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PAIR DIFFERENCE CORDIAL LABELING OF m-COPIES OF SOME GRAPHS

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ABSTRACT. In this paper we investigate the pair difference cordial labeling behaviour of $m-$ copies of K_4, subdivision of star, fan, comb graphs.

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1. Introduction

We consider only finite, undirected and simple graphs. The concept of cordial labeling was introduced by Cahit[1]. Cordial related labeling was studied in [14,15,16,17,18].Ponraj and Parthipan have been defined the pair sum labeling in [12]. Laterly the difference cordial labeling of graphs was introdced in [13]. Motivated by these two concepts we have introduced the pair difference cordial labeling of graphs in [4].The pair difference cordial labeling behaviour of path, cycle, star, wheel,triangular snake,alternate triangular snake, butterfly etc have been investigated in [4,5,6,7,8,9,10,11]. In this paper we investigate the pair difference cordial labeling behaviour of $m-$ copies of K_4, subdivision of star,fan,comb graphs.

2. preliminaries

Definition 2.1. [4]. Let $G = (V, E)$ be a (p, q) graph.
Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm \rho\}$ called the set of labels.
Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and

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repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1.A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph.

Theorem 2.2. [4]. *The star $K_{1,n}$ is pair difference cordial if and only if $3 \leq n \leq 6$.*

Theorem 2.3. [4]. *The wheel W_n is pair difference cordial if and only if n is even.*

Corollary 2.4. [5]. *The complete graph K_p is pair difference cordial if and only if $p \leq 2$.*

Theorem 2.5. [4]. *The comb $Pn \odot K_1$ is a pair difference cordial for all values of n.*

3. Main results

Theorem 3.1. *The m- copies of K_4 is pair difference cordial for all even values of m.*

Proof. Let ${v_i}^{(j)}, 1 \leq i \leq 4$ be the vertices of the j^{th} copy of K_4, $1 \leq j \leq m$.

Consider the first copy K_4. Assign the labels $1, 2, 3, \cdots, n$ respectively to the vertices ${v_1}^{(1)}, {v_2}^{(1)}, {v_3}^{(1)}, \cdots, {v_n}^{(1)}$ and next consider the second copy K_4, assign the labels $(n{+}1), (n{+}2), (n{+}3), \cdots, (2n)$ to the vertices ${v_1}^{(2)}, {v_2}^{(2)}, {v_3}^{(2)}, \cdots, {v_n}^{(2)}$. Next assign the labels $(2n{+}1), (2n{+}2), (2n{+}3), \cdots, (3n)$ respectively to the vertices ${v_1}^{(3)}, {v_2}^{(3)}, {v_3}^{(3)}, \cdots, {v_n}^{(3)}$ of the third copy K_4. Proceeding like this until we reach the vertices of the ${\frac{m}{2}}^{th}$ copy K_4. Note that the vertices ${v_1}^{(\frac{m}{2})}, {v_2}^{(\frac{m}{2})}, {v_3}^{(\frac{m}{2})}, \cdots, {v_n}^{(\frac{m}{2})}$ gets the labels $\frac{mn+2}{2}$, $\frac{mn+4}{2}$, $\frac{mn+6}{2}, \cdots, \frac{mn+2n}{2}$ respectively.

Consider the ${\frac{m+2}{2}}^{th}$ copy K_4. Assign the labels $-1, -2, -3, \cdots, -n$ respectively to the vertices ${v_1}^{(\frac{m+2}{2})}, {v_2}^{(\frac{m+2}{2})}, {v_3}^{(\frac{m+2}{2})}, \cdots, {v_n}^{(\frac{m+2}{2})}$ and assign the labels $-(n{+}1), -(n{+}2), -(n{+}3), \cdots, -(2n)$ to the vertices ${v_1}^{(\frac{m+4}{2})}, {v_2}^{(\frac{m+4}{2})}, {v_3}^{(\frac{m+4}{2})}, \cdots, {v_n}^{(\frac{m+4}{2})}$ of the ${\frac{m+4}{2}}^{th}$ copy K_4. Next assign the labels $-(2n+1), -(2n+2), -(2n+3), \cdots, -(3n)$ respectively to the vertices ${v_1}^{(\frac{m+6}{2})}, {v_2}^{(\frac{m+6}{2})}, {v_3}^{(\frac{m+6}{2})}, \cdots, {v_n}^{(\frac{m+6}{2})}$ of the ${\frac{m+6}{2}}^{th}$ copy K_4. Proceeding this process until we reach the vertices ${v_1}^{(m)}, {v_2}^{(m)}, {v_3}^{(m)}, \cdots, {v_n}^{(m)}$ of the m^{th} copy of K_4. We notice that the vertices ${v_1}^{(m)}, {v_2}^{(m)}, {v_3}^{(m)}, \cdots, {v_n}^{(m)}$ respectively receives the labels $-(\frac{mn+2}{2})$, $-(\frac{mn+4}{2})$, $-(\frac{mn+6}{2}), \cdots, -(\frac{mn+2n}{2})$ respectively.

Here $\Delta_{f_1} = \Delta_{f_1^c} = 3m$.

□

Theorem 3.2. *The m- copies of wheel W_n is pair difference cordial for all even values of m and for all values of $n \geq 3$.*

Proof. Let ${W_n}^{(j)}, 1 \leq i \leq n$ be the j^{th} copy of wheel W_n, $1 \leq j \leq m$.

Let $V({W_n}^{(j)}) = \{{v_i}^{(j)}, v^{(j)} : 1 \leq i \leq n\}$ and $E({W_n}^{(j)}) = \{{v_i}^{(j)}v^{(j)} : 1 \leq i \leq n\} \cup \{v_i^{(j)}{v_{i+1}}^{(j)} : 1 \leq i \leq n-1\} \cup \{{v_1}^{(j)}{v_n}^{(j)}\}$.

Consider the first copy ${W_n}^{(1)}$. Assign the labels $1, 2, 3, \cdots, n$ respectively to the vertices ${v_1}^{(1)}, {v_2}^{(1)}, {v_3}^{(1)}, \cdots, {v_n}^{(1)}$ and next consider the second copy of the wheel , assign the labels $(n+2), (n+3), (n+4), \cdots, (2n+1)$ to the vertices ${v_1}^{(2)}, {v_2}^{(2)}, {v_3}^{(2)}, \cdots, {v_n}^{(2)}$. Next assign the labels $(2n+3), (2n+4), (2n+5), \cdots, (3n+2)$ respectively to the vertices ${v_1}^{(3)}, {v_2}^{(3)}, {v_3}^{(3)}, \cdots, {v_n}^{(3)}$ of the third copy ${W_n}^{(3)}$. Proceeding like this until we reach the vertices of the $\frac{m}{2}^{th}$ wheel ${W_n}^{(\frac{m}{2})}$. Note that the vertices ${v_1}^{(\frac{m}{2})}, {v_2}^{(\frac{m}{2})}, {v_3}^{(\frac{m}{2})}, \cdots, {v_n}^{(\frac{m}{2})}$ gets the labels $\frac{m(n+1)}{2} - n$, $\frac{m(n+1)}{2} - (n-1)$, $\frac{m(n+1)}{2} - (n-2), \cdots, \frac{m(n+1)}{2} - 2, \frac{m(n+1)}{2} - 1$ respectively.

Consider the $\frac{m+2}{2}^{th}$ copy ${W_n}^{(\frac{m+2}{2})}$. Assign the labels $-1, -2, -3, \cdots, -n$ respectively to the vertices ${v_1}^{(\frac{m+2}{2})}, {v_2}^{(\frac{m+2}{2})}, {v_3}^{(\frac{m+2}{2})}, \cdots, {v_n}^{(\frac{m+2}{2})}$ and assign the labels $-(n+2), -(n+3), -(n+4), \cdots, -(2n+1)$ to the vertices ${v_1}^{(\frac{m+4}{2})}, {v_2}^{(\frac{m+4}{2})}$, ${v_3}^{(\frac{m+4}{2})}, \cdots, {v_n}^{(\frac{m+4}{2})}$ of the $\frac{m+4}{2}^{th}$ copy ${W_n}^{(\frac{m+4}{2})}$. Next assign the labels $-(2n+3), -(2n+4), -(2n+5), \cdots, -(3n+2)$ respectively to the vertices ${v_1}^{(\frac{m+6}{2})}, {v_2}^{(\frac{m+6}{2})}$, ${v_3}^{(\frac{m+6}{2})}, \cdots, {v_n}^{(\frac{m+6}{2})}$ of the $\frac{m+6}{2}^{th}$ copy ${W_n}^{(\frac{m+6}{2})}$. Proceeding this process until we reach the vertices ${v_1}^{(m)}, {v_2}^{(m)}, {v_3}^{(m)}, \cdots, {v_n}^{(m)}$ of the m^{th} copy ${W_n}^{(m)}$. We notice that the vertices ${v_1}^{(m)}, {v_2}^{(m)}, {v_3}^{(m)}, \cdots, {v_n}^{(m)}$ respectively receives the labels $-(\frac{m(n+1)}{2} - n)$, $-(\frac{m(n+1)}{2} - (n-1))$, $-(\frac{m(n+1)}{2} - (n-2)), \cdots, -(\frac{m(n+1)}{2} - 2), -(\frac{m(n+1)}{2} - 1)$ respectively.

Finally assign the labels $(n+1), (2n+2), (3n+3), \cdots, \frac{m(n+1)}{2}$ respectively to the vertices $v^{(1)}, v^{(2)}, v^{(3)}, \cdots, v^{(\frac{m}{2})}$ respectively and assign the labels $-(n+1), -(2n+2), -(3n+3), \cdots, -\frac{m(n+1)}{2}$ to the vertices $v^{(\frac{m+2}{2})}, v^{(\frac{m+4}{2})}, v^{(\frac{m+6}{2})}, \cdots, v^{(m)}$.

Clearly $\Delta_{f_1} = \Delta_{f_1^c} = \frac{mn}{2}$.

□

Theorem 3.3. *The m- copies of the subdivision of star $K_{1,n}$ is pair difference cordial for all even values of m and for all values of n.*

Proof. Let $S(K_{1,n}^{(j)},)1 \leq i \leq n$ be the j^{th} copy of the subdivision of star $K_{1,n}$, $1 \leq j \leq m$.

Let $V(K_{1,n}{}^{(j)}) = \{u^{(j)}, g_i{}^{(j)}, v_i{}^{(j)} : 1 \leq i \leq n\}$ and $E(K_{1,n}^{(j)}) = \{u^{(j)}g_i{}^{(j)}, g_i{}^{(j)}v_i{}^{(j)} : 1 \leq i \leq n\}$.

Consider the first subdivision of star. Assign the labels $1, 3, 5 \cdots$, $(2n-1)$ respectively to the vertices $g_1{}^{(1)}, g_2{}^{(1)}, g_3{}^{(1)}, \cdots, g_n{}^{(1)}$ and assign the labels $2, 4, 6, \cdots, (2n)$ to the vertices $v_1{}^{(1)}, v_2{}^{(1)}, v_3{}^{(1)}, \cdots, v_n{}^{(1)}$ respectively. Next consider the second subdivision of star. Assign the labels $(2n+1), (2n+3), (2n+5), \cdots, (4n-1)$ respectively to the vertices $g_1{}^{(2)}, g_2{}^{(2)}, g_3{}^{(2)}, \cdots, g_n{}^{(2)}$ and assign the labels $(2n+2), (2n+4), (2n+6), \cdots, (4n)$ to the vertices $v_1{}^{(2)}, v_2{}^{(2)}, v_3{}^{(2)}, \cdots, v_n{}^{(2)}$ respectively. Now we consider the vertices of the third copy $S(K_{1,n}^{(3)})$. Assign the labels $(4n+1), (4n+3), (4n+5), \cdots, (6n-1)$ respectively to the vertices $g_1{}^{(3)}, g_2{}^{(3)}, g_3{}^{(3)}, \cdots, g_n{}^{(3)}$ and assign the labels $(4n+2), (4n+4), (4n+6), \cdots, (6n)$ to the vertices $v_1{}^{(3)}, v_2{}^{(3)}, v_3{}^{(3)}, \cdots, v_n{}^{(3)}$ respectively. Proceeding like this until we reach the vertices of the $\frac{m}{2}$ copy $S(K_{1,n}^{(\frac{m}{2})})$.

Now we consider the $\frac{m+2}{2}^{th}$ copy $S(K_{1,n}^{(\frac{m+2}{2})})$. Assign the labels $-1, -3, -5 \cdots$, $-(2n-1)$ respectively to the vertices $g_1{}^{(\frac{m+2}{2})}, g_2{}^{(\frac{m+2}{2})}, g_3{}^{(\frac{m+2}{2})}, \cdots, g_n{}^{(\frac{m+2}{2})}$ and assign the labels $-2, -4, -6, \cdots, -2n$ to the vertices $v_1{}^{(\frac{m+2}{2})}, v_2{}^{(\frac{m+2}{2})}, v_3{}^{(\frac{m+2}{2})}, \cdots, v_n{}^{(\frac{m+2}{2})}$ respectively. Next consider the $\frac{m+4}{2}^{th}$ copy $S(K_{1,n}^{(\frac{m+4}{2})})$. Assign the labels $-(2n+1), -(2n+3), -(2n+5), \cdots, -(4n-1)$ respectively to the vertices $g_1{}^{(\frac{m+4}{2})}, g_2{}^{(\frac{m+4}{2})}, g_3{}^{(\frac{m+4}{2})}, \cdots, g_n{}^{(\frac{m+4}{2})}$ and assign the labels $-(2n+2), -(2n+4), -(2n+6), \cdots, -(4n)$ to the vertices $v_1{}^{(\frac{m+4}{2})}, v_2{}^{(\frac{m+4}{2})}, v_3{}^{(\frac{m+4}{2})}, \cdots, v_n{}^{(\frac{m+4}{2})}$. Proceeding this process until we reach the vertices of the m^{th} subdivision of star $S(K_{1,n}^{(m)})$. Note that the vertices $g_1{}^{(m)}, g_2{}^{(m)}, g_3{}^{(m)}, \cdots, g_n{}^{(m)}$ gets the labels $-(mn-(2n-1)), -(mn-(2n-3)), -(mn-(2n-5)), \cdots, -(mn-1)$ respectively and the vertices $v_1{}^{(m)}, v_2{}^{(m)}, v_3{}^{(m)}, \cdots, v_n{}^{(m)}$ respectively receive the labels $-(mn-(2n-2)), -(mn-(2n-4)), -(mn-(2n-6)), \cdots, -(mn)$.

Finally assign the labels $(mn+1), (mn+2), (mn+3), \cdots, (mn-\frac{m}{2})$ respectively to the vertices $u^{(1)}, u^{(2)}, u^{(3)}, \cdots, u^{(\frac{m}{2})}$ and assign the labels $-(mn+1), -(mn+2), -(mn+3), \cdots, -(mn-\frac{m}{2})$ to the vertices $u^{(\frac{m+2}{2})}, u^{(\frac{m+4}{2})}, u^{(\frac{m+6}{2})}, \cdots, u^{(m)}$ respectively.

Here $\Delta_{f_1} = \Delta_{f_1^c} = mn$

□

Theorem 3.4. *The m- copies of the subdivision of star $K_{1,n}$ is pair difference cordial for all odd values of m and for all values of n.*

Proof. Take the vertex set and edge set in thm 3.3.

Consider the first copy $S(K_{1,n}^{(1)})$. Assign the labels $1, 3, 5 \cdots, (2n-1)$ respectively to the vertices $g_1{}^{(1)}, g_2{}^{(1)}, g_3{}^{(1)}, \cdots, g_n{}^{(1)}$ and assign the labels $2, 4, 6, \cdots, (2n)$ to the vertices $v_1{}^{(1)}, v_2{}^{(1)}, v_3{}^{(1)}, \cdots, v_n{}^{(1)}$ respectively. Next consider the second copy $S(K_{1,n}^{(2)})$. Assign the labels $(2n+1), (2n+3), (2n+5), \cdots, (4n-1)$ respectively to the vertices $g_1{}^{(2)}, g_2{}^{(2)}, g_3{}^{(2)}, \cdots, g_n{}^{(2)}$ and assign the labels $(2n+2), (2n+4), (2n+6), \cdots, (4n)$ to the vertices $v_1{}^{(2)}, v_2{}^{(2)}, v_3{}^{(2)}, \cdots, v_n{}^{(2)}$ respectively. Now we consider the vertices of the third copy $S(K_{1,n}^{(3)})$. Assign the labels $(4n+1), (4n+3), (4n+5), \cdots, (6n-1)$ respectively to the vertices $g_1{}^{(3)}, g_2{}^{(3)}, g_3{}^{(3)}, \cdots, g_n{}^{(3)}$ and assign the labels $(4n+2), (4n+4), (4n+6), \cdots, (6n)$ to the vertices $v_1{}^{(3)}, v_2{}^{(3)}, v_3{}^{(3)}, \cdots, v_n{}^{(3)}$ respectively. Proceeding like this until we reach the vertices of the $\frac{m-1}{2}$ copy of the subdivision of star. Note that the vertices $g_1{}^{(\frac{m-1}{2})}, g_2{}^{(\frac{m-1}{2})}, g_3{}^{(\frac{m-1}{2})}, \cdots, g_n{}^{(\frac{m-1}{2})}$ receive the label $(\frac{m-1}{2}n-(2n-1)), (\frac{m-1}{2}n-(2n-3)), (\frac{m-1}{2}n-(2n-5)), \cdots, (\frac{m-1}{2}n-1)$ and assign the labels $(\frac{m-1}{2}n-(2n-2)), (\frac{m-1}{2}n-(2n-4)), (\frac{m-1}{2}n-(2n-6)), \cdots, (\frac{m-1}{2}n)$ to the vertices $v_1{}^{(\frac{m-1}{2})}, v_2{}^{(\frac{m-1}{2})}, v_3{}^{(\frac{m-1}{2})}, \cdots, v_n{}^{(\frac{m-1}{2})}$ respectively.

Secondly we consider the $\frac{m+1}{2}^{th}$ copy $S(K_{1,n}^{(\frac{m+1}{2})})$. Assign the labels $-1, -3, -5, \cdots, -(2n-1)$ respectively to the vertices $g_1{}^{(\frac{m+1}{2})}, g_2{}^{(\frac{m+1}{2})}, g_3{}^{(\frac{m+1}{2})}, \cdots, g_n{}^{(\frac{m+1}{2})}$ and assign the labels $-2, -4, -6, \cdots, -2n$ to the vertices $v_1{}^{(\frac{m+1}{2})}, v_2{}^{(\frac{m+1}{2})}, v_3{}^{(\frac{m+1}{2})}, \cdots, v_n{}^{(\frac{m+1}{2})}$ respectively. Next consider the $\frac{m+3}{2}^{th}$ subdivision of star . Assign the labels $-(2n+1), -(2n+3), -(2n+5), \cdots, -(4n-1)$ respectively to the vertices $g_1{}^{(\frac{m+3}{2})}, g_2{}^{(\frac{m+3}{2})}, g_3{}^{(\frac{m+3}{2})}, \cdots, g_n{}^{(\frac{m+3}{2})}$ and assign the labels $-(2n+2), -(2n+4), -(2n+6), \cdots, -(4n)$ to the vertices $v_1{}^{(\frac{m+3}{2})}, v_2{}^{(\frac{m+3}{2})}, v_3{}^{(\frac{m+3}{2})}, \cdots, v_n{}^{(\frac{m+3}{2})}$. Proceeding this process until we reach the vertices of the $(m-1)^{th}$ copy . Here the vertices $g_1{}^{(m-1)}, g_2{}^{(m-1)}, g_3{}^{(m-1)} \cdots, g_n{}^{(m-1)}$ receive the label $-(\frac{m-1}{2}n-(2n-1)), -(\frac{m-1}{2}n-(2n-3)), -(\frac{m-1}{2}n-(2n-5)), \cdots, -(\frac{m-1}{2}n-1)$ and assign the labels $-(\frac{m-1}{2}n-(2n-2)), -(\frac{m-1}{2}n-(2n-4)), -(\frac{m-1}{2}n-(2n-6)), \cdots, -(\frac{m-1}{2}n)$ to the vertices $v_1{}^{(m-1)}, v_2{}^{(m-1)}, v_3{}^{(m-1)}, \cdots, v_n{}^{(m-1)}$ respectively.

Finally consider the vertices of the m^{th} copy $S(K_{1,n}^{(m)})$. There are two cases arises.

Case 1. n is odd .

Assign the labels $(\frac{m-1}{2}n+1), (\frac{m-1}{2}n+3), (\frac{m-1}{2}n+5), \cdots, (mn-2)$ respectively to the vertices $g_1{}^{(m)}, g_2{}^{(m)}, g_3{}^{(m)}, \cdots, g_{(\frac{n-1}{2})}{}^{(m)}$. Assign the labels $(\frac{m-1}{2}n+2), (\frac{m-1}{2}n+4), (\frac{m-1}{2}n+6), \cdots, (mn-1)$ to the vertices $v_1{}^{(m)}, v_2{}^{(m)}, v_3{}^{(m)}, \cdots,$

${v_{(\frac{n-1}{2})}}^{(m)}$ respectively. Next we assign the labels $-(\frac{m-1}{2}n+1)$,$-(\frac{m-1}{2}n+3)$,$-(\frac{m-1}{2}n+5),\cdots$,$-(mn-2)$ respectively to the vertices ${g_{(\frac{n+1}{2})}}^{(m)}, {g_{(\frac{n+3}{2})}}^{(m)}$, ${g_{(\frac{n+5}{2})}}^{(m)},\cdots$,${g_{(n-1)}}^{(m)}$. Assign the labels $-(\frac{m-1}{2}n+2)$,$-(\frac{m-1}{2}n+4)$,$-(\frac{m-1}{2}n+6),\cdots$,$-(mn-1)$ to the vertices ${v_{(\frac{n+1}{2})}}^{(m)}, {v_{(\frac{n+3}{2})}}^{(m)}$,${v_{(\frac{n+5}{2})}}^{(m)},\cdots$,${v_{(n-1)}}^{(m)}$ respectively. Now we assign the labels $mn+1, mn+2, mn+3,\cdots, mn+\frac{m-1}{2}$ respectively $u^{(1)}, u^{(2)}, u^{(3)},\cdots$,$u^{(\frac{m-1}{2})}$ and assign the labels $-(mn+1), -(mn+2), -(mn+3),\cdots$,$-(mn-\frac{m-1}{2})$ to the vertices $u^{(\frac{m+1}{2})}$,$u^{(\frac{m+3}{2})}$,$u^{(\frac{m+5}{2})},\cdots$,$u^{(m-1)}$ respectively. Also we assign the labels $mn, -mn, mn-1$ respectively to the vertices ${g_n}^{(m)}$,${v_n}^{(m)}$,$u^{(m)}$.

Case 2. n is even .

Assign the labels $(\frac{m-1}{2}n+1)$,$(\frac{m-1}{2}n+3)$,$(\frac{m-1}{2}n+5),\cdots$,$(mn-1)$ respectively to the vertices ${g_1}^{(m)}$,${g_2}^{(m)}, {g_3}^{(m)},\cdots, {g_{(\frac{n}{2})}}^{(m)}$. Assign the labels $(\frac{m-1}{2}n+2)$,$(\frac{m-1}{2}n+4)$,$(\frac{m-1}{2}n+6),\cdots$,$(mn)$ to the vertices ${v_1}^{(m)}$,${v_2}^{(m)}$,${v_3}^{(m)},\cdots$,${v_{(\frac{n}{2})}}^{(m)}$ respectively. Next we assign the labels $-(\frac{m-1}{2}n+1)$,$-(\frac{m-1}{2}n+3)$,$-(\frac{m-1}{2}n+5),\cdots$,$-(mn-1)$ respectively to the vertices ${g_{(\frac{n+2}{2})}}^{(m)}, {g_{(\frac{n+4}{2})}}^{(m)}$,${g_{(\frac{n+6}{2})}}^{(m)},\cdots$, ${g_{(n)}}^{(m)}$. Assign the labels $-(\frac{m-1}{2}n+2)$,$-(\frac{m-1}{2}n+4)$,$-(\frac{m-1}{2}n+6),\cdots$,$-(mn)$ to the vertices ${v_{(\frac{n+2}{2})}}^{(m)}, {v_{(\frac{n+4}{2})}}^{(m)}$,${v_{(\frac{n+6}{2})}}^{(m)},\cdots$,${v_{(n)}}^{(m)}$ respectively. Now we assign the labels $mn+1, mn+2, mn+3,\cdots, mn+\frac{m-1}{2}$ respectively $u^{(1)}, u^{(2)}, u^{(3)},\cdots$, $u^{(\frac{m-1}{2})}$ and assign the labels $-(mn+1), -(mn+2), -(mn+3),\cdots$,$-(mn-\frac{m-1}{2})$ to the vertices $u^{(\frac{m+1}{2})}$,$u^{(\frac{m+3}{2})}$,$u^{(\frac{m+5}{2})},\cdots$,$u^{(m-1)}$ respectively. Also we assign the labels $mn-1$ to the vertex $u^{(m)}$.

Clearly $\Delta_{f_1} = \Delta_{f_1^c} = mn$.

□

Theorem 3.5. *The m- copies of fan F_n is pair difference cordial for all even values of m and for all values of n.*

Proof. Let $F_n^{(j)}, 1 \le i \le n$ be the j^{th} copy of the fan F_n, $1 \le j \le m$.

Let $V({F_n}^{(j)}) = \{{v_i}^{(j)}, v^{(j)} : 1 \le i \le n\}$ and $E({F_n}^{(j)}) = \{{v_i}^{(j)}v^{(j)} : 1 \le i \le n\} \cup \{v_i^{(j)}{v_{i+1}}^{(j)} : 1 \le i \le n-1\}$.

Consider the first fan ${F_n}^{(1)}$. Assign the labels $1, 2, 3,\cdots, n$ respectively to the vertices ${v_1}^{(1)}, {v_2}^{(1)}, {v_3}^{(1)},\cdots, {v_n}^{(1)}$ and next consider the second copy ${F_n}^{(2)}$, assign the labels $(n+2), (n+3), (n+4),\cdots, (2n+1)$ to the vertices ${v_1}^{(2)}, {v_2}^{(2)}, {v_3}^{(2)},\cdots, {v_n}^{(2)}$. Next assign the labels $(2n+3), (2n+4), (2n+5),\cdots, (3n+2)$ respectively to the vertices ${v_1}^{(3)}, {v_2}^{(3)}, {v_3}^{(3)},\cdots, {v_n}^{(3)}$ of the third fan ${F_n}^{(3)}$. Proceeding like this until we reach the vertices of the $\frac{m}{2}^{th}$ copy ${F_n}^{(\frac{m}{2})}$. Note that the vertices ${v_1}^{(\frac{m}{2})}, {v_2}^{(\frac{m}{2})}, {v_3}^{(\frac{m}{2})}, \cdots, {v_n}^{(\frac{m}{2})}$ gets the labels $\frac{m(n+1)}{2} - n$, $\frac{m(n+1)}{2} - (n-1)$, $\frac{m(n+1)}{2} - (n-2), \cdots, \frac{m(n+1)}{2} - 2, \frac{m(n+1)}{2} - 1$ respectively.

Consider the $\frac{m+2}{2}^{th}$ fan ${F_n}^{(\frac{m+2}{2})}$. Assign the labels $-1, -2, -3, \cdots, -(n-1)$ respectively to the vertices ${v_1}^{(\frac{m+2}{2})}, {v_2}^{(\frac{m+2}{2})}, {v_3}^{(\frac{m+2}{2})}, \cdots, {v_{n-1}}^{(\frac{m+2}{2})}$ and assign the labels $-(n+2), -(n+3), -(n+4), \cdots, -(2n)$ to the vertices ${v_1}^{(\frac{m+4}{2})}, {v_2}^{(\frac{m+4}{2})}$, ${v_3}^{(\frac{m+4}{2})}, \cdots, {v_{n-1}}^{(\frac{m+4}{2})}$ of the $\frac{m+4}{2}^{th}$ copy ${F_n}^{(\frac{m+4}{2})}$. Next assign the labels $-(2n+3), -(2n+4), -(2n+5), \cdots, -(3n+1)$ respectively to the vertices ${v_1}^{(\frac{m+6}{2})}, {v_2}^{(\frac{m+6}{2})}, {v_3}^{(\frac{m+6}{2})}, \cdots$,
${v_{n-1}}^{(\frac{m+6}{2})}$ of the $\frac{m+6}{2}^{th}$ copy ${F_n}^{(\frac{m+6}{2})}$. Proceeding this process until we reach the vertices ${v_1}^{(m)}, {v_2}^{(m)}, {v_3}^{(m)}, \cdots, {v_{n-1}}^{(m)}$ of the m^{th} fan ${F_n}^{(mm)}$. We notice that the vertices ${v_1}^{(m)}, {v_2}^{(m)}, {v_3}^{(m)}, \cdots, {v_{n-1}}^{(m)}$ respectively receives the labels $-(\frac{m(n+1)}{2} - n)$, $-(\frac{m(n+1)}{2} - (n-1))$, $-(\frac{m(n+1)}{2} - (n-2)), \cdots, -(\frac{m(n+1)}{2} - 3), -(\frac{m(n+1)}{2} - 2)$ respectively.

Finally assign the labels $(n+1), (2n+2), (3n+3), \cdots, \frac{m(n+1)}{2}$ respectively to the vertices $v^{(1)}, v^{(2)}, v^{(3)}, \cdots, v^{(\frac{m}{2})}$ respectively and assign the labels $-(n+1), -(2n+2), -(3n+3), \cdots, -\frac{m(n+1)}{2}$ to the vertices ${v_n}^{(\frac{m+2}{2})}, {v_n}^{(\frac{m+4}{2})}, {v_n}^{(\frac{m+6}{2})}, \cdots$, ${v_n}^{(m)}$. Also assign the labels $-n, -(2n+1), -(3n+2), \cdots, -(\frac{m(n+1)}{2} - 1)$ respectively to the verices $v^{(\frac{m+2}{2})}, v^{(\frac{m+4}{2})}, v^{(\frac{m+6}{2})}, \cdots, v^{(m)}$.

Clearly $\Delta_{f_1} = \Delta_{f_1^c} = \frac{m(2n-1)}{2}$.

□

Theorem 3.6. *The m- copies of fan F_n is pair difference cordial for all odd values of m and for all values of n.*

Proof. Take the vertex set and edge set from thm 3.5.

Consider the first copy ${F_n}^{(1)}$. Assign the labels $1, 2, 3, \cdots, n$ respectively to the vertices ${v_1}^{(1)}, {v_2}^{(1)}, {v_3}^{(1)}, \cdots, {v_n}^{(1)}$ and next consider the second copy ${F_n}^{(2)}$, assign the labels $(n+2), (n+3), (n+4), \cdots, (2n+1)$ to the vertices ${v_1}^{(2)}, {v_2}^{(2)}, {v_3}^{(2)}, \cdots, {v_n}^{(2)}$. Next assign the labels $(2n+3), (2n+4), (2n+5), \cdots, (3n+2)$ respectively to the vertices ${v_1}^{(3)}, {v_2}^{(3)}, {v_3}^{(3)}, \cdots, {v_n}^{(3)}$ of the third fan ${F_n}^{(3)}$. Proceeding like this until we reach the vertices of the $\frac{m-1}{2}^{th}$ fan ${F_n}^{(\frac{m-1}{2})}$. Note that the vertices ${v_1}^{(\frac{m-1}{2})}, {v_2}^{(\frac{m-1}{2})}, {v_3}^{(\frac{m-1}{2})}, \cdots, {v_n}^{(\frac{m-1}{2})}$ gets the labels $\frac{m(n+1)}{2} - n$, $\frac{m(n+1)}{2} - (n-1)$, $\frac{m(n+1)}{2} - (n-2), \cdots, \frac{m(n+1)}{2} - 2, \frac{m(n+1)}{2} - 1$ respectively.

Consider the $\frac{m+1}{2}^{th}$ copy $F_n{}^{(\frac{m+1}{2})}$. Assign the labels $-1,-2,-3,\cdots,-(n-1)$ respectively to the vertices $v_1{}^{(\frac{m+1}{2})}, v_2{}^{(\frac{m+1}{2})}, v_3{}^{(\frac{m+1}{2})}, \cdots, v_{n-1}{}^{(\frac{m+1}{2})}$ and assign the labels $-(n+2),-(n+3),-(n+4),\cdots,-(2n)$ to the vertices $v_1{}^{(\frac{m+3}{2})}, v_2{}^{(\frac{m+3}{2})}$, $v_3{}^{(\frac{m+3}{2})}, \cdots, v_{n-1}{}^{(\frac{m+3}{2})}$ of the $\frac{m+3}{2}^{th}$ copy $F_n{}^{(\frac{m+3}{2})}$. Next assign the labels $-(2n+3),-(2n+4),-(2n+5),\cdots,-(3n+1)$ respectively to the vertices $v_1{}^{(\frac{m+5}{2})}, v_2{}^{(\frac{m+5}{2})}, v_3{}^{(\frac{m+5}{2})}, \cdots,$
$v_{n-1}{}^{(\frac{m+5}{2})}$ of the $\frac{m+5}{2}^{th}$ fan $F_n{}^{(\frac{m+5}{2})}$. Proceeding this process until we reach the vertices $v_1{}^{(m-1)}, v_2{}^{(m-1)}, v_3{}^{(m-1)}, \cdots, v_{n-1}{}^{(m-1)}$ of the $(m-1)^{th}$ copy $F_n{}^{(m-1)}$. We notice that the vertices $v_1{}^{(m-1)}, v_2{}^{(m-1)}, v_3{}^{(m-1)}, \cdots, v_{n-1}{}^{(m-1)}$ respectively receive the labels $-(\frac{m(n+1)}{2}-n)$, $-(\frac{m(n+1)}{2}-(n-1))$, $-(\frac{m(n+1)}{2}-(n-2)), \cdots,$ $-(\frac{m(n+1)}{2}-3)$,$-(\frac{m(n+1)}{2}-2)$ respectively.

Next assign the labels $(n+1),(2n+2),(3n+3),\cdots,\frac{m(n+1)}{2}$ respectively to the vertices $v^{(1)}, v^{(2)}, v^{(3)}, \cdots, v^{(\frac{m-1}{2})}$ respectively and assign the labels $-(n+1),-(2n+2),-(3n+3),\cdots,-\frac{m(n+1)}{2}$ to the vertices $v_n{}^{(\frac{m+1}{2})}, v_n{}^{(\frac{m+3}{2})}, v_n{}^{(\frac{m+5}{2})}, \cdots,$ $v_n{}^{(m-1)}$. Also assign the labels $-n,-(2n+1),-(3n+2),\cdots,-(\frac{m(n+1)}{2}-1)$ respectively to the verices $v^{(\frac{m+1}{2})}, v^{(\frac{m+3}{2})}, v^{(\frac{m+5}{2})}, \cdots, v^{(m-1)}$.

Now assign the labels to the m^{th} fan $F_n{}^{(m)}$. There are two cases arises.

Case 1. n is even .

Assign the labels $\frac{n(m-1)}{2}+1, \frac{n(m-1)}{2}+2, \frac{n(m-1)}{2}+3, \cdots, \frac{n(m-1)}{2}+\frac{n}{2}$ respectively to the vertices $v_1{}^{(1)}, v_2{}^{(1)}, v_3{}^{(1)}, \cdots, v_n{}^{(\frac{n}{2})}$ and assign the labels $\frac{n(m-1)}{2}+1, \frac{n(m-1)}{2}+2, \frac{n(m-1)}{2}+3, \cdots, \frac{n(m-1)}{2}+\frac{n}{2}$ to the vertices $v_{(\frac{n+2}{2})}{}^{(m)}, v_{(\frac{n+4}{2})}{}^{(m)}$, $v_{(\frac{n+6}{2})}{}^{(m)}, \cdots, v_{(n)}{}^{(m)}$ respectively. Also assign the label $\frac{n(m-1)}{2}+\frac{n}{2}-1$ to the vertex $v^{(m)}$.

Case 2. n is odd .

Assign the labels $\frac{n(m-1)}{2}+1, \frac{n(m-1)}{2}+2, \frac{n(m-1)}{2}+3, \cdots, \frac{n(m-1)}{2}+\frac{n+1}{2}$ respectively to the vertices $v_1{}^{(1)}, v_2{}^{(1)}, v_3{}^{(1)}, \cdots, v_n{}^{(\frac{n+1}{2})}$ and assign the labels $\frac{n(m-1)}{2}+1, \frac{n(m-1)}{2}+2, \frac{n(m-1)}{2}+3, \cdots, \frac{n(m-1)}{2}+\frac{n-1}{2}$ to the vertices $v_{(\frac{n+3}{2})}{}^{(m)}, v_{(\frac{n+5}{2})}{}^{(m)}$, $v_{(\frac{n+7}{2})}{}^{(m)}, \cdots, v_{(n)}{}^{(m)}$ respectively. Also assign the label $\frac{n(m-1)}{2}+\frac{n+1}{2}$ to the vertex $v^{(m)}$

The Table 1 given below establish that this vertex labeling f is a pair difference cordial of the m- copies of the fan F_n for all odd values of m and for all values of n .

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
n is odd	$\frac{m(2n-1)+1}{2}$	$\frac{m(2n-1)-1}{2}$
n is even	$\frac{m(2n-1)-1}{2}$	$\frac{m(2n-1)+1}{2}$

Table 1

□

Theorem 3.7. *The m- copies of the comb $P_n \odot K_1$ is pair difference cordial for all even values of m and for all values of n.*

Proof. Let $P_n \odot {K_1}^{(j)}, 1 \leq i \leq n$ be the j^{th} copy of the comb $P_n \odot K_1$, $1 \leq j \leq m$.

Let $V(P_n \odot {K_1}^{(j)}) = \{{g_i}^{(j)}, {v_i}^{(j)} : 1 \leq i \leq n\}$ and $E(P_n \odot {K_1}^{(j)}) = \{{g_i}^{(j)}{v_i}^{(j)} : 1 \leq i \leq n\} \cup \{g_i^{(j)}{g_{i+1}}^{(j)} : 1 \leq i \leq n-1\}$.

Consider the first copy $P_n \odot {K_1}^{(1)}$. Assign the labels $1, 3, 5 \cdots, (2n-1)$ respectively to the vertices ${g_1}^{(1)}, {g_2}^{(1)}, {g_3}^{(1)}, \cdots, {g_n}^{(1)}$ and assign the labels $2, 4, 6, \cdots, (2n)$ to the vertices ${v_1}^{(1)}, {v_2}^{(1)}, {v_3}^{(1)}, \cdots, {v_n}^{(1)}$ respectively. Next consider the second copy $P_n \odot {K_1}^{(2)}$. Assign the labels $(2n+1), (2n+3), (2n+5), \cdots, (4n-1)$ respectively to the vertices ${g_1}^{(2)}, {g_2}^{(2)}, {g_3}^{(2)}, \cdots, {g_n}^{(2)}$ and assign the labels $(2n+2), (2n+4), (2n+6), \cdots, (4n)$ to the vertices ${v_1}^{(2)}, {v_2}^{(2)}, {v_3}^{(2)}$, $\cdots, {v_n}^{(2)}$ respectively. Now we consider the vertices of the third copy $P_n \odot {K_1}^{(3)}$. Assign the labels $(4n+1), (4n+3), (4n+5), \cdots, (6n-1)$ respectively to the vertices ${g_1}^{(3)}, {g_2}^{(3)}, {g_3}^{(3)}, \cdots, {g_n}^{(3)}$ and assign the labels $(4n+2), (4n+4), (4n+6), \cdots, (6n)$ to the vertices ${v_1}^{(3)}, {v_2}^{(3)}, {v_3}^{(3)}, \cdots, {v_n}^{(3)}$ respectively. Proceeding like this until we reach the vertices of the $\frac{m}{2}$ copy $P_n \odot {K_1}^{(\frac{m}{2})}$.

Secondly we consider the $\frac{m+2}{2}^{th}$ copy $P_n \odot {K_1}^{(\frac{m+2}{2})}$. Assign the labels $-1, -3, -5, \cdots, -(2n-1)$ respectively to the vertices ${g_1}^{(\frac{m+2}{2})}, {g_2}^{(\frac{m+2}{2})}, {g_3}^{(\frac{m+2}{2})}, \cdots, {g_n}^{(\frac{m+2}{2})}$ and assign the labels $-2, -4, -6, \cdots, -2n$ to the vertices ${v_1}^{(\frac{m+2}{2})}, {v_2}^{(\frac{m+2}{2})}, {v_3}^{(\frac{m+2}{2})}$, $\cdots, {v_n}^{(\frac{m+2}{2})}$ respectively. Next consider the $\frac{m+4}{2}^{th}$ copy $P_n \odot {K_1}^{(\frac{m+4}{2})}$. Assign the labels $-(2n+1), -(2n+3), -(2n+5), \cdots, -(4n-1)$ respectively to the vertices ${g_1}^{(\frac{m+4}{2})}, {g_2}^{(\frac{m+4}{2})}, {g_3}^{(\frac{m+4}{2})}, \cdots, {g_n}^{(\frac{m+4}{2})}$ and assign the labels $-(2n+2), -(2n+4), -(2n+6), \cdots, -(4n)$ to the vertices ${v_1}^{(\frac{m+4}{2})}, {v_2}^{(\frac{m+4}{2})}, {v_3}^{(\frac{m+4}{2})}, \cdots$, ${v_n}^{(\frac{m+4}{2})}$. Proceeding this process until we reach the vertices of the m^{th} copy $P_n \odot {K_1}^{(m)}$. Note that the vertices ${g_1}^{(m)}, {g_2}^{(m)}, {g_3}^{(m)}, \cdots, {g_n}^{(m)}$ gets the labels $-(mn-(2n-1)), -(mn-(2n-3)), -(mn-(2n-5)), \cdots, -(mn-1)$ respectively and the vertices ${v_1}^{(m)}, {v_2}^{(m)}, {v_3}^{(m)}, \cdots, {v_n}^{(m)}$ respectively receive the labels $-(mn-(2n-2)), -(mn-(2n-4)), -(mn-(2n-6)), \cdots, -(mn)$.

Clearly $\Delta_{f_1} = \Delta_{f_1^c} = \frac{m(2n-1)}{2}$.

□

Theorem 3.8. *The m- copies of the comb $P_n \odot K_1$ is pair difference cordial for all odd values of m and for all values of n.*

Proof. Take the vertex set and edge set from thm 3.7.

Consider the first comb. Assign the labels $1, 3, 5 \cdots$, $(2n-1)$ respectively to the vertices ${g_1}^{(1)}, {g_2}^{(1)}, {g_3}^{(1)}, \cdots, {g_n}^{(1)}$ and assign the labels $2, 4, 6, \cdots, (2n)$ to the vertices ${v_1}^{(1)}, {v_2}^{(1)}, {v_3}^{(1)}, \cdots, {v_n}^{(1)}$ respectively. Next consider the second copy $P_n \odot {K_1}^{(2)}$. Assign the labels $(2n+1), (2n+3), (2n+5), \cdots, (4n-1)$ respectively to the vertices ${g_1}^{(2)}, {g_2}^{(2)}, {g_3}^{(2)}, \cdots, {g_n}^{(2)}$ and assign the labels $(2n+2), (2n+4), (2n+6), \cdots, (4n)$ to the vertices ${v_1}^{(2)}, {v_2}^{(2)}, {v_3}^{(2)}, \cdots, {v_n}^{(2)}$ respectively. Now we consider the vertices of the third comb $P_n \odot {K_1}^{(3)}$. Assign the labels $(4n+1), (4n+3), (4n+5), \cdots, (6n-1)$ respectively to the vertices ${g_1}^{(3)}, {g_2}^{(3)}, {g_3}^{(3)}, \cdots, {g_n}^{(3)}$ and assign the labels $(4n+2), (4n+4), (4n+6), \cdots, (6n)$ to the vertices ${v_1}^{(3)}, {v_2}^{(3)}, {v_3}^{(3)}, \cdots, {v_n}^{(3)}$ respectively. Proceeding like this until we reach the vertices of the $\frac{m-1}{2}$ copy $P_n \odot {K_1}^{(\frac{m-1}{2})}$. Note that the vertices ${g_1}^{(\frac{m-1}{2})}, {g_2}^{(\frac{m-1}{2})}, {g_3}^{(\frac{m-1}{2})}, \cdots, {g_n}^{(\frac{m-1}{2})}$ receive the label $(\frac{m-1}{2}n-(2n-1)), (\frac{m-1}{2}n-(2n-3)), (\frac{m-1}{2}n-(2n-5)), \cdots, (\frac{m-1}{2}n-1)$ and assign the labels $(\frac{m-1}{2}n-(2n-2)), (\frac{m-1}{2}n-(2n-4)), (\frac{m-1}{2}n-(2n-6)), \cdots, (\frac{m-1}{2}n)$ to the vertices ${v_1}^{(\frac{m-1}{2})}, {v_2}^{(\frac{m-1}{2})}, {v_3}^{(\frac{m-1}{2})}, \cdots, {v_n}^{(\frac{m-1}{2})}$ respectively.

Secondly we consider the $\frac{m+1}{2}^{th}$ comb $P_n \odot {K_1}^{(\frac{m+1}{2})}$. Assign the labels $-1, -3, -5 \cdots$,
$-(2n-1)$ respectively to the vertices ${g_1}^{(\frac{m+1}{2})}, {g_2}^{(\frac{m+1}{2})}, {g_3}^{(\frac{m+1}{2})}, \cdots, {g_n}^{(\frac{m+1}{2})}$ and assign the labels $-2, -4, -6, \cdots, -2n$ to the vertices ${v_1}^{(\frac{m+1}{2})}, {v_2}^{(\frac{m+1}{2})}, {v_3}^{(\frac{m+1}{2})}, \cdots$, ${v_n}^{(\frac{m+1}{2})}$ respectively. Next consider the $\frac{m+3}{2}^{th}$ comb $P_n \odot {K_1}^{(\frac{m+3}{2})}$. Assign the labels $-(2n+1), -(2n+3), -(2n+5), \cdots, -(4n-1)$ respectively to the vertices ${g_1}^{(\frac{m+3}{2})}, {g_2}^{(\frac{m+3}{2})}, {g_3}^{(\frac{m+3}{2})}, \cdots, {g_n}^{(\frac{m+3}{2})}$ and assign the labels $-(2n+2), -(2n+4), -(2n+6), \cdots, -(4n)$ to the vertices ${v_1}^{(\frac{m+3}{2})}, {v_2}^{(\frac{m+3}{2})}, {v_3}^{(\frac{m+3}{2})}, \cdots, {v_n}^{(\frac{m+3}{2})}$. Proceeding this process until we reach the vertices of the $(m-1)^{th}$ comb $P_n \odot {K_1}^{(m-1)}$. Here the vertices ${g_1}^{(m-1)}, {g_2}^{(m-1)}, {g_3}^{(m-1)} \cdots, {g_n}^{(m-1)}$ receive the label $-(\frac{m-1}{2}n-(2n-1)), -(\frac{m-1}{2}n-(2n-3)), -(\frac{m-1}{2}n-(2n-5)), \cdots, -(\frac{m-1}{2}n-1)$ and assign the labels $-(\frac{m-1}{2}n-(2n-2)), -(\frac{m-1}{2}n-(2n-4)), -(\frac{m-1}{2}n-(2n-6)), \cdots, -(\frac{m-1}{2}n)$ to the vertices ${v_1}^{(m-1)}, {v_2}^{(m-1)}, {v_3}^{(m-1)}, \cdots, {v_n}^{(m-1)}$ respectively.

Finally consider the vertices of the m^{th} copy $P_n \odot {K_1}^{(m)}$. There are two cases arises.

Case 1. n is odd .

Assign the labels $(\frac{m-1}{2}n+1)$,$(\frac{m-1}{2}n+3)$,$(\frac{m-1}{2}n+5), \cdots ,(mn-2)$ respectively to the vertices ${g_1}^{(m)}$,${g_2}^{(m)}, {g_3}^{(m)}, \cdots , {g_{(\frac{n-1}{2})}}^{(m)}$. Assign the labels $(\frac{m-1}{2}n+2)$,$(\frac{m-1}{2}n+4)$,$(\frac{m-1}{2}n+6), \cdots ,(mn-1)$ to the vertices ${v_1}^{(m)}$,${v_2}^{(m)}$,${v_3}^{(m)}, \cdots$, ${v_{(\frac{n-1}{2})}}^{(m)}$ respectively. Next we assign the labels $-(\frac{m-1}{2}n+1)$,$-(\frac{m-1}{2}n+3)$,$-(\frac{m-1}{2}n+5), \cdots ,-(mn-2)$ respectively to the vertices ${g_{(\frac{n+1}{2})}}^{(m)}, {g_{(\frac{n+3}{2})}}^{(m)}$, ${g_{(\frac{n+5}{2})}}^{(m)}, \cdots ,{g_{(n-1)}}^{(m)}$. Assign the labels $-(\frac{m-1}{2}n+2)$,$-(\frac{m-1}{2}n+4)$,$-(\frac{m-1}{2}n+6), \cdots ,-(mn-1)$ to the vertices ${v_{(\frac{n+1}{2})}}^{(m)}, {v_{(\frac{n+3}{2})}}^{(m)}$,${v_{(\frac{n+5}{2})}}^{(m)}, \cdots ,{v_{(n-1)}}^{(m)}$ respectively. Also we assign the labels $mn, -mn$, respectively to the vertices ${g_n}^{(m)}$,${v_n}^{(m)}$.

Case 2. n is even .

Assign the labels $(\frac{m-1}{2}n+1)$,$(\frac{m-1}{2}n+3)$,$(\frac{m-1}{2}n+5), \cdots ,(mn-1)$ respectively to the vertices ${g_1}^{(m)}$,${g_2}^{(m)}, {g_3}^{(m)}, \cdots , {g_{(\frac{n}{2})}}^{(m)}$. Assign the labels $(\frac{m-1}{2}n+2)$,$(\frac{m-1}{2}n+4)$,$(\frac{m-1}{2}n+6), \cdots ,(mn)$ to the vertices ${v_1}^{(m)}$,${v_2}^{(m)}$,${v_3}^{(m)}, \cdots ,{v_{(\frac{n}{2})}}^{(m)}$ respectively. Next we assign the labels $-(\frac{m-1}{2}n+1)$,$-(\frac{m-1}{2}n+3)$,$-(\frac{m-1}{2}n+5), \cdots ,-(mn-1)$ respectively to the vertices ${g_{(\frac{n+2}{2})}}^{(m)}, {g_{(\frac{n+4}{2})}}^{(m)}$,${g_{(\frac{n+6}{2})}}^{(m)}, \cdots$, ${g_{(n)}}^{(m)}$. Assign the labels $-(\frac{m-1}{2}n+2)$,$-(\frac{m-1}{2}n+4)$,$-(\frac{m-1}{2}n+6), \cdots ,-(mn)$ to the vertices ${v_{(\frac{n+2}{2})}}^{(m)}, {v_{(\frac{n+4}{2})}}^{(m)}$,${v_{(\frac{n+6}{2})}}^{(m)}, \cdots ,{v_{(n)}}^{(m)}$ respectively.

The Table 2 given below establish that this vertex labeling f is a pair difference cordial of the m- copies of the comb $P_n \odot K_1$ for all odd values of m and for all values of n .

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
n is odd	$\frac{m(2n-1)+1}{2}$	$\frac{m(2n-1)-1}{2}$
n is even	$\frac{m(2n-1)-1}{2}$	$\frac{m(2n-1)+1}{2}$

TABLE 2

□

4. conclusion

Presently, it is difficult to investigate the pair difference cordial labeling behaviour of cylinder graphs and twisted cylinder graphs. In this paper we have studied the pair difference cordial labeling behaviour of $m-$ copies of some graphs like wheel, subdivision of star, fan graphs. The pair difference cordial labeling behaviour of Petersen graph , Generalized Petersen graph, Durer Graph, Generalized Durer graphs, $m-$ copies of helm,web ,closed helm,closed web are the possible future directions of research work.

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