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A Study on Optimum Value of Design Parameter of Multivariate EWMA and CUSUM charts for Monitoring Dispersion Matrix

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Abstract

Properties and comparison of multivariate CUSUM and EWMA charts for monitoring Σ of multivariate normal $N(\underline{\mu}, \Sigma)$ process has considered. Comparison of the performances of the considered charts, the numerical values are obtained by simulation with 10,000 iteration in terms of ATS, ANSS and ANSW. We found that EWMA chart with small values of smoothing constant more effectively detects the process changes than with large smoothing constant. And we also found that CUSUM chart with small value of reference value is more effectively detecting the process change than with large reference value. If a process engineer has interest in detecting small amount of shift rather than large shift, he/she can be recommended to use small smoothing constant in EWMA chart and small reference value in CUSUM chart.

Keywords: ANSW, ATS, LRT, reference value, smoothing constant

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1. Introduction

A control chart is continuously used and focused to detect accidental or random causes of variation so that these causes should be found and eliminated, when the process parameters of the production has changed. During the control process, a process engineer wishes to detect any departure from in-control state as quickly as possible and identify which characteristics are responsible for the out-of-control state.

The quality of the a product is usually determined by several related variables or characteristics. Many situations in quality control involve a vector of measurements of multiple related quality characteristics rather than a single characteristic. Especially when some correlation exists between the quality characteristics, we could obtain better sensitivity by using multivariate control procedure than separate control procedures for each of process parameters or characteristics. Before recent years, application of multivariate quality procedures in quality control was restricted by the lack of adequate computational resources.

Hotelling[1] first introduced the multivariate control chart procedures. Jackson[2] and Ghare and Togersen[3] presented multivariate Shewhart charts based on Hotelling's T^2 statistic. Woodall and Ncube[[4] studied a single multivariate CUSUM chart for monitoring the means of multivariate normal process $N(\underline{\mu}, \Sigma)$. A multivariate EWMA (exponentially weighted moving average) chart for monitoring $\underline{\mu}$ of $N(\underline{\mu}, \Sigma)$ using accmulate-combine method was presented by Lowry *et al.*[5]. By simulation, they showed that the efficiencies of the multivariate EWMA chart performs better than the multivariate CUSUM chart of Crosier[6] and Pignatiello and Runger[7], and the chart performs roughly the same if small shift in $\underline{\mu}$ has occurred.

The operation of a control chart to detect process shifts can be described by a control statistic and two disjoint regions, the signal (or out-of-control) region and the in-control region. In each sampling time, a control

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statistic is located within the control limits, the process is considered to be in-control state and allowed to continue for the next sample. On the other hand, if a control statistic is located outside the control limits, the chart then give signal and correcting action in production process is needed to identify the cause of shifts and bring the production process back into in-control state.

The Shewhart control chart, first introduced by Shewhart[8], is simple to understand and easy to construct. Shewhart chart is effective to detect large process change, but ineffective at small change. The reason is that the basic Shewhart chart only uses the sample information at the time of inspection, and as the result its efficiency is reducing in detecting small or moderate shifts in process.

Most of the studies on multivariate control chart have been focused on controlling $\underline{\mu}$ of multivariate normal process. In this paper, we considered VSI (variable sampling intervals) EWMA and CUSUM charts for monitoring the variance-covariance matrix Σ with p-variate normal process $N_p(\underline{\mu}, \Sigma)$. At some selected smoothing constants for EWMA and reference values for CUMSUM chart, we has evaluated and compared the efficiency of EWMA and CUSUM charts. We found that EWMA chart with small values of smoothing constant λ is more efficient for detecting small or moderate changes in EWMA charts. And we also found that CUSUM chart with small values of reference value k is more efficient for detecting small or moderate changes.

2. Control Statistics for Dispersion Matrix and Shewhart Chart

In many industrial quality control, there are many situations in which the quality of an output or constancy of a production process is generally determined by some joint levels of several correlated quality variables.

In this paper, we assume that the quality of an output has $p(p\geq 2)$ quality variables $\underline{X}=(X_1, X_2, \cdots, X_p)'$ and that the p-variate has a multivariate normal distribution $N(\underline{\mu}, \Sigma)$ with $(\underline{\mu}_0, \Sigma_0)$, the known target process parameters of $(\underline{\mu}, \Sigma)$.

At each sampling occasion i ($i = 1, 2, \cdots$), we take a sequence of $\underline{X}_i' = (\underline{X}_{i1}', X_{i2}', \cdots, \underline{X}_{in}')$ in the production process where $\underline{X}_{ij}' = (X_{ij1}, X_{ij2}, \cdots, X_{in})$. Then the jkth element of $\underline{X}_i$, $\underline{X}_{i,jk}$ is the jth observation for kth quality variable at every sampling time i ($j = 1, 2, \cdots, n$; $k = 1, 2, \cdots, p$) where $\underline{X}_i$ is an $np\times 1$ column vector. It is also assumed that the sequential observation vectors between and within samples are iid (independent and identically distributed).

The general statistical quality control procedures can be regarded as a series of sequential significant tests. And so a control statistic for controlling variance-covariance matrix $\Sigma_{p\times p}$ can be obtained from the LRT (likelihood ratio test) statistic for testing H_0: $\Sigma = \Sigma_0$ vs H_1: $\Sigma\neq\Sigma_0$ where target mean vector $\underline{\mu}_0$ is known.

For the ith sample at the ith sample ($i = 1, 2, \cdots$), the likelihood ratio λ can be expressed as

$$\lambda = n^{-\frac{np}{2}} \cdot \left|A_i\Sigma_0^{-1}\right|^{\frac{n}{2}} \cdot \exp\left[-\frac{1}{2}tr(\Sigma_0^{-1}A_i)+\frac{1}{2}np\right].$$

If we let control statistic TV_i be $-2\ln\lambda$, then

$$TV_i=tr(A_i\Sigma_0^{-1})-n\ln|A_i|+n\ln|\Sigma_0|+np\ln n-np . \quad (2.1)$$

Hence, the statistic TV_i can be used as the control statistic for monitoring Σ and the region above the UCL (upper control limit) corresponds to the LRT rejection region.

A multivariate Shewhart chart based on LRT statistic TV_i signals whenever

$$TV_i \geq h_{TV(S)} \quad (2.2)$$

where $h_{TV(S)}$ can be obtained to satisfy a specified in-control ATS (average time to signal) or ANSS (average number of samples to signal) by simulation.

Since it is not easy to know the exact distribution of LRT statistic TV_i, UCL $h_{TV(S)}$ and numerical performances of FSI Shewhat chart are obtained by simulations with 10,000 iterations.

For the VSI Shewhart chart based on control statistic TV_i with sampling interval values d_1, d_2 ($0 < d_1 < d_2$), let

$$d_1 \text{ be used when } TV_i \in (g_{TV}, h_{TV}], \quad (2.3)$$
$$d_2 \text{ be used when } TV_i \in (0, g_{TV}].$$

The process parameters g_{TV} and h_{TV} can be obtained by simulation.

3. FSI and VSI Control Charts

In traditional quality control chart, RL (run length) is defined by the number of random samples required to signal on a chart, and ARL (average run length) the expected value of RL. Therefore, the expected time to signal in FSI chart is defined by the product of the ARL and the length of the fixed sampling interval d, so the ARL can be considered as the same as ATS. In FSI, the length of the sampling interval $|t_i - t_{i-1}|$ is constant for all i ($i = 1, 2, \cdots$).

Following the definitions of Reynolds *et al.*[9], the number of samples to signal (NSS) is the number of samples tested from the beginning of the process until the chart signals, and ANSS is the expected value of NSS. Also, they defined that the time to signal (TS) is the time from the beginning of the process until the chart signals and ATS is the expected value of TS.

The basic idea of VSI chart is that when there is a certain sign of process change the sampling interval might be short and when there is no sign of process change the sampling interval might be long. And when the sign of a process change is strong enough, the VSI chart gives signal as the FSI chart does.

Arnold[10] first introduced VSI procedures. Many researches on control charts showed that the performance of VSI chart was better than FSI's in terms of required time to signal. But, one disadvantage of VSI scheme with η sampling intervals is that it switches frequently between different sampling intervals $d_1, d_2, \cdots, d_n$ ($0 < d_1 < d_2 < \cdots < d_n$), and as the result it requires more costs and efforts to operate it than FSI chart does. Amin and Letsinger[11] studied the switching behavior of VSI charts. For applying and comparing different VSI charts, we need to define the number of switches (NSW), the number of switches made from the beginning until the chart signals, and ANSW, the expected value of NSW.

4. CUSUM Chart

The efficiency of Shewhart chart known that when small and moderate changes are occurred in process parameters its detection is slow. The CUSUM chart is better than the Shewhart chart when the detection of small shifts is important. Page[12] first introduced CUSUM (cumulative sum) control chart. And Barnard[13] suggested the CUSUM procedure as a sequential LRT for testing H_0: $\underline{\mu} = \underline{\mu}_0$ vs H_1: $\underline{\mu} \neq \underline{\mu}_0$. Brook and Evans[14] originally developed a RL distribution for a discrete one-sided CUSUM chart by Markov chain approach.

For FSI CUSUM chart based on LRT statistic TV_i is given by

$$Y_{TV,i} = \max\{Y_{TV,i-1}, 0\} + (TV_i - k), \tag{4.1}$$

where $Y_{TV,i} = \omega_{TV} \cdot I_{(\omega_{TV} \geq 0)}$ and the reference value $k \geq 0$. This multivariate CUSUM chart signals whenever $Y_{TV,i} \geq h_{(C)}$. And for VSI CUSUM chart with two sampling intervals d_1, d_2 ($0 < d_1 < d_2$) based on LRT statistic TV_i, suppose that the sampling interval;

$$\begin{aligned} &d_1 \text{ is used when } Y_{TV,i} \in (g_{TV(C)}, h_{TV(C)}], \\ &d_2 \text{ is used when } Y_{TV,i} \in (-k, g_{TV(C)}], \end{aligned} \tag{4.2}$$

where $g_{TV(C)} \leq h_{TV(C)}$. The design parameters $g_{TV(C)}$ and $h_{TV(C)}$ can be obtained to satisfy a desired ATS or ANSS. Since it is not easy to get the exact distribution of the chart statistic in (4.1) and (4.2), the performances of the charts can be evaluated by simulation under the process in-control or out-of-control states.

5. EWMA Chart

Like the multivariate Shewhart chart, the EWMA chart is also easy to implement and interpret. In EWMA scheme, the more weight is assigned to the recent sample information and the less weight to the older sample information. Roberts[15] reviewed that the EWMA chart is effective in detecting small shifts in a process but is not effective at large shifts.

The EWMA control chart is also a good alternative when process engineers are interested in detecting small shifts of a process. The ability of the EWMA chart is known approximately equal to the CUSUM chart's, and EWMA chart is more easier to operate and interpret than the CUSUM chart does.

For FSI multivariate EWMA chart based on LRT statistic TV_i is given by

$$Y_{TV,i} = (1-\lambda) Y_{TV,i-1} + \lambda TV_i, \tag{5.1}$$

where $Y_{TV,0} = \omega(\omega \geq 0)$ and smoothing constant λ ($0 < \lambda \leq 1$). This chart signals whenever $Y_{TV,i} > h_{TV(E)}$. When the smoothing constant λ in (5.1) is 1.0, this EWMA chart changes to Shewhart chart.

For VSI EWMA chart with two sampling intervals d_1, d_2 ($0 < d_1 < d_2$) based on LRT statistic TV_i, suppose the sampling interval;

d_1 is used when $TV_i \in (g_{TV(E)}, h_{TV(E)}]$, (5.2)
d_2 is used when $TV_i \in (0, g_{TV(E)}]$

where $g_{TV(E)} \le h_{TV(E)}$. The process parameters ($g_{TV(C)}$, $h_{TV(C)}$) was obtained when the process is in-control sates, and numerical performances when the porcess is out-of-control states was also obtained, by simulation with 10,000 iterations.

6. Numerical Performances and Concluding Remarks

Shewhart, CUSUM and EWMA charts, which are considered in this research, have the same ANSS and ATS at in-control states, and so we need some kinds of standards for comparison of their performances and efficiencies.

For some sort of simplicity in our numerical computation, we assumed that all diagonal elements of Σ_0 σ_{i0}^2 ($i = 1, 2, \cdots, p$) are 1.0 and off-diagonal elements of Σ_0 are 0.30. The numerical results were obtained for the ANSS and ATS of the in-control state being approximately equal to 200.0, $d_0 = 1$, $d_1 = 0.1$, $d_2 = 1.9$ and the sample size for each variable was five for p =2, 2, 4.

Since the performance of the considered charts depends on the every components of variance-covariance matrix Σ, it is impossible to investigate all of the possible changes in which Σ could take. Hence, we considered the following typical shifts for comparison as follows.

(1) V_i: σ_{10} of Σ_0 is increased to $(1.0 + 0.1i)$
(2) C_i: ρ_{120} and ρ_{210} of Σ_0 are changed to $(0.30 + 0.1i)$
(3) (V_i, C_i) for $i = 1, 2, \cdots, 6$

After the design parameters *h's* and *g's* of the considered Shewhart, SUSUM and EWMA charts, the ANSS, ATS and ANSW values of the considered shifts were obtained through simulation.

The numerical results from the simulation are given in Table 1 through Table 5. The simulation results shows the following properties. It showed that when the amount of process change is not large, Shewhart chart detects the change faster than CUSUM and EWMA chart.

Table 1. Numerical performances of multivariate CUSUM and Shewhart charts (p=2)

	CUSUM chrt (k=4.0)			CUSUM chrat (k=4.5)			CUSUM chart (k=5.0)			Shewhart
shifts	ANSS	ATS	ANSW	ANSS	ATS	ANSW	ANSS	ATS	ANSW	ATS
in-control	200.03	200.00	34.71	200.03	199.99	53.81	199.99	199.97	68.60	200.00
V_1	150.93	140.90	25.82	156.31	148.44	41.50	161.07	154.24	54.72	168.75
V_2	36.68	23.99	5.97	38.77	26.58	9.17	42.28	30.47	12.77	47.55
V_3	13.10	7.40	2.54	12.35	6.78	2.93	12.57	7.03	3.48	12.61
V_4	7.06	3.99	1.79	6.27	3.37	1.75	6.08	3.22	1.80	4.81
V_5	4.66	2.69	1.42	4.09	2.28	1.30	3.87	2.14	1.25	2.63
V_6	3.40	2.08	1.17	2.98	1.79	1.03	2.80	1.68	0.96	1.81
C_1	165.37	157.60	28.35	172.47	166.19	45.99	176.38	171.02	60.22	183.03
C_2	93.39	76.29	15.38	105.35	89.46	26.98	115.38	100.61	38.12	135.13
C_3	43.80	27.55	6.68	50.72	33.80	11.67	59.09	41.55	17.78	83.63
C_4	19.94	9.88	3.03	21.35	10.08	4.14	25.01	12.26	6.00	38.88
C_5	9.83	4.33	1.87	9.21	3.48	1.81	9.67	3.39	1.94	12.16
C_6	4.90	1.94	1.26	4.24	1.54	1.11	4.03	1.43	1.05	2.24
(V_1, C_1)	136.71	124.23	23.22	143.40	133.42	37.90	149.52	140.81	50.59	159.49
(V_2, C_2)	32.27	20.42	5.27	34.16	22.40	7.96	37.40	25.83	11.08	44.11
(V_3, C_3)	11.68	6.33	2.32	10.91	5.65	2.58	11.12	5.81	2.99	10.80
(V_4, C_4)	6.11	3.31	1.62	5.42	2.73	1.53	5.21	2.58	1.53	3.74
(V_5, C_5)	3.83	2.07	1.23	3.32	1.74	1.08	3.13	1.62	1.01	1.79
(V_6, C_6)	2.53	1.40	0.92	2.19	1.23	0.76	2.05	1.18	0.68	1.17

Table 2. Numerical performances of multivariate EWMA and Shewhart charts (p=2)

	EWMA chrt (λ=0.1)			EWMA chrat (λ=0.2)			EWMA chart (λ=0.3)			Shewhart
shifts	ANSS	ATS	ANSW	ANSS	ATS	ANSW	ANSS	ATS	ANSW	ATS
in-control	200.04	200.01	27.47	200.03	200.01	39.90	200.03	200.01	49.37	200.00
V_1	153.89	146.08	20.65	158.40	149.94	31.49	160.28	152.56	39.45	168.75
V_2	45.07	39.10	5.28	43.28	32.20	7.81	44.76	32.72	10.27	47.55
V_3	18.64	18.73	2.61	15.04	11.42	2.95	14.42	9.51	3.30	12.61
V_4	10.86	11.81	2.13	7.99	6.73	2.08	7.17	5.04	2.00	4.81
V_5	7.34	8.27	1.92	5.21	4.64	1.73	4.57	3.48	1.55	2.63
V_6	5.39	6.14	1.75	3.84	3.54	1.46	3.30	2.63	1.24	1.81
C_1	167.64	160.96	22.77	173.35	166.22	34.48	175.21	168.79	43.22	183.03
C_2	103.12	88.19	13.00	110.75	92.47	21.24	118.16	100.26	28.49	135.13
C_3	53.52	42.10	5.79	57.65	38.52	9.72	64.74	43.09	14.04	83.63
C_4	27.62	23.12	2.89	26.74	15.25	3.76	30.30	14.74	5.16	38.88
C_5	15.00	14.45	2.17	12.24	7.84	2.19	12.64	5.85	2.19	12.16
C_6	8.11	8.54	2.00	5.76	4.38	1.93	5.17	2.99	1.67	2.24
(V_1, C_1)	139.73	130.35	18.63	146.41	135.53	28.99	150.24	140.20	36.96	159.49
(V_2, C_2)	40.75	35.21	4.72	38.93	28.06	6.88	40.68	28.43	9.12	44.11
(V_3, C_3)	16.97	17.00	2.49	13.51	10.06	2.68	12.91	8.15	2.93	10.80
(V_4, C_4)	9.59	10.44	2.06	6.97	5.80	1.95	6.23	4.25	1.81	3.74
(V_5, C_5)	6.15	6.86	1.86	4.37	3.81	1.62	3.76	2.74	1.37	1.79
(V_6, C_6)	4.09	4.52	1.67	2.87	2.51	1.28	2.45	1.83	1.00	1.17

Table 3. Numerical performances of multivariate CUSUM and Shewhart charts (p=4)

	CUSUM chrt (k=16.0)			CUSUM chrat (k=16.6)			CUSUM chart (k=17.0)			Shewhart
shifts	ANSS	ATS	ANSW	ANSS	ATS	ANSW	ANSS	ATS	ANSW	ATS
in-control	200.00	199.99	25.93	200.01	199.98	34.07	200.00	200.02	42.16	200.00
V_1	177.28	171.53	22.91	178.99	173.84	30.36	180.25	175.76	37.85	190.22
V_2	80.01	61.79	10.03	82.63	64.96	13.35	85.72	69.09	17.10	126.21
V_3	33.25	20.46	4.27	32.47	19.53	5.00	32.81	19.90	6.00	54.55
V_4	17.46	9.90	2.69	16.09	8.61	2.79	15.51	8.04	2.99	19.83
V_5	11.00	6.11	2.14	9.87	5.08	2.07	9.20	4.59	2.07	8.04
V_6	7.69	4.25	1.82	6.83	3.56	1.72	6.30	3.17	1.65	4.10
C_1	182.96	178.49	23.68	184.14	180.33	31.24	186.78	183.35	39.22	193.01
C_2	138.39	124.84	17.62	143.14	130.63	23.89	148.12	136.71	30.58	173.81
C_3	88.07	68.78	10.87	92.66	73.85	14.81	98.07	80.07	19.38	144.07
C_4	49.08	31.80	5.79	50.42	32.80	7.46	53.87	35.75	9.74	104.48
C_5	26.37	14.65	3.26	25.65	13.28	3.66	26.21	13.25	4.23	61.25
C_6	13.33	6.72	2.13	12.15	5.39	2.06	11.63	4.70	2.01	20.91
(V_1, C_1)	166.43	158.27	21.45	169.07	161.69	28.54	171.29	164.87	35.80	185.84
(V_2, C_2)	68.36	50.22	8.46	70.88	52.86	11.24	74.02	56.85	14.45	114.74
(V_3, C_3)	27.58	16.31	3.64	26.63	15.11	4.12	26.58	15.11	4.79	43.99
(V_4, C_4)	14.21	7.86	2.37	12.94	6.61	2.37	12.31	6.04	2.42	14.40
(V_5, C_5)	8.58	4.60	1.87	7.64	3.81	1.77	7.07	3.35	1.71	5.11
(V_6, C_6)	5.47	2.85	1.53	4.83	2.35	1.40	4.41	2.06	1.30	2.11

Table 4. Numerical performances of multivariate EWMA and Shewhart charts (p=4)

	EWMA chrt (λ=0.1)			EWMA chrat (λ=0.2)			EWMA chart (λ=0.3)			Shewhart
shifts	ANSS	ATS	ANSW	ANSS	ATS	ANSW	ANSS	ATS	ANSW	ATS
in-control	200.00	200.00	27.29	199.99	199.99	40.39	199.97	200.00	50.28	200.00
V_1	181.08	176.98	24.49	183.98	179.50	37.03	185.98	181.92	46.71	190.22
V_2	94.76	83.45	11.28	101.07	83.69	19.19	108.35	90.38	26.20	126.21
V_3	44.61	41.62	4.51	41.73	30.60	6.85	44.50	30.47	9.60	54.55
V_4	26.06	27.85	2.78	20.51	16.20	3.36	19.93	13.22	4.08	19.83
V_5	17.68	20.82	2.32	12.53	11.16	2.46	11.23	8.16	2.63	8.04
V_6	13.22	16.37	2.13	8.81	8.54	2.15	7.42	5.96	2.13	4.10
C_1	186.38	183.03	25.23	189.22	185.75	38.15	190.02	186.92	47.76	193.01
C_2	150.54	139.78	19.62	158.34	146.00	31.47	164.24	152.81	40.97	173.81
C_3	104.01	89.94	12.34	113.93	93.63	21.54	123.35	102.76	29.73	144.07
C_4	64.36	53.85	6.40	69.73	48.75	11.58	80.14	55.39	17.51	104.48
C_5	37.69	34.39	3.26	36.88	22.98	4.86	42.72	22.85	7.34	61.25
C_6	21.25	22.93	2.21	16.79	12.15	2.32	17.52	8.91	2.53	20.91
(V_1, C_1)	172.63	166.62	23.17	176.56	170.13	35.45	179.69	173.71	45.09	185.84
(V_2, C_2)	83.37	72.91	9.59	88.79	71.04	16.46	97.01	77.69	23.09	114.74
(V_3, C_3)	38.29	36.67	3.78	34.58	25.06	5.46	36.86	23.96	7.59	43.99
(V_4, C_4)	22.02	24.45	2.50	16.61	13.49	2.79	15.76	10.36	3.21	14.40
(V_5, C_5)	14.52	17.63	2.14	9.88	9.17	2.18	8.56	6.38	2.20	5.11
(V_6, C_6)	9.97	12.65	2.00	6.40	6.44	1.95	5.26	4.34	1.85	2.11

Table 5. Numerical performances of multivariate Shewhart charts with different p

	Shewhart chart (p=2)			Shewhart chart (p=3)			Shewhart chart (p=4)		
shifts	ANSS	ATS	ANSW	ANSS	ATS	ANSW	ANSS	ATS	ANSW
in-control	199.97	200.00	100.03	199.98	200.03	100.12	200.02	200.00	99.94
V_1	172.61	168.75	86.33	186.20	183.21	93.24	192.32	190.22	96.09
V_2	55.79	47.55	27.17	98.79	87.12	48.59	138.28	126.21	68.46
V_3	17.66	12.61	7.91	35.16	26.24	16.21	68.66	54.55	32.52
V_4	7.64	4.81	3.11	13.91	8.90	5.82	29.14	19.83	12.70
V_5	4.37	2.63	1.65	6.93	4.05	2.66	13.63	8.04	5.38
V_6	2.92	1.81	1.04	4.21	2.41	1.51	7.51	4.10	2.74
C_1	186.27	183.03	93.14	191.42	189.15	95.81	194.64	193.01	97.20
C_2	146.42	135.13	72.63	168.20	159.43	83.98	180.21	173.81	89.92
C_3	103.35	83.63	49.35	133.72	116.63	65.58	157.83	144.07	78.19
C_4	61.53	38.88	25.63	95.67	71.33	44.05	126.61	104.48	61.00
C_5	30.78	12.16	8.39	56.76	31.21	21.25	89.96	61.25	39.26
C_6	10.36	2.24	1.20	23.93	6.64	4.24	48.52	20.91	14.50
(V_1, C_1)	164.50	159.49	82.18	181.20	176.97	90.71	188.92	185.84	94.36
(V_2, C_2)	53.21	44.11	25.68	89.70	77.15	43.81	128.42	114.74	63.32
(V_3, C_3)	16.16	10.80	6.98	30.36	21.37	13.52	58.27	43.99	26.99
(V_4, C_4)	6.78	3.74	2.49	11.69	6.67	4.47	23.39	14.40	9.57
(V_5, C_5)	3.58	1.79	1.10	5.46	2.69	1.76	10.30	5.11	3.49
(V_6, C_6)	2.09	1.17	0.55	2.95	1.41	0.81	5.00	2.11	1.38

We have found that EWMA chart detects the process change more effectively at small values of smoothing constant than at large or moderate values. In addition we have also found that CUSUM chart detects the process change better at small reference values than at large or moderate reference values. When a process engineer want to detect small shift of process rather than large or moderate shift, we recommand to use small smoothing value in EWMA chart and small reference value in CUSUM chart.

However, in the process change detection the optimum value of the smoothing constant λ and reference value k can depend on the amount of process change, which an engineer want to detect as quickly as possible, sample size and the number of main quality characteristics, which effects on the product quality. Therefore if a process engineer is to find a optimum design parameterr λ or k, then considering this aspect, he/she will be recommanded to manage the product process with exploring optimum design parameters.

References

[1] H. Hotelling, “Multivariate Quality Control”, Techniques of Statistical Analysis, McGraw-Hill, New York, pp. 111-184, 1947.
[2] J. S. Jackson, “Quality control methods for several related variables”, Technometrics, Vol. 1, pp. 359-377, 1959.
[3] P. H. Ghare and P. E. Torgersen, “The Multicharacteristic Control Chart”, Journal of Industrial Engineering, Vol. 19, pp. 269-272, 1968.
[4] W. H. Woodall and M. M. Ncube, “Multivariate CUSUM Quality Control Procedure”, Technometrics, Vol. 27, pp. 285-292, 1985.
[5] C. A. Lowry, W. H. Woodall, C. W. Champ and S. E. Rigdon. “A Multivariate Exponentially Weighted Moving Average Control Charts”, Technometrics, Vol. 34, pp 46-53. 1992.
[6] R. B. Crosier, “Multivariate Generalization of Cumulative Sum Quality-Control Scheme”, Technometrics, Vol. 30, pp 291-303, 1988.
[7] J. J. Jr. Pignatiello and G. C. Runger, “Comparisons of Multivariate CUSUM Charts”, Journal of Quality Technology, Vol. 22, pp. 173-186, 1990.
[8] W. A. Shewhart, “Economic control of quality manufactured Production”, Van Nostrand, New York, 1931.
[9] M. R. Jr. Reynolds, R. W. Amin, J. C. Arnold, and J. A Nachlas, “$\overline{X}$-charts with variable sampling intervals”, Technometrics, Vol. 30, pp. 181-192, 1988.
[10] J. C. Arnold, “A Markovian Sampling Policy Applied to Quality Monitoring of Streams”, Biometrics, Vol. 26, pp. 739-747, 1970.
[11] R. W. Amin and W. C. Letsinger, “Improved switching rules in control procedures using variable sampling intervals”, Communications in Statistics-Theory and Methods, Vol. 20, pp. 205-230, 1991.
[12] E. S. Page, “Continuous inspection schemes”, Biometrika, Vol. 41, pp. 110-114, 1954.
[13] G. A. Barnard, “Control Charts and Stochastic Process”, Journal of Royal Statistical Society, Ser. B, Vol. 21, pp. 239-257, 1959.
[14] D. Brook and D. A. Evans, “An Approach to the Probability Distribution of CUSUM Run Length”, Biometrika, Vol. 59, pp. 539-549, 1972.
[15] S. W. Roberts, “Control Chart Tests Based on Geometric Moving Averages”, Technometrics, Vol. 1, pp. 239-250, 1959.