

# Inter-Factor Determinants of Return Reversal Effect with Dynamic Bayesian Network Analysis: Empirical Evidence from Pakistan

Abdul HAQUE<sup>1</sup>, Marriam RAO<sup>2</sup>, Muhammad Ali Jibran QAMAR<sup>3</sup>

Received: November 15, 2021 Revised: January 23, 2022 Accepted: February 15, 2022

## Abstract

Bayesian Networks are multivariate probabilistic factor graphs that are used to assess underlying factor relationships. From January 2005 to December 2018, the study examines how Dynamic Bayesian Networks can be utilized to estimate portfolio risk and return as well as determine inter-factor relationships among reversal profit-generating components in Pakistan's emerging market (PSX). The goal of this article is to uncover the factors that cause reversal profits in the Pakistani stock market. In visual form, Bayesian networks can generate causal and inferential probabilistic relationships. Investors might update their stock return values in the network simultaneously with fresh market information, resulting in a dynamic shift in portfolio risk distribution across the networks. The findings show that investments in low net profit margin, low investment, and high volatility-based designed portfolios yield the biggest dynamical reversal profits. The main triggering aspects related to generation reversal profits in the Pakistan market, in the long run, are net profit margin, market risk premium, investment, size, and volatility factor. Investors should invest in and build portfolios with small companies that have a low price-to-earnings ratio, small earnings per share, and minimal volatility, according to the most likely explanation.

**Keywords:** Return Reversal, Bayesian Networks, Portfolio Risk, Contrarian Strategy

**JEL Classification Code:** C11, C58, G11

## 1. Introduction

In the stock market, the long-run reversal effect in stock returns is a well-known phenomenon. When an investor invests to earn a return, but a certain set of conditions emerge over time that has a significant impact on the pattern of return, a reversal happens. In the long run, the existence of consistent important elements that induce reversal anomaly causes loser portfolios to become winners and vice versa. Even with the most prominent traditional regression-based

risk-return methodologies given by Fama and French (2015), Hou et al. (2015), Malin and Bornholt (2013), and Pastor and Stamburg (2003), the problem of identifying the driving factors of abnormal stock returns that result in a return reversal effect in the stock market remains difficult to examine.

All of these studies provide us with useful information, but they are not without flaws. To quantify the variance in a beta of asset price risk variables in explaining stock returns, the famous linear risk-return models (Fama & French, 2015) rely on the selection of a valid set of factors. Traditional portfolio models are static rather than dynamic (Hou et al., 2015). There are no systematic means to update results as new information becomes available.

Analysts can use a dynamic Bayesian network model of portfolio return to incorporate new information, see the impact of that information on the entire network's return distribution, and visualize the distribution of returns rather than simply the summary statistics. Furthermore, standard regression models are incapable of establishing a link between independent variables. To investigate the deep path, a comprehensive neural network of factor sets that form that path for reversal effect must be evaluated. The chosen model

<sup>1</sup>First Author and Corresponding Author. Department of Economics, COMSATS University Islamabad, Lahore Campus, Pakistan [Postal Address: Defence Road, Off Raiwind Road, Lda Avenue Phase 1, Lahore, Punjab 54000, Pakistan] Email: ahaque@cuilahore.edu.pk  
<sup>2</sup>Department of Management Sciences, COMSATS University Islamabad, Lahore Campus, Pakistan.  
Email: marriamrao11@gmail.com  
<sup>3</sup>Business Division, Higher Colleges of Technology, Abu Dhabi, United Arab Emirates. Email: majqamar@googlemail.com

© Copyright: The Author(s)  
This is an Open Access article distributed under the terms of the Creative Commons Attribution Non-Commercial License (<https://creativecommons.org/licenses/by-nc/4.0/>) which permits unrestricted non-commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

must be able to anticipate the entire underlying stock market system for this method to work. The focus has shifted to non-linear mathematical models like Artificial Neural Networks (ANN) and Dynamical Bayesian Network Analysis (DBNA) (Ticknor, 2001; De Oliveira et al., 2013).

The purpose of this study is to show how Dynamic Bayesian Networks can be used to model portfolio risk and return to find linkages of reversal anomaly-causing elements in Pakistan's emerging market (PSX) from January 2005 to December 2018. Identifying inter-factor relationships Bayesian Networks, which are based on Artificial Neural Network techniques, have evolved into a tool for revealing the course, structure, and inferences of underlying models. The entire neural network structure might be constructed, just like the human body neural network, where each neuron transfers information to other neurons. Bayesian is more powerful as it outperforms the failure of many models due to its handling of non-linearity, time-varying, and boundless problems in one unified modeling framework. Bayesian is particularly used to determine the dominant inter-relationships amongst the variables (Scutari, 2009).

The Pakistan Stock Exchange (PSX) is one of the most established stock exchanges in emerging nations. In 2003 and 2006, the market was ranked first and third, respectively, in terms of turnover ratio. There is little doubt that international investors would like to relocate their assets to Pakistan's burgeoning market, given the market's growing size and prospects. Political instabilities, social unrest, questionable accounting standards, and unstable currencies all increased market risk in emerging markets. The larger the relative market risk, the higher the possible gains. Profit booms in fast-growing emerging economies, as well as diversification benefits, motivate investors to invest in emerging markets. Academicians worked to explain anomalies in expected stock return in cross-sectional variation analysis of asset price risk factors in linear regression models of (Fama & French, 2015) and non-linear neural networks of (Wang et al., 2008).

The academicians established stock market asset price risk factors based on linear regression models which directly cause an impact on expected stock returns are evidenced and explained by researchers in emerging and developed markets (Hou et al., 2015; Ho et al., 2008; Fama & French, 2015). These established asset price driving risk factors studied and tested by researchers include the three, four, and five driving factor models of Fama and French (2012), Carhart (1997), and Pastor and Stamburg (2003). They are 1. The excess market return (MKT), 2. The size factor (SMB), 3. The valuation factor (HML), 4. The momentum factor (MOM) and 5. The liquidity factor (LIQ). The other established asset price risk factors include 6. Profitability factor (RMW), 7. Investment factor (CMA), 8. The short-term reversal factor (STR),

9. The long-term reversal factor (LTR) was established by Fama and French (2015), and Hou et al. (2015). The market gearing factor (UMD) was established by Ho et al. (2008).

The goal of this study is to first create a loser minus winner (LMW) asset price risk factor for the Pakistan Stock Exchange market, and then to incorporate and examine all asset price risk factors studied by experts and explained above in a dynamic Bayesian network model to identify dynamic causal effects, inter-factor connections, and associations with the reversal factor (LMW). Table 1 shows the operationalization of established asset price driving risk indicators that have been examined and tested by researchers and used in our analysis.

The rest of the paper is distributed as follows. The literature review is divided into two sections. In section one, we briefly discuss traditional portfolio risk-return models and reversal detection methodologies. In section two we explain the semantics of Bayesian Networks. Next, the methodology is discussed where Bayesian Network Parameters and structure learning methodologies are defined. In the end, we, discuss the Results discussion with limitations and conclusion.

## 2. Literature Review

### 2.1. Traditional Risk Return Models and Reversal Detection Methodologies

Academics and scholars in the field of finance have spent a lot of time looking into the factors that cause the return reversal effect. Sharpe (1964), a pioneer in the study of anomalous stock returns, worked on asset pricing risk factors to investigate abnormal stock returns. Sharpe, Linter, and Black (SLB), who discovered market risk premium as the primary factor leading to variation in stock returns, used the phrase "asset pricing model" to describe studies on stock returns based on various asset pricing risks components involved. The capital asset pricing model (CAPM) gave researchers a place to look into the stock market's stock return anomalies. The postulated relationship in the standard CAPM model is that the expected return on a security is a positive linear function of the market rate of return and an asset-specific factor, as shown in Equation 1.

$$R_i = \alpha_i + b_i R_m + E_i \quad (1)$$

Where  $R_i$  denote return on asset  $i$ ,  $\alpha_i$  and  $b_i$  are constants,  $R_m$  denotes return on market index and  $E_i$  denotes an uncertain variable related to asset price risk factors. Following the SLB model, many studies followed to examine the CAPM model's existence in different markets with different characteristics. These studies indicated that some specific

identifiable components of risk are not fully accounted for by just the market index. Other industry-specific, country-specific, and many other components account for correlation among individual securities. Merton (1973) and Ross (1976) were the pioneers in the development of the multifactor model. The multifactor model can be a representation of the expanded version of Equation 1.

$$R_i = \alpha_i + b_{i1} F_1 + b_{i2} F_2 + b_{i3} F_3 + E_i \quad (2)$$

Because of the mixed results, researchers used other parameters instead of the market risk premium to investigate driving forces in the short and long term. According to Banz (1981), size is a critical component that determines variation in stock returns and initiates the return reversal effect, among other variables that contribute to the difference in returns. According to the findings of the study, equities with a lower market capitalization (smaller stocks) usually have better regular returns. In a three-factor asset pricing model, Fama and French (2012) looked at the explanatory power of cross-sectional fluctuation of variables. They document SIZE and Book to Market Equity as the powerful measures of asset price risk factors in cross-sectional variation of average returns in addition to the market risk premium. The finding indicates that small-size firms with high book-to-market equity contribute toward higher expected returns in long run. They list a number of probable reasons. Small businesses are in the early stages of innovation and growth, therefore they earn larger returns by doing their best. Second, enterprises with low stock prices and high book-to-market equity ratios are penalized with greater capital costs than firms with strong prospects. The high cost of capital entails a high level of risk, resulting in larger predicted stock returns in the future. Due to their modest market capitalization, small-cap stocks have a higher cost of financing, but large-cap stocks do not carry as much risk because they already have strong growth potential.

In short, returns on tiny stocks are more sensitive to size factor risk than returns on large stocks (SMB), and returns on high book to market equity stocks capture more risk than returns on low book market equity companies (HML). This stresses the importance of the size risk factor (SMB) and the value risk component in the long run return reversal effect's productivity (HML). Hou et al. (2015) worked on a neoclassical investment Q-theory. In multifactor asset pricing models, they used Q-factors. Nearly 80 abnormalities in cross-section variation of abnormal returns were investigated. They show that, in addition to market risk premium (MKT), size (SMB), and valuation (HML), asset pricing risk factors investment factor (CMA), and profitability factor (RMW) are the most important contributors to anomalies and return reversal. They also demonstrated and demonstrated that the Q-factor model is superior and powerful in explaining

anomalous returns when compared to the Fama and French (2012) 3-factor model and Carhart's (1997) 4-factor model. Cakici et al. (2013) investigated Carhart's four-factor model, finding that document size, value, and momentum are the main drivers of return reversal in emerging markets.

Ayub (2017) investigated the long-term association between return reversal determinants in Pakistan's, India's, and China's rising stock markets. With a sample of 1198 enterprises across three developing marketplaces from 2001 to 2013, the study used Quantile Regression methods to include firm-specific risk indicators. In addition to market effect (MKT), these elements include size (SMB), value (HML), reversal (LMW), and growth volatility (UMD). Market reversal and volatility factor (UMD) were found to be major contributors to the short-term return reversal effect. They also discovered that in emerging economies, growth companies beat value stocks over time. Their research, however, was limited to a robustness test of the Fama French five-factor model. They also failed to account for inter-factor relationships among the driving forces, as well as non-linearity in real-world market data.

De Bondt and Thaler (1985) proposed the Loser Minus Winner (LMW) reversal factor based on their Contrarian mean test methodology to detect reversal effect in the short and long run. Portfolios are divided into losers and winners based on their previous J-month performance. These portfolios are then held for a future k-month return to look for a reversal effect in the stock market index over a longer period of time. Concerned about Asian markets, Reddy et al. (2019) used Malin and Bornholt's (2013) late-stage contrarian technique in Chinese stock markets from March 2011 to 2016. With a portfolio construction time of 24 months and holding periods of 6, 9, 12, and 24 months, the data indicated the presence of a long-term reversal effect. They further reasoned that the development of excess abnormal returns is explained by small portfolios with lower book-to-market ratios. They also looked at the asset price risk factor loser minus winner (LMW) in the well-known Fama French Five-factor model and discovered that it was a significant driver to stock return anomalies.

## 2.2. Dynamic Bayesian Networks Semantics

From January 2005 to December 2018, the current study aims to investigate inter-factor determinants of return reversal effect by combining neural network of asset price risk factors originating from economic theories of tested asset pricing models into a single combined framework of Dynamical Bayesian Network for Pakistan's emerging markets. A causal data-driven method with a dynamic systematic approach is the dynamical Bayesian factor approach (Wang et al., 2008), which is a subclass of the Bayesian network (Pearl, 2003). The Bayesian factor model is a cause-and-

effect network model that can calculate the probabilistic certainty of the cause given the resource. This strategy aids in the comprehension of conditional independence and probabilistic interactions between variables. The model is based on a joint probability distribution system, which represents the uncertainty of the data-driven mechanism's results. The relationship is strengthened by the model's provision of conditional independence among the variables of the return reversal effect, which provides causality and an inference mechanism for conditional probability tables.

Previous research investigated the constraints of regression models and showed why Bayesian Networks outperformed multiple linear regression models, which are commonly employed in Fama and French risk-return models. It showed that multiple linear regression models suffer from an over-parameterization problem and that the models are unable to handle huge data sets due to the risk of bias in linear regression models. When big variable components are added to the problem, the explanatory power of the model is reduced due to the heteroscedasticity problem. The research also reveals that when looking for inter-factor correlations, regression does not work. Regression models are used to determine the influence of one variable while controlling for all other variables. In the real-world market, we can't assume that changing one variable would keep all other variables constant.

Regression works in the same way as a scientific experiment does, with specific variables being controlled. If we consider a pond and a stone, the place where the stone hits the pond has the most impact, as opposed to a distance away from where the stone has a minor impact. Similarly, variables that are close and strongly reliant on one another have a greater effect than variables that are far apart but still have an effect. If one variable changes, there's a risk that other variables with the same element may change as well, which is why regression doesn't work. Variables have interdependencies between them. The interrelationships differ from one variable to the next.

Bayesian Networks outperform regression in terms of expressing the interrelationships between all variables. It also describes how changing one element in each variable affects the other, as well as the overall impact of all variables on the target variable as variables change over time. While determining the effect of one variable with its probabilistic graphical conditional independence property between each other, Bayesian Networks take into account all variables' dependencies. Regressions and Bayesian network approaches were contrasted by Zong et al. (2013). The results show that Bayesian Network (BIC) goodness of fit is better than Regression models in accident severity testing modeling.

Because the stock market is dynamic, time variable, and devoid of boundaries, the best way for predicting return reversal is to use a whole set of factors and investigate their

quantitative inter-factor interactions in a probabilistic factor graph that changes over time. This prompted academics to focus on data-driven solutions rather than model-driven solutions to be able to anticipate stock market movements with a large enough data set and impartial factor selection. In "Dynamical Bayesian Factor Graphs Network Approach," the current paper fills this gap by presenting a method for solving for dynamics, time variation, and boundary-less real-world market data (Scutari, 2009; Wang et. al., 2008). (Scutari, 2009; Wang et. al., 2008). In emerging markets, there are just a few research that looked at inter-factor interactions of stock return drivers using a dynamical Bayesian factor graphs network technique.

Wang et al. (2008) chose the US stock market to investigate and analyze the key components that cause the return reversal effect. They developed a technique called dynamical Bayesian factor graph to apply to reversal effects. They concluded that the liquidity effect had a consistent impact on the reversal effect. They investigated a multivariate large factor set in a dynamical Bayesian Network to deal with multivariable interrelationships in a single integrated model by dealing with all potential real-world scenarios. It outlines the relationships between all factors and relative efficacy. Liquidity has a constant impact on stock reversals, according to the results of their research in the US market. According to their findings, high turnover stocks have a higher chance of experiencing a reversal impact. This creates a research gap in emerging markets, where prediction and investigation of inter-factor linkages between driving components of return reversal impact lag.

### 3. Methodology

#### 3.1. Data Description

The data for this study comes from monthly closing stock returns for a sample of all non-financial companies in the PSX index. The study's sample duration is 14 years, from January 2005 to December 2018. Financial companies are excluded since their accounting period ends in December, whereas non-financial companies' accounting cycle ends in June. As a result, comparing the many variables employed in this study at a single point in time is not conceivable. Furthermore, the capital structures of the financial and non-financial sectors (businesses) differ. Financial companies usually have a higher percentage of debts in their capital structures while non-financial firms usually have a higher percentage of equity.

We also do not include service sectors due to their small sample size as the total service firm's ratio represents only 18 percent of the total listed companies on the PSX index. The study used the ASM program of STATA software for building portfolios. In terms of data filtration only such

firms are selected which are: (1) equity by nature, (2) traded in the local currency (3) listed on the domestic stock exchange. (4) Continuous trading companies are selected for the reason of building balanced and overlapping panel portfolios. To address delisting and new-listing company data errors for our sample period, company data before 2005 is not included in the composition of balanced panel portfolios. As a result, our sample runs from January 2005 to December 2018. Stocks with a high market capitalization are regularly traded. The reason for selecting stocks based on market capitalization is to avoid inactive stocks and to ignore stock market microstructure issues. In the PSX index, there are 327 non-financial businesses listed in 40 categories. 273 businesses were chosen from a total of 327 based on the availability of continuous trade data from 2005 to 2018 to create balanced portfolios for determining the reversal effect in the Pakistan stock market. The PSX-Index has 273 firms in its final sample data filtration.

A time series of emergent networks over a specific time would be generated to evaluate dynamical change and evolution of factor relationships to adapt Bayesian in complicated time variable systems. The following steps would be included in the procedure as a whole: 1) Parameters are determined using a Bayesian parameter learning technique, in which parameters are chosen based on the highest log-likelihood and least Bayesian Information criterion to produce the most mutual information and least entropy. 2. Acyclical Graph Class Factor for Directed Acyclical Graphs (Return Reversal would be defined). 3. A Constraint-Based Algorithm (PC algorithm) of model Structure learning of Bayesian networks based on evidence data is used to learn the network structure. 4. New information entered as evidence is used to update the time series networks. 5. Inter-factor links with dynamical change in posterior distributions are explored using new data throughout a time period to find inter-factor relationships with dynamical change in posterior distributions.

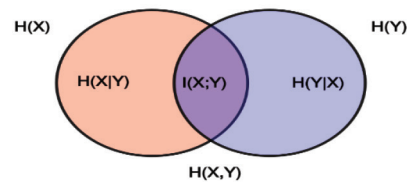
The Inference method is used to construct the marginal and conditional probabilities tables. Each main driver's function influence might be revealed by comparing conditional probabilities. 6. The consistently influential factors across time would be thoroughly explored using the maximum probable explanation approach (MPE). This would give us vital information on the things that have the most and least impact. This would provide us with information on the neural network of inter-factor relationships that causes market reversal anomalies. In terms of institutional structure, economic instability, and liquidity, Pakistan's growing stock market is distinct from that of the United States. This research focuses on the inter-factor causes of return reversal effects in the Pakistan stock market index.

### 3.2. Bayesian Factor Network

Bayesian Factor network graphs is a subclass of Bayesian network where the edges in a factor reflect inter-factor relationships, including **causality, relevance, and conditional independence**. A Bayesian factor is a directed acyclic (DAG) where the joint distribution of  $k$  factors,  $V = \{X_1, X_2, \dots, X_d\}$ , is encoded. Nodes represent the factors, while the CPD (conditional independence probability tables) represent the quantitative information between the factors. Two unconnected nodes imply that corresponding factors are conditionally independent. If there exists an edge from node  $X_i$  to  $X_j$ , then  $X_i$  is called a parent of  $X_j$ , and  $X_j$  is a child of  $X_i$ . With the parent node set  $X_p$ , the whole factor graph can be represented as  $G = \{G_1, G_2, \dots, G_d\}$ . The joint probability of  $X$  given  $G$  could be given in Equation 3 as:

$$p(X/G) = p(X_1, \dots, X_d/G) = \prod_{i=1}^d p(X_i/X_G) \quad (3)$$

The structure of a Bayesian Network is unknown at first and must be learned based on  $X$ 's observations. To learn graph structures, we would use the incremental association Markov blanket (IAMB) method, which is one of the improved versions of the inductive causation algorithm. The IAMB Incremental Association (Constraint-Based Algorithm) is a two-step process. With a Markov blanket of factors, the undirected structure (skeleton) is learned first. Second, in light of the  $V$  structure, the directions are set with d-separation. In an acyclic directed graph, there are three types of basic structures:  $XYZ$ ,  $XZY$ , and  $XYZ$ . The first two represent the same conditional independence restrictions, namely that  $X$  and  $Z$  are conditionally independent in the presence of  $Y$ . Based on observational data, they are comparable and indistinguishable, according to Pearl et al. (2001). The latter, often known as a  $V$ -structure, denotes that  $X$  is only minimally independent of  $Z$ . As a result, it is distinct from the other two structures and may be identified. Mutual information of two factors would be estimated as a measure of factor association. The mutual information (MI) of two random variables (Informational theoretic distance measure) is a measure of the mutual dependency between the two variables. It measures the "amount of information received about one random variable through observation of another random variable."



$$MI(X; Y | Z) = \sum_{x \in X} \sum_{y \in Y} \sum_{z \in Z} p_{x,y,z}(x, y, z) \log \frac{p_Z(z) p_{X,Y,Z}(x, y, z)}{p_{X,Z}(x, z) p_{Y,Z}(y, z)} \tag{4}$$

$(X; Y) = H(X) + H(Y) - H(X, Y)$ ,  
then  $MI = 0$  (Rule of Independence)

Where,  $p_{x,y,z}(x, y, z)$ ,  $p_{x,z}(x, z)$ ,  $p_{y,z}(y, z)$  is for the joint probability distribution  $p_Z(z)$  is the marginal probability density function of Z. It is always true that  $MI(X; Y|Z) \geq 0$  and  $MI(X; Y|Z) = MI(Y; X|Z)$ . With the Common effect, V structure, the direction of edges is set with a method called *d*-Separation (Geiger et. al, 1991). In the last, we would add directions to other edges to meet the acyclic restriction of a factor graph.

The marginal probabilities of a node in a factor graph reflect the possibilities that the node will take conceivable values if no information is provided, whereas the CPT conditioned on the key drivers reveals the chances that the node will take those values if the key drivers offer information. Assume we're interested in which values of the key drivers are most likely to cause the node to take a particular value. We can search for the conditional probabilities that are higher than the marginal probabilities by first finding the marginal and conditional probabilities that correspond to the value. The values of the desired key driver can thus be intuitively revealed in this way.

### 3.3. Asset Price Risk Factors including Reversal Factor as Class Factor

Based on traditional risk-return methodologies of Fama French we incorporate all established 11 asset price risk factors in the multifactor model given below:

$$R_i - R_f = \alpha_1 + b_1 (MKT)_{it} + b_2 (SMB)_{it} + b_3 (HML)_{it} + b_4 (NPM)_{it} + b_5 (INV)_{it} + b_6 (VOL)_{it} + b_7 (EPS)_{it} + b_8 (PER)_{it} + b_9 (ROE)_{it} + b_{10} (DIV)_{it} + b_{11} (LMW)_{it} + E_{it} \tag{5}$$

Where  $R_i$  = Stock returns of *i* company at time *t*,  $R_f$  = Risk-free Rate, LMW = Loser minus winner in terms of past *t*-60 to *t*-2 cumulative returns. The remaining factors are described in Table 1 given below.

### 3.4. Class Factor/Target Variable Factor

Long Term Reversal Stock Factor (LMW) is used as a class factor to create a directed acyclic graph (DAG). The DAG aims to identify a long-term reversal factor by focusing on all driving factors such as causality, relevance, influence, and interconnectivity (either as input or output variables) (LMW). According to Fama and French's (2015) reversal factor technique, equal-weighted cumulative abnormal returns are calculated and sorted in ascending order from month *t*-60 to *t*-2 of at least the previous five years for the built of (LMW) reverse factor.

**Table 1:** Potential Driving Factors; Bivariate Sorts Market Capitalization and Testing Factor

Sr.	Factors	Name	Description
1	Reversal Factor	LMW	Returns of loser portfolio–Returns of winner portfolio based on ( <i>t</i> -60, <i>t</i> -2)
2	Market Factor	MKT	Returns of loser portfolio–Returns of winner portfolio
3	SIZE Factor	SMB	Returns of small size portfolio–Returns of big size portfolio
4	Valuation Factor	HML	Returns of high book to market equity portfolio–Returns of the low book to a market equity portfolio
5	Profitability Factor	NPM	Returns of high profitability portfolio–Returns of low profitability portfolio
6	Investment Factor	INV	Returns of a high investment portfolio–Returns of the low investment portfolio
7	Earnings Per Share Factor	EPS	Returns of high earnings per share portfolio–Returns of low earnings per share portfolio
8	Price to Earnings Factor	PER	Returns of high Price to Earnings portfolio–Returns of low Price to Earnings portfolio
9	Return on Equity Factor	ROE	Returns of a high return on equity portfolio–Returns of low return on equity portfolio
10	Volatility Factor	VOL	Returns of high volatility portfolio–Returns of low volatility portfolio
11	Dividends Factor	DIV	Returns of a high dividend-paying portfolio–Returns of low dividend-paying portfolio

Similarly, cumulative anomalous returns of stocks are classified according to size, or market capitalization; that is, returns are sorted from small to large firms. Portfolios are created based on size and five-year cumulative returns ( $5 * 5 = 25$ ). The long-term reversal risk premium factor, commonly known as the loser minus winner (LMW) factor, is the difference between the equal-weighted average stock returns of the lowest loser portfolios (P1) and the equal-weighted average stock returns of the highest winner portfolios (P25).

When data values are discretized, Bayesian Networks perform better. We divided each risk component into two bins based on discretization: 0 and 1. 0 denoted low risk and 1 denoted high risk. Table 2 shows the bins and values assigned to each factor. We use the class element  $IsReversal_{it}$ , which can be either 1 or 0, to indicate whether or not return reversal occurs at  $t + 1$ . A reversal occurs when the value of  $IsReversal_{it}$  equals 1. If the following limitation is met,  $IsReversal_{it}$  will be 1; otherwise, it will be 0.

**Table 2:** Discretized Bins for Bayesian Network Analysis

Sr.	Factors	Values	Bins
1	LMW	0	No Reversal
		1	IS Reversal
2	MKT	0	Low
		1	High
3	SMB	0	BIG
		1	Small
4	HML	0	Low
		1	High
5	NPM	0	Low
		1	High
6	INV	0	Low
		1	High
7	EPS	0	Low
		1	High
8	PER	0	Low
		1	High
9	VOL	0	Low
		1	High
10	ROE	0	Low
		1	High
11	DIV	0	Low
		1	High

Loser Portfolio Return – Winner Portfolio Return  $> 0$ , Is Reversal, bin 1.

Loser Portfolio Return – Winner Portfolio Return  $< 0$ , No Reversal, bin 0.

#### 4. Results and Discussion

The summary statistics of all excess stock returns driving elements are shown in Table 3. The lowest and largest excess returns of portfolios classified on ( $5 * 5$ ) quintiles based on size and previous t-60 to t-12 returns are P1 and P25, respectively. The risk-return spread between the loser portfolio P1 and the winner portfolio p25 is known as the loser minus winner factor. LMW risk spread is  $-3.396$  on average, with a minimum of  $-7.226$  and a top of  $0.43$ . The excess stock return  $R_i - R_f$ , on the other hand, has a mean of  $0.412$ , a minimum of  $-1.506$ , and a maximum of  $1.593$ . Outliers and missing data can both be handled by Bayesian statistics.

For given values of unknown parameters, the log-likelihood assesses the goodness of fit of a statistical model to a sample of data. When evidence (input data) is fed into a Dynamic Bayesian network, the probability (likelihood) of that evidence can be determined, also known as  $P(e)$ . The probability of evidence  $P(e)$  reflects the likelihood that the data was created by the network. The greater the value, the more probable the model will provide high goodness of fit power. On the other hand, the Bayesian information criterion is a model selection criterion that favors the model with the lowest BIC among a finite set of models.

**Table 3:** Summary Statistics

	Mean	Std. Dev.	Min	Max
$R_i - R_f$	0.412959	0.763368	-1.5069	1.593709
MKT	0.309469	0.997503	-1.71892	3.041909
SMB	0.679198	1.606451	-1.67451	4.752403
HML	0.264226	1.432584	-2.63571	3.003781
VOL	1.336474	1.321395	-0.48119	5.054519
EPS	0.182842	1.064503	-1.85873	2.231468
DIV	0.114246	0.919372	-1.58278	2.234512
PER	-0.15654	0.598024	-1.67692	0.878412
INV	0.220102	0.862384	-1.48663	2.2475
NPM	0.028405	0.961831	-2.06397	1.655131
LIQ	-0.28301	1.644344	-2.99891	3.825176
LMW	-3.39673	2.164664	-7.22614	0.43293
P1	-0.85784	1.48691	-5.95484	-0.16907
P25	2.273896	1.37911	0.489241	5.538971





share ( $EPS = 1$ ), and high volatility ( $VOL = 1$ ). The size of the firm's factor returns is low at 69.88 percent with  $SMB = 0$ , which is a noteworthy statistic to note. The liquidity component has a direct relationship with  $SMB$ , while the volatility factor has a direct relationship with  $LMW$ . This suggests that the Pakistani market is characterized by high volatility and little liquidity, which has a negative impact on its results. Furthermore, the investment element and the net profit margin factor are both affected by volatility.

In the third phase, we created a Bayesian Network model without using the most likely explanation or an evidence set, i.e. without adding any additional data. Without entering any new data, the findings show a reversal effect ( $LMW = 1$ ; 69.86%), significant volatility ( $VOL = 1$ ; 82.14%), and a high return of 78.28 percent for small enterprises. The investment, net profit margin, and market risk premium ( $R_m - R_f$ ) are all high, with a probability of value = 1 exceeding 70%.

In the final stage, we updated the Bayesian Network Structure with new data (input data). For example, if data indicates that a reversal has happened  $LMW = 1$  and returns are also high  $R_i - R_f = 1$ , what influence does this have on the information output obtained from other driving factors? A dynamical change in network information is formed as a result of this new information. Over the time span, reversal had happened, as we know from MPE. Given this data, we calculated proof excess returns are high  $R_i - R_f = 1$ , where Reversal factor is likewise high, indicating that a looser portfolio suggests reversal;  $LMW = 1$ . We also included the fact that the loser portfolio has a large return ( $LMW = 1$ ;  $R_i - R_f = 1$ ). Based on the value of the information table; Net profit margin (NPM), Investments (INV), and Volatility are the highest dependent influencers on excess returns.

As a result, we look for conditional probabilities where evidence is entered for Net Profit Margin ( $NPM = 1$ ), Investments ( $INV = 1$ ), and Volatility ( $VOL = 1$ ) among other things. The first direct nodes from excess returns are NPM, INV, and  $R_m - R_f$ . The reversal factor is likewise a child of the investment factor, which is a child of the volatility factor. The Pakistani market is characterized by excessive volatility and a lack of liquidity, which has a negative impact on the index. Then, based on the most likely explanation, we entered evidence that volatility is high. The new data provides us with conditional probabilities that lead to a strong reversal anomaly in the market and high excess returns. This also highlights how inter-factor relationships can be used to investigate the deep path that leads to an increase in market anomaly over time.

To simplify the process of inference and interpretation, we generated Table 5 based on the Bayesian network structures obtained from the input data. All joint and conditional probabilities of each state (high = 0 or low = 1) occurrence are represented in table 5 with distinct

evidence set information provided. At first, the most probable explanation (MPE) states are mentioned. Second, probabilities densities are entered before evidence; evidence = 0. The remaining columns show the dynamic change that occurs after fresh evidence or information is entered. Based on the evidence provided, we represented conditional probabilities distributions. The percentage change is denoted by the symbol percent, which denotes the difference between distributions with and without evidence. In other words, this is the dynamical difference or difference between CPDs after evidence has been entered and CPDs without evidence having been entered into the data-driven model process.

Table 5 shows the outcomes of the Bayesian Inference Mechanism with and without evidence (input data entered) (no data entered). When the evidence (input data) is  $R_i - R_f = 1$ , we want to know how the conditional probabilities of network factor associations change when the market return reversal effect occurs. This means we're looking for clues as to what triggers the occurrence of high excess returns in the market. The biggest dynamic change in Market risk premium (MKT), Size (SMB), Liquidity (LIQ), and Earnings per share (EPS) is shown in Table 5. (EPS). With evidence entered  $R_i - R_f = 1$ , for example, small business stock returns increased to 78.28 percent from 63.33 percent, a dynamic shift of 14.95 percent.

The stock returns of large corporations have been reduced to 21.72 percent from 36.67 percent, a decrease of -14.95 percent. High market risk premiums also increased by 18.33 percent to 88.33 percent. The poor liquidity component rose to 68.76 percent, an increase of 12.09 percent. In addition, the low Earnings per Share component state increased by 10.38 percent to 54.47 percent. In essence, portfolios should be created around high  $SMB$  enterprises with little liquidity, large market risk premiums, and low earnings per share to achieve abnormal stock returns over time. These findings are congruent with Banz (1981), who discovered that size is a critical component that determines variation in stock returns and initiates the return reversal effect, among other variables that contribute to the difference in returns.

The findings are likewise in line with those of Shahrudin et al., (2020) and Fama and French (2012), who found that stocks with a lower market capitalization (smaller stocks) have better regular returns. This could be because small businesses are still in the early stages of innovation and growth, thus they make more money by doing their best. Second, firms with low stock prices that the market believes have bad prospects are penalized with greater capital costs than enterprises with strong prospects. The high cost of capital entails a high level of risk, resulting in larger predicted stock returns in the future.

In other words, the returns on small stocks are more sensitive to the size factor's risk than the returns on large

**Table 5:** Inference Mechanism; Dynamical Change Output Results after New Evidence

Factors	States	MPE	Marginal	$R_i - R_f = 1$ ; High		LMW = 1		$R_i - R_f = 1$ ; LMW = 1	
			Evidence = 0	After	$\Delta\%$	After	$\Delta\%$	After	$\Delta\%$
			Before						
$R_i - R_f$	Low	Low	22.67%	0		23.27	0.6%	0	
	High		77.33	100		76.73	-0.6	100	
LMW	No-Reversal	Is Reversal	30	30.54	0.54	0		0	
	Is-Reversal		70	69.46	-0.54	100		100	
MKT	Low	Low	30	11.67	-18.33	31.34	1.34	12.64	-17.36
	High		70	88.33	18.33	68.66	-1.34	87.36	17.36
SMB	Low	Low	36.67	21.72	-14.95	38.47	1.8	23.28	-13.39
	High		63.33	78.28	14.95	61.53	-1.8	76.72	13.39
HML	Low	Low	43.33	36.25	-7.08	46.83	3.5	39.55	-3.78
	High		56.67	63.75	7.08	53.17	-3.5	60.45	3.78
VOL	Low	High	16.67	17.86	1.19	11.9	-4.77	12.86	-3.81
	High		83.33	82.14	-1.19	88.1	4.77	87.14	3.81
PER	Low	High	56.67	68.97	12.3	55.19	-1.48	67.69	11.02
	High		43.33	31.03	-12.3	44.81	1.48	32.31	-11.02
EPS	Low	High	44.09	54.47	10.38	42.04	-2.05	52.24	8.15
	High		55.91	45.53	-10.38	57.96	2.05	47.76	-8.15
DIV	Low	High	50	53.84	3.84	48.1	-1.9	52.05	2.05
	High		50	46.16	-3.84	51.9	1.9	47.95	-2.05
INV	Low	High	36.67	39.78	3.11	30.95	-5.72	33.84	-2.83
	High		63.33	60.22	-3.11	69.05	5.72	66.16	2.83
NPM	Low	High	38.7	47.99	9.29	34.62	-4.08	43.09	4.39
	High		61.3	52.01	-9.29	65.38	4.08	56.91	-4.39
LIQ	Low	High	56.67	68.76	12.09	54.35	-2.32	66.56	9.89
	High		43.33	31.24	-12.09	45.65	2.32	33.44	-9.89

ones (SMB). Liquidity and earnings per share indicate the largest change in impact, which is another crucial piece of information collected. This underlines the importance of a high size risk factor (SMB), a low earnings per share (EPS) risk factor, and a low liquidity factor (LIQ) factor in the long-run return reversal effect's productivity. With evidence entry of LMW = 1, we want to know if reversal has occurred and if big changes in effects have occurred. Net profit margin and investments, which are also parents of excess returns  $R_i - R_f$ , are the most important determinants. When two pieces of information are entered into the model, a clearer but slightly different picture emerges. For example, we'd like to know if the reversal occurs with the highest returns when  $R_i - R_f = 1$  and LMW = 1 evidence is entered, as this results in a

more visible change in delta. Although the overall results are comparable, the price-to-profits factor shows the greatest difference of 11.02 percent. Furthermore, excessive volatility has been enhanced by 3% from 1%. The findings are similar to those of Shoaib and Siddiqui (2017) and Hongsakulvasu and Liamukda (2020).

Table 5 represents Bayesian Inference Mechanism results based on evidence. For the inference mechanism, the Variable Elimination algorithm is utilized. 1<sup>st</sup> Column represents driving factors. The second Column shows the states according to discretized bins. MPE means the most probable explanation given all evidence provided. The fourth column represents marginal probabilities driven from data. This is accounted as before evidence input

factors certainty. The conditional probabilities representing inference mechanism is generated after adding evidence are demonstrated following with the dynamical % change between before evidence and after adding evidence. The evidence inputs are at first where individual stock market excess returns are high ( $R_i - R_f = 1$ ), a reversal occurs with the high state shown with Is-reversal equals 1 (LMW = 1) and third evidence is entered where is-reversal equals 1 as well stock excess returns are also high [ $(R_i - R_f) = 1$ ; LMW = 1].

Table 6 represents Bayesian Inference Mechanism results based on evidence. For the inference mechanism, the Variable Elimination algorithm is utilized. 1<sup>st</sup> Column represents driving factors. The second Column shows the

states according to discretized bins. MPE means the most probable explanation given all evidence provided. The fourth column represents marginal probabilities driven from data. This is accounted as before evidence input factors certainty. The conditional probabilities representing inference mechanism is generated after adding evidence are demonstrated following with the dynamical % change between before evidence and after adding evidence. The evidence inputs are where Net Profit Margins are low (NPM = 0), Investment is low (INV = 0) and Volatility is high (VOL = 1) with low price to earnings (PER = 0).

Table 6 shows new data collected from evidence on net profit margin, investments, and volatility. When evidence

**Table 6:** Inference Mechanism; Dynamical Change After Entering New Evidence

Factors	States	MPE	Marginal	NPM = 1		INV = 1		PER = 0; VOL= 1	
			Evidence = 0	After	Δ%	After	Δ%	After	Δ%
			Before						
$R_i - R_f$	Low	Low	22.67%	4.12	-18.55	16.1	-6.57	5.88	-16.79
	High		77.33	95.88	18.55	83.9	6.57	94.12	16.79
LMW	Low	High	30	37.38	7.38	40.91	10.91	27.08	-2.92
	High		70	62.62	-7.38	59.09	-10.91	72.92	2.92
MKT	Low	Low	30	3.86	-26.14	15.11	-14.89	5.15	-24.85
	High		70	96.14	26.14	84.89	14.89	94.85	24.85
SMB	Low	Low	36.67	10.29	-26.38	16.72	-19.95	3.38	-33.29
	High		63.33	89.71	26.38	83.28	19.95	96.62	33.29
HML	Low	Low	43.33	13.73	-29.6	4.55	-38.78	30.24	-13.09
	High		56.67	86.27	29.6	95.45	38.78	69.76	13.09
VOL	Low	High	16.67	33.06	16.39	40.91	24.24	0	
	High		83.33	66.94	-16.39	59.09	-24.24	100	
PER	Low	High	56.67	78.37	21.7	73.08	16.41	100	
	High		43.33	21.63	-21.7	26.92	-16.41	0	
EPS	Low	High	44.09	77.47	33.38	66.9	22.81	55.43	11.34
	High		55.91	22.53	-33.38	33.1	-22.81	44.57	-11.34
DIV	Low	High	50	66.07	16.07	71.06	21.06	57.11	7.11
	High		50	33.93	-16.07	28.94	-21.06	42.89	-7.11
INV	Low	High	36.67	79.49	42.82	100		35.25	-1.42
	High		63.33	20.51	-42.82	0		64.75	1.42
NPM	Low	High	38.7	100		83.9	45.2	44.26	5.56
	High		61.3	0		16.1	-45.2	55.74	-5.56
LIQ	Low	High	56.67	84.25	27.58	82.29	25.62	81.99	25.32
	High		43.33	15.75	-27.58	17.71	-25.62	18.01	-25.32

is entered, the most dynamical change is observed: low net profit margin, low investments, and significant volatility. Based on the most likely explanation, we rated the evidence as low. In the Pakistan market, these three risk spreads of return were found to cause reversal anomalies. When  $NPM = 0$  was originally entered, the reversal factor LMW was reduced to 62.62 percent, a change of 7.38 percent. The risk spread shift for high excess market return climbed to 26.14 percent. The high size risk spread of high-return small businesses increased by 26.38 percent. The risk spread for low volatility increased by 16.39%. The risk spread on low dividends increased by 16.07 percent. The spread between low and high investment risk has widened by 42.82 percent. These findings are similar to those of Imran (2017) and Abhyankar et al. (1997), who argued that size, market risk premium, volatility, reversal factor, and price to earnings ratio are all factors that contribute to stock return anomalies in the market. We may also deduce that a low net profit margin leads to modest investments, and small enterprises with strong returns have a high book-to-market equity ratio. These findings are in line with those of Liammukda et al. (2020).

Net profit margin, Market Risk Premium, Investment, Size, and Volatility Factor are the main triggering elements for reversal of stock returns in the Pakistan market when it comes to inter-factor relationships. The next piece of evidence is that investment is entered as  $INV = 1$ , and the rest of the results are the same, except that HML is dependent on INV, Liquidity is dependent on HML, Size is dependent on LIQ, and Size is dependent on  $R_m - R_f$ , which is a parent of  $R_i - R_f$ . This results in an unusual chain, with cheap investment and high book-to-market equity providing a cushion for small businesses to produce excess profits over time. If evidence of poor investment is present, we can witness the biggest dynamical shift in book to market equity of 38.78 percent in Table 6.

Volatility = 0 is entered as the last piece of evidence. This shows the highest price-to-earnings ratio change of 21.61 percent and earnings-per-share ratio change of 29.32 percent. Volatility is another important source of market reversal anomalies. Substantial or low volatility can lead to the largest changes in PER and EPS, which can lead to high excess returns for loser portfolios with small market capitalization firms.

## 5. Conclusion

Investors can now create and manage risk-return portfolios in dynamic ways because of recent technology and methodological developments. In this setting, static, non-dynamic linear models are insufficient to determine the return trigger aspects of an investor's portfolio. Flexible methods that can quickly adapt to changing market conditions

and allow investors to use a variety of information and methodologies are now required. In visual form, Bayesian networks can generate causal and inferential probabilistic relationships. Investors may update their stock return values in the network at the same time as new information is received, resulting in a dynamic shift in portfolio risk spread.

The study makes a contribution by proposing Bayesian Networks as a mechanism for analyzing and modeling portfolio risk and returns for the Pakistan Stock Exchange Index from January 2005 to December 2018. Dynamic Bayesian Networks are possible to model the interdependence of many elements that affect portfolio returns and cause reversal anomalies. As well as the recent marginal and conditional probabilities posterior distributions had allowed us to compute posterior distributions of portfolio returns even with multivariate probability models with many variables. The Bayes rule may be used to dynamically update the marginal and conditional probabilities portfolio returns, and the key driver trigger factors can be identified changes in distributions before and after adding data. In the long run, a reversal happens when a losing portfolio becomes a winner and a winning portfolio becomes a loser. For determining the existence of reversal abnormalities, relatively few studies exist that examined and studied inter-factor correlations among the factors that cause reversal anomalies, particularly in Pakistan's emerging market.

In summary, in Pakistan the most likely condition of reversal factor is  $R_i - R_j = 1$ . In Pakistan, we search for an interesting fact. Excess returns ( $R_i - R_j = 1$ ) of 69.88 percent indicate the occurrence of reversal. The excess returns of 69.88 percent are due to large net profit margins ( $NPM = 1$ ), high investment in these firms ( $INV = 1$ ), high earnings per share ( $EPS = 1$ ), and high volatility ( $VOL = 1$ ). The size of the firm's factor returns is low at 69.88 percent with  $SMB = 0$ , which is a noteworthy statistic to note. The liquidity component has a direct relationship with  $SMB$ , while the volatility factor has a direct relationship with  $LMW$ . This suggests that the Pakistani market is characterized by high volatility and little liquidity, which has a negative impact on its results. Further, the volatility component influences the investment and net profit margin factors. When evidence is entered, the most dynamical change is observed: low net profit margin, low investments, and significant volatility. Based on the most likely explanation, we rated the evidence as low. Talking about inter-factor linkages in Pakistan, Net profit margin, Market Risk Premium, Investment, Size, and volatility factors are the most triggering reasons for the reversal of stock returns in the Pakistan market.

## References

- Abhyankar, A., Copeland, L. S., & Wong, W. (1997). Uncovering nonlinear structure in real-time stock-market indexes: The S&P

- 500, the DAX, the Nikkei 225, and the FTSE-100. *Journal of Business & Economic Statistics*, 15(1), 1–14. <https://doi.org/10.2307/1392068>
- Ayub, Q. M. Y. (2017). The stock market reaction to extreme events: The evidence from Pakistan. *Business and Management*, 9(4), 111–127. <https://doi.org/10.21251/1479263>
- Banz, R. W. (1981). The relationship between return and market value of common stocks. *Journal of Financial Economics*, 9(1), 3–18. [https://doi.org/10.1016/0304-405X\(81\)90018-0](https://doi.org/10.1016/0304-405X(81)90018-0)
- De Bondt, W. F., & Thaler, R. H. (1985). Does the stock market overreact? *The Journal of Finance*, 40(3), 793–805. <https://doi.org/10.1111/j.1540-6261.1985.tb05004.x>
- De Oliveira, F. A., Nobre, C. N., & Zarate, L. E. (2013). Applying artificial neural networks to prediction of stock price and improvement of the directional prediction index: A case study of PETR4, Petrobras, Brazil. *Expert Systems with Applications*, 40(18), 7596–7606. <http://doi.org/10.1016/j.eswa.2013.06.071>
- Cakici, N., Fabozzi, F. J., & Tan, S. (2013). Size, value, and momentum in emerging market stock returns. *Emerging Markets Review*, 16, 46–65. <https://doi.org/10.1016/j.ememar.2013.03.001>
- Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of Finance*, 52(1), 57–82. <https://doi.org/10.1111/j.1540-6261.1997.tb03808.x>
- Fama, E. F., & French, K. R. (2012). Size, value, and momentum in international stock returns. *Journal of Financial Economics*, 105(3), 457–472. <https://doi.org/10.1016/j.jfineco.2012.05.011>
- Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, 116(1), 1–22. <https://doi.org/10.1016/j.jfineco.2014.10.010>
- Geiger, D., & Yuille, A. (1991). A common framework for image segmentation. *International Journal of Computer Vision*, 6(3), 227–243.
- Ho, R. Y. W., Strange, R., & Piesse, J. (2008). Corporate financial leverage and asset pricing in the Hong Kong market. *International Business Review*, 17(1), 1–7. <https://doi.org/j.ibr2008.17.1.17>
- Hongsakulvasu, N., & Liammukda, A. (2020). Asian stock markets analysis: The new evidence from a time-varying coefficient autoregressive model. *Journal of Asian Finance, Economics, and Business*, 7(9), 95–104. <https://doi.org/10.13106/jafeb.2020.vol7.no9.095>
- Hou, K., Xue, C., & Zhang, L. (2015). Digesting anomalies: An investment approach. *The Review of Financial Studies*, 28(3), 650–705. <https://doi.org/10.1093/rfs/hhu068>
- Liammukda, A., Khamkong, M., Saenchan, L., & Hongsakulvasu, N. (2020). The time-varying coefficient Fama-French five-factor model: A case study in the return of Japan portfolios. *The Journal of Asian Finance, Economics, and Business*, 7(10), 513–521. <https://doi.org/10.13106/jafeb.2020.vol7.no10.513>
- Malin, M., & Bornholt, G. (2013). Long-term return reversal: Evidence from international market indices. *Journal of International Financial Markets, Institutions, and Money*, 25, 1–17. <https://doi.org/10.1016/J.INTFIN.2013.01.002>
- Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica*, 41(5), 867–887. <https://doi.org/10.2307/1913811>
- Pastor, L., & Stambaugh, R. F. (2003). Liquidity risk and expected stock returns. *Journal of Political Economy*, 111(3), 642–685. <http://doi.org/10.1086/374184>
- Pearl, J. (2003). Causality: Models, reasoning, and inference. *Econometric Theory*, 19, 675–685. <https://doi.org/10.1017/S0266466603004109>
- Pearl, J., Geiger, D. D., & Verma, D. T. (2001). Identifying independence in bayesian networks. *Networks*, 20(5), 507–534. <https://doi.org/10.1002/net.3230200504>
- Reddy, K., Qamar, M. A. J., & Rao, M. (2019). Return reversal effect in Shanghai: Share market. *Managerial Finance*, 45(6), 698–715. <https://doi.org/10.1108/MF-04-2018-0140>
- Ross, S. A. (1976). Options and efficiency. *The Quarterly Journal of Economics*, 90(1), 75–89. <https://doi.org/10.2307/1886087>
- Schwarz, G. (1978). Estimating the dimension of a model. *The Annals of Statistics*, 6(2), 461–464. <https://www.jstor.org/stable/2958889>
- Scutari, M. (2009). Learning Bayesian networks with the bnlearn R package. *Journal of Statistical Software*, 35(3), 1–22. <https://arxiv.org/abs/0908.3817>
- Shaharuddin, S. S., Lau, W. Y., & Ahmad, R. (2018). Is the Fama French three-factor model relevant? Evidence from Islamic unit trust funds. *The Journal of Asian Finance, Economics, and Business*, 5(4), 21–34. <https://doi.org/cite/10.13106/jafeb.2018.vol5.no4.21>
- Shoaib, A., & Siddiqui, M. A. (2017). Long-run adjustment of size, value, momentum, and growth premium in equity returns: Evidence from South Asian emerging markets. *Investment Analyst's Journal*, 46(2), 97–116. <https://doi.org/10.1080/10293523.2016.1275427>
- Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, 19(3), 425–442. <https://doi.org/10.1111/j.1540-6261.1964.tb02865.x>
- Ticknor, J. L. (2013). A Bayesian regularized artificial neural network for stock market forecasting. *Expert Systems with Applications*, 40(14), 5501–5506. <http://doi.org/10.1016/j.eswa.2013.04.013>
- Wang, Z., Wang, L., & Tan, S. (2008). Emergent and spontaneous computation of factor relationships from a large factor set. *Journal of Economic Dynamics and Control*, 32(12), 3939–3959. <https://doi.org/10.1016/j.jedc.2008.04.005>
- Zong, F., Xu, H., & Zhang, H. (2013). Prediction for traffic accident severity: comparing the Bayesian network and regression models. *Mathematical Problems in Engineering*, 20(3), 15–27. <https://doi.org/10.1155/2013/475194>