0	-0.000015	I	Liu et al. (2016)
57,007.6716	$\pm 0.0002$	13055.0	+0.000737	I	Nelson (2015)
57,016.0399	$\pm 0.0002$	13077.0	+0.000104	I	Liu et al. (2016)
57,254.9347	$\pm 0.0001$	13705.0	+0.000029	I	Nelson (2016)
57,383.5114	$\pm 0.0001$	14043.0	-0.000358	I	Hübscher (2017)
57,989.4971	$\pm 0.0010$	15636.0	+0.000373	I	Pagel (2018)
58,460.05588	$\pm 0.00008$	16873.0	-0.000057	I	SOAO (This Paper)
58,461.00690	$\pm 0.00010$	16875.5	-0.000044	II	SOAO (This Paper)
58,884.96524	$\pm 0.00007$	17990.0	-0.000134	I	SOAO (This Paper)
59,147.06264	$\pm 0.00101$	18679.0	+0.000252	I	SOAO (This Paper)
59,192.13997	$\pm 0.00018$	18797.5	-0.000144	II	SOAO (This Paper)
59,249.96141	$\pm 0.00013$	18949.5	-0.000020	II	SOAO (This Paper)

**Table 3**  
 Parameters for the quadratic *plus* two-LTT ephemeris of V608 Cas.

Parameter	Quadratic <i>plus</i> Two-LTT		Unit
$T_0$	2,452,041.4737(2)		HJD
$P$	0.38040305(2)		day
$A$	$2.08(1) \times 10^{-10}$		day
$a_{12} \sin i_{3,4}$	$9.28(7) \times 10^8$	$9.25(8) \times 10^8$	km
$e$	0.66(1)	0.52(1)	
$\omega$	297.8(6)	895.8(5)	deg
$n$	0.06169(7)	0.03752(1)	deg d <sup>-1</sup>
$T$	2,447,937(10)	2,416,274(15)	HJD
$P_{3,4}$	15.98(2)	26.27(1)	year
$K_{3,4}$	0.0341(3)	0.0305(3)	day
$f(m_{3,4})$	0.934(7)	0.342(3)	$M_{\odot}$
$M_{3,4} \sin i_{3,4}$	2.195(14)	1.267(8)	$M_{\odot}$
$dP/dt$	$3.99(2) \times 10^{-7}$		d yr <sup>-1</sup>
$dM/dt$	$1.51 \times 10^{-7}$		$M_{\odot}$ yr <sup>-1</sup>



**Figure 4.** Eclipse timing  $O-C$  diagram of V608 Cas. In the top panel, constructed with the linear terms of the quadratic *plus* two-LTT ephemeris, the dashed and solid curves represent the quadratic term and the full ephemeris, respectively. The second and third panels represent the LTT orbits of  $\tau_3$  and  $\tau_4$ , respectively. The bottom panel shows the residuals from the complete ephemeris.

thirteen parameters of the ephemeris were calculated by applying the Levenberg-Marquart method (Press 1992), and summarized in Table 3, including related quantities. These and subsequent calculations have

used our absolute parameters given in Table 1. The  $O-C$  diagram of V608 Cas is plotted in the top panel of Figure 4, where the dashed and solid curves represent the quadratic term and the full contribution,

**Table 4**  
Applegate parameters for possible magnetic activities of V608 Cas.

Parameter	Short-term ( $\tau_3$ )		Long-term ( $\tau_4$ )		Unit
	Primary	Secondary	Primary	Secondary	
$\Delta P$		1.2050		0.6557	s
$\Delta P/P$		$3.67 \times 10^{-5}$		$1.99 \times 10^{-5}$	
$\Delta Q$	$-1.87 \times 10^{50}$	$-6.17 \times 10^{49}$	$-1.02 \times 10^{50}$	$-3.36 \times 10^{49}$	$\text{g cm}^2$
$\Delta J$	$6.70 \times 10^{47}$	$3.23 \times 10^{47}$	$3.65 \times 10^{47}$	$1.76 \times 10^{47}$	$\text{g cm}^2 \text{ s}^{-1}$
$I_s$	$7.46 \times 10^{53}$	$9.11 \times 10^{52}$	$7.46 \times 10^{53}$	$9.11 \times 10^{52}$	$\text{g cm}^2$
$\Delta \Omega$	$8.98 \times 10^{-7}$	$3.54 \times 10^{-6}$	$4.89 \times 10^{-7}$	$1.93 \times 10^{-6}$	$\text{s}^{-1}$
$\Delta \Omega/\Omega$	$4.70 \times 10^{-3}$	$1.85 \times 10^{-2}$	$2.56 \times 10^{-3}$	$1.01 \times 10^{-2}$	
$\Delta E$	$1.20 \times 10^{42}$	$2.29 \times 10^{42}$	$3.57 \times 10^{41}$	$6.77 \times 10^{41}$	erg
$\Delta L_{rms}$	$7.50 \times 10^{33}$	$1.42 \times 10^{34}$	$1.35 \times 10^{33}$	$2.56 \times 10^{33}$	$\text{erg s}^{-1}$
	1.95	3.70	0.35	0.67	$L_\odot$
	2.22	10.01	0.40	1.80	$L_{1,2}$
	1.02	1.49	0.27	0.46	mag
$B$	22.6	33.1	13.0	19.0	kG

respectively. The two panels in the middle represent the LTT orbits of  $\tau_3$  and  $\tau_4$ , respectively. The bottom panel shows the residuals for the complete ephemeris, and these are listed as  $O-C_{\text{full}}$  in the fourth column of Table 2. As seen in this figure, the quadratic *plus* two-LTT ephemeris gives us satisfactory representation for the eclipse timing variation. The LTT orbits have periods of  $P_3 = 16.0$  yr and  $P_4 = 26.3$  yr, semi-amplitudes of  $K_3 = 0.0341$  d and  $K_4 = 0.0305$  d, and eccentricities of  $e_3 = 0.66$  and  $e_4 = 0.52$ , respectively. The mass functions of the circumbinary objects become  $f(m_3) = 0.934 M_\odot$  and  $f(m_4) = 0.342 M_\odot$ , and their estimated minimum masses are  $M_3 = 2.2 M_\odot$  and  $M_4 = 1.27 M_\odot$ , respectively.

The quadratic term ( $A$ ) in Equation (1) denotes a continuous period increase of  $+3.99 \times 10^{-7}$  d yr $^{-1}$ , which is a rate corresponding to a fractional period change of  $+1.09 \times 10^{-9}$ . As the most common explanation, a period increase can be interpreted as a mass transfer from the less massive secondary component to the primary star, in which the transfer rate is calculated as  $1.51 \times 10^{-7} M_\odot \text{ yr}^{-1}$ .

## 5. SUMMARY AND DISCUSSION

In this study, we presented the photometric properties of V608 Cas, through detailed analyses of our new  $VR$  light curves and the eclipse timing  $O-C$  diagram. We summarized our results as follows:

1. Our light curve synthesis shows that V608 Cas is an overcontact binary with a temperature difference of 155 K and a mass ratio of 0.328, and in a shallow contact of 26%. The absolute parameters were estimated to be  $M_1 = 0.88 M_\odot$  and  $M_2 = 0.29 M_\odot$ ,  $R_1 = 1.15 R_\odot$  and  $R_2 = 0.70 R_\odot$ , and  $L_1 = 0.88 L_\odot$  and  $L_2 = 0.37 L_\odot$ , respectively.
2. The orbital period of V608 Cas has not varied in a monotonous way but through a combination of an upward parabola and two periodic variations,

with period lengths of  $P_3 = 16.0$  yr and  $P_4 = 26.3$  yr, and semi-amplitudes of  $K_3 = 0.0341$  d and  $K_4 = 0.0305$  d, respectively. The continuous period increase of  $+3.99 \times 10^{-7}$  d yr $^{-1}$  may be caused by the mass transfer of about  $1.51 \times 10^{-7} M_\odot \text{ yr}^{-1}$  from the secondary component to the primary star.

3. The periodic variations can be interpreted as a pair of LTT effects due to the presence of two circumbinary objects around V608 Cas. If the third and fourth companions are assumed to be the main sequence stars, their minimum masses are calculated as  $M_3 = 2.20 M_\odot$  and  $M_4 = 1.27 M_\odot$ , respectively. Because the  $\ell_3$  contributions found from the light curves analysis are very low compared to the estimated masses, each additional object could be a very faint compact object.

As shown in Figure 4, all eclipse minima of V608 Cas coincide with our quadratic *plus* LTT ephemeris, but there is a possibility of periodic deformations due to magnetic activity cycle of the components (Applegate 1992; Lanza et al. 1998). Therefore, we calculated the model parameters for each component by applying the periods ( $P_{3,4}$ ) and amplitudes ( $K_{3,4}$ ) listed in Table 3 to the Applegate formula. The results are listed in Table 4, where the rms luminosity changes ( $\Delta m_{\text{rms}}$ ) were converted to magnitude scale (Kim et al. 1997). Our results are that the variations of the gravitational quadrupole moment ( $\Delta Q$ ) are two orders of magnitude smaller than typical values ( $10^{51} \sim 10^{52}$ ) for contact binaries (Lanza & Rodono 1999), and the rms luminosity variations ( $\Delta L_{\text{rms}}$ ) represent that the Applegate mechanism cannot function in both components. Consequentially, there is currently no other alternative to account for the sinusoidal variations than the LTT effects, so they are most likely caused by two circumbinary objects around the eclipsing pair. The putative circumbinary companions would have made an important contribution to the formation of the binary systems, and these

will evolve into a single star in the future through angular momentum loss.

We suggested the possibility of the circumbinary companions around V608 Cas. The eclipse timing observations spanning 23 yr have covered only 1.4 cycle and 87% of the two periodic variations 16.0 yr and 26.3 yr. Future high-precision timing measurements are important to identify the LTT effects and to understand their characteristics.

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