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A NEW ITERATION METHOD FOR FIXED POINT OF NONEXPANSIVE MAPPING IN UNIFORMLY CONVEX BANACH SPACE

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Abstract. The aim of this paper is to introduce a new iterative process and show that our iteration scheme is faster than other existing iteration schemes with the help of numerical examples. Next, we have established convergence and stability results for the approximation of fixed points of the contractive-like mapping in the framework of uniformly convex Banach space. In addition, we have established some convergence results for the approximation of the fixed points of a nonexpansive mapping.

1. Introduction

Fixed point theory always has an important role in the field of analysis. Banach contraction principle become milestone in the fixed point theory always inspire researchers to obtain fixed point of different mappings over different spaces. The origin of fixed point theory lies in the method of successive approximation used for proving existence of solution of differential equation introduced independently by Joseph Liouville in 1837 and Charles Picard in 1890.

In 1953, Mann [27], introduced an iterative scheme. Later in 1974, Ishikawa [22], introduced an iterative scheme which was two step iterative scheme. In 2000, Noor [15], introduced a three-step iterative scheme for approximating fixed point problems. Later several researchers modified Mann, Ishikawa and Noor iterations.

In 2007, Agarwal et al. [20], developed a new iteration method and proved that this iteration process converges faster than Mann iteration for contraction mapping. In 2014, Abbas and Nazir [14] introduced an new iterative scheme which is converges faster than Agarwal et al. [20]. In 2016, Thakur et al. [3] proposed a new iteration method for suzuki generalized nonexpansive mapping and proved that this iteration process converges faster than previous process. In 2018, Ullah et al. [10] developed new iteration scheme. Let M be a nonempty closed convex bounded subset of uniformly

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Banach space K for each $u_0 \in M$ construct a sequence $\{u_n\}$ by

$$(1.1) \qquad \begin{cases} u_{n+1} = Tv_n, \\ v_n = T((1-\alpha_n)w_n + \alpha_n Tw_n) \\ w_n = (1-\beta_n)u_n + \beta_n Tu_n \end{cases}$$

$\forall\, n \geq 0$ and $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $(0, 1)$. This iteration process is converges faster than previous iteration process.

In 2020 Hassan et al. [21] developed new iteration process. Let M be a nonempty closed convex bounded subset of uniformly Banach space K for each $u_0 \in M$ construct a sequence $\{u_n\}$ by

$$(1.2) \qquad \begin{cases} u_{n+1} = T((1-\alpha_n)v_n + \alpha_n Tv_n) \\ v_n = T((1-\beta_n)w_n + \beta_n Tw_n) \\ w_n = T((1-\gamma_n)x_n + \gamma_n Tx_n) \\ x_n = T((1-\zeta_n)u_n + \zeta_n Tu_n) \end{cases}$$

for all $n \geq 0$ and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\zeta_n\}$ are sequences in $(0, 1)$. This iteration process is converges faster than previous iteration process.

Our aim is to introduce a new faster iteration process than those mentioned above and to prove the convergence results for nonexpansive mappings in uniformly convex Banach space.

2. Preliminaries

In this section, we recall some definitions and results to be used in establishing the main results.

Definition 2.1. [9] A Banach space K is said to be uniformly convex if for each $\epsilon \in (0, 2]$ there is a $\delta > 0$ such that $x, y \in K$

$$\begin{cases} ||x|| \leq 1, \\ ||y|| \leq 1, \\ ||x-y|| \geq \epsilon \end{cases} \Rightarrow ||\frac{x+y}{2}|| \geq \delta.$$

Definition 2.2. [28] A Banach space K is said to satisfy Opial's property if for each sequence $\{x_n\}$ in K converging weakly to $x \in K$, we have

$$\limsup_{n\to\infty} ||x_n - x|| < \limsup_{n\to\infty} ||x_n - y||,$$

for all $y \in K$ s.t. $x \neq y$.

Definition 2.3. [4] Let K be a Banach space and let $T : K \to K$ be a self map. The mapping T is called contractive like mapping if there exist a constant $\delta \in [0, 1)$ and a strictly increasing and continuous function $\xi : [0, \infty) \to [0, \infty)$ with $\xi(0) = 0$ such that for all $x, y \in K$,

$$(2.1) \qquad ||Tx - Ty|| \leq \delta||x - y|| + \xi(||x - Tx||).$$

Definition 2.4. Let K be a Banach space and M be any nonempty subset of K. Let $T : M \to M$ be said to be nonexpansive if for each $x, y \in K$

$$||Tx - Ty|| \leq ||x - y||.$$

Definition 2.5. [7] A mapping $T : K \to K$ is said to satisfy condition A, if $\exists$ a non decreasing function $f : [0, \infty) \to [0, \infty)$ with $f(0) = 0$ and $f(c) > 0$ for all $c > 0$ s.t. $||x - Tx|| \geq f(d(x_n, F(T)))$, for all $x \in K$, where $d(x, F(T)) = \inf\{||x - x^*|| : x^* \in F(T)\}$.

The following definition is about the rate of convergence due to Berinde [26] which is used to verify that our iteration process (3.1) convergence faster than the other existing iteration process.

Definition 2.6. [26] Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be two sequence of positive numbers such that converge to a and b respectively. Assume that there exists

$$\lim_{n\to\infty} \frac{|a_n - a|}{|b_n - b|} = l.$$

(i) If $l = 0$, then the sequence $\{a_n\}$ converges faster than the sequence $\{b_n\}$.
(ii) If $0 < l < \infty$, then we say that the sequence $\{a_n\}$ and $\{b_n\}$ have the same rate of converges.
(iii) If $l = \infty$, then the sequence $\{b_n\}$ converges faster than sequence $\{a_n\}$.

Definition 2.7. [2] Let $\{t_n\}$ be any arbitrary sequence in K. Then an iteration procedure $x_{n+1} = f(T, x_n)$, converging to fixed point p, is said to T-stable, if for $\epsilon_n = ||t_{n+1} - f(T, t_n)||$, $\forall n \in N$, we have $\lim\limits_{n\to\infty} \epsilon_n = 0$ if and only if $\lim\limits_{n\to\infty} t_n = p$.

Lemma 2.8. [26] *Suppose that for two fixed point iteration processes $\{u_n\}$ and $\{v_n\}$ both converging to the same fixed point x^*, the error estimates*

$$||u_n - x^*|| \leq a_n \quad n \geq 1,$$
$$||v_n - x^*|| \leq b_n \quad n \geq 1$$

are available where $\{a_n\}$ and $\{b_n\}$ are two sequences of positive numbers converging to zero. If $\{a_n\}$ converges faster than $\{b_n\}$, then $\{u_n\}$ converges faster than $\{v_n\}$ to x^*.

Lemma 2.9. [25] *If λ is a real number such that $0 \leq \lambda < 1$ and $\{\epsilon_n\}$ is the sequence of positive numbers such that*

$$\lim_{n\to\infty} \epsilon_n = 0,$$

then for an sequence of positive numbers v_n satisfying

$$v_{n+1} \leq \lambda v_n + \epsilon_n, \quad for\ n = 1, 2, ...,$$

we have

$$\lim_{n\to\infty} v_n = 0.$$

Lemma 2.10. [8] *Let K be a uniformly convex Banach space and $0 < p \leq t_n \leq q < 1$ $\forall n \in N$. Let $\{x_n\}$ and $\{y_n\}$ be two sequences of K s.t. $\limsup_{n\to\infty} ||x_n|| \leq a$, $\limsup_{n\to\infty} ||y_n|| \leq a$ and $\limsup_{n\to\infty} ||t_n x_n + (1 - t_n) y_n|| = a$ hold for some $a \geq 0$. Then $\lim\limits_{n\to\infty} ||x_n - y_n|| = 0$.*

Lemma 2.11. [9] *Let K be a uniformly convex Banach space and M be any nonempty closed convex subset of K. Let T be a nonexpansive mapping on K. Then, $I - T$ is demiclosed at zero.*

3. Main result

3.1. New Iteration Scheme. In this section, we introduce a new iteration scheme. Let K be uniformly convex Banach space and $\phi \neq M$ be closed and convex subset of K. Let $T : M \to M$ be any nonlinear mapping and for each $u_0 \in M$ construct the sequence $\{u_n\}$

$$\begin{cases} u_{n+1} = T((1-\alpha_n)v_n + \alpha_n T v_n) \\ v_n = T((1-\beta_n)w_n + \beta_n T w_n) \\ w_n = T((1-\gamma_n)x_n + \gamma_n T x_n) \\ x_n = T((1-\zeta_n)y_n + \zeta_n T y_n) \\ y_n = T((1-\eta_n)u_n + \eta_n T u_n) \end{cases} \tag{3.1}$$

For all $n \geq 1$ and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\zeta_n\}$ and $\{\eta_n\}$ are sequences in $(0, 1)$.

3.2. Convergence and Stability Results for Contractive like Mapping. In this section we establish convergence and stability results for new iteration process (3.1).

Theorem 3.1. *Let $\phi \neq M \subset K$ be closed and convex, where K be uniformly convex Banach space. Let $T : M \to M$ with satisfying equation (2.1) and x^* be a fixed point of T. Suppose that $\{u_n\}$ generated by (3.1) and $\sum_{n=0}^{\infty} \alpha_n = \infty$ or $\sum_{n=0}^{\infty} \beta_n = \infty$ or $\sum_{n=0}^{\infty} \gamma_n = \infty$ or $\sum_{n=0}^{\infty} \zeta_n = \infty$ or $\sum_{n=0}^{\infty} \eta_n = \infty$. Then $\{u_n\}$ converges strongly to a unique fixed point of T.*

Proof. Using iteration (3.1) and definition (2.1), we have

$$\begin{aligned} ||y_n - x^*|| &= ||(T((1-\eta_n)u_n + \eta_n T u_n)) - x^*|| \\ &\leq \delta(1-\eta_n)||u_n - x^*|| + \delta\eta_n||Tu_n - x^*|| \\ &\leq \delta(1-\eta_n)||u_n - x^*|| + \delta^2\eta_n||u_n - x^*|| \\ &= \delta(1-(1-\delta)\eta_n)||u_n - x^*||, \end{aligned} \tag{3.2}$$

$$\begin{aligned} ||x_n - x^*|| &= ||(T((1-\zeta_n)y_n + \zeta_n T y_n)) - x^*|| \\ &\leq \delta(1-\zeta_n)||y_n - x^*|| + \delta\zeta_n||Ty_n - x^*|| \\ &\leq \delta(1-\zeta_n)||y_n - x^*|| + \delta^2\zeta_n||y_n - x^*|| \\ &= \delta(1-(1-\delta)\zeta_n)||y_n - x^*||, \end{aligned} \tag{3.3}$$

$$\begin{aligned} ||w_n - x^*|| &= ||(T((1-\gamma_n)x_n + \gamma_n T x_n)) - x^*|| \\ &\leq \delta(1-\gamma_n)||x_n - x^*|| + \delta\gamma_n||Tx_n - x^*|| \\ &\leq \delta(1-\gamma_n)||x_n - x^*|| + \delta^2\gamma_n||x_n - x^*|| \\ &= \delta(1-(1-\delta)\gamma_n)||x_n - x^*||, \end{aligned} \tag{3.4}$$

$$
\begin{aligned}
||v_n - x^*|| &= ||(T((1-\beta_n)w_n + \beta_n T w_n)) - x^*|| \\
&\leq \delta(1-\beta_n)||w_n - x^*|| + \delta\beta_n||Tw_n - x^*|| \\
&\leq \delta(1-\beta_n)||w_n - x^*|| + \delta^2\beta_n||w_n - x^*|| \\
&= \delta(1-(1-\delta)\beta_n)||w_n - x^*||, \qquad (3.5)
\end{aligned}
$$

$$
\begin{aligned}
||u_{n+1} - x^*|| &= ||(T((1-\alpha_n)v_n + \alpha_n T v_n)) - x^*|| \\
&\leq \delta(1-\alpha_n)||v_n - x^*|| + \delta\alpha_n||Tv_n - x^*|| \\
&\leq \delta(1-\alpha_n)||v_n - x^*| + \delta^2\alpha_n||v_n - x^*|| \\
&= \delta(1-(1-\delta)\alpha_n)||v_n - x^*||, \qquad (3.6)
\end{aligned}
$$

From equation (3.2), (3.3), (3.4), (3.5) and (3.6), we obtain

$$
\begin{aligned}
||u_{n+1} - x^*|| &\leq \delta^5(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)(1-(1-\delta)\zeta_n)(1-(1-\delta)\eta_n)||u_n - x^*|| \\
&\leq \delta^{5+5}\prod_{k=n-1}^{n}(1-(1-\delta)\alpha_k)(1-(1-\delta)\beta_k)(1-(1-\delta)\gamma_k)(1-(1-\delta)\zeta_k)(1-(1-\delta)\eta_k)||u_{n-1} - x^*|| \\
&\vdots \\
&\leq \delta^{5(n+1)}\prod_{k=0}^{n}(1-(1-\delta)\alpha_k)(1-(1-\delta)\beta_k)(1-(1-\delta)\gamma_k)(1-(1-\delta)\zeta_k)(1-(1-\delta)\eta_k)||u_0 - x^*|| \qquad (3.7)
\end{aligned}
$$

Since $\delta \in [0,1)$ and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\zeta_n\}$ and $\{\eta_n\}$ are sequences in $(0,1)$. Using inequality $1-z \leq e^{-z}\ \forall\ z \in [0,1]$, thus From equation (3.7), we have

$$
||u_{n+1} - x^*|| \leq \frac{\delta^{5(n+1)}||u_0 - x^*||}{e^{(1-\delta)\sum_{k=0}^{\infty}\alpha_k + \sum_{k=0}^{\infty}\beta_k + \sum_{k=0}^{\infty}\gamma_n + \sum_{k=0}^{\infty}\zeta_n + \sum_{k=0}^{\infty}\eta_n}} \qquad (3.8)
$$

Taking limits on both sides

$$
\begin{aligned}
\lim_{n\to\infty}||u_{n+1} - x^*|| &\leq \lim_{n\to\infty}\frac{\delta^{5(n+1)}||u_0 - x^*||}{e^{(1-\delta)\sum_{k=0}^{\infty}\alpha_k + \sum_{k=0}^{\infty}\beta_k + \sum_{k=0}^{\infty}\gamma_n + \sum_{k=0}^{\infty}\zeta_n + \sum_{k=0}^{\infty}\eta_n}} \\
&\leq 0.
\end{aligned}
$$

$\{u_n\}$ is strongly convergent to x^*. Next we have to show that x^* is unique. Let x^*, $x^{**} \in F(T)$, such that $x^* \neq x^{**}$. Now

$$
||x^* - x^{**}|| = ||Tx^* - Tx^{**}|| \qquad (3.9)
$$

using equation (2.1), we have

$$
\begin{aligned}
||Tx^* - Tx^{**}|| &\leq \delta||x^* - x^{**}|| + \xi(||x^* - Tx^*||) \\
&\leq ||x^* - x^{**}|| \qquad (3.10)
\end{aligned}
$$

From equation (3.9) and (3.10), we have

$$
||x^* - x^{**}|| \leq ||x^* - x^{**}||.
$$

Clearly we have that $||x^* - x^{**}|| = ||x^* - x^{**}||$. Hence $x^* = x^{**}$ □

THEOREM 3.2. *Let $\phi \neq M \subset K$ be closed and convex, where K is a uniformly convex Banach space. Let $T : M \to M$ with satisfying equation* (2.1) *and x^* be a fixed point of T. Suppose that $\{u_n\}$ is generated by iteration process* (3.1)*. Then iteration process* (3.1) *is T-stable.*

Proof. Let $\{p_n\}$ be an arbitrary sequence in K and the sequence generated by (3.1) is $u_{n+1} = f(T, u_n)$ converging to a unique fixed point x^* and $\epsilon_n = ||p_{n+1} - f(T, p_n)||$. We have to show that $\lim_{n\to\infty} \epsilon_n = 0$ iff $\lim_{n\to\infty} p_n = x^*$. Let $\lim_{n\to\infty} \epsilon_n = 0$ and

$$\begin{aligned}
||p_{n+1} - x^*|| &= ||p_{n+1} - f(T,p_n) + f(T,p_n) - x^*|| \\
&\leq ||p_{n+1} - f(T,p_n)|| + ||f(T,p_n) - x^*|| \\
&= \epsilon_n + ||T((1-\alpha_n)q_n + \alpha_n T q_n) - x^*|| \\
&\leq \epsilon_n + \delta(1-(1-\delta)\alpha_n)||q_n - x^*|| \\
&= \epsilon_n + \delta(1-(1-\delta)\alpha_n)||T((1-\beta_n)r_n + \beta_n T r_n) - x^*|| \\
&\leq \epsilon_n + \delta^2(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)||r_n - x^*|| \\
&= \epsilon_n + \delta^2(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)||T((1-\gamma_n)s_n + \gamma_n T s_n) - x^*|| \\
&\leq \epsilon_n + \delta^3(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)||s_n - x^*|| \\
&= \epsilon_n + \delta^3(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)||T((1-\zeta_n)t_n + \zeta_n T t_n) - x^*|| \\
&\leq \epsilon_n + \delta^4(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)(1-(1-\delta)\zeta_n)||t_n - x^*|| \\
&= \epsilon_n + \delta^4(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)(1-(1-\delta)\zeta_n)||T((1-\eta_n)p_n + \eta_n T p_n) - x^*|| \\
&\leq \epsilon_n + \delta^5(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)(1-(1-\delta)\zeta_n)(1-(1-\delta)\eta_n)||p_n - x^*||,
\end{aligned}$$

Since $\delta \in [0, 1)$ and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\zeta_n\}$ and $\{\eta_n\}$ are sequences in $(0, 1)$, $\delta^5(1 - (1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)(1-(1-\delta)\zeta_n)(1-(1-\delta)\eta_n) \in (0, 1)$. Hence by Lemma 2.2, we have $\lim_{n\to\infty} ||p_n - x^*|| = 0$, which gives $\lim_{n\to\infty} p_n = x^*$. On the other hand, suppose that $\lim_{n\to\infty} p_n = x^*$. Then,

$$\begin{aligned}
\epsilon_n &= ||p_{n+1} - f(T,p_n)|| \\
&\leq ||p_{n+1} - x^*|| + ||f(T,p_n) - x^*|| \\
&= ||p_{n+1} - x^*|| + ||T((1-\alpha_n)q_n + \alpha_n T q_n) - x^*|| \\
&\leq ||p_{n+1} - x^*|| + \delta(1-(1-\delta)\alpha_n)||q_n - x^*|| \\
&= ||p_{n+1} - x^*|| + \delta(1-(1-\delta)\alpha_n)||T((1-\beta_n)r_n + \beta_n T r_n) - x^*|| \\
&\leq ||p_{n+1} - x^*|| + \delta^2(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)||r_n - x^*|| \\
&= ||p_{n+1} - x^*|| + \delta^2(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)||T((1-\gamma_n)s_n + \gamma_n T s_n) - x^*|| \\
&\leq ||p_{n+1} - x^*|| + \delta^3(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)||s_n - x^*|| \\
&= ||p_{n+1} - x^*|| + \delta^3(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)||T((1-\zeta_n)t_n + \zeta_n T t_n) - x^*|| \\
&\leq ||p_{n+1} - x^*|| + \delta^4(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)(1-(1-\delta)\zeta_n)||t_n - x^*|| \\
&= ||p_{n+1} - x^*|| + \delta^4(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)(1-(1-\delta)\zeta_n)||T((1-\eta_n)p_n + \eta_n T p_n) - x^*|| \\
&\leq ||p_{n+1} - x^*|| + \delta^5(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)(1-(1-\delta)\zeta_n)(1-(1-\delta)\eta_n)||p_n - x^*||,
\end{aligned}$$

Taking limit both sides, we have

$$\lim_{n\to\infty} \epsilon_n = 0.$$

Hence iteration process (3.1) is T-stable. □

3.3. Comparison Result. In this section, we comparing the new iteration scheme (3.1) and iteration scheme (1.2) for contractive mappings due to Berinde [26] :

THEOREM 3.3. *Let $\phi \neq M \subset K$ be closed and convex, where K is a uniformly convex Banach space. Let $T : M \to M$ with satisfying equation* (2.1) *and x^* be a fixed point of T. Supoose that $\{u_n\}$ and $\{r_n\}$ are sequences defined by iteration process* (3.1) *and* (1.2) *and $\sum_{n=0}^{\infty} \eta_n = \infty$. Then $\{u_n\}$ converges faster than $\{r_n\}$.*

Proof. From equation (3.7) in Theorem 3.1, we have

(3.11)

$$||u_{n+1} - x^*|| \leq \delta^{5(n+1)} \prod_{k=0}^{n} (1-(1-\delta)\alpha_k)(1-(1-\delta)\beta_k)(1-(1-\delta)\gamma_k)(1-(1-\delta)\zeta_k)(1-(1-\delta)\eta_k)||u_0 - x^*||$$

Now using iteration process (1.2) and equation (2.1), we have

$$\begin{aligned} ||s_n - x^*|| &= ||T(1-\zeta_n)r_n + \zeta_n T r_n) - x^*|| \\ &\leq \delta(1-\zeta_n)||r_n - x^*|| + \delta\zeta_n||Tr_n - x^*|| \\ &= \delta(1-(1-\delta)\zeta_n)||r_n - x^*||, \quad (3.12) \\ ||q_n - x^*|| &= ||T((1-\gamma_n)s_n + \gamma_n T s_n) - x^*|| \\ &\leq \delta(1-\gamma_n)||s_n - x^*|| + \delta\gamma_n||Ts_n - x^*|| \\ &= \delta(1-(1-\delta)\gamma_n)||s_n - x^*|| \quad (3.13) \\ ||p_n - x^*|| &= ||T((1-\beta_n)q_n + \beta_n T q_n) - x^*|| \\ &\leq \delta(1-\beta_n)||q_n - x^*|| + \delta\beta_n||Tq_n - x^*|| \\ &= \delta(1-(1-\delta)\beta_n)||q_n - x^*|| \quad (3.14) \\ ||r_{n+1} - x^*|| &= ||T((1-\alpha_n)p_n + \alpha_n T p_n) - x^*|| \\ &\leq \delta(1-\alpha_n)||p_n - x^*|| + \delta\alpha_n||Tp_n - x^*|| \\ &= \delta(1-(1-\delta)\alpha_n)||p_n - x^*|| \quad (3.15) \end{aligned}$$

From equation (3.12), (3.13), (3.14) and (3.15), we have

$$\begin{aligned} ||r_{n+1} - x^*|| &\leq \delta^4(1-(1-\delta)\alpha_n)(1-(1-\delta)\beta_n)(1-(1-\delta)\gamma_n)(1-(1-\delta)\zeta_n)||r_n - x^*|| \\ &\vdots \\ &\leq \delta^{4(n+1)} \prod_{k=0}^{\infty}(1-(1-\delta)\alpha_k)(1-(1-\delta)\beta_k)(1-(1-\delta)\gamma_k)(1-(1-\delta)\zeta_k)||r_0 - x^*|| \end{aligned}$$

Now

$$\begin{aligned} &\lim_{n\to\infty} \frac{||u_{n+1} - x^*||}{||r_{n+1} - x^*||} \\ &\leq \lim_{n\to\infty} \frac{\delta^{5(n+1)} \prod_{k=0}^{n}(1-(1-\delta)\alpha_k)(1-(1-\delta)\beta_k)(1-(1-\delta)\gamma_k)(1-(1-\delta)\zeta_k)(1-(1-\delta)\eta_k)||u_0 - x^*||}{\delta^{4(n+1)} \prod_{k=0}^{\infty}(1-(1-\delta)\alpha_k)(1-(1-\delta)\beta_k)(1-(1-\delta)\gamma_k)(1-(1-\delta)\zeta_k)||r_0 - x^*||} \\ &\leq \lim_{n\to\infty} \frac{\delta^{(n+1)} \prod_{k=0}^{\infty}(1-(1-\delta)\zeta_k)||u_0 - x^*||}{||r_0 - x^*||} \end{aligned}$$

Using inequality $1 - z \leq e^{-z}\ \forall\ z \in [0,1]$, we have

$$\begin{aligned} \lim_{n\to\infty} \frac{||u_{n+1} - x^*||}{||r_{n+1} - x^*||} &\leq \lim_{n\to\infty} \frac{||u_0 - x^*||\delta^{(n+1)}}{||r_0 - x^*||e^{(1-\delta)\sum_{n=0}^{\infty}\eta_n}} \\ &\leq 0. \end{aligned}$$

Hence by definition 2.6 and lemma 2.8, iteration process (3.1) converges to x^* faster than iteration process (1.2). □

3.4. Numerical Example.

Example 3.4. Let $K = R$ and $M = [0, 60]$ and $T : M \to M$ be a mapping defined by $T(x) = \sqrt{(x^2 - 6x + 48)}$, for all $x \in M$ for $x_0 = 100$ and $\alpha_n = \beta_n = \gamma_n = \zeta_n = \eta_n = 2/3$, $n = 1, 2, 3, \ldots$ From Table 1 we can see that all the iteration procedure are converging to $x^* = 8$ clearly, our iteration process requires the least number of iteration as compared to other iteration scheme.

The convergence behaviour of these iteration process are represented in the Figure 1.

It No.	New	Hassan	Ullah	Thakur	Abass	Agrwal	Noor	Ishikawa
0	100.00000000	100.00000000	100.0000000	100.00000000	100.00000000	100.00000000	100.00000000	100.00000000
1	76.88489427	81.46543216	90.69448347	94.09902562	93.89214394	95.95936500	95.44593195	96.89242398
2	54.45329523	63.31482922	81.46533656	88.22654404	87.81490958	91.93153627	90.90869431	93.79238794
3	33.39538742	45.79841313	72.33045956	82.38639575	81.77259116	87.91763715	86.38999096	90.70039161
4	15.97752996	29.48754556	63.31455234	76.58322320	75.77041826	83.91894046	81.89178870	87.61698448
5	8.55219930	15.97752996	54.45286163	70.82270235	69.81483834	79.93689555	77.41637248	84.54277243
6	8.01354766	9.08714700	45.79773758	65.11186097	63.91391009	75.97316133	72.96641528	81.47842546
7	8.00030721	8.06121826	37.43070552	59.45952389	58.07786208	72.02964733	68.54506760	78.42468678
8	8.00000695	8.00298149	29.48579681	53.87694799	52.31990248	68.10856483	64.15607319	75.38238351
9	8.00000016	8.00014401	22.19559369	48.37874621	46.65741883	64.21249133	59.80392033	72.35243933
10	8.00000000	8.00000695	15.97253226	42.98426007	41.11379562	60.34445234	55.49404180	69.33588953
11	8.00000000	8.00000034	11.45495317	37.71963945	35.72122535	56.50802617	51.23308243	66.33389899
12	8.00000000	8.00000002	9.08200038	32.62104378	30.52511930	52.70747968	47.02926101	63.34778373
13	8.00000000	8.00000000	8.26820505	27.73959064	25.59100874	48.94794547	42.89286528	60.37903703
14	8.00000000	8.00000000	8.06076331	23.14881556	21.01481733	45.23565581	38.83693502	57.42936099
15	8.00000000	8.00000000	8.01344474	18.95476278	16.93536486	41.57825411	34.87820942	54.50070518
16	8.00000000	8.00000000	8.00295873	15.30471974	13.53809028	37.98521309	31.03843705	51.59531397
17	8.00000000	8.00000000	8.00065033	12.37662020	11.01303065	34.46839920	27.34615248	48.71578499
18	8.00000000	8.00000000	8.00014291	10.30990262	9.42537361	31.04283503	23.83895687	45.86514160
19	8.00000000	8.00000000	8.00003140	9.07881940	8.60168214	27.72772040	20.56604605	43.04692319
20	8.00000000	8.00000000	8.00000690	8.46084622	8.23715727	24.54776342	17.58985797	40.26529812
21	8.00000000	8.00000000	8.00000152	8.18727724	8.09048406	21.53480474	14.98376119	37.52520521
22	8.00000000	8.00000000	8.00000033	8.07435771	8.03406046	18.72948889	12.8201608	34.83253115
23	8.00000000	8.00000000	8.00000007	8.02923577	8.01275409	16.18214452	11.14469090	32.19433187
24	8.00000000	8.00000000	8.00000002	8.01144958	8.00476633	13.95082100	9.94733639	29.61910606
25	8.00000000	8.00000000	8.00000000	8.00447699	8.00177989	12.09285757	9.15623528	27.11712591
26	8.00000000	8.00000000	8.00000000	8.00174951	8.00066448	10.64692213	8.66634550	24.70082122
27	8.00000000	8.00000000	8.00000000	8.00068351	8.00024804	9.61028643	8.37671638	22.38519001
28	8.00000000	8.00000000	8.00000000	8.00026701	8.00009259	8.92895137	8.21052211	20.18816228
29	8.00000000	8.00000000	8.00000000	8.00010430	8.00003456	8.51487210	8.11685900	18.13075417
30	8.00000000	8.00000000	8.00000000	8.00004074	8.00001290	8.27788669	8.06462079	16.23670723
31	8.00000000	8.00000000	8.00000000	8.00001592	8.00000482	8.14759647	8.03565798	14.53113577
32	8.00000000	8.00000000	8.00000000	8.00000622	8.00000180	8.07768589	8.01965293	13.03764143
33	8.00000000	8.00000000	8.00000000	8.00000243	8.00000067	8.04068727	8.01082464	11.7731536
34	8.00000000	8.00000000	8.00000000	8.00000095	8.00000025	8.02125334	8.00595995	10.74534560
35	8.00000000	8.00000000	8.00000000	8.00000037	8.00000009	8.01108640	8.00328084	9.94319427

TABLE 1. Comparison table

3.5. Convergence Results for Nonexpansive Mapping. In this section we establish some convergence results for nonexpansive mappings;

LEMMA 3.5. *Let $\phi \neq M \subset K$ be closed and convex, where K is a uniformly convex Banach space. Let $T : M \to M$ be a nonexpansive mapping with $F(T) \neq \phi$. Suppose that $\{u_n\}$ is generated by (3.1). Then $\lim_{n\to\infty} ||u_n - x^*||$ exists $\forall x^* \in F(T)$.*

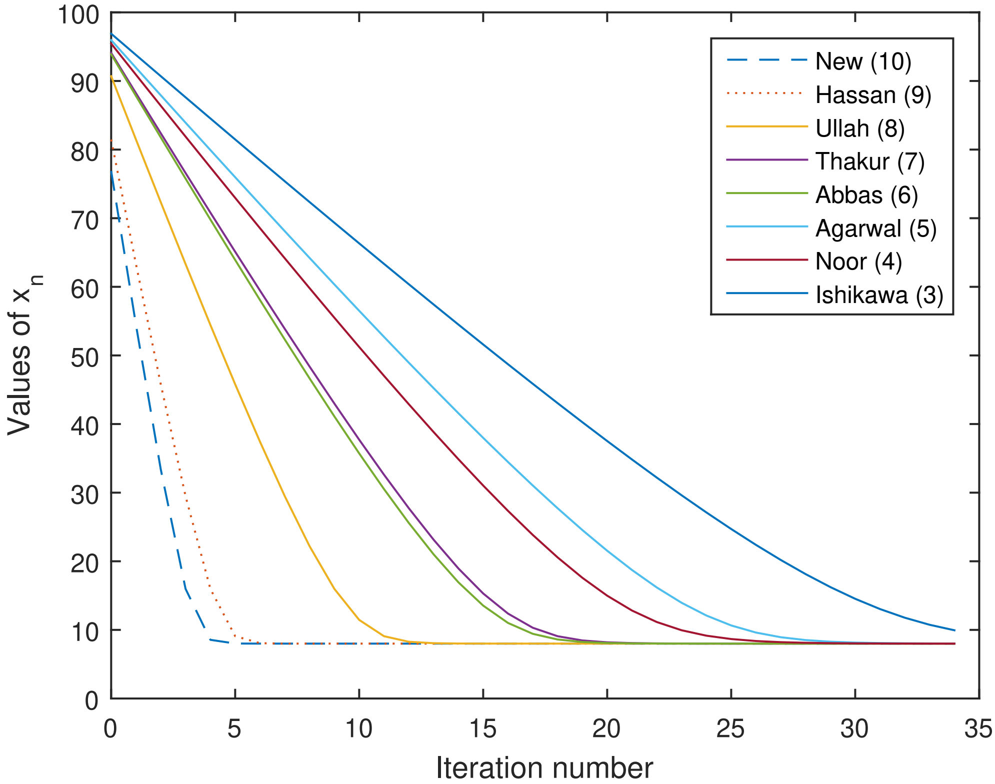

FIGURE 1. Comparison Plot

Proof. Let $x^* \in F(T)\ \forall\ n \in N$. Using iteration process (3.1), we have

$$\begin{aligned}
||y_n - x^*|| &= ||T((1-\eta_n)u_n + \eta_n T u_n) - x^*|| \\
&\leq (1-\eta_n)||u_n - x^*|| + \eta_n||Tu_n - x^*|| \\
&\leq ||u_n - x^*||, \quad (3.16) \\
||x_n - x^*|| &= ||T((1-\zeta_n)y_n + \zeta_n T y_n) - x^*|| \\
&\leq (1-\zeta_n)||y_n - x^*|| + \zeta_n||Ty_n - x^*|| \\
&\leq ||y_n - x^*|| \\
&\leq ||u_n - x^*||, \quad (3.17) \\
||w_n - x^*|| &= ||T((1-\gamma_n)x_n + \gamma_n T x_n) - x^*|| \\
&\leq (1-\gamma_n)||x_n - x^*|| + \gamma_n||Tx_n - x^*|| \\
&\leq ||x_n - x^*|| \\
&\leq ||u_n - x^*||, \quad (3.18) \\
||v_n - x^*|| &= ||T((1-\beta_n)w_n + \beta_n T w_n) - x^*|| \\
&\leq (1-\beta_n)||w_n - x^*|| + \beta_n||Tw_n - x^*|| \\
&\leq ||w_n - x^*|| \\
&\leq ||u_n - x^*||, \quad (3.19)
\end{aligned}$$

Thus

$$\begin{aligned} ||u_{n+1} - x^*|| &= ||T((1-\alpha_n)v_n + \alpha_n Tv_n) - x^*|| \\ &\leq (1-\alpha_n)||v_n - x^*|| + \alpha_n||Tv_n - x^*|| \\ &\leq ||v_n - x^*|| \\ &\leq ||u_n - x^*||, \end{aligned} \tag{3.20}$$

Hence $\lim_{n\to\infty} ||u_n - x^*|| = exists$ for all $x^* \in F(T)$. □

Lemma 3.6. *Let $\phi \neq M \subset K$ be closed and convex, where K is a uniformly convex Banach space. Let $T : M \to M$ be a nonexpansive mapping with $F(T) \neq \phi$. Suppose $\{u_n\}$ is generated by (3.1). Then $\lim\limits_{n\to\infty} ||Tu_n - u_n|| = 0$.*

Proof. Let $x^* \in F(T)$ and let $x^* \in M$. Then by Lemma 3.3

$$\lim_{n\to\infty} ||u_n - x^*|| = exists.$$

Let $\lim_{n\to\infty} ||u_n - x^*|| = a$.
Case I $a = 0$, we are done.
Case II $a > 0$. From equation (3.11) in Lemma 3.3, we have

$$\begin{aligned} ||y_n - x^*|| &\leq ||u_n - x^*|| \\ \limsup_{n\to\infty} ||y_n - x^*|| &\leq \limsup_{n\to\infty} ||u_n - x^*|| = a \end{aligned} \tag{3.21}$$

Since T is nonexpansive mapping then $||Ty_n - x^*|| \leq ||y_n - x^*||$. If follows that

$$\limsup_{n\to\infty} ||Ty_n - x^*|| \leq a. \tag{3.22}$$

$$\begin{aligned} ||u_{n+1} - x^*|| &= ||T((1-\alpha_n)v_n + \alpha_n Tv_n) - x^*|| \\ &\leq ||v_n - x^*|| \\ &= ||T((1-\beta_n)w_n + \beta_n Tw_n) - x^*|| \\ &\leq ||w_n - x^*|| \\ &= ||T((1-\gamma_n)x_n + \gamma_n Tx_n) - x^*|| \\ &\leq ||x_n - x^*|| \\ &= ||T((1-\zeta_n)y_n + \zeta_n Ty_n) - x^*|| \\ &\leq (1-\zeta_n)||u_n - x^*|| + \zeta_n||Ty_n - x^*|| \\ &= ||u_n - x^*|| - \zeta_n||u_n - x^*|| + \zeta_n||y_n - x^*||, \end{aligned} \tag{3.23}$$

This implies that

$$\frac{||u_{n+1} - x^*|| - ||u_n - x^*||}{\zeta_n} \leq ||y_n - x^*|| - ||u_n - x^*||$$

$$\begin{aligned} ||u_{n+1} - x^*|| - ||u_n - x^*|| &\leq \frac{||u_{n+1} - x^*|| - ||u_n - x^*||}{\zeta_n} \\ &\leq ||y_n - x^*|| - ||u_n - x^*||, \\ ||u_{n+1} - x^*|| &\leq ||y_n - x^*|| \\ a &\leq \liminf_{n\to\infty} ||y_n - x^*|| \\ \lim_{n\to\infty} ||y_n - x^*|| &= a \\ \lim_{n\to\infty} ||T((1-\eta_n)u_n + \eta_n T u_n) - x^*|| &= a \\ \lim_{n\to\infty} ||(1-\eta_n)(u_n - x^*) + \eta(Tu_n - x^*)|| &= a \end{aligned}$$

Hence by Lemma 2.10, we have

$$\lim_{n\to\infty} ||Tu_n - u_n|| = 0.$$

□

THEOREM 3.7. *Let $\phi \neq M \subset K$ be closed and convex, where K is a uniformly convex Banach space with satisfy opial's property. Let $T : M \to M$ be a nonexpansive mapping with $F(T) \neq \phi$. Suppose that $\{u_n\}$ is generated by* (3.1). *Then $\{u_n\}$ converges weakly to a fixed point of T.*

Proof. Let $x^* \in F(T)$. By Lemma 3.3, then $\lim_{n\to\infty} ||u_n - x^*||$ exists. We prove that $\{u_n\}$ has a unique weak subsequential limit in $F(T)$. Let p and q be weak limits of the subsequences $\{u_m\}$ and $\{u_k\}$ of $\{u_n\}$ respectively. From Lemma 3.4, we have $\lim_{n\to\infty} ||u_n - Tu_n|| = 0$ and $I - T$ is demiclosed with respect to zero by Lemma 2.11, we have that $Tp = p$. Similar to prove that $q \in F(T)$. Next we have to show that uniqueness. From Lemma 3.3, we have $\lim_{n\to\infty} ||u_n - q||$ exist. Now suppose that $p \neq q$, then by opial's condition,

$$\begin{aligned} \lim_{n\to\infty} ||u_n - p|| &= \lim_{m\to\infty} ||u_m - p|| \\ &< \lim_{m\to\infty} ||u_m - q|| \\ &= \lim_{n\to\infty} ||u_n - q|| \\ &= \lim_{k\to\infty} ||u_k - q|| \\ &\leq \lim_{k\to\infty} ||u_k - p|| \\ &= \lim_{n\to\infty} ||u_n - p||. \end{aligned}$$

which is contradiction, so $p = q$. Hence $\{u_n\}$ converges weakly to a fixed point of $F(T)$. □

THEOREM 3.8. *Let $\phi \neq M \subset K$ be closed and convex, where K is a uniformly convex Banach space. Let $T : M \to M$ be a nonexpansive mapping with $F(T) \neq \phi$. Suppose that $\{u_n\}$ is generated by* (3.1).*Then the sequence $\{u_n\}$ converges to a fixed* *point of T iff* $\liminf_{n\to\infty} d(u_n, F(T)) = 0$*, where* $d(u, F(T)) = inf\{||u - x^*|| : x^* \in F(T)\}$.

Proof. Let $\{u_n\}$ converges to x^*, then $\lim_{n\to\infty} d(u_n, x^*) = 0$.It follows that $\lim_{n\to\infty} d(u_n, F(T)) = 0$. Therefore, $\liminf_{n\to\infty} d(u_n, F(T)) = 0$.
Conversely: Suppose that $\lim_{n\to\infty} d(u_n, F(T)) = 0$. It follows from Lemma 3.3, that $\lim\limits_{n\to\infty} ||u_n - x^*||$ exists and that $\liminf_{n\to\infty} d(u_n, F(T))$ exists for all $x^* \in F(T)$. But by our assumption $\liminf_{n\to\infty} d(u_n, F(T)) = 0$, therefore we have $\lim\limits_{n\to\infty} d(u_n, F(T)) = 0$. We will show that $\{u_n\}$ is a cauchy sequence in M. Since $\lim\limits_{n\to\infty} d(u_n, F(T)) = 0$ for given $\epsilon > 0$, $\exists$, $n_0 \in N$ s.t. $\forall n \geq n_0$,

$$d(u_n, F(T)) < \frac{\epsilon}{2}.$$

In particular, $\inf\{||u_{n_0} - x^*|| : x_1^* \in F(T)\} < \frac{\epsilon}{2}$.
Hence $\exists x_1^* \in F(T)$ s.t.$||u_{n_0} - x_1^*|| < \frac{\epsilon}{2}$.
Now for $m, n \geq n_0$,

$$\begin{aligned} ||u_{n+m} - u_n|| &\leq ||u_{n+m} - x_1^*|| + ||u_n - x_1^*|| \\ &\leq 2||u_{n_0} - x_1^*|| \\ &< \epsilon. \end{aligned}$$

Hence $\{u_n\}$ is a cauchy sequence in M. Since M is closed in a uniformly Banach space K, so that $\exists$ a point $x^* \in M$ s.t. $\lim\limits_{n\to\infty} u_n = x^*$. Now $\lim\limits_{n\to\infty} d(u_n, F(T)) = 0$, gives that $d(x^*, F(T)) = 0$, since $F(T)$ is closed, $x^* \in F(T)$. □

Theorem 3.9. *Let $\phi \neq M \subset K$ be closed and convex, where K is a uniformly convex Banach space. Let $T : M \to M$ be a nonexpansive mapping with $F(T) \neq \phi$. Suppose that $\{u_n\}$ is generated by* (3.1)*. Let T satisfy condition A, then $\{u_n\}$ converges strongly to a fixed point of T.*

Proof. By using Lemma 3.4, we have

$$\lim_{n\to\infty} ||u_n - Tu_n|| = 0. \tag{3.24}$$

From condition A and equation (3.24), we have

$$\begin{aligned} \lim_{n\to\infty} f(d(u_n, F(T))) &\leq \lim_{n\to\infty} ||u_n - Tu_n|| \\ &\Rightarrow \lim_{n\to\infty} f(d(u_n, F(T))) = 0. \end{aligned}$$

Since $f : [0, \infty) \to [0, \infty)$ is a non decreasing function satisfying $f(0) = 0$ and $f(c) > 0$ for all $c \in (0, \infty)$, therefore, we have

$$\lim_{n\to\infty} d(u_n, F(T)) = 0.$$

By Theorem 3.6, the sequence $\{u_n\}$ strongly converges to a fixed point of $F(T)$. □

4. Conclusion

The conclusion of our work as follows:

- We have introduced a new iteration process and proved it to be faster than other existing processes in this paper with the help of numerical examples.

- We have discussed the convergence and stability results for the approximation of fixed points of the contractive-like mapping.
- We have established some convergence results for the approximation of fixed points of a nonexpansive mapping.

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