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가중치를 갖는 그래프신호를 위한 샘플링 집합 선택 알고리즘

김윤학*

Sampling Set Selection Algorithm for Weighted Graph Signals

Yoon Hak Kim*

요 약

그래프신호가 각각의 가중치를 갖고 발생하는 경우 그래프상의 최적의 샘플링 노드집합을 선택하는 탐욕알고리즘에 대해 연구한다. 이를 위해 가중치를 반영한 복원오차를 비용함수로 사용하고 여기에 QR 분해를 적용하여 단순한 형태로 전개한다. 이렇게 도출된 가중치 복원오차를 최소화하기 위해 다양한 수학적 증명을 통해 반복적으로 노드를 선택할 수 있는 수학적 결과식을 유도한다. 이러한 결과식에 기반하여, 노드를 선택하는 샘플링 집합 선택알고리즘을 제안한다. 성능평가를 위해 다양한 그래프에서 발생하는 가중치를 갖는 그래프신호에 적용하여 기존 샘플링 선택 기술대비, 복잡도를 유지하면서 가중치 신호의 복원성능이 우수함을 보인다.

ABSTRACT

A greedy algorithm is proposed to select a subset of nodes of a graph for bandlimited graph signals in which each signal value is generated with its weight. Since graph signals are weighted, we seek to minimize the weighted reconstruction error which is formulated by using the QR factorization and derive an analytic result to find iteratively the node minimizing the weighted reconstruction error, leading to a simplified iterative selection process. Experiments show that the proposed method achieves a significant performance gain for graph signals with weights on various graphs as compared with the previous novel selection techniques.

키워드

Graph Signal Processing, Sampling Set Selection, Greedy Algorithm, Weighted Reconstruction Error
그래프 신호 처리, 샘플링 집합 선택, 탐욕 알고리즘, 가중치 복원오차

Ⅰ. Introduction

Graphs provide a powerful tool to represent geometric, irregular structured and high-dimensional data generated from emerging applications such as neural, energy, transportation, social and sensor networks. Specifically, the data samples on the network nodes are regarded as graph signals defined on the nodes or vertices of the graph and the similarity between data samples on the two

* 교신저자 : 조선대학교 전자공학부
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• Corresponding Author : Yoon Hak Kim
Dept. Electronic Engineering, Chosun University,
Email : yhk@chosun.ac.kr

nodes are described by using the weight of the edge connecting them [1, 2].

One crucial task among signal processing techniques for such graph signals is the sampling set selection, the goal of which is to select a subset of vertices of the graph such that the original graph signals can be recovered from those signals on the sampled nodes by minimizing the reconstruction error. To this end, efficient sampling set selection techniques have been proposed in [3-6] where the worst case of the reconstruction error is minimized by taking a greedy selection strategy which allows us to choose one node at each iteration that minimizes the metric [3-5] and the local uncertainty principle can be employed to perform a non-uniform sampling which selects more samples in areas of lower uncertainty [6]. To expediate the selection process, algorithms without eigendecomposition were presented in [7, 8]. A metric related to signal variation was employed to produce a fast selection process [9]. A greedy selection algorithm based on the QR factorization was recently proposed to yield a competitive performance [10].

In this work, we consider the scenario where graph signals are generated with weights and seek to find the best sampling set that minimizes the weighted reconstruction error, not the reconstruction error previously regarded as the metric to be minimized. This scenario can happen, depending upon applications. For instance, in a certain application, some nodes of graph can be considered to generate more relevant information with respect to the application objective. In this case, higher weights should be assigned to the signal values on those nodes such that the metric with such weights should be minimized to achieve better performance. Hence, we assume that graph signals are bandlimited with weights and formulate the weighted reconstruction error as a metric to be minimized. We then seek to simplify the metric by regarding the nodes with higher weights as those with lower noise level. We further manipulate the weighted metric by applying the QR factorization and exploit the analytic results derived in [10] to present a simple criterion which enables an iterative selection process. Finally, we evaluate the performance of the proposed method through experiments and demonstrate a significant performance gain of the proposed technique over previous sampling set selection methods for various graphs with weights.

This paper is organized as follows. The problem is formulated in Section II. The proposed algorithm is explained and summarized in Section III. The performance of the proposed algorithm is demonstrated by experiments for various graphs in Section IV and the conclusion given in Section V.

II. Problem formulation

We assume that a graph $G(V,E)$ is undirected and weighted with a set of N vertices denoted by $V=\{1,\cdots,N\}$ and edges $E=\{i,j,e_{ij}\}$ where e_{ij} indicates an edge weight connecting node i with node j. A graph signal $\boldsymbol{f}=[f_1\ \cdots\ f_N]^T\in\boldsymbol{R}^{\boldsymbol{N}}$ with f_i the signal value on the i-th vertex is defined on V. Variation operators such as the combinatorial graph Laplacian, the normalized Laplacian or any suitable operators can be constructed from the connectivity of the graph to describe signal variations caused by the irregular structure of the graph [2, 4]. It is assumed that the variation operator $\boldsymbol{L}$, $N\times N$ matrix, has eigenvalues $|\lambda_1|\leq\cdots\leq|\lambda_N|$ and corresponding orthonormal eigenvectors $\boldsymbol{u}_1\cdots\boldsymbol{u}_N$. Then, the graph signal $\boldsymbol{f}$ can be represented by $\boldsymbol{f}=\boldsymbol{Uc}$ where $\boldsymbol{U}=[\boldsymbol{u}_1\cdots\boldsymbol{u}_N]$ is the eigenvector matrix and $\boldsymbol{c}=\boldsymbol{U}^{-1}\boldsymbol{f}=\boldsymbol{U}^T\boldsymbol{f}$ the graph Fourier transform (GFT) of $\boldsymbol{f}$. If the graph signal $\boldsymbol{f}$ is ω-bandlimited with its GFT entries $c_j=0,|\lambda_j|>\omega,\ \forall_{j>r}$, then

$$\boldsymbol{f}=\sum_{i=1}^{r} c_i \boldsymbol{u_i}=\boldsymbol{U_{VR}}\boldsymbol{c_R} \tag{1}$$

where $\boldsymbol{c_R}$ is an $r\times 1$ column vector with the entries of $\boldsymbol{c}$ indexed by $R=\{1,...,r\}$ and $\boldsymbol{U_{VR}}$ a submatrix of the eigenvector matrix $\boldsymbol{U}$ with rows indexed by V and columns by R, respectively.

For the space of the ω-bandlimited signals, a sampling set S is defined as the uniqueness set if the ω-bandlimited noise-free graph signal $\boldsymbol{f}$ can be perfectly reconstructed from the sampled signal $\boldsymbol{f_S}$ with signal values $f_j, j\in S$ and the uniqueness set can be constructed by choosing r independent row vectors of $\boldsymbol{U_{VR}}$: that is, the j-th node is selected by choosing the j-th row [4]. Hence, construction of the $r\times r$ matrix $\boldsymbol{U_{SR}}$ consisting of r independent rows of $\boldsymbol{U_{VR}}$ indexed by the sampling set S produces the sampled signal $\boldsymbol{f_S}$ expressed by $\boldsymbol{f_S}=\boldsymbol{U_{SR}}\boldsymbol{c_R}$. Then the least square estimate for $\boldsymbol{c_R}$ is given by $\hat{c}_R=\left(\boldsymbol{U_{SR}^T}\boldsymbol{U_{SR}}\right)^{-1}\boldsymbol{U_{SR}^T}\boldsymbol{f_S}$ and the reconstructed signal $\hat{\boldsymbol{f}}$ can be given by

$$\hat{\boldsymbol{f}}=\boldsymbol{U_{VR}}\hat{c}_R=\boldsymbol{U_{VR}}\boldsymbol{U_{SR}^+}\boldsymbol{f_S} \tag{2}$$

where $\boldsymbol{U_{SR}^+}=\left(\boldsymbol{U_{SR}^T}\boldsymbol{U_{SR}}\right)^{-1}\boldsymbol{U_{SR}^T}$ is the pseudo-inverse of $\boldsymbol{U_{SR}}$ and $\hat{\boldsymbol{f}}=\boldsymbol{f}$ for noise-free bandlimited graph signals $\boldsymbol{f}$.

In this work, we assume that the graph signal $\boldsymbol{f}$ is ω-bandlimited and the signal value f_i at the i-th node is weighted by $w_i>0$. Then, we seek to minimize the weighted reconstruction error denoted by MSE_W

$$MSE_W=\sum_{i=1}^{N} w_i^2\left|\hat{f}_i-f_i\right|^2 \tag{3}$$

where $\hat{\boldsymbol{f}}=[\hat{f}_1,...,\hat{f}_N]^T$ is obtained by (2). Instead of directly minimizing MSE_W, we formulate the new metric by regarding signal values with higher weight as those corrupted by lower noise level: specifically, we assume that signal value $f_i, i\in V$ is corrupted by an additive uncorrelated noise $\boldsymbol{n}$ with zero mean and the variance σ_i^2 equal to the function $g(w_i^2)$ which is determined to take values inversely proportional to the weight w_i^2. Then we obtain the error vector from (2):

$$\boldsymbol{e}=\hat{\boldsymbol{f}}-\boldsymbol{f}=\boldsymbol{U_{VR}}\boldsymbol{U_{SR}^+}(\boldsymbol{f_S}+\boldsymbol{n_S})-\boldsymbol{f}=\boldsymbol{U_{VR}}\boldsymbol{U_{SR}^+}\boldsymbol{n_S} \tag{4}$$

where $\boldsymbol{n_S}$ is the $|S|\times 1$ column vector with entries of $\boldsymbol{n}$ indexed by S. Note that the function $g(w^{2_i})$ can be experimentally determined, depending upon the type of graphs.

Hence, the reconstruction mean squared error (MSE) is given by

$$MSE=tr\left[E\boldsymbol{e}\boldsymbol{e^T}\right] \tag{5}$$

$$=tr\left[\boldsymbol{U_{VR}}\boldsymbol{U_{SR}^+}E\boldsymbol{n_S}\boldsymbol{n_S^T}(\boldsymbol{U_{SR}^+})^T\boldsymbol{U_{VR}^T}\right] \tag{6}$$

$$=tr\left[\boldsymbol{U_{VR}}\boldsymbol{U_{SR}^+}\boldsymbol{C_S}(\boldsymbol{U_{SR}^+})^T\boldsymbol{U_{VR}^T}\right] \tag{7}$$

$$=tr\left[(\boldsymbol{U_{SR}^T}\boldsymbol{C_S^{-1}}\boldsymbol{U_{SR}})^+\right] \tag{8}$$

where (8) follows from the notion that $\boldsymbol{U_{VR}}$ has orthonormal columns and $\boldsymbol{C_S}=E\boldsymbol{n_S}\boldsymbol{n_S^T}$ is the diagonal covariance matrix with entries $\sigma_{(i)}^2=g(w_{(i)}^2)$ where the subscript (i) indicates the number of the node selected at the i-th iteration.

In this work, we conduct a greedy selection process by choosing one node at each iteration and thus focus on minimizing the intermediate reconstruction error at the i-th iteration given by $MSE_i=tr\left[(\boldsymbol{U_{S_iR}^T}\boldsymbol{C_{S_i}^{-1}}\boldsymbol{U_{S_iR}})^+\right]$ where S_i is the set of i nodes selected. Note that the selection at the (i+1)-th iteration is made over the nodes in the set of the remaining vertices in V denoted by $S_i^C\equiv(V-S_i)$. More specifically,

$$j^* = \arg \min_{S_{i+1} = S_i + \{j\}, j \in S_i^C} tr\left[(U^T_{S_{i+1}R} C^{-1}_{S_{i+1}} U_{S_{i+1}R})^+ \right] \tag{9}$$

$$S^*_{i+1} = S_i + \{j^*\}, i = 0,...,|S|-1. \tag{10}$$

where $U_{S_{i+1}R}$ is constructed at the (i+1)-th iteration by choosing one row from $U_{S^C_iR}$:

$$U^T_{S_{i+1}R} = [u^{(1)} \cdots u^{(i+1)}] \tag{11}$$

where $(u^{(i)})^T$ denotes the independent row vector selected at the i-th iteration from U_{VR}. The greedy selection in (9) and (10) are r times repeated with S_i replaced by S^*_{i+1} at the next iteration. In the following section, we provide an analytic result to select the next node minimizing the metric in (9).

Ⅲ. Sampling set selection algorithm

We first conduct the QR factorization of $U^T_{S_{i+1}R} = Q^{i+1}R^{i+1}$ in (9) where Q^{i+1} is the $r \times (i+1)$ matrix with $i+1$ orthonormal columns $q_1,...,q_{i+1}$ and R^{i+1} the $(i+1)\times(i+1)$ upper triangular matrix with columns $r_1,...,r_{i+1}$. Specifically, we have

$$MSE_{i+1} = tr\left[\left(Q^{i+1} R^{i+1} C^{-1}_{S_{i+1}} (R^{i+1})^T (Q^{i+1})^T \right)^+ \right]$$

$$= tr\left[Q^{i+1} (R^{i+1} C^{-1}_{S_{i+1}} (R^{i+1})^T)^{-1} (Q^{i+1})^T \right] \tag{12}$$

$$= tr\left[\left(R^{i+1} C^{-1}_{S_{i+1}} (R^{i+1})^T \right)^{-1} \right] \tag{13}$$

where (12) follows from $((Q^{i+1})^T)^+ = Q^{i+1}$ and $(Q^{i+1})^+ = (Q^{i+1})^T$ and (13) from the notion that Q^{i+1} has orthonormal columns. We continue to manipulate the metric in (13) by using the analytic result in [10] which is given as follows:

$$(R^{i+1})^{-1} = \begin{bmatrix} (R^i)^{-1} & -\frac{(R^i)^{-1}b}{d} \\ 0_{1\times i} & \frac{1}{d} \end{bmatrix} \tag{14}$$

where $0_{1\times i}$ is the $1 \times i$ matrix with entries of all zeros, $b = [q_1^T u^{(i+1)} \cdots q_i^T u^{(i+1)}]^T$ and $d = q^T_{i+1} u^{(i+1)}$. Then, denoting $R^i C^{-1}{}_{S_i} (R^i)^T$ by P^i_w for a simple notation and plugging (14) into (13) yields

$$(P^{i+1}_w)^{-1} = ((R^{i+1})^T)^{-1} \begin{bmatrix} C_{S_i} & 0_{i\times 1} \\ 0_{1\times i} & g(w^2_{(i+1)}) \end{bmatrix} (R^{i+1})^{-1}$$

$$= \begin{bmatrix} (P^i_w)^{-1} & -\frac{(P^i_w)^{-1}b}{d} \\ -\frac{b^T(P^i_w)^{-1}}{d} & \frac{b^T(P^i_w)^{-1}b}{d^2} + \frac{g(w^2_{(i+1)})}{d^2} \end{bmatrix} \tag{15}$$

Thus, we obtain the metric to be minimized at the (i+1)-th iteration as follows:

$$MSE_{i+1} = tr[(P^{i+1}_w)^{-1}]$$

$$= tr[(P^i_w)^{-1}] + \frac{b^T(P^i_w)^{-1}b}{d^2} + \frac{g(w^2_{(i+1)})}{d^2} \tag{16}$$

$$\propto \frac{b^T(P^i_w)^{-1}b}{d^2} + \frac{g(w^2_{(i+1)})}{d^2} \tag{17}$$

where (17) follows since the first term in (16) is irrelevant in finding the minimizing row denoted by $u^{(i+1)*}$ at the (i+1)-th iteration. Hence, we can find the row $u^{(i+1)*}$ that minimizes the reconstruction error at the (i+1)-th iteration:

$$u^{(i+1)*} = \arg \min_{u^{(i+1)} \in U_{S^C_i}, d \neq 0} \left(\frac{b^T(P^i_w)^{-1}b}{d^2} + \frac{g(w^2_{(i+1)})}{d^2} \right) \tag{18}$$

Note that the minimizing row $u^{(1)*}$ at the first iteration is found with $\boldsymbol{Q}^1 = \dfrac{\boldsymbol{u}^{(1)}}{\| \boldsymbol{u}^{(1)} \|}$ as follows:

$$\boldsymbol{u}^{(1)*} = \arg \min_{\boldsymbol{u}^{(1)} \in \boldsymbol{U}_{VR}, d \neq 0} \left(\frac{g(w_{(1)}^2)}{d^2} \right), d^2 = \| \boldsymbol{u}^{(1)} \|^2 \quad (19)$$

If the selected row at the first iteration is the j^*-th row, then we have

$$(\boldsymbol{P}_w^1)^{-1} = \frac{g(w_{j^*}^2)}{d^2}, \ d^2 = \| \boldsymbol{u}_{j^*} \|^2$$
$$\boldsymbol{Q}^1 = \frac{\boldsymbol{u}_{j^*}}{\| \boldsymbol{u}_{j^*} \|}, \ \boldsymbol{R}^1 = \| \boldsymbol{u}_{j^*} \| \quad (20)$$

The proposed algorithm is summarized as follows:

Step 1: initialize $S_o = \varnothing$

Step 2: compute $\boldsymbol{u}^{(1)*}$ and $(\boldsymbol{P}_w^1)^{-1}$ by using (19) and (20), respectively.

Step 3: $S_1 = \{j^*\}$ if $\boldsymbol{u}^{(1)*}$ is the j^*-th row in $\boldsymbol{U}_{VR}$.

Step 4: For $i = 1$ to $r-1$ do

for each $\boldsymbol{u}^{(i+1)}$ selected from $\boldsymbol{U}_{S_i^c R}$

1) Given $\boldsymbol{Q}^i, \boldsymbol{R}^i$ and $(\boldsymbol{P}_w^i)^{-1}$, compute $\boldsymbol{b} = \left[\boldsymbol{q}_1^T \boldsymbol{u}^{(i+1)} \cdots \ \boldsymbol{q}_i^T \boldsymbol{u}^{(i+1)} \right]^T$ and $d = \boldsymbol{q}_{i+1}^T \boldsymbol{u}^{(i+1)}$.

2) compute $\boldsymbol{u}^{(i+1)*}$ by (18).

3) $S_{i+1} = S_i + \{j^*\}$ if $\boldsymbol{u}^{(i+1)*}$ is the j^*-th row in $\boldsymbol{U}_{S_i^c R}$.

4) for the selected $\boldsymbol{u}^{(i+1)*}$, compute $\boldsymbol{Q}^{i+1}, \boldsymbol{R}^{i+1}$ and $(\boldsymbol{P}_w^{i+1})^{-1}$ from the QR factorization and (15), respectively.

Step 5: end for

Step 6: Return $S = S_r$

IV. Simulation results

In the experiments, we examine three different graphs given below for evaluation of the various selection methods:

1) Random sensor graph (RSS) with N=1000

2) Random regular graph (RRG) with each vertex connected to six vertices and N=1000

3) Minnesota road network graph (MRNG) with N=2642.

For each of three graphs, we generate 30 graph realizations with N vertices and compute the eigenvector matrix $\boldsymbol{U}$ and GFT $\boldsymbol{c}$ by employing the combinatorial Laplacian matrix $\boldsymbol{L}$ as a variation operator. We construct uniqueness sampling sets S with size r=|S| from 30 to 80 based on three different techniques, denoted by efficient sampling method (ESM) [4], QR factorization-based method (QRM) [10], and our proposed method, respectively. We test the case of noisy non-bandlimited signals since graph siganls are non-bandlimited or approximately bandlimited in a real situation. We generate random graph signals from the Gaussian joint distribution as follows:

$$p(\boldsymbol{f}) \propto \exp\left(-\boldsymbol{f}^T \boldsymbol{K}^{-1} \boldsymbol{f}\right) = \exp\left(-\boldsymbol{f}^T (\boldsymbol{L} + \delta \boldsymbol{I}) \boldsymbol{f}\right) \quad (21)$$

where $\boldsymbol{L} + \delta \boldsymbol{I}$ corresponds to the inverse covariance matrix and δ is assumed to take a small value (=0.01) for the existence of the inverse matrix. We assume that graph signals are noise-corrupted by an iid additive noise drawn from $N(0, \sigma^2)$. In evaluating the selection methods, we compare the weighted reconstruction error in (3) which is averaged over 100 graph signal values at each node. In the experiments, weights w_i are generated from uniform distribution over the interval (0 2) and the function $g(w_i^2) = \dfrac{1}{w_i^2}$ is used for RSG and MRNG and $g(w_i^2) = \dfrac{1}{\log(w_i^2 + \alpha)}$ for RRG where α is adjusted for $\log(w_i^2 + \alpha) > 0, \forall w_i$. Graphs and their attributes (i.e., $\boldsymbol{L}, \boldsymbol{U}, e_{ij}$) are created from the graph signal processing toolbox (GSPBox) for Matlab [11].

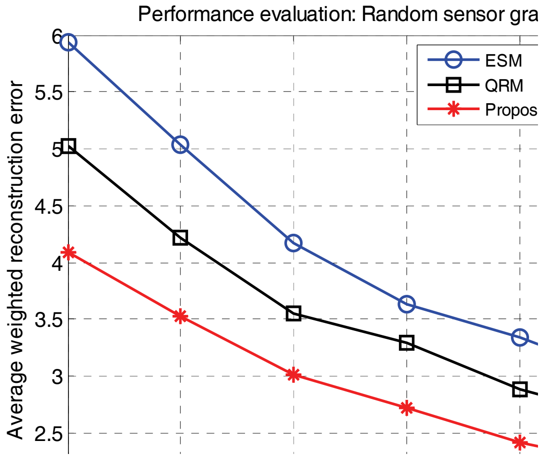

Fig. 1 Performance evaluation of different sampling methods for RSG by varying sample size with noise level $\sigma = 0.1$.

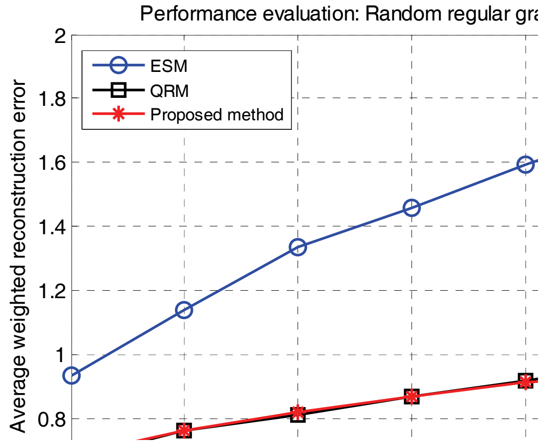

Fig. 2 Performance evaluation of different sampling methods for RRG by varying sample size with noise level $\sigma = 0.1$.

We evaluate the reconstruction performance for the case of noisy non-bandlimited graph signals with $\sigma = 0.1$ by varying the sample size $|S|=r$ from 30 to 80. As seen in Figure 1, 2 and 3, the proposed algorithm achieves the better reconstruction performance than the other methods since ESM and QRM are devised to optimize the metric related to the reconstruction error without weights.

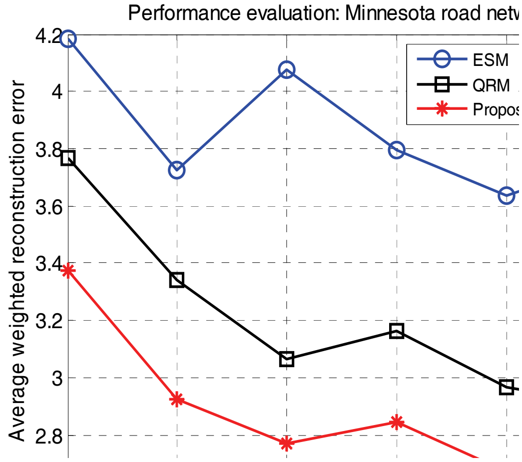

Fig. 3 Performance evaluation of different sampling methods for MRNG by varying sample size with noise level $\sigma = 0.1$.

We also investigate the complexity by examining the execution time of the methods in the same condition in Figure 4. It can be seen that the proposed algorithm runs as fast as QRM while maintaining better reconstruction accuracy than QRM when graph signals are generated with weights. Note that ESM operates much faster than the others at the cost of performance degradation.

V. Conclusion

We studied the sampling set selection problem when each node generates a signal value with its weight. To minimize the weighted reconstruction error, we assume that signal values with high weights are corrupted by lower noise variance. Then, we simplified the reconstruction error based on the QR factorization and presented an analytic solution which enables a simple iterative selection process. We conducted experiments for performance evaluation of various sampling methods, resulting in a competitive performance of the proposed method.

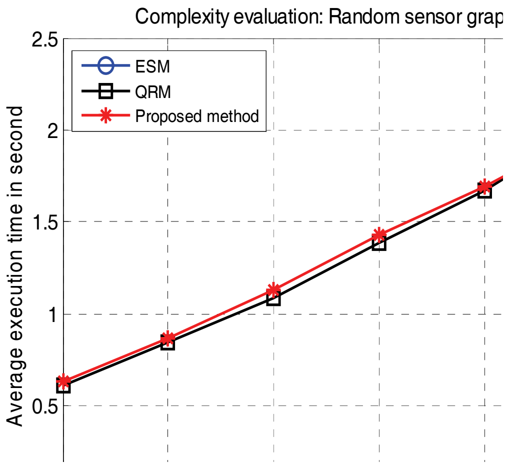

Fig. 4 Complexity evaluation of different sampling methods for RSG by varying sample size with noise level $\sigma=0.1$.

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BIOGRAPHY

Yoon Hak Kim

1992: BS degree in Electronic Engineering, Yonsei University
1994: MS degree in Electronic Engineering, Yonsei University
2007: Ph.D. degree in Electrical Engineering, University of Southern California.
2012 - Present: Associate professor, Chosun Univeristy