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MOBILE APP FOR COMPUTING OPTION PRICE OF THE FOUR-UNDERLYING ASSET STEP-DOWN ELS

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ABSTRACT. We present the user-friendly graphical user interface design and implementation of Monte Carlo simulation (MCS) for computing option price of the four-underlying asset step-down equity linked securities (ELS) using the Android platform. The ELS has been one of the most important and influential financial products in South Korea. Most ELS products are based on one-, two-, and three-underlying assets. However, currently there is a demand for higher coupon payment from ELS products because of the increased interest rate in financial market. In order to allow the investors to have higher coupon payment, it is necessary to design a multi-asset ELS such as four-asset step-down ELS. We conduct the computational experiments to demonstrate the performance of the Android platform for pricing four-asset step-down ELS. Furthermore, we perform a comparison test with a three-asset step-down ELS.

1. INTRODUCTION

A step-down equity-linked securities (ELS) derivative with four underlying assets have product structures that can satisfy the needs of consumers who want to receive higher coupon rates in a short period time for market conditions with a relatively low interest rate. In other words, if we want to receive more coupon rates from the perspective of customers who purchase derivatives, it is better to purchase products with four underlying assets than products with three underlying assets. As one of the major issues in a financial field, multi-asset option pricing is an important problem. To solve this problem, numerical studies have been conducted to set fair prices for financial derivatives such as European multi-asset options, European swaptions, applying the Monte Carlo method [1, 2]. In recent years, numerical studies were performed to calculate the fair prices of step-down ELS derivative with four underlying assets, applying

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the Monte Carlo method [3] or finite difference method (FDM) [4]. In the FDM, we use the four-dimensional Black–Scholes (BS) partial differential equation (PDE):

$$\frac{\partial u}{\partial t} + 0.5(\sigma_x x)^2 \frac{\partial^2 u}{\partial x^2} + 0.5(\sigma_y y)^2 \frac{\partial^2 u}{\partial y^2} + 0.5(\sigma_z z)^2 \frac{\partial^2 u}{\partial z^2} + 0.5(\sigma_w w)^2 \frac{\partial^2 u}{\partial w^2}$$
$$+ \rho_{xy}\sigma_x\sigma_y xy \frac{\partial^2 u}{\partial x \partial y} + \rho_{xz}\sigma_x\sigma_z xz \frac{\partial^2 u}{\partial x \partial z} + \rho_{xw}\sigma_x\sigma_w xw \frac{\partial^2 u}{\partial x \partial w}$$
$$+ \rho_{yz}\sigma_y\sigma_z yz \frac{\partial^2 u}{\partial y \partial z} + \rho_{yw}\sigma_y\sigma_w yw \frac{\partial^2 u}{\partial y \partial w} + \rho_{zw}\sigma_z\sigma_w zw \frac{\partial^2 u}{\partial z \partial w}$$
$$+ rx\frac{\partial u}{\partial x} + ry\frac{\partial u}{\partial y} + rz\frac{\partial u}{\partial z} + rw\frac{\partial u}{\partial w} - ru = 0,$$

where $u(x, y, z, w, t)$ is the option price. In addition, $u(x, y, z, w, T)$ is the final condition. The parameters used are volatilities of each underlying asset σ_x, σ_y, σ_z, σ_w, correlation values between two underlying assets ρ_{xy}, ρ_{xz}, ρ_{xw}, ρ_{yz}, ρ_{yw}, ρ_{zw}, and interest rate r. Here, the subscript notations represent the corresponding asset. For more details about the governing equations for the four-asset ELS and their FDM, please refer to [4].

To estimate the financial parameter values such as volatility or correlation coefficients, etc, the practitioners in the financial market may apply various approach models such as the autoregressive conditional heteroskedasticity (ARCH) type models [5, 6, 7, 8], exponentially weighted moving averages (EWMA) type models [9, 10], stochastic alpha beta rho (SABR) model [11], lognormal-mixture dynamics [12], dynamic conditional correlation (DCC) [13, 14] and others. For some related application examples, please see articles [15, 16] and references therein.

As the Fintech field [17, 18] has developed, banking-, stock-, and tax processing-related business are in the palm of our hand. In this work, using the Android platform, we shall develop a calculator using Monte Carlo computational algorithm that can conveniently calculate the fair prices of four-underlying asset step-down ELS derivative with given conditions. This study is an extension of the previous one-, two-, and three-asset ELS [19, 20].

The contents of this paper are as follows. The numerical solution algorithm for pricing the four-asset ELS with step-down structure is presented in Section 2. In Section 3, we perform computational tests using the proposed Android platform. In Section 4, we conclude this paper.

2. Numerical solution algorithm

For pricing a step-down ELS derivative with four underlying assets, we set the following parameters: early redemption dates T_1, T_2, T_3, T_4, T_5, T_6, strike price percentages K_1, K_2, K_3, K_5, K_6, coupon rates $c_1, c_2, c_3, c_4, c_5, c_6$, volatilities $\sigma_1, \sigma_2, \sigma_3, \sigma_4$, face value $F = 10000$, knock-in barrier percentage $kib = 50$, dummy rate $d = 0.3$, $r = 0.01$ and temporal step size $\Delta t = 1/365$. In Table 1, the parameters values used are given.

TABLE 1. Parameter values.

Dates	$T_1 = 1/2$	$T_2 = 1$	$T_3 = 3/2$	$T_4 = 2$	$T_5 = 5/2$	$T_6 = 3$
Strikes	$K_1 = 85$	$K_2 = 80$	$K_3 = 75$	$K_4 = 70$	$K_5 = 65$	$K_6 = 60$
Rates	$c_1 = 0.05$	$c_2 = 0.1$	$c_3 = 0.15$	$c_4 = 0.2$	$c_5 = 0.25$	$c_6 = 0.3$

To define correlated random numbers among the given four-underlying assets, let us consider Cholesky factorization [21, 22] of (4×4) size of correlation coefficients matrix $\mathbf{\Sigma}$:

$$\mathbf{\Sigma} = \begin{pmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{12} & 1 & \rho_{23} & \rho_{24} \\ \rho_{13} & \rho_{23} & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{pmatrix},$$

where ρ_{12}, ρ_{13}, ρ_{14}, ρ_{23}, ρ_{24} and ρ_{34} are the correlation coefficients among the four-underlying assets. The correlated random numbers Z_1^*, Z_2^*, Z_3^* and Z_4^* can be calculated from combinations of standard multivariate normal distributions and correlation coefficients:

$$\begin{aligned}
Z_1^* &= Z_1, \\
Z_2^* &= \rho_{12} Z_1 + \sqrt{1-\rho_{12}^2} Z_2, \\
Z_3^* &= \rho_{13} Z_1 + \frac{(\rho_{23}-\rho_{12}\rho_{13})}{\sqrt{1-\rho_{12}^2}} Z_2 + \sqrt{1-\rho_{13}^2 - \frac{(\rho_{23}-\rho_{12}\rho_{13})^2}{1-\rho_{12}^2}} Z_3, \\
Z_4^* &= \rho_{14} Z_1 + \frac{\rho_{24}-\rho_{12}\rho_{14}}{\sqrt{1-\rho_{12}^2}} Z_2 + \frac{\rho_{34}-\rho_{13}\rho_{14} - \frac{(\rho_{23}-\rho_{12}\rho_{13})(\rho_{24}-\rho_{12}\rho_{14})}{1-\rho_{12}^2}}{\sqrt{1-\rho_{13}^2 - \frac{(\rho_{23}-\rho_{12}\rho_{13})^2}{1-\rho_{12}^2}}} Z_3 \\
&\quad + \sqrt{1-\rho_{14}^2 - \frac{(\rho_{24}-\rho_{12}\rho_{14})^2}{1-\rho_{12}^2} - \frac{\left[\rho_{34}-\rho_{13}\rho_{14} - \frac{(\rho_{23}-\rho_{12}\rho_{13})(\rho_{24}-\rho_{12}\rho_{14})}{1-\rho_{12}^2}\right]^2}{1-\rho_{13}^2 - \frac{(\rho_{23}-\rho_{12}\rho_{13})^2}{1-\rho_{12}^2}}} Z_4,
\end{aligned}$$

where Z_1, Z_2, Z_3 and Z_4 are identically independent standard normal distributions. Discrete paths for correlated four assets can be created using the following formula and Monte Carlo method:

$$\begin{aligned}
S_1(t_{j+1}) &= S_1(t_j) e^{(r-0.5\sigma_1^2)\Delta t + \sigma_1 \sqrt{\Delta t} Z_1^*(t_j)}, \\
S_2(t_{j+1}) &= S_2(t_j) e^{(r-0.5\sigma_2^2)\Delta t + \sigma_2 \sqrt{\Delta t} Z_2^*(t_j)}, \\
S_3(t_{j+1}) &= S_3(t_j) e^{(r-0.5\sigma_3^2)\Delta t + \sigma_3 \sqrt{\Delta t} Z_3^*(t_j)}, \\
S_4(t_{j+1}) &= S_4(t_j) e^{(r-0.5\sigma_4^2)\Delta t + \sigma_4 \sqrt{\Delta t} Z_4^*(t_j)},
\end{aligned}$$

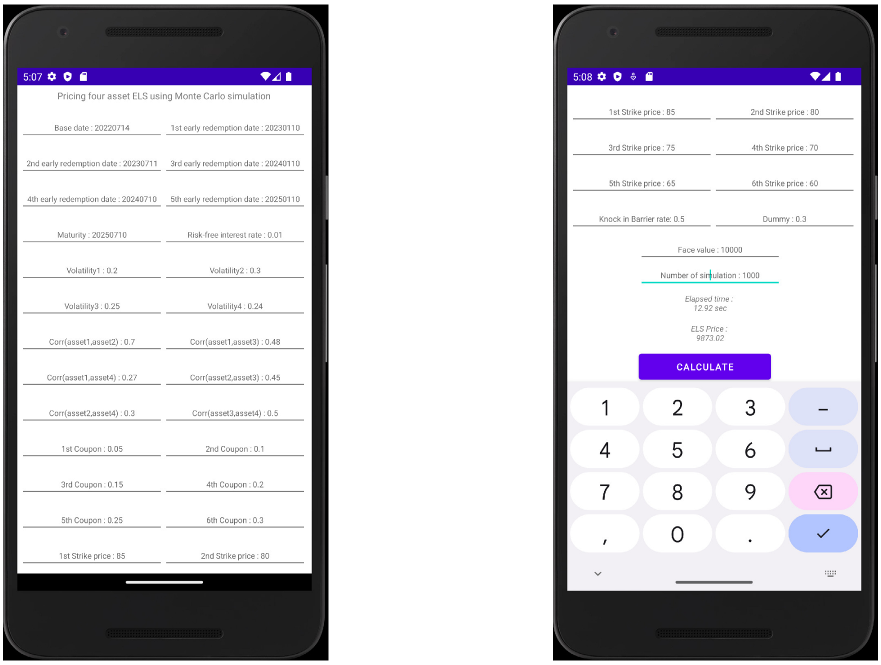
where $t_j = j\Delta t$.

The minimum value $W(t_j)$ among discrete four-asset paths is defined as

$$W(t_j) = \min(S_1(t_j), S_2(t_j), S_3(t_j), S_4(t_j)).$$

We present the pseudo algorithm of pricing four-asset step-down ELS in Algorithm 1.

In Fig. 1, we can see that snapshots of calculator for pricing four-asset ELS with step-down structure using MCS. All parameters in step-down four-asset ELS can be changed by touching the screen of the proposed calculator and inputting values of parameters with the keyboard.



(a) (b)

FIGURE 1. Calculator for pricing four-asset ELS using MCS, in the Android platform. (a) Top screen of the calculator, (b) bottom screen of the calculator and computational result.

3. Numerical experiments

We now conduct computational tests for four-asset ELS with step-down structure. The computational results are obtained from the proposed calculator of Android platform in Samsung Galaxy Wide 5 on a 2.2GHz Octa-core with 6GB RAM.

The convergence for computational fair prices of four-asset step-down ELS can be seen in Fig. 2(a). The computational results are presented for each 10 simulations. As the number of sample paths increases from 10^3 to 10^6, the convergence radius of the computational fair prices decreases and converges to a specific price. Next, we check the elapsed times for calculating

Algorithm 1 MCS procedure for four-asset step-down ELS

Require:

Current underlying prices $S_1^*(0)$, $S_2^*(0)$, $S_3^*(0)$, and $S_4^*(0)$
Maturity time T
Checking days N_c
Sample paths N_p
Total time step N_T
Temporal step size $\Delta t = T/N_T$
Face value F
Volatilities of four underlying assets σ_1, σ_2, σ_3, and σ_4
Correlation coefficients matrix

$$\mathbf{\Sigma} = \begin{pmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{12} & 1 & \rho_{23} & \rho_{24} \\ \rho_{13} & \rho_{23} & 1 & \rho_{34} \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{pmatrix}$$

Interest rate r
Early redemption dates T_1, T_2, T_3, T_4, T_5, and T_6
Coupon rates c_1, c_2, c_3, c_4, c_5, and c_6
Strike price percentages K_1, K_2, K_3, K_4, K_5, and K_6
Dummy d
Knock-in barrier kib
Initialize payoff $M_i = 0$, for $i = 1, \ldots, N_c$, $\quad N_c$ is the number of coupon rates
Scale the underlying assets with the initial prices $S_1(t) = S_1^*(t)/S_1^*(0)$, $S_2(t) = S_2^*(t)/S_2^*(0)$, $S_3(t) = S_3^*(t)/S_3^*(0)$, and $S_4(t) = S_4^*(t)/S_4^*(0)$

▷ Cholesky decomposition of (4×4) correlation matrix Σ, $U = chol(\Sigma)$, where U is upper triangular matrix

for $k = 1$ to N_p **do**
  ▷ Generate daily discrete stock paths for t_j using Monte Carlo method
  for $j = 0$ to $N_T - 1$ **do**
    $\big(B_1,\ B_2,\ B_3,\ B_4\big)' = U' \times \big(Z_1,\ Z_2,\ Z_3,\ Z_4\big)'$, $\quad Z_1, Z_2, Z_3, Z_4 \sim N(0,1)$, $\quad '$ is transpose matrix
    $S_1(t_{j+1}) = S_1(t_j)\exp((r - 0.5\sigma_1^2)\Delta t + \sigma_1\sqrt{\Delta t}B_1(t_j))$
    $S_2(t_{j+1}) = S_2(t_j)\exp((r - 0.5\sigma_2^2)\Delta t + \sigma_2\sqrt{\Delta t}B_2(t_j))$
    $S_3(t_{j+1}) = S_3(t_j)\exp((r - 0.5\sigma_3^2)\Delta t + \sigma_3\sqrt{\Delta t}B_3(t_j))$
    $S_4(t_{j+1}) = S_4(t_j)\exp((r - 0.5\sigma_4^2)\Delta t + \sigma_4\sqrt{\Delta t}B_4(t_j))$
  end for
  ▷ Define worst performer value
  $W = \min(S_1, S_2, S_3, S_4)$
  ▷ Check the values of discrete stock paths at checking days
  if $W(T_1) \geq K_1$ **then** $M_1 = M_1 + (1 + c_1)F$
  else if $W(T_2) \geq K_2$ **then** $M_2 = M_2 + (1 + c_2)F$
  $\vdots$
  else if $W(T_{N_c}) \geq K_{N_c}$ **then** $M_{N_c} = M_{N_c} + (1 + c_{N_c})F$
  else if $\min\limits_{1 \leq j \leq N_T} \{W(t_j)\} \leq kib$ **then** $M_{N_c} = M_{N_c} + W(t_{N_T})$
  else
    $M_{N_c} = M_{N_c} + (1 + d)F$
  end if
end for
▷ Discount to present price and take average.

$$V^0 = \sum_{i=1}^{N_c} e^{-rT_i} M_i / N_p$$

computational fair prices of four-asset step-down ELS with increasing number of sample paths. Computational results for elapsed times are presented in Fig. 2(b).

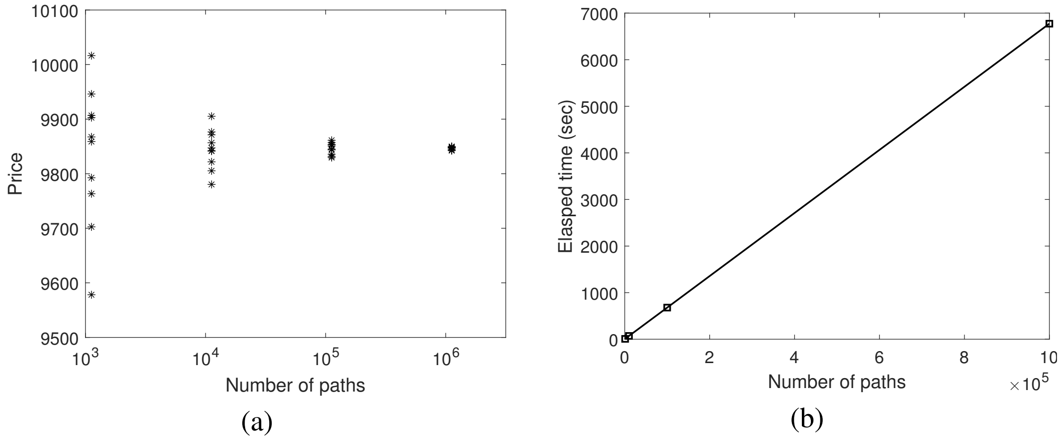


FIGURE 2. (a) Convergence of computational fair prices of four-asset step-down ELS. (b) Elapsed times of main computational algorithm part with respect to increasing number of sample paths.

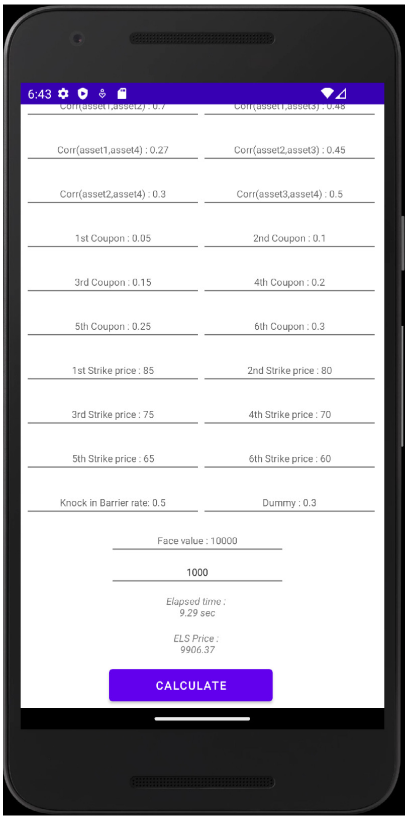
As shown in Fig. 3 and Table 2, the computational values for pricing four-asset step-down ELS using the proposed computational algorithm converge to specific price value as the number of sample paths increases from 10^3 to 10^6. Furthermore, we can see that the elapsed times of main iteration are strictly increased about a factor of 10, as the number of sample paths increases by a factor of 10.

TABLE 2. Checking elapsed times (in seconds) of proposed calculator for the four-asset step-down ELS with the following parameters values: $K_1 = 85,\ K_2 = 80,\ K_3 = 75,\ K_4 = 70,\ K_5 = 65,\ K_6 = 60$, knock-in barrier percentage $kib = 50$, volatilities $\sigma_1 = 0.2,\ \sigma_2 = 0.3,\ \sigma_3 = 0.25,\ \sigma_4 = 0.3$, the correlation coefficients $\rho_{12} = 0.7,\ \rho_{13} = 0.48,\ \rho_{14} = 0.27,\ \rho_{23} = 0.45,\ \rho_{24} = 0.3,\ \rho_{34} = 0.5$, and $r = 0.01$.

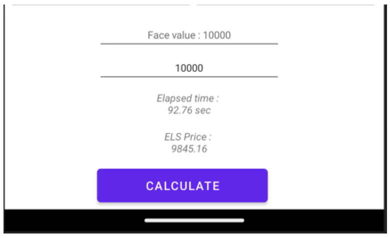
Number of sample paths	10^3	10^4	10^5	10^6
Elapsed time	9.29	92.76	917.04	9115.22
ELS Price	9906.37	9845.16	9841.96	9846.28

Next, we compare pricing values between the three-asset and four-asset step-down ELS using the proposed Android platform calculator. In Fig. 4 and Table 3, the fair values are shown for the three-asset and four asset step-down ELS.

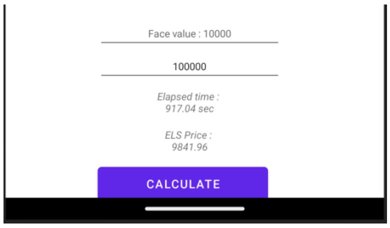
Under the same parameter values setting conditions as shown in Table 3, the fair value of the step-down ELS derivative with four-underlying assets is lower than that of the step-down



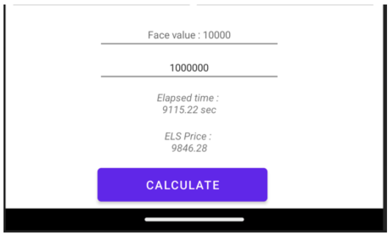
(a) $N_p = 10^3$



(b) $N_p = 10^4$



(c) $N_p = 10^5$



(d) $N_p = 10^6$

FIGURE 3. Pricing four-asset step-down ELS in the Android platform calculator.

TABLE 3. Comparison pricing values between three-asset and four-asset step-down ELS with the following parameters values: $K_1 = 85,\ K_2 = 80,\ K_3 = 75,\ K_4 = 70,\ K_5 = 65,\ K_6 = 60$, knock-in barrier percentage $kib = 50$, volatilities $\sigma_1 = 0.3,\ \sigma_2 = 0.3,\ \sigma_3 = 0.3,\ \sigma_4 = 0.3$, the correlation coefficients $\rho_{12} = 0.5,\ \rho_{13} = 0.5,\ \rho_{14} = 0.5,\ \rho_{23} = 0.5,\ \rho_{24} = 0.5,\ \rho_{34} = 0.5$, and $r = 0.01$.

Three-asset step-down ELS price	9623.94
Four-asset step-down ELS price	9348.01

ELS derivative with three-underlying assets. In Fig. 5 (a), we can observe that discrete stock paths for four-underlying asset $S_1(t_j)$, $S_2(t_j)$, $S_3(t_j)$ and $S_4(t_j)$. As shown in Fig. 5(b), we can intuitively see that the expectation value of four-asset worst performer is lower than

(a) (c)

(b) (d)

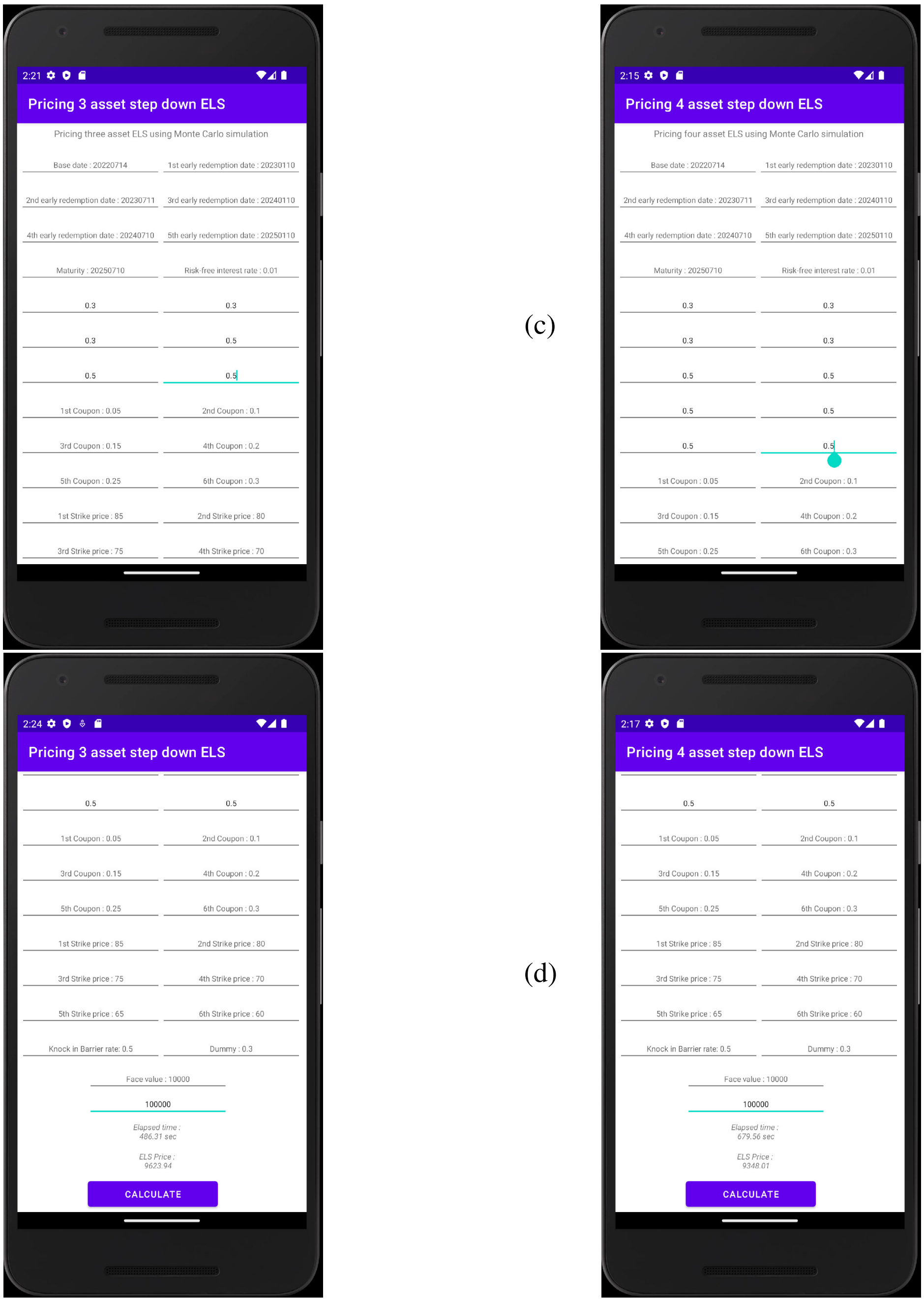


FIGURE 4. Comparison pricing between (a), (b) three-asset and (c), (d) four-asset step-down ELS in the Android platform calculator.

the expectation value of three-asset worst performer because the former is always less than or equal to the latter.

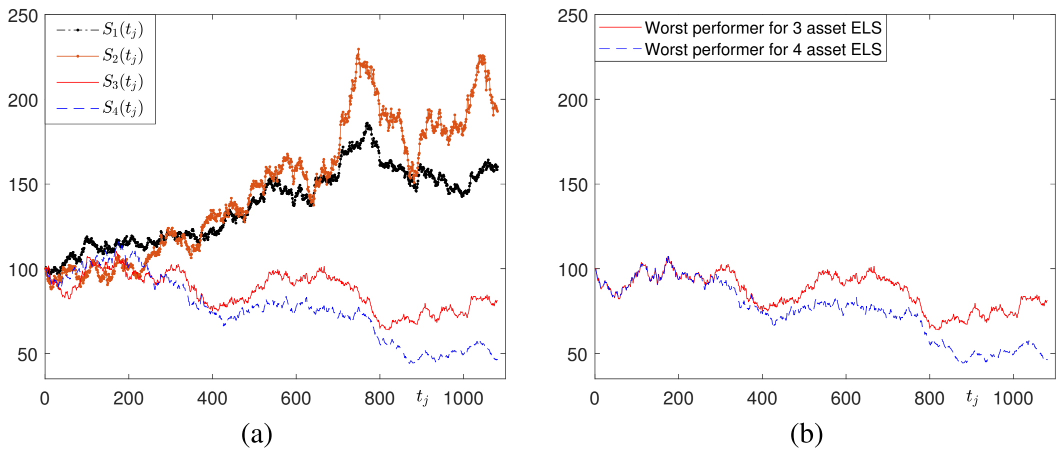


FIGURE 5. (a) Discrete stock paths for $S_1(t_j)$, $S_2(t_j)$, $S_3(t_j)$ and $S_4(t_j)$, (b) comparison values between worst performer of three-asset ELS and four-asset ELS.

Therefore, we can conclude that higher coupon rates can be provided in the structure of four-asset step-down ELS derivative.

4. CONCLUSION

In the era of the popularity of smartphones, the development of the Fintech field is inevitable. As the Fintech field develops, this work presented the mobile implementation for the previous four-asset step-down ELS pricing. Most ELS products are based on one-, two-, and three-underlying assets. However, currently there is a demand for higher coupon payment from ELS products because of the increased interest rate in financial market. To allow the investors to have higher coupon payment, it is necessary to design a multi-asset ELS such as four-asset step-down ELS. In addition, we performed a comparison test with a three-asset step-down ELS. In the future work, we will study much faster pricing for four-asset step-down ELS using the Brownian bridge method and the Monte Carlo simulation in the Android platform.

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