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COMPLETE CONTROLLABILITY OF SEMILINEAR STOCHASTIC INTEGRO-DIFFERENTIAL EQUATIONS WITH INFINITE DELAY AND POISSON JUMPS

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Abstract. This manuscript deals with the exact (complete) controllability of semilinear stochastic differential equations with infinite delay and Poisson jumps utilizing some basic and readily verified conditions. The results are obtained by using fixed-point approach and by using advance phase space definition for infinite delay part. We have used the axiomatic definition of the phase space in terms of stochastic process to consider the time delay of the system. An infinite delay along with the Poisson jump is the new investigation for the given stochastic system. An example is given to illustrate the effectiveness of the results.

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1. Introduction

Controllability concepts play a vital role in deterministic control theory. It is well known that controllability of deterministic equations is widely used in many fields of science and technology. But in many practical systems such a fluctuating stock prices or physical system subject to thermal fluctuations, population dynamics etc, some randomness appear, so the system should be modeled by a stochastic form.

In setting of deterministic systems: Kalman [13] introduced the concept of controllability for finite-dimensional deterministic linear control systems. The basic concepts of control theory in finite and infinite-dimensional spaces have been introduced in [2]. In [28] Naito established sufficient conditions for approximate controllability of deterministic semi-linear control system dominated by the linear part using Schauder's fixed point theorem. Balachandran and Dauer

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[3] studied the controllability of nonlinear systems in Banach spaces. However, in many cases, some kind of randomness can appear in the problem, so that the system should be modeled by a stochastic form. Only few authors have studied the extensions of deterministic controllability concepts to stochastic control systems [14, 20, 22, 24].

In setting of stochastic systems: In [4] Bashirov et al. provides some concepts for controllability of linear stochastic systems. Using these concepts, Mahmudov [24] established sufficient conditions for controllability of linear stochastic systems in Hilbert spaces. In [21]-[26], Mahmudov et al. established results for controllability of linear and semi-linear stochastic systems in Hilbert spaces. In [34] Sukavanam et al. obtained some results for stochastic controllability of an abstract first order semi-linear control system using Schauder's fixed point theorem. Sakthivel et al. [33] studied the controllability of nonlinear stochastic systems in finite-dimensional spaces using Banach fixed-point theorem.

Now, in the last few decades, stochastic differential equations with Poisson jumps have witnessed a growing interest. To be more precise, in [30] Sakthivel established results for complete controllability of stochastic evolution equations with jumps in a separable Hilbert space. Recently, Shukla and Sukavanam et al. [31] studied the complete controllability of semi-linear stochastic system with delay using Banach fixed point theorem. Diop, et.al [8] studied the stability results for a partial impulsive stochastic integrodifferential equations with infinite delay; Dimplekumar et.al [6] studied the Approximate controllability of impulsive fractional neutral evolution equations with infinite delay in Banach spaces. Anguraj and Ramkumar [1] discussed approximate controllability of semi-linear stochastic integrodifferential system with nonlocal conditions through Sadovskii's fixed point theorem.

Moreover, Numerous practical systems (such as sudden price variations [jumps] like earthquakes, market crashes, hurricanes and so on) may undergo some jump type stochastic perturbations. For examples if a system jumps from a "normal state" to a "bad state" the paths are not being continuous then it is seize to consider stochastic processes with jumps in describing such models. Stochastic differential equations with Poisson jumps are examined by several authors [19, 29, 27].

Also, it has been observed that the existence or the controllability results proved by different authors are through an axiomatic definition of the phase space given by Hale and Kato [11]. However, as remarked by Hino, Murakami, and Naito [12], it has come to our attention that these axioms for the phase space are not correct for the systems with infinite time or state dependent delay.

Motivated by these facts, our main purpose in this paper is to study the complete controllability of semi-linear stochastic differential equations with delay and Poisson jumps. However, to the best of our knowledge, there are no results on the complete controllability of semi-linear stochastic differential equations with infinite delay and Poisson jumps as treated in the current paper.

Highlights:

(1) Complete controllability of semi-linear stochastic differential equations with time dependent delay and Poisson jumps has been studied. No literature is reported so far in this direction.
(2) Advanced definition of phase space has been used particularly for infinite time delay part of the system. Researchers are using the phase space defined by Hall and Kato [11] for infinite delay but we claim that it is wrong due to Hino, Murakami, and Naito [12]. For more detail pl refer to [5].
(3) An example is given to illustrate the theory. Detailed future work is mentioned in the conclusion part.

Consider a stochastic differential equations with infinite time dependent delay and Poisson jumps given in the form :

$$\begin{aligned} dy(t) &= \left[Ay(t) + Bu(t) + \int_0^t Q(t-s)y(s)ds + f(t, y(t-h))\right] dt \\ &+ \sigma(t, y(t-h))dw(t) + \int_{\mathcal{Z}} g(t, y(t-h), z)\tilde{N}(dt, dz),\ t \in J = [0, T] \quad (1) \\ y(t) &= \psi(t) \in \mathcal{C}_b,\ t \in (-\infty, 0],\ y(0) = y_0 = \psi(0) \text{ (say)} \quad (2) \end{aligned}$$

where $A : D(A) \subset \mathbb{H} \to \mathbb{H}$ is the infinitesimal generator of a strongly continuous semi-group of bounded linear operators $R(t),\ t \geq 0$ on Hilbert space $\mathbb{H}$. The control function $u(.)$ takes values in $u \in \mathcal{L}_J^2(J, U)$, the space of admissible control functions, U is a Hilbert space, B is a bounded linear operator from U into $\mathbb{H}$ and $Q(t)$ is a closed linear operator with domain $D(Q(t)) \supset D(A)$. The functions $f : J \times \mathcal{C}_b \to \mathbb{H}$; $\sigma : J \times \mathcal{C}_b \to \mathcal{L}_2^0$ and $g : J \times \mathcal{C}_b \times \mathcal{Z} \to \mathbb{H}$ are nonlinear suitable functions. $\mathcal{C}_b$ is defined later. For simplicity of considerations, we generally assume that the set of admissible controls is $U_{ad} = \mathcal{L}_{\Im}^2(J, U)$.

2. Preliminaries

Let $(\Omega, \Im, \mathbb{P})$ be a complete probability space equipped with a normal filtration $\Im_t,\ t \in J = [0, T]$. Let $\mathbb{K}$ be the separable Hilbert space with norm $\|.\|_{\mathbb{K}}$. and $\mathcal{W}$ is a Q-Wiener process on $(\Omega, \Im_t, \mathbb{P})$ with the covariance operator Q such that $trQ < \infty$. We use same notation $\|.\|$ for the norm of $\mathcal{L}(\mathbb{K}, \mathbb{H})$, where $\mathcal{L}(\mathbb{K}, \mathbb{H})$ denotes the space of all bounded linear operators from $\mathbb{K}$ into $\mathbb{H}$, simply $\mathcal{L}(\mathbb{H})$ if $\mathbb{K} = \mathbb{H}$. We assume that there exists a complete orthonormal system e_n in $\mathbb{K}$, a bounded sequence of non-negative real numbers λ_n such that $Qe_n = \lambda_n e_n$, $n = 1, 2, 3, \cdots$ and a sequence β_n of independent Brownian motions such that

$$\mathcal{W}(t) = \sum_{n=1}^{\infty} \sqrt{\lambda_n}\beta_n(t)e_n,\ t \in J = [0, T]$$

and $\Im_t = \Im_t^{\omega}$, where $\Im_t^{\omega}$ is the σ-algebra generated by $\mathcal{W}$. Let $\mathcal{L}_2^0 = \mathcal{L}_2(Q^{1/2}\mathbb{K}; \mathbb{H})$ be the space of all Hilbert-Schmidt operators from $Q^{1/2}\mathbb{K}$ to $\mathbb{H}$. Then the space

$\mathcal{L}_2^0$ is a separable Hilbert space equipped with the norm $\|\zeta\|^2_{\mathcal{L}_2^0} = tr(\zeta Q \zeta^*)$. Let $\mathcal{L}_2^{\Im}(J, \mathbb{H})$ be the space of all $\Im_t$-adapted, $\mathbb{H}$-valued measurable square integrable processes on $J \times \Omega$. Let $C([0,T]; \mathcal{L}^2(\Im, \mathbb{H}))$ be the Banach space of continuous maps from $[0,T]$ into $\mathcal{L}^2(\Im, \mathbb{H})$ satisfying the condition $\sup_{t\in J} \mathbf{E}\|y(t)\|^2 < \infty$. Let $\mathbb{H}_2$ is the closed subspace of $C([0,T]; \mathcal{L}^2(\Im, \mathbb{H}))$ consisting of measurable and $\Im_t$-adapted $\mathbb{H}$ valued processes $\psi \in C([0,T]; \mathcal{L}^2(\Im, \mathbb{H}))$ endowed with the norm

$$\|\psi\|_{\mathbb{H}} = \left(\sup_{t\in[0,T]} \mathbf{E}\|\psi(t)\|^2_{\mathbb{H}} \right)^{1/2}.$$

Let $\{q = (q(t)), t \in D_q\}$, be a stationary $\Im_t$-Poisson point process with characteristic measure λ. Let $\mathcal{N}(dt, dz)$ be the Poisson counting measure associated with q. Thus we have $\mathcal{N} = \sum_{s\in D_q, s\le t} I_Z(q(s))$ with a measurable set $Z \in \mathcal{B}(\mathbb{K} - \{0\})$, which denotes the Borel σ field of $\mathbb{K} - \{0\}$. Let $\tilde{\mathcal{N}}(dt, dz) = \mathcal{N}(dt, dz) - dt\lambda(dz)$ be the compensated Poisson measure that is independent of $\mathcal{W}(t)$. Let $\mathbb{P}^2([0,T] \times \mathcal{Z}; \mathbb{H})$ be the space of all predictable mappings $g : [0,T] \times \mathcal{Z} \times \Omega \to \mathbb{H}$ for which

$$\int_0^T \int_{\mathcal{Z}} \mathbf{E}\|g(t,z)\|^2_{\mathbb{H}} \, dt\lambda(dz) < \infty.$$

Then, we can define the $\mathbb{H}$-valued stochastic integral $\int_0^T \int_{\mathcal{Z}} g(t,z)\tilde{N}(dt,dz)$, which is a centered square-integrable martingale. Now, we define the abstract phase space $\mathcal{C}_b$ [12]. Assume that $b : (-\infty, 0] \to (0, +\infty)$ is a continuous function satisfying $l = \int_{-\infty}^0 b(t)dt < +\infty$. The Banach space $(\mathcal{C}_b, \|.\|_{\mathcal{C}_b})$ induced by the function b is defined as: $\mathcal{C}_b = \big\{\psi : (-\infty, 0] \to \mathbb{H}$, for any $a > 0, \mathbf{E}(|\psi(\theta)|^2)^{1/2}$ is a bounded and measurable function on $[-a, 0]$ and $\int_{-\infty}^0 b(s) \sup_{s\le\theta\le 0} \mathbf{E}(|\psi(\theta)|^2)^{1/2} ds < +\infty\big\}$. If $\mathcal{C}_b$ is endowed with norm $\|\psi\|_{\mathcal{C}_b} = \int_{-\infty}^0 b(s) \sup_{s\le\theta\le 0} \mathbf{E}(|\psi(\theta)|^2)^{1/2} ds$. $C((-\infty, v], \mathbb{H})$ denote the space of all continuous $\mathbb{H}$- valued stochastic process $\{\xi(t),\ t \in (-\infty, v]\}$. Let $\mathcal{C}_v = \{y;\ y \in C((-\infty, v], \mathbb{H}),\ y_0 = \psi \in \mathcal{C}_b\}$.

Set $\|.\|_v$ be a semi-norm defined by

$$\|x\|_v = \|x_0\|_{\mathcal{C}_b} + \sup_{s\in[0,t]} \mathbf{E}|(x(s)|^2)^{1/2},\ x \in \mathcal{C}_v$$

Now, the corresponding linear system with respect to (1)-(2) is given by the equation

$$dx(t) = \left[Ax(t) + Bu(t) + \int_0^t C(t-s)x(s)ds \right] dt \tag{3}$$

$$x(0) = x_0 \tag{4}$$

Definition 2.1. A resolvent operator for (3)-(4) is a bounded linear operator valued function $R(t) \in \mathcal{L}(\mathbb{X})$ for $t \ge 0$, having the following properties:
(i) $R(0) = I$ and $|R(t)| \le \lambda e^{\beta t}$ for some constants λ and β.
(ii) For each $x \in \mathbb{X}$, $R(t)x$ is strongly continuous for $t \ge 0$.

(iii) $R(t) \in \mathcal{L}(\mathbb{Y})$ for $t \geq 0$. For $x \in \mathbb{Y}$, $R(\cdot)x \in C^1([0,+\infty);\mathbb{X}) \cap C([0,+\infty);\mathbb{Y})$ and

$$\begin{aligned} R^{'}(t)x &= AR(t)x + \int_0^t B(t-s)R(s)xds \\ &= R(t)Ax + \int_0^t R(t-s)B(s)xds \text{ for } t \geq 0. \end{aligned}$$

The resolvent operator plays an important role to study the existence of solutions and to give a variation of constants formula for nonlinear systems. We need to know that the linear system (3)-(4) has a resolvent operator. For more details on resolvent operators, we refer to [10].

Definition 2.2. A stochastic process $\{y(t), t \in (-\infty, v]\}$ is a mild solution of (1)-(2) if $y_0 = \psi \in \mathcal{C}_b$ and for each $u \in \mathcal{L}^2_{\Im}([0,T],U)$, it satisfies the following integral equation:

$$\begin{aligned} y(t;y_0,u) &= R(t)y_0 + \int_0^t R(t-s)[Bu(s) + f(s,y(s-h))]ds \\ &+ \int_0^t R(t-s)\sigma(s,y(s-h))dw(s) \\ &+ \int_0^t \int_{\mathcal{Z}} R(t-s)g(s,y(s-h),z)\tilde{N}(ds,dz), \\ y(t;y_0,u) &= \psi(t) \in \mathcal{C}_b \text{ for } t \in (-\infty,0]. \end{aligned} \tag{5}$$

Let us introduce the following operators and sets (see [33]):
$\mathcal{L}_T \in \mathbf{L}(U_{ad}, \mathcal{L}_2(\Omega, \Im_T, \mathbb{H}))$ defined by

$$\mathcal{L}_T u = \int_0^T R(T-s)Bu(s)ds.$$

Then its adjoint operator $\mathcal{L}_T^* : \mathcal{L}_2(\Omega, \Im_T, \mathbb{H}) \to U_{ad}$ is given by

$$\mathcal{L}_T^* z = B^* R^*(T-s)\mathbf{E}\{z|\Im_t\}.$$

The set of all states reachable in time T from initial state $y(0) = y_0 \in \mathcal{L}_2(\Omega, \Im_T, \mathbb{H})$, using admissible controls is defined as

$$\mathcal{R}_T(U_{ad}) = \{y(T;y_0,u) \in \mathcal{L}_2(\Omega, \Im_T, \mathbb{H}) : u \in U_{ad}\},$$

where

$$\begin{aligned} y(T;y_0,u) &= R(T)y_0 + \int_0^T R(T-s)Bu(s)ds + \int_0^T R(T-s)f(s,y(s-h))ds \\ &+ \int_0^T R(t-s)\sigma(s,y(s-h))dw(s) \\ &+ \int_0^T \int_{\mathcal{Z}} R(T-s))g(s,y(s-h),z)\tilde{N}(ds,dz) \end{aligned}$$

Let us introduce the linear controllability operator $\Psi_0^T \in \mathbf{L}\big(\mathcal{L}_2(\Omega, \Im_T, \mathbb{H}), \mathcal{L}_2(\Omega, \Im_T, \mathbb{H})\big)$ as follows:

$$\begin{aligned}\Psi_0^T\{\cdot\} &= \mathcal{L}_T(\mathcal{L}_T)^*\{\cdot\}\\ &= \int_0^T R(T-t)BB^*R^*(T-t)\mathbf{E}\{.|\Im_t\}\,dt\end{aligned}$$

The corresponding controllability operator for deterministic model is

$$\begin{aligned}\Gamma_s^T &= \mathcal{L}_T(s)\mathcal{L}_T^*(s)\\ &= \int_0^T R(T-t)BB^*R^*(T-t))dt\end{aligned}$$

Definition 2.3. The stochastic dynamic system (1)-(2) is said to be completely controllable on $[0,T]$ if

$$\overline{\mathcal{R}_T(U_{ad})} = \mathbb{L}_2(\Omega, \Im_T, \mathbb{H})$$

i.e., all points in $\mathcal{L}_2(\Omega, \Im_T, \mathbb{H})$ can be reached from the point y_0 in time T.

Lemma 2.4. *Let $\sigma : [0,T]\times\Omega \to \mathcal{L}_2^0$ be a strongly measurable mapping such that $\int_0^T \mathbf{E}\,\|\sigma(t)\|^p\,dt < \infty$. Then*

$$\mathbf{E}\left\|\int_0^t \sigma(s)dw(s)\right\|^p \leq L_\sigma \int_0^t \mathbf{E}\,\|\sigma(s)\|^p\,ds, \tag{6}$$

Lemma 2.5. (Schwartz inequality): *Let $\phi_1(x)$ and $\phi_2(x)$ be any two square-integrable real functions in $[a,b]$, then*

$$\left[\int_a^b \phi_1(x)\phi_2(x)dx\right]^2 \leq \int_a^b [\phi_1(x)]^2\,dx \int_a^b [\phi_2(x)]^2\,dx$$

3. Main results

Lemma 3.1. *Assume that the operator Ψ_0^T is invertible. Then for arbitrary $y_T \in \mathcal{L}_2(\Omega, \Im_T, \mathbb{H})$, $f(\cdot) \in \mathcal{L}_2([0,T], \mathbb{H})$, $\sigma(\cdot) \in \mathcal{L}_2([0,T], \mathbb{H})$ and $g(\cdot) \in \mathcal{L}_2([0,T], \mathbb{H})$, the control defined as*

$$u(t) = B^*R^*(T-t)\mathbf{E}\left\{(\Psi_0^T)^{-1}p(y)|\Im_t\right\}, \tag{7}$$

where

$$\begin{aligned}p(y) &= y_T - R(t)y_0 - \int_0^T R(T-s))f(s, y(s-h))ds\\ &+ \int_0^T R(T-s))\sigma(s, y(s-h))dw(s)\\ &+ \int_0^T\int_{\mathcal{Z}} R(T-s))g(s, y(s-h), z)\tilde{N}(dt, dz)\end{aligned}$$

transfers the system (1)-(2) from $y_0 \in \mathbb{H}$ to the final state y_T at time T, provided the system (1)-(2) has a solution.

Proof. By substituting (5) in (3), we can easily obtain,

$$
\begin{aligned}
y(t;y_0,u) &= R(t)y_0 + \int_0^t R(t-s)BB^*R^*(T-s)\mathbf{E}\left\{(\Psi_0^T)^{-1}p(y)|\Im_s\right\}ds \\
&+ \int_0^t R(t-s)f(s,y(s-h))ds + \int_0^t R(t-s)\sigma(s,y(s-h))dw(s) \\
&+ \int_0^t \int_{\mathcal{Z}} R(t-s))g(s,y(s-h),z)\tilde{N}(dt,dz).
\end{aligned}
$$

Hence, for a given final time $t=T$, we simply have the following equality:

$$
\begin{aligned}
y(T;y_0,u) &= R(T)y_0 + \int_0^T R(T-s))(BB^*R(T-s))\mathbf{E}\Bigg\{(\Psi_0^T)^{-1} \\
&\times \Bigg(y_T - R(T)y_0 - \int_0^T R(T-s))f(s,y(s-h))ds \\
&+ \int_0^T R(T-s))\sigma(s,y(s-h))ds \\
&+ \int_0^T \int_{\mathcal{Z}} R(T-s)g(s,y(s-h),z)\tilde{N}(ds,dz)\Bigg)\Bigg\} \mid \Im_s ds \\
&+ \int_0^T R(T-s))f(s,y(s-h))ds \\
&+ \int_0^T R(T-s)\sigma(s,y(s-h))dw(s) \\
&+ \int_0^T \int_{\mathcal{Z}} R(T-s)g(s,y(s-h),z)\tilde{N}(ds,dz)
\end{aligned}
$$

Thus, taking into account the form of the operator Ψ_0^T, we have

$$
\begin{aligned}
y(T;y_0,u) &= R(T)y_0 + (\Psi_0^T)(\Psi_0^T)^{-1}\Bigg(y_T - R(T)y_0 \\
&- \int_0^T R(T-s)f(s,y(s-h))ds \\
&+ \int_0^T R(T-s)\sigma(s,y(s-h))dw(s) \\
&+ \int_0^T \int_{\mathcal{Z}} R(T-s)h(s,y(s-h),z)\tilde{N}(ds,dz)\Bigg) \\
&+ \int_0^T R(T-s)f(s,y(s-h))ds \\
&+ \int_0^T R(T-s)\sigma(s,y(s-h))dw(s)
\end{aligned}
$$

$$\begin{aligned} &+ \int_0^T \int_{\mathcal{Z}} R(T-s)g(s,y(s-h),z)\tilde{N}(ds,dz) \\ &= y_T. \end{aligned}$$

Therefore, we see that the control $u(t)$ transfers the system (1)-(2) from the initial state $y_0 \in \mathcal{L}_2(\Omega, \Im_T, \mathbb{H})$ to the final state $y_T \in \mathcal{L}_2(\Omega, \Im_T, \mathbb{H})$ at time T. □

Now we assume the following hypotheses:

(H1) f, σ and g satisfy the Lipschitz condition with respect to y. i.e.,

$$\|f(t,y_1)-f(t,y_2)\|_{\mathbb{H}}^2 + \|\sigma(t,y_1)-\sigma(t,y_2)\|_{\mathbb{H}}^2 + \int_{\mathcal{Z}} \|g(t,y_1,z)-g(t,y_2,z)\|_{\mathbb{H}}^2 v(dz) \le C\|y_1-y_2\|_{\mathcal{C}_b}^2$$

(H2) f, σ and g is continuous on $[0,T]\times\mathbb{H}$ and satisfies

$$\|f(t,y)\|_{\mathbb{H}}^2 + \|\sigma(t,y)\|_{\mathbb{H}}^2 + \int_{\mathcal{Z}} \|g(t,y,z)\|_{\mathbb{H}}^2 v(dz) \le C(1+\|y\|_{\mathcal{C}_b}^2)$$

(H3) There exists a number $\tilde{C}_0 > 0$ such that for any arbitrary $y_1, y_2 \in \mathcal{C}_b$,

$$\begin{aligned} \int_{\mathcal{Z}} \|h(t,y_1,z)-h(t,y_2,z)\|_{\mathbb{H}}^4 v(dz) &\le C_0\left(\|y_1-y_2\|_{\mathcal{C}_b}^4\right), \\ \int_{\mathcal{Z}} \|h(t,y,z)\|_{\mathbb{H}}^4 v(dz) &\le C_0(1+\|y\|_{\mathcal{C}_b}^4) \end{aligned}$$

(H4) The linear system corresponding to (1)-(2) is exactly controllable.

Let us define the nonlinear operator $\mathbf{S} : \mathbb{H}_2 \to \mathbb{H}_2$ for $t \in (-\infty, 0]$ as follows:

$$\begin{aligned} (\mathbf{S}_\alpha \mathbf{y})(t) &= R(t)y_0 + \int_0^t R(t-s)Bu(s)ds + \int_0^t R(t-s)f(s,y(s-h))ds \\ &+ \int_0^t R(t-s)\sigma(s,y(s-h))dw(s) \\ &+ \int_0^t \int_{\mathcal{Z}} R(t-s)g(s,y(s-h),z)\tilde{N}(ds,dz) \\ y(t) &= \psi(t) \text{ for } t \in (-\infty,0] \end{aligned}$$

From Lemma 3.1, the control $u(t)$ transfers the system (1)-(2) from the initial state y_0 to the final state y_T provided that the operator $\mathbf{S}$ has a fixed point. So, if the operator $\mathbf{S}$ has a fixed point then the system (1)-(2) is exactly controllable. Now for convenience, let us introduce the notation

$$n_1 = \max\left\{\|R(t)\|^2 : t \in [0,T]\right\},\ n_2 = \|B\|^2,$$
$$n_3 = \mathbf{E}\|y_T\|^2,\ M = \max\left\|\Pi_0^T\right\|^2$$

Lemma 3.2. *For every $v \in \mathcal{L}_2(\Omega, \Im_T, \mathbb{H})$, there exists a process $\varphi(\cdot) \in \mathbb{L}_2([0,T], \mathbb{H})$ such that*

$$\begin{aligned} v &= \mathbf{E}v + \int_0^T \varphi(s)dw(s) \\ \Psi_0^T v &= \Gamma_0^T \mathbf{E}v + \int_0^T \Gamma_s^T \varphi(s)dw(s) \end{aligned}$$

Moreover,

$$\begin{aligned} \mathbf{E}\left\|\Psi_0^T v\right\|^2 &\leq M\mathbf{E}\left\|\mathbf{E}\{v|\Im_T\}\right\|^2 \\ &\leq M\mathbf{E}\left\|v\right\|^2, \ v \in \mathbb{L}_2(\Omega, \Im_T, \mathbb{H}). \end{aligned}$$

Note that if the hypotheses (H4) holds, then for some $\delta > 0$

$$\mathbf{E}\left\langle \Psi_0^T v, v\right\rangle \geq \delta \mathbf{E}\left\|v\right\|^2, \ \textit{for all } v \in \mathbb{L}_2(\Omega, \Im_T, \mathbb{H})$$

(see Mahumudov [20]*) and consequently*

$$\mathbf{E}\left\|(\Psi_0^T)^{-1}\right\|^2 \leq \frac{1}{\delta} = n_4.$$

Theorem 3.3. *System (1)-(2) is completely controllable if the conditions (H1), (H2), (H3) and (H4) are satisfied.*

Proof. As mentioned above, to prove the complete controllability it is enough to show that $\mathbf{S}$ has a fixed point in $\mathbb{H}_2$. To do this, we use the contraction mapping principle. To apply the contraction mapping principle, first we show that $\mathbf{S}$ maps $\mathbb{H}_2$ into itself. Now by Lemmas 2.1 and 2.2, we have

$$\begin{aligned} \mathbf{E}\left\|(\mathbf{S}_\alpha y)(t)\right\|^2 &= \mathbf{E}\Big\|\psi(t) + R(t)y_0 + \Psi_0^T\Bigg[R^*(T-t))(\Psi_0^T)^{-1} \times \Bigg(y_T - R(T)y_0 \\ &- \int_0^T R(T-s)f(s, y(s-h))ds \\ &- \int_0^T R(T-s)\sigma(s, y(s-h))dw(s) \\ &- \int_0^T \int_{\mathcal{Z}} R(T-s)g(s, y(s-h), z)\tilde{N}(ds, dz)\Bigg)\Bigg] \\ &+ \int_0^t R(t-s)f(s, y(s-h))ds + \int_0^t R(t-s)\sigma(s, y(s-h))dw(s) \\ &+ \int_0^t \int_{\mathcal{Z}} R(t-s)g(s, y(s-h), z)\tilde{N}(ds, dz)\Big\|^2 \\ &\leq 6\left\|\psi\right\|^2 + 6n_1\left\|y_0\right\|^2 \\ &+ 6\mathbf{E}\Psi_0^t\Bigg[R^*(T-t)(\Psi_0^T)^{-1} \times \Bigg(y_T - R(T)y_0 \\ &- \int_0^T R(T-s)f(s, y(s-h))ds \end{aligned}$$

$$
\begin{aligned}
&- \int_0^T R(T-s)\sigma(s,y(s-h))dw(s) \\
&- \int_0^t \int_{\mathcal{Z}} R(t-s)g(s,y(s-h),z)\tilde{N}(ds,dz)\Bigg)\Bigg] \\
&+ 6n_1 t \int_0^t \mathbf{E}\left\|f(s,y(s-h))\right\|^2 ds + 6n_1 L_\sigma \int_0^t \mathbf{E}\left\|\sigma(s,y(s-h))\right\|^2 ds \\
&+ 6n_1 \int_0^t \int_{\mathcal{Z}} \mathbf{E}\left\|g(s,y(s-h),z)\right\|^2 v(dz)ds \\
&+ 6n_1 \left(\int_0^t \int_{\mathcal{Z}} \mathbf{E}\left\|g(s,y(s-h),z)\right\|^4 v(dz)ds\right)^{1/2} \\
&\leq 6\left\|\psi\right\|^2 + 6n_1\left\|y_0\right\|^2 + 30Mn_1n_3n_4 + 30Mn_1^2n_4\left\|y_0\right\|^2 \\
&+ 30Mn_1^2n_4T \int_0^T \mathbf{E}\left\|f(s,y(s-h))\right\|^2 ds \\
&+ 30Mn_1^2n_4L_\sigma \int_0^T \mathbf{E}\left\|\sigma(s,y(s-h))\right\|^2 ds \\
&+ 30Mn_1^2n_4 \int_0^T \int_{\mathcal{Z}} \mathbf{E}\left\|g(s,y(s-h),z)\right\|^2 v(dz)ds \\
&+ 30Mn_1^2n_4 \left(\int_0^T \int_{\mathcal{Z}} \mathbf{E}\left\|g(s,y(s-h),z)\right\|^4 v(dz)ds\right)^{1/2} \\
&+ 6n_1 T \int_0^t \mathbf{E}\left\|f(s,y(s-h))\right\|^2 ds + 6n_1 L_\sigma \int_0^t \mathbf{E}\left\|\sigma(s,y(s-h))\right\|^2 ds \\
&+ 6n_1 \int_0^t \int_{\mathcal{Z}} \mathbf{E}\left\|g(s,y(s-h),z)\right\|^2 v(dz)ds \\
&+ 6n_1 \left(\int_0^t \int_{\mathcal{Z}} \mathbf{E}\left\|g(s,y(s-h),z)\right\|^4 v(dz)ds\right)^{1/2} \\
&\leq B_1 + \left(30Mn_1^2n_4 + 6n_1\right)\Bigg[T \int_0^t \mathbf{E}\left\|f(s,y(s-h))\right\|^2 ds \\
&+ L_\sigma \int_0^t \mathbf{E}\left\|\sigma(s,y(s-h))\right\|^2 ds \\
&+ \int_0^t \int_{\mathcal{Z}} \mathbf{E}\left\|g(s,y(s-h),z)\right\|^2 v(dz)ds \\
&+ \left(\int_0^t \int_{\mathcal{Z}} \mathbf{E}\left\|g(s,y(s-h),z)\right\|^4 v(dz)ds\right)^{1/2}\Bigg] \\
&\leq B_1 + B_2\Bigg[T \int_0^t \mathbf{E}\left\|f(s,y(s-h))\right\|^2 ds \\
&+ L_\sigma \int_0^t \mathbf{E}\left\|\sigma(s,y(s-h))\right\|^2 ds \\
&+ \int_0^t \int_{\mathcal{Z}} \mathbf{E}\left\|g(s,y(s-h),z)\right\|^2 v(dz)ds
\end{aligned}
$$

$$+ \quad \left(\int_0^t \int_{\mathcal{Z}} \mathbf{E} \, \| g(s, y(s-h), z) \|^4 \, v(dz) ds \right)^{1/2} \Bigg]$$

where $B_1 > 0$ and $B_2 > 0$ are suitable constants. It follows from the above and the condition (H2) and (H3) that there exists K_1 such that

$$\begin{aligned} \mathbf{E} \, \| (\mathbf{S}_\alpha y)(t) \|^2 &\leq K_1 \left(1 + \int_0^T \mathbf{E} \, \| y(r-h) \|^2 \, dr \right) \\ &\leq K_1 \left(1 + T \sup_{-\infty \leq t \leq T} \mathbf{E} \, \| y(t) \|^2 \right) \end{aligned}$$

for all $t \in [-\infty, T]$. Therefore, $\mathbf{S}$ maps $\mathbb{H}_2$ into itself. Second, we show that $\mathbf{S}$ is a contraction mapping, indeed.

$$\begin{aligned} &\mathbf{E} \, \| (\mathbf{S}_\alpha y_1)(t) - \mathbf{S}_\alpha y_2)(t) \|^2 \\ &= \mathbf{E} \Big\| \Psi_0^t \Bigg[R^*(T-t))(\Psi_0^T)^{-1} \\ &\quad \times \Bigg(\int_0^T R(T-s) \, [f(s, y_1(s-h)) - f(s, y_2(s-h))] \, ds \\ &\quad + \int_0^T R(T-s) \, [\sigma(s, y_1(s-h)) - \sigma(s, y_2(s-h))] \, dw(s) \\ &\quad + \int_0^T \int_{\mathcal{Z}} R(T-s) \, [g(s, y_1(s-h), z) - g(s, y_2(s-h), z)] \, \tilde{N}(ds, dz) \Bigg) \Bigg] \\ &\quad + \int_0^t R(T-s) \, [f(s, y_1(s-h)) - f(s, y_2(s-h))] \, ds \\ &\quad + \int_0^t R(T-s) \, [\sigma(s, y_1(s-h)) - \sigma(s, y_2(s-h))] \, dw(s) \\ &\quad + \int_0^t \int_{\mathcal{Z}} R(T-s) \, [g(s, y_1(s-h), z) - g(s, y_2(s-h), z)] \, \tilde{N}(ds, dz) \Big\|^2 \\ &\leq 6 M n_1^2 n_4 \Bigg[T \int_0^T \mathbf{E} \, \| f(s, y_1(s-h)) - f(s, y_2(s-h)) \|^2 \, ds \\ &\quad + L_\sigma \int_0^T \mathbf{E} \, \| \sigma(s, y_1(s-h)) - \sigma(s, y_2(s-h)) \|^2 \, ds \\ &\quad + \int_0^T \int_{\mathcal{Z}} \mathbf{E} \, \| g(s, y_1(s-h), z) - g(s, y_2(s-h), z) \|^2 \, v(dz) ds \\ &\quad + \left(\int_0^T \int_{\mathcal{Z}} \mathbf{E} \, \| g(s, y_1(s-h), z) - g(s, y_2(s-h), z) \|^4 \, v(dz) ds \right)^{1/2} \Bigg] \\ &\quad + 6 n_1 \Bigg[T \int_0^t \mathbf{E} \, \| f(s, y_1(s-h)) - f(s, y_2(s-h)) \|^2 \, ds \\ &\quad + L_\sigma \int_0^t \mathbf{E} \, \| \sigma(s, y_1(s-h)) - \sigma(s, y_2(s-h)) \|^2 \, ds \end{aligned}$$

$$+\int_0^t\int_{\mathcal{Z}}\mathbf{E}\left\|g(s,y_1(s-h),z)-g(s,y_2(s-h),z)\right\|^2 v(dz)ds$$
$$+\left(\int_0^t\int_{\mathcal{Z}}\mathbf{E}\left\|g(s,y_1(s-h),z)-g(s,y_2(s-h),z)\right\|^4 v(dz)ds\right)^{1/2}\Bigg]$$
$$\leq 6Mn_1^2n_4\left[CT+CL_\sigma+C+\sqrt{C_0}\right]\int_0^T\mathbf{E}\left\|y_1(s-h)-y_2(s-h)\right\|^2 ds$$
$$+6n_1\left[CT+CL_\sigma+C+\sqrt{C_0}\right]\int_0^t\mathbf{E}\left\|y_1(s-h)-y_2(s-h)\right\|^2 ds$$
$$\leq 6n_1\left[Mn_1n_4+1\right]\left[C\Big(T+L_\sigma+1\Big)+\sqrt{C_0}\right]\int_0^T\mathbf{E}\left\|y_1(s-h)-y_2(s-h)\right\|^2 ds$$

This proves that

$$\sup_{t\in[-h,T]}\mathbf{E}\left\|(\mathbf{S}_\alpha y_1)(t)-\mathbf{S}_\alpha y_2)(t)\right\|^2$$
$$\leq 6n_1\left[Mn_1n_4+1\right]\left[C\Big(T+L_\sigma+1\Big)+\sqrt{C_0}\right]T$$
$$\times\sup_{t\in[-\infty,T]}\mathbf{E}\left\|y_1(t)-y_2(t)\right\|^2$$

Therefore, for every $\alpha>0$, there exists $\eta(\alpha)>0$ such that

$$\mathbf{E}\left\|(\mathbf{S}_\alpha y_1)(t)-\mathbf{S}_\alpha y_2)(t)\right\|_{\mathbb{H}}\leq t\eta(\alpha)\left\|y_1-y_2\right\|^2_{\mathcal{C}_b}$$

Moreover,

$$\begin{aligned}\mathbf{E}\left\|\mathbf{S}_\alpha(x_1)(t)-\mathbf{S}_\alpha(x_2)(t)\right\|^2_{\mathbb{H}} &\leq \eta(\alpha)\int_0^t\mathbf{E}\left\|\mathbf{S}_\alpha(x_1)(s)-\mathbf{S}_\alpha(x_2)(s)\right\|^2\\ &\leq \eta(\alpha)\int_0^t s\eta(\alpha)\mathbf{E}\left\|x_1(s)-x_2(s)\right\|^2 ds\\ &= \eta^2(\alpha)\frac{t^2}{2}\left\|x_1-x_2\right\|^2_{\mathcal{C}_b}\end{aligned}$$

Using mathematical induction, one can get

$$\begin{aligned}\mathbf{E}\left\|\mathbf{S}_\alpha(y_1)(t)-\mathbf{S}_\alpha(y_2)(t)\right\|^2_{\mathbb{H}} &\leq \eta(\alpha)\int_0^t\mathbf{E}\left\|\mathbf{S}_\alpha^{\mathbf{n-1}}(y_1)(s)-\mathbf{S}_\alpha^{\mathbf{n-1}}(y_2)(s)\right\|^2\\ &\leq \frac{(t\eta(\alpha))^n}{n}\left\|y_1-y_2\right\|^2_{\mathcal{C}_b}\end{aligned}$$

In general,

$$\left\|\mathbf{S}_\alpha(y_1)-\mathbf{S}_\alpha(y_2)\right\|^2_{\mathbb{H}} \leq \frac{(T\eta(\alpha))^n}{n!}\left\|y_1-y_2\right\|^2_{\mathcal{C}_b}$$

So every $\alpha>0$, there exists n such that $\frac{(T\eta(\alpha))^n}{n!}<1$. It follows that $\mathbf{S}_\alpha^{\mathbf{n}}$ is a contraction mapping for sufficiently large n. Now, by the contraction mapping principle, the operator $\mathbf{S}_\alpha$ has a unique fixed point x_α in $\mathbb{H}_2$, which is the mild solution of (1)-(2). Thus the system (1)-(2) is completely controllable . So, the theorem is proved. □

4. Example

Consider a control system with a semi-linear stochastic integro-differential equations with delay and Poisson jumps of the form:

$$\begin{cases} dx(t,v) = \left[\frac{\partial^2}{\partial v^2}x(t,v) + \int_{-\infty}^{t} \tilde{Q}(t-s)x(s)ds + \tilde{B}(t,v) + \tilde{f}(t,x(t-h),v)\right]dt \\ +\tilde{\sigma}(t,x(t-h),v)dw(t) \\ + \int_{\mathcal{Z}} \Big(\int_{-\infty}^{t} \tilde{g}(t,v(t-h),z)ds\Big)\tilde{N}(dt,dz),\ t \in [0,T], \\ x(t,0) = x(t,\pi) = 0,\ t \in [0,T], \\ x(0,v) + \int_0^{\pi} q_1(v,y)z(t,y)dy = \psi(t,v),\ t \in (-\infty,0]. \end{cases} \tag{8}$$

Let $\mathbb{H} = \mathcal{L}^2[0,\pi]$ and $U = \mathcal{L}^2[0,T]$. Here $q_1(v,y) \in \mathcal{L}^2[0,\pi]$ and $W(t),\ t \geq 0$ is a real standard Brownian motion and $\tilde{N}(.,.)$ is a compensated Poisson measure on $[1,\infty)$ with parameter $v(dz)ds$ such that

$$\int_1^{\infty} v(ds) < \infty.$$

Let $A : \mathbb{H} \to \mathbb{H}$ be an operator defined by $Av = v^{''}$ with domain

$$D(A) = \Big\{ w \in \mathbb{X} : w \text{ and } w^{'} \text{ are absolutely continuous, } w^{''} \in \mathbb{H}, w(0) = w(\pi) = 0 \Big\}$$

Then

$$Aw = \sum_{n=1}^{\infty} n^2 \langle w, e_n \rangle e_n,\ w \in D(A),$$

where $e_n(v) = (\frac{2}{\pi})^{1/1}\sin\ nv,\ 0 \leq v \leq \pi,\ n = 1,2,...$ is the orthogonal set of eigenvectors of A. If A is the infinitesimal generator of a semi-group $T(t),\ t > 0$, in $\mathbb{H}$ and given by

$$T(t)w = \sum_{n=1}^{\infty} e^{n^2 t} \langle w, e_n \rangle e_n,\ w \in \mathbb{H}.$$

Now, we present a special case $\mathcal{C}_b$. Let $b(s) = e^{2s},\ s < 0$, then $l = \int_{-\infty}^{0} b(s)ds = 1/2$. Let $\|\psi\|_{\mathcal{C}_b} = \int_{\infty}^{0} b(s)\sup_{s\leq\theta\leq 0}(E|\psi(\theta)|^2)^{1/2}ds$, then $(\mathcal{C}_b, \|.\|_{\mathcal{C}_b})$ is a Banach space. For $(t,\psi) \in [0,T] \times \mathcal{C}_b$,, where $\psi(\theta)(v) = \psi(\theta,v),\ (\theta,v) \in (-\infty,0] \times [0,\pi]$, and we define the functions $f : [0,T] \times \mathcal{C}_b \to \mathbb{H}$, $\sigma : [0,T] \times \mathcal{C}_b \to L_Q(\mathbb{H})$ and $g : [0,T] \times \mathcal{C}_b \times \mathcal{Z} \to \mathbb{H}$ for infinite delay as follows:

$$f(t,\psi)(v) = \int_{-\infty}^{0} \tilde{f}(t,x,\theta)\psi(\theta)(v)d\theta$$

$$\sigma(t,\psi)(v) = \int_{-\infty}^{0} \tilde{\sigma}(t,x,\theta)\psi(\theta)(v)d\theta$$

$$g(t,\psi)(v) = \int_{-\infty}^{0} \tilde{g}(t,v,\theta)\psi(\theta)(v)d\theta$$

Let $Bu : J \to \mathbb{H}$ be defined by

$$Bu(t)(v) \quad = \quad \tilde{B}(t,v), \quad 0 \le v \le \pi, \quad u \in J.$$

Assume that the operator L_0^T be defined by

$$(L_0^T u)(v) \quad = \quad \int_0^T e^{-n^2(T-s)}\tilde{B}(t,v)ds.$$

On the other hand, it is known that the linear system corresponding to (8) is exactly controllable. Hence, all conditions in Theorem 3.1 are satisfied. Therefore, the system (8) can be written in the abstract formulation (1)-(2). By Theorem 3.1, system (8) is completely controllable on $[0,T]$.

5. Conclusion

This paper deals with the complete controllability of semi-linear stochastic differential equations with infinite delay and Poisson jumps under some basic and readily verified conditions. The results are obtained by using fixed-point approach. This motivatse the future research work such as the exact (complete) controllability of semi-linear stochastic differential equations with infinite delay driven by a fractional Brownian motion. One can extend the same work for second order system/inclusion. Complete controllability of semi-linear stochastic fractional order differential equations with infinite time dependent delay/ state dependent delay and Poisson jumps under some basic and readily verified conditions of Riemann Louville derivative and Caputo derivative would be interesting. Complete controllability of semi-linear stochastic fractional order differential equations with infinite delay and Poisson jumps using recently developed Atangana Baleanu Caputo (ABC) [18] derivative.

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